African Journal of Agricultural Research Vol. 7(24), pp. 3506-3512, 26 June, 2012
Available online at http://www.academicjournals.org/AJAR
DOI: 10.5897/AJAR11.1704
ISSN 1991-637X ©2012 Academic Journals

Full Length Research Paper

Reproduction and culling effect on the number of ewes and lambs in two types of breeding − Mathematical model

Milan Krajinović[1], Snežana Matić-Kekić[2], Nebojša Dedović[2*], Ivan Pihler[1], Mirko Simikić[2], Vladislav Simin[1] and Lazar Savin[2]

[1]Department of Animal Husbandry, University of Novi Sad, Faculty of Agriculture, 21000 Novi Sad, Republic of Serbia.
[2]Department of Agricultural Engineering, University of Novi Sad, Faculty of Agriculture, 21000 Novi Sad, Republic of Serbia.

Accepted 18 May, 2012

This study presents the development of mathematical model for determining the potential number of sheep and lambs in a ten-year period. The model enables prediction of the number of female sheep and lambs, plans for future feeding, care and breeding costs. Two types of sheep breeding (traditional and intensive) were considered, assuming the following: 1) the initial herd contained *S* pregnant ewes; 2) new ewes or female lambs were not bought; 3) female lambs obtained by reproduction of the livestock unit were not sold; 4) male and female lambs and ewes which did not satisfy the selection and health criteria for further reproduction, were sold. Considering the reproductive cycles of Württemberg, Ile d' France, Suffolk and domestic Tsigai (from Serbia) sheep breeds, we established the number of ewes and lambs for sale, after n years from the herd establishment. It depends on the following parameters: the initial size of herd *S*, average percentage p of new, two year old ewes reproduced from the herd, and average percentage r of non culled sheep. A recursive formula for the number of ewes is given, as well as the number of lambs for sale after n years. Proposed formulas could be used for numerous additional financial analyses of sheep breeding. General model for evaluation of yearly and cumulative income for both types of sheep breeding has been presented. The obtained results represent the first step in deciding which breed and which type of breeding should be accepted in order to gain the highest possible profit.

Key words: Culling and reproductive, mathematical model, sheep breeding.

INTRODUCTION

Reproductive parameters such as fertility index (number of lambs per ewe per year), sex ratio (proportion of male to female offspring born) and percentage of lambing have important influence on herd size increasing speed as well as quantity and quality of meat and milk (Janssens et al., 2004; Ochoa-Cordero et al., 2007). Population studies for sheep breeding have generally been based on an expected sex ratio 0.5: 0.5, which is accepted in this study. This ratio is rarely different for sheep breeding (Napier and Mullaney, 1974; Lindström et al., 2002), but not for cattle-breeding (Demiral et al., 2007; Silva et al., 2007), otherwise, other reproductive parameters will not be fixed and their variability can affect meat or milk quality and quantity. For example, heat stress has affected the reproductive function of Suffolk ewes (Tabarez-Rojas et al., 2009), while the number of lambs per ewe has effect on sheep milk composition (Ochoa-Cordero et al., 2007). Shearing at housing, grass silage feed value and extended grazing during pregnancy also

*Corresponding author. E-mail: dedovicn@polj.uns.ac.rs. Tel: +381 21 485 3 292.

has effect on meat quantity (Keady and Hanrahan, 2009).

Economic analysis of the animal husbandry was performed by methods that use spreadsheets. These methods, among other things, can determine the size of herds, but input data must be entered for each year if size of the herds after 5 years need to be known. This typically holds for free “Breedcow and Dynama” software made by Holmes (2005). It is designed to plan, evaluate and improve the profitability and financial management of extensive beef cattle enterprises and projected herd composition, cash flow, debt, gross margin, net worth and net income for up to 10 years. Although, this software has great potential, it does not apply to sheep breeding.

The aim of this study was to derive general formulas for the number of ewes $S_{\mathbf{n}}$ and lambs $L_{\mathbf{n}}$ for sale, after **n** years from the herd establishment. The suggested new formulas are applicable to any sheep breed. In this study, they were applied and analyzed for the following breeds: Suffolk, Ile d’ France, Württemberg and domestic Tsigai, including traditional and intensive sheep breeding.

MATERIALS AND METHODS

The intensive sheep breeding involves growing sheep under strictly controlled conditions of nutrition, care and keeping of animals. The goal is to increase the reproduction of sheep to lamb three times in two years and aim to maximize profit. Fertility per sheep is higher and the mortality of lambs decreased. The opposite is the traditional sheep breeding. Animals are often left to care on their own, diet is based mostly on pasture during the growing season and winter sheep feed. One cannot expect a big economic gain, but otherwise no major investment. Fertility per head is low with high mortality of lambs.

