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ARTICLE

Adams completion and symmetric algebra

M. Routaray and A. Behera

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Review

Adams completion and symmetric algebra

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Deleanu, Frei and Hilton have developed the notion of generalized Adams completion in a categorical context. In this paper, the symmetric algebra of a given algebra is shown to be the Adams completion of the algebra by considering a suitable set of morphisms in a suitable category.

Key words: Category of fraction, calculus of left fraction, symmetric algebra, tensor algebra, Adams completion.

INTRODUCTION

The notion of generalized completion (Adams completion) arose from a categorical completion process suggested by Adams (1973, 1975). Originally this was considered for admissible categories and generalized homology (or cohomology) theories. Subsequently, this notion has been considered in a more general framework by Deleanu et al. (1974) where an arbitrary category and an arbitrary set of morphisms of the category are considered; moreover they have also suggested the dual notion, namely the completion (Adams completion) of an object in a category.

The notion of Let \mathcal{C} be an arbitrary category and S a set of morphisms of \mathcal{C} . Let $\mathcal{C}[S^{-1}]$ denote the category of fractions of \mathcal{C} with respect S and $F: \mathcal{C} \rightarrow \mathcal{C}[S^{-1}]$ be the canonical function. Let \mathcal{S} denote the category of sets and functions. Then for a given object Y of \mathcal{C} ,

$$\mathcal{C}[S^{-1}](Y, -) : \mathcal{C} \rightarrow \mathcal{S}$$

defines a covariant function. If this function is representable by an object Y_S of \mathcal{C} , that is, $\mathcal{C}[S^{-1}](Y, -) \cong \mathcal{C}(Y_S, -)$. then Y_S is called the (generalized) Adams completion of Y with respect to the set of morphisms S or simply the S -cocompletion of Y . We shall often refer to Y_S as the completion of Y (Deleanu et al., 1974). Given a set S of morphisms of \mathcal{C} , the saturation \bar{S} of S is defined as the set of all morphisms u in \mathcal{C} such that $F(u)$ is an isomorphism in $\mathcal{C}[S^{-1}]$. S is said to be saturated if $S = \bar{S}$ (Deleanu et al., 1974).

Theorem 1

Behera and Nanda (1987) Let \mathcal{C} be a complete small \mathcal{U} -category (\mathcal{U} is a fixed Grothendieck universe) and S a set of morphisms of \mathcal{C} that admits a calculus of left

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fractions. Suppose that the following compatibility condition with co-product is satisfied. If each $s_i : X_i \rightarrow Y_i$, $i \in I$, is an element is of \mathcal{U} , then

$$\prod_{i \in I} s_i : \prod_{i \in I} X_i \rightarrow \prod_{i \in I} Y_i$$

is an element of S . Then every object X of \mathcal{C} has an Adams completion X_S with respect to the set of morphisms S .

Theorem 2

Let S be a set of morphisms of \mathcal{C} admitting a calculus of left fractions. Then an object Y_S of \mathcal{C} is the S -completion of the object Y with respect to S if and only if there exists a morphism $e : Y \rightarrow Y_S$ in \bar{S} which is co-universal with respect to morphisms of S : given a morphism $s : Y \rightarrow Z$ in S there exists a unique morphism $t : Z \rightarrow Y_S$ in \bar{S} such that $ts = e$. In other words, the following diagram is commutative (Deleanu et al., 1974):

$$\begin{array}{ccc} Y & \xrightarrow{e} & Y_S \\ s \downarrow & \nearrow t & \\ Z & & \end{array}$$

Theorem 3

Let S be a set of morphisms in a category \mathcal{C} admitting a calculus of left fractions. Let $e : Y \rightarrow Y_S$ be the canonical morphism as defined in Theorem 2, where Y_S is the S -completion of Y . Furthermore, let S_1 and S_2 be sets of morphisms in the category \mathcal{C} which have the following properties (Behera and Nanda, 1987b):

S_1 and S_2 are closed under composition,
 $fg \in S_1$ implies that $g \in S_1$,
 $fg \in S_2$ implies that $f \in S_2$,
 $S = S_1 \cap S_2$.
 Then $e \in S$.

