

Full Length Research Paper

Cost efficient and practical design of water supply network using harmony search

Zong Woo Geem¹, Jin-Hong Kim^{2*} and Soon-Hyuk Jeong³

¹Environmental Planning and Management Program, Johns Hopkins University, 11833 Skylark Road, Clarksburg, Maryland 20871, USA.

²Department of Civil and Environmental Engineering, Chung-Ang University, 221 Heukseok-dong, Dongjak-gu, Seoul, Korea.

³Department of Water Resources, Dongho, Incorporation, 1588-8 Gwanyang-dong, Dongan-gu, Anyang-si, Korea.

Accepted 30 May, 2011

Water supply network is a key infrastructure in urban civilization and agricultural irrigation. In order to save the design cost of a network, which contains a nonlinear relationship between hydraulic energy loss and water flowrate, researchers have traditionally used mathematical optimization approaches. However, they have been more interested in meta-heuristic approaches recently because (1) these approaches directly provide commercial discrete diameters instead of impractical continuous ones; (2) they do not require complex gradient derivatives and free from divergence; (3) they do not require starting feasible vector and have more chance to find global optimum, etc. This study presents a new real-world example for the water network design, hoping to be a good literature. Furthermore, this study newly considers a more practical constraint (flow velocity range) that can avoid a critical problem of water hammer or pipe choking by sedimentation. Then, the numerical example is solved using a meta-heuristic algorithm named harmony search, and the results are compared with those from a mathematical approach.

Key words: Water distribution network, linear programming, harmony search, optimization, velocity constraint.

INTRODUCTION

Water network can be one of the key elements in any civilization as we observed in Roman aqueduct. In addition, a recent survey by the British Medical Journal claimed that the idea of piping water into homes is the best medical advance among various candidate advances including the discovery of antibiotics and the development of vaccines. According to a vote of more than 11,000 people worldwide, engineering protection against health hazards is often the best way to improve population health, since inadequate sanitation still remains a problem in the developing world, contributing to millions of deaths (BBC, 2007). Also, the water network can be a critical irrigation tool for quality harvest in agricultural field.

Basic idea of water network design comes from a hydraulic equilibrium. If there are two parallel pipelines

between water origin (for example, reservoir) and water destination (for example, a house), the amount of water in each pipeline can be determined based on the concept of equal hydraulic head loss in each pipeline. Here, the relationship between hydraulic head loss and water amount is nonlinear from Darcy-Weisbach or Hazen-Williams equation as follows (May and Tung, 1992):

$$h_L = f \frac{L V^2}{D 2g} = K_{DW} Q^2 \quad (1)$$

$$h_L = \frac{KLQ^{1.852}}{C_{HW} D^{4.87}} = K_{HW} Q^{1.852} \quad (2)$$

Where h_L is the hydraulic head loss; f is the friction factor; L is the pipe length; D is the pipe diameter; V is the water velocity; g is the gravity acceleration; K_{DW}

*Corresponding author. E-mail: jinhkim@cau.ac.kr Tel: +82-2-820-5893.

is the constant value for Darcy-Weisbach equation; Q is the water amount (flowrate); K is the coefficient related with unit; C_{HW} is the Hazen-Williams roughness coefficient; K_{HW} is the constant value for Hazen-Williams equation. By expanding the aforementioned hydraulic equilibrium concept of two parallel pipes, a more complicated hydraulic network, which has multiple reservoirs and demand nodes can be designed.

Traditionally, hydraulic engineers have designed the water network structure based on their engineering sense. Once water demand at each node is forecasted, the link layout of water network is determined. If certain link requires high volume of water, its diameter correspondingly becomes large. However, they could not consider the optimality in this process.

The optimal design of water distribution networks has started since 1960s (Goulter, 1992). Most of these approaches have utilized linear and nonlinear techniques and their decision variables (pipe diameters) are assumed to be continuous. However, the obtained continuous diameters are not practical because commercial diameters are normally discrete. Although engineers converted the continuous values into discrete ones by round-off, this operation could worsen the solution quality and might not even guarantee the feasibility of the solution (Geem, 2006)