The notation that is used in this article includes: S - the initial size of herd (number of pregnant healthy ewes); $S_{\mathbf{n}}$ - number of ewes in herd after **n** years from the herd establishment; $L_{\mathbf{n}}$ - number of lambs for sale in **n**-th year from the herd establishment; p_1 - average percentage of lambing; p_2 - average annual percentage of number of female lambs per ewe; p_3 - average annual percentage of withhold female lambs in herd (lambs for further reproduction); p_4 - average annual fertility index (number of lambs per ewe in one year); **p** - average percentage of new female sheep in closed farming; **p̲** - average annual percentage of lambs for sale (male lambs and culled female lambs); **r** - average annual percentage of sheep which are not culled from herd. Here, closed farming means that after the herd establishment new ewes or female lambs are not bought, and the ones obtained by reproduction of livestock unit are not sold. However, male lambs, culled female lambs and ewes are sold. Only rams can be replaced according to reproductive demands.

In this study, analysis of input parameters (**p** and **r**) for mathematical formulas for the number of ewes $S_{\mathbf{n}}$ in closed farming and analysis of input parameter **p̲** for the number of lambs $L_{\mathbf{n}}$ for sale were carried out. For that purpose, a recursive formula was created and symbolic calculation of program package Mathematica 6.0 (Wolfram, 2008) was used. It should be mentioned that software Mathematica is widely applicable to many other problems related to agriculture (Bodroža-Pantić et al., 2008; Nikolić et al., 2009; Babić et al., 2011a, b; Dedović et al., 2011; Tomić et al., 2011) and optimization (Matić-Kekić and Acketa, 1997; Acketa et al., 2000; Acketa and Matić-Kekić, 2000).

Relation between percentages p, p̲ and parameters p_1, p_2, p_3 and p_4

In general (model 5), for $S_{\mathbf{n}}$ variable **p** is the one of three input parameters (**p**, **r** and S), and, as would be shown, it depends on p_1, p_2, p_3 and p_4. Values of **p** for considered breeds were analyzed by changing the minimum, average and maximum values of breeds reproduction parameters p_1, p_2, p_3 and p_4 (Table 1), according to some papers cited below. Those studies mostly present researches which include large number of livestock units. The choice of breeds in this study was motivated by the presence of these breeds in Serbia, UK and Germany. The Suffolk breed originates from UK, a region where sheep breeding is extremely developed, so it would be interesting to compare the data obtained for the same breed in Serbia. The most common sheep breed in Germany is Württemberg breed, and comparison with the data from Serbia for that breed will be given too. It should be pointed out that Tsigai, Württemberg, Suffolk and Ile d’ France breeds hold an important place in sheep breeding in Serbia.

Prior to giving the formula for average percentage of new female sheep in closed farming (**p**), the formulas of average annual percentage of number of female lambs per ewe (p_2) should be given. More precisely, since the mortality rate at lambing is 6% for Tsigai breed and 4% for other considered breeds, and under the assumption that 50% of lambs are female lambs (Mekić et al., 2007), it can be concluded that the average annual percentage of number of female lambs per ewe p_2 is equal to:

$$p_2 = 0.5\cdot 0.94\cdot p_4 \text{ for Tsigai breed and} \qquad (1)$$
$$p_2 = 0.5\cdot 0.96\cdot p_4 \text{ for other considered breeds}$$

Where p_4 is the annual fertility index (from 1.1 to 1.92, (Table 1). In traditional sheep breeding, the number of lambs per ewe per year (fertility index p_4) for Ile d’ France breed is about 1.3 (Plieninger and Wilbrand, 2001), while in intensive sheep breeding it ranges from 1.23 to 2.11 (Cognie et al., 1980). These results are given for the flock of 400 ewes in case of traditional breeding (Plieninger and Wilbrand, 2001), and for 2000 ewes in case of intensive breeding (Cognie et al., 1980). For the Suffolk breed, average number of lambs born per ewe lambing is 1.65 ± 0.57 in traditional breeding, based on 3885 ewes. In intensive breeding, based on 1124 ewes, that number is 1.69 ± 0.61 (Janssens et al., 2004). In Serbia, in traditional sheep breeding, for Tsigai and Ile d’ France breed, average fertility indices are 1.28 ± 0.06 and 1.50 ± 0.07, respectively (Petrović et al., 2009).Some authors have conducted research on the input values p_1, p_3 and p_4 for parameters **p** and **p̲** (Table 1) for the considered breeds in Serbia. Stančić (2006) obtained $p_3 = 0.8$ and $p_4 = 1.25$ for Tsigai breed. For Württemberg breed, the number of lambs per ewe in one year is 1.35, while with intensive breeding (three lambings in two years) the fertility index can be 1.75 (Mekić et al., 2007). According to Krajinović (2006), Suffolk breed has p_4 ranging from 1.5 to 1.9, while Ile d’ France breed has p_4 ranging from 1.5 to 2.2, depending on the breeding type. As regards the Tsigai breed, only traditional sheep breeding is applied. Thus, the formula for percentage **p** of new ewes obtained from herd is:

$$\mathbf{p} = p_1\cdot p_2\cdot p_3 \qquad (2)$$

Table 1. Relation between percentage of lambs for sale (**p̲**), new ewes obtained from herd (**p**), average percentage of non culled sheep (**r**), annual percentage of lambing (p_1), female lambs (p_2), withhold female lambs (p_3) and fertility index (p_4) for Suffolk (S), Ile d' France (F), Württemberg (W) and domestic Tsigai (T) breeds of sheep.