Symmetric algebra

Let K be a commutative ring. Let M be a K -module and $T(M)$ be the *tensor algebra* of M over K . $T(M)$ is a graded K -algebra with the graded piece of degree $n \geq 0$ being the additive subgroup $M^{\otimes n}$, which we denote by $T^n(M)$. The map $M^{\otimes n} \rightarrow T^n(M)$ defined by $m_1 \otimes \cdots \otimes m_n \rightarrow (0, \dots \otimes m_1 \otimes \cdots \otimes m_n, 0, \dots)$ is a morphism of K -modules, which gives an isomorphism of K -modules of M with its image $T^n(M)$ (Murfet, 2006). Let $A^n(M)$ denote the n -th symmetric algebra (Grinberg, 2013). The map $\rho_n : M^{\otimes n} \rightarrow A^n(M)$ is a subjective K -module homomorphism. We prove the following for our need.

Theorem 4

Let K be a commutative ring with unit 1. Let $f : M^{\otimes n} \rightarrow N^{\otimes n}$ be K -module isomorphism. Then f has the following property: given a module isomorphism $g : M^{\otimes n} \rightarrow T^n(M)$, there exists a unique module isomorphism $\theta : N^{\otimes n} \rightarrow T^n(M)$ such that $g = \theta f$, that is, the following diagram is commutative :

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{f} & N^{\otimes n} \\ g \downarrow & \swarrow \theta & \\ T^n(M) & & \end{array}$$

Proof 1

For $n_1 \otimes n_2 \otimes \cdots \otimes n_n \in N^{\otimes n}$, define $\theta : N^{\otimes n} \rightarrow T^n(M)$ by the rule $\theta(n_1 \otimes n_2 \otimes \cdots \otimes n_n) = gf^{-1}(n_1 \otimes n_2 \otimes \cdots \otimes n_n)$.

Clearly θ is well-defined, homomorphism, one-one and onto. We have

$$\begin{aligned} & (m_1 \otimes m_2 \otimes \cdots \otimes m_n) \\ &= \theta(f(m_1 \otimes m_2 \otimes \cdots \otimes m_n)) \\ &= gf^{-1}(f(m_1 \otimes m_2 \otimes \cdots \otimes m_n)) \\ &= g(m_1 \otimes m_2 \otimes \cdots \otimes m_n), \end{aligned}$$

showing $g = \theta f$.

For showing the uniqueness of θ let there exist another $\theta': N^{\otimes n} \rightarrow T^n(M)$ such that $g = \theta' f$. Then

$$\begin{aligned} & \theta(n_1 \otimes n_2 \otimes \dots \otimes n_n) \\ &= g f^{-1}(n_1 \otimes n_2 \otimes \dots \otimes n_n) \\ &= \theta' f f^{-1}(n_1 \otimes n_2 \otimes \dots \otimes n_n) \\ &= \theta'(n_1 \otimes n_2 \otimes \dots \otimes n_n). \end{aligned}$$

This completes the proof.

Theorem 5

Let K be a commutative ring with unit 1. Let $M^{\otimes n}$ and $N^{\otimes n}$ be free K -modules and let $f: M^{\otimes n} \rightarrow N^{\otimes n}$ be a free K -module subjective homomorphism. Then f has the following property: given a free K -module subjective homomorphism $\rho_M: M^{\otimes n} \rightarrow A^n(M)$, there exists a unique free K -module subjective homomorphism $\varphi: N^{\otimes n} \rightarrow A^n(M)$ such that $\rho_M = \varphi f$, that is, the following diagram is commutative:

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{f} & N^{\otimes n} \\ \rho_M \downarrow & \swarrow \varphi & \\ A^n(M) & & \end{array}$$

Proof 2

Theorem 4, there exists a unique K -module isomorphism $\psi: N^{\otimes n} \rightarrow T^n(M)$ such that $\psi f = g$:

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{f} & N^{\otimes n} \\ g \downarrow & \swarrow \psi & \\ T^n(M) & & \end{array}$$

Where $g: M^{\otimes n} \rightarrow T^n(M)$ is a K -module isomorphism. Consider the diagram

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{f} & N^{\otimes n} \\ g \downarrow & \swarrow \psi & \downarrow \varphi \\ T^n(M) & \xrightarrow{h} & A^n(M) \end{array}$$

Let $h = \rho_M g^{-1}$. For each $n \in N^{\otimes n}$, define $\varphi: N^{\otimes n} \rightarrow A^n(M)$ by the rule $\varphi(n) = h\psi(n)$. Then for each $m \in M^{\otimes n}$ we have

$$\varphi f(m) = h\psi(f(m)) = \rho_M g^{-1}\psi(f(m)) = \rho_M f^{-1}\psi^{-1}\psi(f(m)) = \rho_M f^{-1}(f(m)) = \rho_M(m)$$