In order to overcome the aforementioned shortcomings of mathematical approaches in water network optimization, since 1990s, researchers have turned their interests into meta-heuristic algorithms (Geem, 2006) whose term was derived from the Greek word "Eureka" for "discover". Meta-heuristic approaches stochastically find better solutions, while mathematical approaches deterministically find better solutions using vertex visiting, gradient measuring, or tree pruning. The advantages of meta-heuristic approaches over mathematical approaches include several major points: (1) The former does not require gradient-based derivatives that are sometimes very difficult to obtain; however, certain meta-heuristic algorithm has different-type stochastic derivative, for example, the harmony search algorithm utilizes human experience-based stochastic derivatives (Geem, 2008); also, it is free from divergence (Geem, 2006); (2) the former does not require an initial vector to start with, thus, it does not require the effort to find feasible starting vector (Geem, 2006) and has higher chance to avoid local optima and to find global optimum; (3) the former is able to consider not only continuous variables but also discrete variables without additional process. Also, it can consider not only functions but also data tables easily.

Moreover, the meta-heuristic algorithms, such as genetic algorithm (Broad et al., 2005, tabu search (Cunha and Ribeiro, 2004), ant colony optimization (Maier et al., 2003), shuffled frog-leaping (Eusuff and Lansey, 2003) and harmony search (Geem, 2009), have obtained much better designs than mathematical approach (hybrid

method using linear programming and dynamic programming (Shaake and Lai, 1969) when they were applied to the design of New York City water network. Results showed that all of meta-heuristic algorithms have saved design cost more than 50% of the cost obtained by the mathematical approach.

If there is multiple global optima in linear programming problem, meta-heuristic algorithms may reach most of them, while simplex method finds only one of them and terminates the search (Geem, 2007). When the location of global optimum (or near-global optima) is hard to find because of complicated constraints, meta-heuristic algorithms may find a better solution without enumerating too many candidate solutions when compared with branch and bound method (Geem, 2005). Although problem size is too big (for example, total enumeration = 10^{454}), meta-heuristic algorithms found good solutions within reasonable time and memory usage (Geem, 2009).

Thus, this paper intends to present the advantages of meta-heuristic algorithms in water network design by showing a new numerical example. Also, this study proposes a more realistic formulation than previous researches by including a flow velocity range constraint.

METHODS

Optimization formulation for water network design

Optimal design problem of water distribution network can have the following objective function:

$$\text{Minimize Cost} = \sum_i f(D_i, L_i) \quad (3)$$

Where $f(\cdot)$ is cost function or cost table; D_i is diameter in each link i ; and L_i is length of each link i .

The problem has technical constraints, such as mass conservation, energy conservation, pressure head range, and flow velocity range (Mays and Tung, 1992). Mass conservation constraint can be expressed as follows:

$$\sum Q_j^{in} - \sum Q_j^{out} = d_j \quad (4)$$

Where Q_j^{in} is each flow into node j ; Q_j^{out} is each flow out of node j ; and d_j is nodal water demand.

Energy conservation constraint can be expressed as follows:

$$\sum_i h_i^{loss} = 0, \quad i \in k \quad (5)$$

Where h_i^{loss} is head loss along link i which forms loop k .

Pressure head range constraint can be expressed as follows:

$$h_j^{\min} \leq h_j \leq h_j^{\max} \quad (6)$$

where h_j is pressure head at node j ; and h_j^{\min} and h_j^{\max} are lower and upper limits of pressure head at node j (this study sets only lower limit of 10 m).

Flow velocity range constraint can be expressed as follows:

$$v_i^{\min} \leq v_i \leq v_i^{\max} \quad (7)$$

Where v_i is flow velocity in link i ; and v_i^{\min} and v_i^{\max} are lower and upper limits of flow velocities in link i .

Pipe diameter should be discrete and commercially available:

$$D_i \in \Phi_i \quad (8)$$

Where Φ_i is candidate diameter set for link i .

In order to satisfy mass conservation constraint in Equation 4 and energy conservation constraint in Equation 5, a popular hydraulic simulator EPANET (Rossman, 2000), which calculates a set of nonlinear equations by establishing a matrix, can be adopted as a module. EPANET tracks the flow rate in each pipe and the pressure at each node. Thus, the optimizer in this study only checks pressure head range and flow velocity range because mass conservation and energy conservation are automatically satisfied by the hydraulic simulator. Although the flow velocity constraint is a very reasonable one, previous researches have not vigorously considered it in their formulations.