Breeds	Parameters	p_4 (p_4^*)	p_1 (%)	p_2, p_2* (%)	p_3 (%)	p, p* (%)	p̲, p̲* (%)	r, r* (%)
	Min	1.2 (1.4)	70	57.6, 67.2	60	24.2, 28.2	56.4, 65.9	60, 60
W	Average	1.35 (1.75)	82	64.8, 84	80	42.5, 55.1	63.8, 82.7	82, 76
	Max	1.5 (1.9)	95	72, 91.2	90	61.6, 78.0	75.2, 95.3	90, 90
	Min	1.5 (1.8)	70	72, 86.4	60	30.2, 36.3	70.6, 84.7	70, 70
S	Average	1.71 (2.2)	82	82, 105.6	80	53.8, 69.3	80.8, 103.9	82, 76
	Max	1.9 (2.35)	95	91.2, 112.8	90	78.0, 96.4	95.3, 117.9	90, 90
	Min	1.55 (1.7)	70	74.4, 81.6	60	31.2, 34.3	72.9, 80.0	70, 65
F	Average	1.64 (2.1)	81	78.7, 100.8	80	51.0, 65.3	76.5, 98.0	81, 76
	Max	1.92 (2.3)	95	92.2, 110.4	90	78.8, 94.4	96.3, 115.4	90, 90
	Min	1.1	60	51.7	60	18.6	43.4	60
T	Average	1.25	75	58.7	80	35.2	52.9	75
	Max	1.54	95	72.4	90	61.9	75.6	90

* denotes calculated values for intensive sheep breeding.

Since p_1 is an average percentage of lambing, p_2 is an average annual percentage of number of female lambs per ewe and p_3 is an average annual percentage of withhold female lambs in herd (lambs for further reproduction), this implies that $p_1 \cdot p_2 \cdot p_3$ is the annual percentage of female lambs per ewe. After two years, these lambs will become new ewes in herd. Percentage p_3 of lambs that are not culled from the herd is about 60 to 90% for all considered breeds, while average percentage p_1 of ewes that had lambed is 75% for Tsigai breed and about 82% for others (Krajinović, 2006; Petrović et al., 2009). Under the assumption that all male lambs and those female lambs, which are not intended for reproduction for health reasons, will be sold, it follows that average annual percentage **p̲** of lambs for sale is calculated as:

$$\underline{\mathbf{p}} = p_1 \cdot p_2 + p_1 \cdot p_2 \cdot (1 - p_3) \qquad (3)$$

where $p_1 \cdot p_2$ is percentage of male lambs and $p_1 \cdot p_2 \cdot (1-p_3)$ is percentage of female lambs which are culled from the herd. It can be observed that the p_2 (eq. 1) is the percentage of average annual number of female lambs per ewe, but p_2 is also the average annual percentage of a male lambs per ewe, because of the assumption that the secondary sex ratio is 0.5:0.5.

Finally, after the first year, $\mathbf{p} \cdot S$ represents the expected total number of female lambs that will remain on the farm. This will serve as a base for herd increase, while $\underline{\mathbf{p}} \cdot S$ will represent the expected total number of lambs for sale in first year. Since $S_\mathbf{n}$ is the number of ewes in **n**-th year, it follows that the total number $L_\mathbf{n}$ of lambs for sale in **n**-th year since the herd establishment is calculated by:

$$L_\mathbf{n} = \underline{\mathbf{p}} \cdot S_\mathbf{n} \qquad (4)$$

Table 1 shows the input values (annual percentage of lambing p_1, percentage of withhold female lambs p_3 and fertility index p_4) and the output values (percentage p_2 of female lambs (Equation 1), percentage p of new ewes obtained from herd (Equation 2) and percentage p̲ of lambs for sale (Equation 3). An average annual percentage r of sheep which are not culled from herd also represents the input data.

Annual percentage r of ewes which are not culled from a herd

Based on Durand et al. (2009), it was assumed that ewes' life expectancy is from 4 to 10 years. This implies that average annual percentage **r** of ewes which are not culled from herd, varies from 0.75 to 0.9. For Ile d' France breed, average useful life of an ewe is 6 to 7 years in case of traditional sheep breeding (Plieninger and Wilbrand, 2001). In intensive breeding in Serbia (three lambings in two years), annual percentage of sheep culled from herd is 24% (Krajinović, 2006; Radulović et al., 1987), whereby **r** is 0.76.