Showing $\varphi f = \rho_M$. Clearly φ is subjective. For the uniqueness suppose there exists another $\varphi': N^{\otimes n} \rightarrow A^n(M)$ such that $\rho_M = \varphi' f$. For any each $n \in N^{\otimes n}$ let $n = f(m)$, $m \in M^{\otimes n}$; thus $\varphi(n) = \varphi f(m) = \rho_M(m) = \varphi' f(m) = \varphi'(n)$, showing $\varphi = \varphi'$. This completes the proof.

The category \mathcal{M}

Let \mathcal{U} be a fixed Grothendieck universe (Schubert, 1972). Let \mathcal{M} denote the category of all free K -modules and free module homomorphisms where K is a commutative ring with unit 1. We assume that the underlying sets of the elements of \mathcal{M} are elements of \mathcal{U} . Let S_n denote the set of all free K -module homomorphisms $f: M^{\otimes n} \rightarrow N^{\otimes n}$ such that f is subjective.

Proposition

Let $\{s_i: X_i^{\otimes n} \rightarrow Y_i^{\otimes n}, i \in I\}$ be a subset of S_n ; where the index set I is an element of \mathcal{U} , then

$$\bigvee_{i \in I} s_i: \bigvee_{i \in I} X_i^{\otimes n} \rightarrow \bigvee_{i \in I} Y_i^{\otimes n}$$

is an element of S_n .

Proof 3

The proof is trivial.

We will show that the set S_n of free K -module homomorphisms of the category \mathcal{M} of free K -modules and free K -modules homomorphisms admits a calculus of left fraction.

Proposition

S_n admits a calculus of left fractions.

Proof 4

Since S_n consists of all subjective K -module homomorphisms in \mathcal{M} ; clearly S_n is a closed family of morphisms of the category \mathcal{M} . We shall verify conditions (i) and (ii) of Theorem 1.3 ([6], p. 67). Let $M^{\otimes n}$, $N^{\otimes n}$ and $P^{\otimes n}$ be in \mathcal{M} . Let $u : M^{\otimes n} \rightarrow N^{\otimes n}$ and $v : N^{\otimes n} \rightarrow P^{\otimes n}$ be two free K -module homomorphisms of the category \mathcal{M} . We show that if $vu \in S_n$ and $u \in S_n$ then $v \in S_n$. Since $vu \in S_n$ and $u \in S_n$ we have $vu(M^{\otimes n}) = P^{\otimes n}$ and $u(M^{\otimes n}) = N^{\otimes n}$. Then $v(N^{\otimes n}) = v(u(M^{\otimes n})) = P^{\otimes n}$. So v is surjective. Hence condition (i) of Theorem 2 (Deleanu et al., 1974) holds. In order to prove condition (ii) of Theorem 2 (Deleanu et al., 1974) consider the diagram

$$\begin{array}{ccc} A^{\otimes n} & \xrightarrow{f} & B^{\otimes n} \\ s \downarrow & & \\ C^{\otimes n} & & \end{array}$$

in \mathcal{M} with $s \in S_n$. We assert that the above diagram can be embedded to a weak push-out diagram

$$\begin{array}{ccc} A^{\otimes n} & \xrightarrow{f} & B^{\otimes n} \\ s \downarrow & & \downarrow t \\ C^{\otimes n} & \xrightarrow{g} & D^{\otimes n} \end{array}$$

in \mathcal{M} with $t \in S_n$. Let $D^{\otimes n} = (B^{\otimes n} \oplus C^{\otimes n})/N^{\otimes n}$

Where $N^{\otimes n}$ is a sub-module of $B^{\otimes n} \oplus C^{\otimes n}$ generated by

$$\{(f(a_1 \otimes a_2 \otimes \cdots \otimes a_n), -s(a_1 \otimes a_2 \otimes \cdots \otimes a_n)) : a_1 \otimes a_2 \otimes \cdots \otimes a_n \in A^{\otimes n}\}.$$