RESULTS AND DISCUSSION

Formulas for the number of ewes and lambs for sale

Let S be a number of pregnant healthy ewes at the beginning (zero year) which are going to lamb in the spring. $S_\mathbf{n}$ denotes the number of ewes after **n** years (zero year is not included in n years). It is assumed that healthy female lambs are kept in the herd and can be fertilized at the age of 9 to 16 months. Then, the first lambing would be at the age of about 1.5 years. It should be noted that **p** stands for the average percentage of new ewes in closed farming, and **r** denotes average percentage of ewes which are not culled from the herd.

Table 2. Indices of herd size $\mathbf{k_n}$, $\mathbf{n}$ = 1, 2,...,10 in $\mathbf{n}$-th year from herd's foundation for Suffolk (S), Ile d' France (F), Württemberg (W) and domestic Tsigai (T) breeds of sheep. Coefficients $\mathbf{k_n}$ depend on percentage $\mathbf{p}$ of new ewes obtained from herd ($\mathbf{p}$ depends on: p_1-annual percentage of lambing, p_3-withhold female lambs and p_4-fertility index), and average annual percentage $\mathbf{r}$ of sheep which are not culled from the herd.

Breeds	p_1, p_3, p_4	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	p	r
	Min	0.60	0.60	0.51	0.45	0.39	0.34	0.30	0.26	0.23	0.20	24.2	60
W	Average	0.82	1.10	1.25	1.49	1.75	2.07	2.44	2.88	3.40	4.02	42.5	82
	Max	0.90	1.43	1.84	2.53	3.41	4.63	6.26	8.48	11.5	15.6	61.6	90
	Min	0.70	0.79	0.77	0.78	0.77	0.78	0.78	0.78	0.78	0.78	30.2	70
S	Average	0.82	1.21	1.43	1.83	2.27	2.85	3.56	4.45	5.56	6.96	53.8	82
	Max	0.90	1.59	2.13	3.16	4.51	6.52	9.38	13.5	19.5	28.1	80.0	90
	Min	0.70	0.80	0.78	0.80	0.80	0.81	0.82	0.83	0.83	0.84	31.2	70
F	Average	0.81	1.17	1.36	1.69	2.07	2.54	3.11	3.81	4.67	5.73	51.0	81
	Max	0.90	1.60	2.15	3.19	4.56	6.62	9.56	13.8	20.0	28.9	78.8	90
	Min	0.60	0.55	0.44	0.37	0.30	0.25	0.21	0.17	0.14	0.12	18.6	60
T	Average	0.75	0.92	0.95	1.04	1.11	1.20	1.29	1.39	1.5	1.61	35.2	75
	Max	0.90	1.43	1.84	2.54	3.43	4.66	6.32	8.57	11.6	15.8	61.9	90

Recursive formula for $S_{\mathbf{n}}$, $\mathbf{n} \geq 2$, is:

$$S_{\mathbf{n}} = \mathbf{r} \cdot S_{\mathbf{n-1}} + \mathbf{p} \cdot S_{\mathbf{n-2}} \qquad (5)$$

Here the addend $\mathbf{r}\ S_{\mathbf{n-1}}$ denotes the number of non culled ewes from the previous year, while the addend $\mathbf{p} \cdot S_{\mathbf{n-2}}$ stands for the number of new ewes obtained from female lambs born two years earlier. The similar recursive formula was used in dairy production and cow-cattle system (Matić-Kekić et al., 2011). The initial data are $S_0 = S$ and $S_1 = \mathbf{r}\ S$. Using (5), one can derive number of ewes $S_{\mathbf{n}}$ in the $\mathbf{n}$-th year for $\mathbf{n}$=0, 1…10:

$$S_0 = S = k_0 \cdot S$$
$$S_1 = \mathbf{r} \cdot S = k_1 \cdot S$$
$$S_2 = (\mathbf{r}^2 + \mathbf{p}) \cdot S = k_2 \cdot S$$
$$S_3 = \mathbf{r} \cdot (\mathbf{r}^2 + 2\mathbf{p}) \cdot S = k_3 \cdot S$$
$$S_4 = (\mathbf{r}^4 + 3\mathbf{p}\cdot\mathbf{r}^2 + \mathbf{p}^2) \cdot S = k_4 \cdot S$$
$$S_5 = \mathbf{r} \cdot (\mathbf{r}^4 + 4\mathbf{p}\cdot\mathbf{r}^2 + 3\mathbf{p}^2) \cdot S = k_5 \cdot S$$
$$S_6 = (\mathbf{r}^6 + 5\mathbf{p}\cdot\mathbf{r}^4 + 6\mathbf{p}^2\cdot\mathbf{r}^2 + \mathbf{p}^3) \cdot S = k_6 \cdot S$$
$$S_7 = \mathbf{r} \cdot (\mathbf{r}^6 + 6\mathbf{p}\cdot\mathbf{r}^4 + 10\mathbf{p}^2\cdot\mathbf{r}^2 + 4\mathbf{p}^3) \cdot S = k_7 \cdot S$$
$$S_8 = (\mathbf{r}^8 + 7\mathbf{p}\cdot\mathbf{r}^6 + 15\mathbf{p}^2\cdot\mathbf{r}^4 + 10\mathbf{p}^3\cdot\mathbf{r}^2 + \mathbf{p}^4) \cdot S = k_8 \cdot S$$
$$S_9 = \mathbf{r} \cdot (\mathbf{r}^8 + 8\mathbf{p}\cdot\mathbf{r}^6 + 21\mathbf{p}^2\cdot\mathbf{r}^4 + 20\mathbf{p}^3\cdot\mathbf{r}^2 + 5\mathbf{p}^4) \cdot S = k_9 \cdot S$$
$$S_{10} = (\mathbf{r}^{10} + 9\mathbf{p}\cdot\mathbf{r}^8 + 28\mathbf{p}^2\cdot\mathbf{r}^6 + 35\mathbf{p}^3\cdot\mathbf{r}^4 + 15\mathbf{p}^4\cdot\mathbf{r}^2 + \mathbf{p}^5) \cdot S = k_{10} \cdot S \qquad (6)$$

For example, formula for S_4, without software support, can be derived by iteratively applying (Equation 5):

$$S_4 = \mathbf{r}\cdot S_3 + \mathbf{p}\cdot S_2 = \mathbf{r}\cdot(\mathbf{r}\cdot S_2 + \mathbf{p}\cdot S_1) + \mathbf{p}\cdot(\mathbf{r}\cdot S_1 + \mathbf{p}\cdot S_0)$$
$$= \mathbf{r}^2\cdot(\mathbf{r}\cdot S_1 + \mathbf{p}\cdot S_0) + \mathbf{r}\cdot\mathbf{p}\cdot S_1 + \mathbf{p}\cdot\mathbf{r}\cdot S_1 + \mathbf{p}^2\cdot S_0$$
$$= \mathbf{r}^3 S_1 + \mathbf{r}^2\cdot\mathbf{p}\cdot S_0 + 2\mathbf{r}\cdot\mathbf{p}\cdot S_1 + \mathbf{p}^2\cdot S_0 = (\mathbf{r}^3 + 2\mathbf{r}\cdot\mathbf{p})S_1 + (\mathbf{r}^2\cdot\mathbf{p} + \mathbf{p}^2)S_0.$$

After replacing S_1 and S_0 with $\mathbf{r}\cdot S$ and S, respectively, it follows that:

$$S_4 = (\mathbf{r}^3 + 2\mathbf{r}\cdot\mathbf{p})\ \mathbf{r}\cdot S + (\mathbf{r}^2\cdot\mathbf{p} + \mathbf{p}^2)\ S = (\mathbf{r}^4 + 2\mathbf{r}^2\cdot\mathbf{p} + \mathbf{r}^2\cdot\mathbf{p} + \mathbf{p}^2)\ S = k_4 \cdot S.$$

In (Equation 6) the coefficients which multiply S with $k_{\mathbf{n}}$, $\mathbf{n}$=0, 1…10 are given. Then:

$k_0 = 1$, $k_1 = r$, $k_2 = (r^2 + p)$, $k_3 = r \cdot (r^2 + 2p)$,..., $k_{10} = (r^{10} + 9p\cdot r^8 + 28p^2\cdot r^6 + 35p^3\cdot r^4 + 15p^4\cdot r^2 + p^5)$.

The coefficients $k_{\mathbf{n}}$, $\mathbf{n}$ = 1,2,…,10, are given in Table 2 for Württemberg, Ile d' France, Suffolk and domestic Tsigai breeds in case of minimum, average and maximum values of initial data p_1, p_3, p_4 and $\mathbf{r}$. In comparison to the initial herd, number of sheep, for minimal values of p_1, p_3, p_4 and $\mathbf{r}$ for Tsigai and Württemberg breeds, after ten years, drastically decreased by 88 and 80%, respectively (coefficient k_{10} in Table 2). Among all considered cases, the largest herd increase (almost 30 times) that could be observed after ten years was for Ile d' France breed, with maximum values of: lambing (p_1 = 95%), withhold female lambs (p_3 = 90%), fertility index (p_4 = 1.92) and withhold ewes ($\mathbf{r}$ = 90%). For Suffolk breed, number of sheep is doubled between 4-th and 5-th year after the herd establishment. For Ile d' France breed, 5 years is a sufficient period of time to double the number of sheep.

Table 3. Average percentage **p** of new female sheep in closed farming for different values of average annual percentage **r** of sheep which are not culled from herd, in order to have herd size greater than initial size of herd *S*, after ten years time.

r	**0.9**	**0.82**	**0.81**	**0.75**	**0.7**	**0.6**
p ≥	0.1116	0.2015	0.2128	0.2806	0.3373	0.4513

Table 4. Number of ewes and lambs for sale in **n**-th year for Württemberg breed for traditional and intensive sheep breeding (rows with sign*).