Define $t : B^{\otimes n} \rightarrow D^{\otimes n}$ by the rule

$$t(b_1 \otimes b_2 \otimes \cdots \otimes b_n) = (b_1 \otimes b_2 \otimes \cdots \otimes b_n, 0) + N$$

and $g : C^{\otimes n} \rightarrow D^{\otimes n}$ by the rule

$$g(c_1 \otimes c_2 \otimes \cdots \otimes c_n) = (0, c_1 \otimes c_2 \otimes \cdots \otimes c_n) + N$$

Clearly, the two maps are well defined and homomorphisms. For any

$a_1 \otimes a_2 \otimes \cdots \otimes a_n \in A^{\otimes n}$, we have

$$\begin{aligned} tf(a_1 \otimes a_2 \otimes \cdots \otimes a_n) &= (f(a_1 \otimes a_2 \otimes \cdots \otimes a_n), 0) \\ &= (0, s(c_1 \otimes c_2 \otimes \cdots \otimes c_n)) \\ &= gs(a_1 \otimes a_2 \otimes \cdots \otimes a_n); \end{aligned}$$

Thus $tf = gs$. Hence the diagram is commutative. In order to show t is surjective, take an element $d_1 \otimes d_2 \otimes \cdots \otimes d_n + N \in D^{\otimes n}$,

where

$$d_1 \otimes d_2 \otimes \cdots \otimes d_n = (b_1 \otimes b_2 \otimes \cdots \otimes b_n, c_1 \otimes c_2 \otimes \cdots \otimes c_n).$$

. Then

$$\begin{aligned} d_1 \otimes d_2 \otimes \cdots \otimes d_n + N &= (b_1 \otimes b_2 \otimes \cdots \otimes b_n, c_1 \otimes c_2 \otimes \cdots \otimes c_n) + N \\ &= (b_1 \otimes b_2 \otimes \cdots \otimes b_n, 0) + (0, c_1 \otimes c_2 \otimes \cdots \otimes c_n) + N \\ &= t(b_1 \otimes b_2 \otimes \cdots \otimes b_n) + g(c_1 \otimes c_2 \otimes \cdots \otimes c_n) \\ &= t(b_1 \otimes b_2 \otimes \cdots \otimes b_n) + g(s(a_1 \otimes a_2 \otimes \cdots \otimes a_n)) \\ &= t(b_1 \otimes b_2 \otimes \cdots \otimes b_n) + tf(a_1 \otimes a_2 \otimes \cdots \otimes a_n) \\ &= t((b_1 \otimes b_2 \otimes \cdots \otimes b_n) + f(a_1 \otimes a_2 \otimes \cdots \otimes a_n)) \end{aligned}$$

Thus t is an epimorphism. So $t \in S_n$.

Next let $u : B^{\otimes n} \rightarrow X^{\otimes n}$ and $v : C^{\otimes n} \rightarrow X^{\otimes n}$ be in category \mathcal{M} such that $uf = vs$. Consider the following diagram

$$\begin{array}{ccc} A^{\otimes n} & \xrightarrow{f} & B^{\otimes n} \\ s \downarrow & & \downarrow t \\ C^{\otimes n} & \xrightarrow{g} & D^{\otimes n} \\ & & \theta \searrow \\ & & X^{\otimes n} \end{array} \quad \begin{array}{l} \searrow u \\ \searrow v \end{array}$$

Define $\theta : D^{\otimes n} \rightarrow X^{\otimes n}$ by the rule

$$\theta((d_1 \otimes d_2 \otimes \dots \otimes d_n) + N) = u(b_1 \otimes b_2 \otimes \dots \otimes b_n) + v(c_1 \otimes c_2 \otimes \dots \otimes c_n)$$

Where $d_1 \otimes d_2 \otimes \dots \otimes d_n = ((b_1 \otimes b_2 \otimes \dots \otimes b_n), (c_1 \otimes c_2 \otimes \dots \otimes c_n))$.

It is easy to show that θ is well defined and also a homomorphism. Next we show the two triangles are commutative. For any $b_1 \otimes b_2 \otimes \dots \otimes b_n \in B^{\otimes n}$, we have

$$\begin{aligned} &\theta t(b_1 \otimes b_2 \otimes \dots \otimes b_n) \\ &= \theta((b_1 \otimes b_2 \otimes \dots \otimes b_n), 0) + N \\ &= u(b_1 \otimes b_2 \otimes \dots \otimes b_n) \end{aligned}$$

and for any $c_1 \otimes c_2 \otimes \dots \otimes c_n \in C^{\otimes n}$, we have

$$\begin{aligned} &\theta g(c_1 \otimes c_2 \otimes \dots \otimes c_n) \\ &= \theta(0, (c_1 \otimes c_2 \otimes \dots \otimes c_n)) + N \\ &= v(c_1 \otimes c_2 \otimes \dots \otimes c_n). \end{aligned}$$

So $\theta t = u$ and $\theta g = v$. Clearly θ is unique. This completes the proof. The following result is a well-known.