Year (n)		**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$S_{\mathbf{n}}$ = Number of ewes		100	82	110	125	149	175	207	244	288	340	402
$S_{\mathbf{n}}$ = Number of ewes	*	100	76	113	128	159	191	233	283	343	417	506
$L_{\mathbf{n}}$ = Lambs for sale		64	52	70	80	95	112	132	156	184	217	256
$L_{\mathbf{n}}$ = Lambs for sale	*	83	63	93	106	132	158	193	234	284	344	418

Finally, for Württemberg breed, number of sheep will be doubled after 6 years from herd establishment. These calculations are based on average values of p_1, p_3, p_4 and r.

Furthermore, the conditions for the increase of $S_{\mathbf{10}}$ are presented. One year after herd establishment, number of female sheep is $S_{\mathbf{1}}$. This number is less than S because some sheep will be culled from the herd If:

$$k_2 = \mathbf{r}^2 + \mathbf{p} > 1,$$

Then two years after the herd establishment, number of ewes will be greater than S and herd increase will be continued. If $k_{\mathbf{2}}<1$, herd size may grow (7th and 11th row in Table 2), may not change (4th row in Table 2), or it may decrease (1st and 10th row in Table 2). At the end of the 10th year, the number of female sheep will be greater than S, if the following equation holds:

$$k_{\mathbf{10}} = \mathbf{r}^{10} + 9\mathbf{p}\cdot\mathbf{r}^8 + 28\mathbf{p}^2\cdot\mathbf{r}^6 + 35\mathbf{p}^3\cdot\mathbf{r}^4 + 15\mathbf{p}^4\cdot\mathbf{r}^2 + \mathbf{p}^5 > 1 \quad (7)$$

Equation 7 is satisfied for the values of **p** (Table 3, 2nd row) and **r** (Table 3, 1st row). Number of lambs $L_{\mathbf{n}}$ for sale is:

$$L_{\mathbf{n}} = \underline{\mathbf{p}} \cdot k_{\mathbf{n}} \cdot S \quad (8)$$

where $\underline{\mathbf{p}}$ is an average annual percentage of lambs for sale (Equation 3) and $k_{\mathbf{n}}\,S$ is the number of ewes at n-th year (Equation 5). If the initial number of ewes is S = 100, then number of lambs for sale $L_{\mathbf{n}}$, for **n** = 1, 2,..., 10 in closed farming, are given in Table 4 for Württemberg breed (only average values of p_1, p_3, p_4 and **r** for traditional and intensive sheep breeding are considered, only). Calculations for other breeds can be easily repeated.

By applying Equation 8 to different breeds and different types of breeding (Figure 1), after ten years from herd establishment, one can conclude that in the case of traditional breeding, number of lambs for sale is the largest for Suffolk breed and smallest for domestic Tsigai breed. In the case of intensive breeding, the number of lambs for sale is the largest for Ile d' France breeds and the smallest for Württemberg breed. Ile d' France breed has the largest difference in the number of lambs for sale depending on the type of breeding, while the Württemberg breed has the smallest one. It means that by changing the type of breeding from traditional to intensive, Ile d' France breed has the largest benefit in number of lambs for sale.

Conclusions

The fastest herd growth could be observed for Suffolk breed. More precisely, in ten years time, herd size will be increased almost seven times. Domestic breed showed considerably small increase. Input parameters (p_1, p_3, p_4 and **r**) variability from minimum to the maximum value, caused drastic differences in herd size. The proposed formulas can be used for numerous additional financial analyses of both examined types of sheep breeding. For example, if the grace period for credit repayment is 5 years, and if a herd of 100 ewes can bring enough profit for the payment of annuities, it is easy to determine the starting fund S in order to have 100 ewes at the

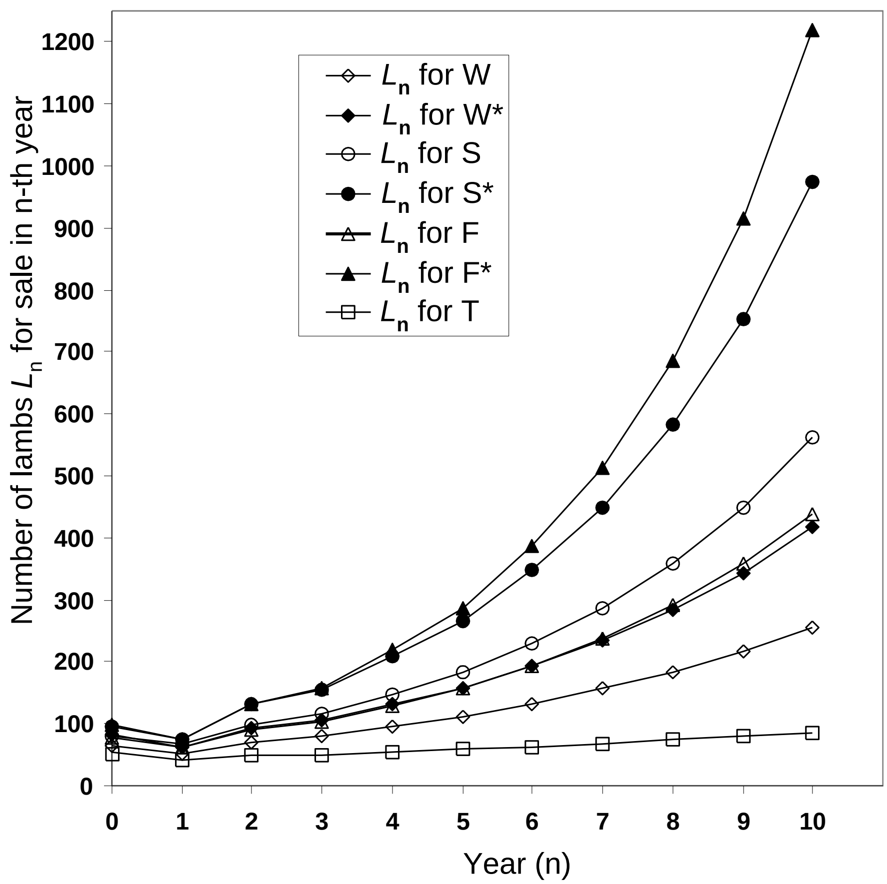

Figure 1. Number of lambs L_n (see (4) and (8)) for sale in **n**-th year for Suffolk (S), Ile d' France (F), Württemberg (W) and domestic Tsigai (T) sheep breeds. Initial herd contains 100 ewes. Notation: p_1-average percentage of lambing, p_3-average percentage of withhold female lambs, p_4-average percentage of fertility index, **r**-average percentage of withhold ewes. With sign * it is denoted calculated values for intensive sheep breeding.

beginning of credit repayment. Then, after 5 years, if S_5 =100 ewes, for average reproductive values of Württemberg breed, initial fund should be $S = S_5 / k_5 = 57$ ewes for traditional breeding, or 52 ewes for intensive breeding (three lambing in two years). On the basis of Equations 6 and 7 the number of female sheep and lambs can be predicted, which enables the planning of future feeding, care and breeding expenses.

Testing the next model would also be interesting since it could provide us with the answers to the following questions: Is there any profit at all? Which year is expected to have the maximum profit? Which type of sheep breeding brings higher profit under given circumstances? Which market conditions cause replacement in leadership? So, let B denote the seed capital needed for herd establishment with S, I_m yearly income per ewe, I_c yearly income from lamb sale, I_r yearly income from culling of ewes, and I_e yearly expenses for livestock servicing per each ewe. Let D_i be the total income up to **i**-th year (including the **i**-th year), **i** = 0, 1,…, 10. Then:

$$D_{\mathbf{i}} = \sum_{\mathbf{n}=0}^{\mathbf{i}} S_{\mathbf{n}} \cdot (I_{\mathbf{m}} - I_{\mathbf{c}} + (1-\mathbf{r}) \cdot I_{\mathbf{r}} + \underline{\mathbf{p}} \cdot I_{\mathbf{c}}) - B^{\cdot}$$

Many of these questions can be answered using software which projects gross margin, net worth and net income for sheep and goat (Millear et al., 2005). On the other hand, because of the recursiveness of (Equation 5) and assumption in the presented model that input reproductive parameters p_1, p_3 and p_4 for a particular breed are stable, it is sufficient to enter the data once in order to get S_n and L_n for arbitrary number of years **n**. It is enough to predict herd size for every subsequent year. “Breedcow and Dynama” software is spreadsheet-based and all input data must be inserted for each year. The proposed model in this study is not spreadsheet based, but a recursive one. Additional advantage of this model, which is not explicitly given in “Breedcow and Dynama” software, is direct calculation of the required number of ewes for the initial herd; that is, if it is desired to have a herd with *e* ewes after *y* years. Values of *e* and *y* are arbitrary. Similar studies have not been found.

ACKNOWLEDGEMENT

This work was supported by the Republic of Serbia (Projects: 2199/2011, TR-31046 and TR-37017).

REFERENCES

Acketa DM, Matić-Kekić S (2000). Asymptotics for the maximal number of edges of a digital convex arc. Indian J. Pure Appl. Math., 31(6): 583- 590.

Acketa DM, Mudrinski V, Matić-Kekić S (2000). A large collection of 2-designs from a wreath product on 21 points. Ars Combinatoria, 54: 109-118.

Babić Lj, Babić M, Turan J, Matić-Kekić S, Radojčin M, Mehandžić-Stanišić S, Pavkov I, Zoranović M (2011a). Physical and stress-strain properties of wheat (*Triticum aestivum*) kernel. J. Sci. Food Agric., 91(7): 1236-1243.

Babić Lj, Matić-Kekić S, Dedović N, Babić M, Pavkov I (2011b). Surface area and volume eq.ing of the Williams pear (*Pyrus communis*). Int. J. Food Prop., doi: 10.1080/10942912.2010.506020.

Bodroža-Pantić O, Matić-Kekić S, Jakovljev B, Marković Đ (2008). On MTE-eq. of mathematics Teaching: Studying the Problem Related to a Plane Division Using the MTE-eq.. Int. J. Math. Educ. Sci. Technol., 39(2): 197-213.