Theorem 6

The category \mathcal{M} is complete. From Theorems 6, 7 and 8 we see that all the conditions of the Theorem 1 (Deleanu, 1975) are satisfied. So from the Theorem 1 (Deleanu et al., 1974) hence we have the following result.

Theorem 7

Every object $M^{\otimes n}$ of the category \mathcal{M} has an Adams completion M_{S_n} with respect to the set of morphisms S_n . Furthermore, there exists a morphism $e : M^{\otimes n} \rightarrow M_{S_n}$ in \bar{S}_n which is couniversal with respect to the morphisms in S_n : given a morphism $s : M^{\otimes n} \rightarrow N^{\otimes n}$ in S_n there exists a unique morphism $t : N^{\otimes n} \rightarrow M_{S_n}$ in \bar{S}_n that $ts = e$. In other words the following diagram is commutative:

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{e} & M_{S_n} \\ s \downarrow & \nearrow t & \\ N^{\otimes n} & & \end{array}$$

Theorem 8

The free K -module homomorphism $e : M^{\otimes n} \rightarrow M_{S_n}$ is in S_n .

Proof 5

Let $S_n^1 = \{f : M^{\otimes n} \rightarrow N^{\otimes n} \text{ in } \mathcal{M} \mid f \text{ is a subjective free } K\text{-module homomorphism}\}$

and

$S_n^2 = \{f : M^{\otimes n} \rightarrow N^{\otimes n} \text{ in } \mathcal{M} \mid f \text{ is a free } K\text{-module homomorphism}\}$.

Clearly,

- (a) $S_n = S_n^1 \cap S_n^2$ and
- (b) S_n^1 and S_n^2 satisfy all the conditions of Theorem 3.

Hence $e_n \in S_n$. This completes the proof.

Result

We show that the n th term of symmetric algebra $A^n(M)$ of a free K -module M , is precisely the Adams completion M_{S_n} of $M^{\otimes n}$.

Theorem 9

$$A^n(M) \cong M_{S_n}$$

Proof 6

Consider the following diagram:

$$\begin{array}{ccc} M^{\otimes n} & \xrightarrow{e} & M_{S_n} \\ \rho_M \downarrow & & \swarrow \varphi \end{array}$$

$A^n(M)$

By Theorem 5, there exists a unique morphism $\varphi : M_{S_n} \rightarrow A^n(M)$ in S_n such that $\varphi e = \rho_M$.

Next consider the following diagram:

$$\begin{array}{ccc}
 M^{\otimes n} & \xrightarrow{e} & M_{S_n} \\
 \rho_M \downarrow & \nearrow \psi & \\
 A^n(M) & &
 \end{array}$$

By Theorem 7, there exists a unique morphism $\psi : A^n(M) \rightarrow M_{S_n}$ in S_n such that $\psi\rho_M = e$. Consider the following diagram:

$$\begin{array}{ccc}
 M^{\otimes n} & \xrightarrow{e} & M_{S_n} \\
 & & \nearrow \psi \\
 e \downarrow & A^n(M) & \\
 & \nearrow \varphi & \nearrow 1_{M_{S_n}} \\
 & & M_{S_n}
 \end{array}$$

We have $\psi\varphi e = \psi\rho_M = e$. By the uniqueness condition of the co-universal property of e , we conclude that $\psi\varphi = 1_{M_{S_n}}$. Next consider the following diagram:

$$\begin{array}{ccc}
 M^{\otimes n} & \xrightarrow{\rho_M} & A^n(M) \\
 & & \nearrow \varphi \\
 \rho_M \downarrow & M_{S_n} & \\
 & \nearrow \psi & \nearrow 1_{A^n(M)} \\
 & & A^n(M)
 \end{array}$$

We have $\varphi\psi\rho_M = \varphi e = \rho_M$. By the uniqueness condition of the property of ρ_M , we conclude that $\varphi\psi = 1_{A^n(M)}$. Thus $A^n(M) \cong M_{S_n}$. This completes the proof.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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