Cognie Y, Perret G, Oldham CM (1980). Reproductive aspects of intensive sheep breeding. In Proceedings of Australian Society for Animal Production, Sidney, pp. 305-308.

Dedović N, Matić-Kekić S, Ponjican O, Karadzic B (2011). New approach to border line evaluations for whole sample of Williams pear (*Pyrus communis*). Comput. Electron. Agric., 79(1): 94-101.

Demiral O, Mustafa U, Murat A, Tayfur B (2007). The effect of artificial insemination timing on the sex ratio of offspring and fertility in dairy cows. Turkish J. Veterinary Anim. Sci., 31: 21-24.

Durand B, Martinez MJ, Calavas D, Ducrot C (2009). Comparison of strategies for substantiating freedom from scrapie in a sheep flock. BMC Veterinary Res., 5: 16 doi:10.1186/1746-6148-5-16.

Holmes WE (2005). Breedcow and Dynama Herd Budgeting Software Package, Version 5.04 DPIF Townsville, Training Series QE9902, ISSN 0812-0005, ISBN 0 73454 1143 2, AGDEX 821, Version 5.05 available at: http://www.dpi.qld.gov.au/16_6886.htm.

Janssens S, Vandepitte W, Bodin L (2004). Genetic parameters for litter size in sheep: natural versus hormone-induced oestrus. Genet. Sel. Evol., 36: 543–562.

Keady TWJ, Hanrahan JP (2009). Effects of shearing at housing, grass silage feed value and extended grazing herbage allowance on ewe and subsequent lamb performance. Anim., 3(1): 143-151.

Krajinović M (2006) Sheep Breeding and Goat Farming (in Serbian), Faculty of Agriculture, Novi Sad, Serbia, p. 256.

Lindström J, Coulson T, Kruuk L, Forchhammer MC, Coltman DW, Clutton-Brock T (2002). Sex-ratio variation in Soay sheep. Behav. Ecol. Sociobiol., 53: 25-30.

Matić-Kekić S, Acketa DM (1997). Non-greedy optimal digital convex polygons. Indian J. Pure Appl. Math., 28(4): 455-470.

Matić-Kekić S, Dedović N, Trivunović S (2011). Recursive formula for number of cows in milk production. Contemp. Agric., 61(1): 7-14.

Mekić C, Latinović D, Grubić G (2007). Breeding, Reproduction, Selection and Feeding Sheep (in Serbian), University of Belgrade, Belgrade, Serbia, p. 328.

Millear G, Conway A, Mills T (2005). http://www2.dpi.qld.gov.au/sheep/5340.html.

Napier KM, Mullaney PD (1974). Sex ratio in sheep. J. Reprod. Fertil. 39: 391-392.

Nikolić Lj, Pajević S, Ljevnaić B (2009). Primary production dynamics of dominant hydrophytes in Lake Provala (Serbia). Cent. Eur. J. Biol., 4(2): 250-257.

Ochoa-Cordero M, Torres-Hernández AG, Mandeville PB, Díaz-Gómez MO (2007). Effects of physiological and management factors on the milk composition of Rambouillet ewes. Agrociencia, 41: 263-270.

Petrović MP, Ružić-Muslić D, Maksimović N (2009). Evaluation of genetic potential of sheep in different production systems. Biotechnol. Anim. Husbandry, 25: 421-429.

Plieninger T, Wilbrand C (2001). Land use, biodiversity conservation, and rural development in the dehesas of Cuatro Lugares, Spain. Agroforestry Syst., 51: 23–34.

Radulović N, Milovanović S, Djordjević G, Miladinović Z (1987). Estrus synchronization and off-season lambing of sheep with intention of intensive lamb production. Anim. Physiol. Reprod., 3: 42-44, 1987.

Silva Del Rio N, Stewart S, Rapnicki P, Chang YN, Fricke PM (2007). An observational analysis of twin births, calf sex ratio, and calf mortality in Holstein dairy cattle. J. Dairy Sci., 90: 1255-1264.

Stančić B (2006). Reproduction of Sheep's – manual (in Serbian). Faculty of Agriculture, Novi Sad, Serbia, p. 223.

Tabarez-Rojas A, Porras-Almeraya A, Vaquera-Huerta H, Hernández-Ignacio J, Valencia J, Rojas-Maya S, Hernández-Cerón J (2009). Embryonic development in pelibuey and Suffolk ewes under heat stress conditions. Agrociencia, 43: 671-680.

Tomić M, Matić-Kekić S, Savin L, Desnica E, Dedović N, Simikić M, Ponjičan O, Ašonja A (2011). Optimization of the locations of overhaul capacities for agricultural engineering in Serbia by applying integer programming, Afr. J. Agric. Res., 18: 3346-3354.

Wolfram S (2008). Mathematica: A System for Doing Mathematics by Computer, Fourth Edition. Addison-Wesley Publishing Company, Inc., Wolfram Research, Illinois, p. 584.