Harmony search algorithm

Various meta-heuristic algorithms, such as genetic algorithm (Broad et al., 2005), ant colony optimization (Maier et al., 2003), and particle swarm optimization (Geem, 2009) have been applied to the optimal design of water distribution network in recent years. As mentioned in the Introduction, meta-heuristic algorithms do not require round-off process, but directly obtain discrete diameters. Among assorted meta-heuristic algorithms, the performance of the harmony search (HS) algorithm is one of the best (Geem, 2006; Geem, 2009). When three popular benchmark network problems (two-loop, Hanoi, and New York) were considered, HS outperformed other algorithms (genetic algorithm, simulated annealing, tabu search, ant colony optimization, shuffled frog-leaping, cross entropy, scatter search) in terms of design cost or computation amount. Thus, this study adopts HS for optimizing the water network.

As a meta-heuristic algorithm, HS is similar to other meta-heuristic algorithms such as genetic algorithm and particle swarm algorithm because they manage a group of solutions in a population rather than a single solution with gradient information. However, HS has its own uniqueness. For example, while genetic algorithm creates next chromosomes using one (mutation) or two (crossover) existing ones, HS makes full use of all the solutions (Geem, 2009).

The harmony search algorithm, which was inspired by music improvisation (Geem, 2001), starts with memory matrix named harmony memory (HM) as follows:

$$HM = \begin{bmatrix} D_1^1 & D_2^1 & \dots & D_n^1 & f(\mathbf{D}^1) \\ D_1^2 & D_2^2 & \dots & D_n^2 & f(\mathbf{D}^2) \\ \vdots & \vdots & \dots & \vdots & \vdots \\ D_1^{HMS} & D_2^{HMS} & \dots & D_n^{HMS} & f(\mathbf{D}^{HMS}) \end{bmatrix} \quad (9)$$

where HMS is harmony memory size (number of vectors in the matrix).

In Step 1, HM is initially filled with randomly generated vectors as many as HMS. Here, each variable of each solution vector should have one value out of candidate discrete diameters as described in Equation 8.

In Step 2, a new vector \mathbf{D}^{New} is generated based on three operations as follows:

$$D_i^{New} \leftarrow \begin{cases} D_i, & D_i \in \Phi_i & w.p. & (1-HMCR) \\ D_i, & D_i \in HM & w.p. & HMCR(1-PAR) \\ D_i + \Delta, & D_i \in HM & w.p. & HMCR \cdot PAR \end{cases} \quad (10)$$

Where $HMCR$ is harmony memory considering rate; PAR is pitch adjusting rate; and Δ is neighboring distance ($= D_i(k+1) - D_i(k)$ or $D_i(k-1) - D_i(k)$ if $D_i \in \Phi_i = \{D_i(1), \dots, D_i(k), \dots, D_i(K_i)\}$).

The diameter of

each pipe in the new vector \mathbf{D}^{New} can be determined based on one of three operations (random selection, memory consideration, or pitch adjustment). In random selection, the diameter is randomly selected by considering all candidate ones Φ_i with probability of $(1-HMCR)$; in memory consideration, the diameter is randomly selected by considering all diameters stored in HM with probability of $HMCR(1-PAR)$; and, in pitch adjustment, the diameter is obtained by slightly modifying the diameter, which is originally selected in memory consideration.

In Step 3, if the new vector \mathbf{D}^{New} is better than worst vector \mathbf{D}^{Worst} in HM, they are swapped:

$$\mathbf{D}^{New} \in HM \wedge \mathbf{D}^{Worst} \notin HM \quad \text{if } f(\mathbf{D}^{New}) \text{ is better than } f(\mathbf{D}^{Worst}) \quad (11)$$

In Step 4, termination criterion is checked. Until termination criterion is satisfied, new vectors are repeatedly generated and HM is updated.

For better demonstration, the HS algorithm is described using a flowchart as shown in Figure 1. Also, the reference (Geem, 2006) contains the details of water network design process using HS. Here, Equation (10) can be rewritten in the format of the novel stochastic derivative of HS (Geem, 2008) as follows:

$$\left. \frac{\partial f}{\partial D_i} \right|_{D_i=D_i(l)} = \frac{1}{K_i} \cdot (1-HMCR) + \frac{n(D_i(l))}{HMS} \cdot HMCR \cdot (1-PAR) + \frac{n(D_i(l \mp 1))}{HMS} \cdot HMCR \cdot PAR \quad (12)$$

The value of the stochastic derivative stands for the probability with which certain candidate diameter $D_i(l)$ is selected for the new vector.

RESULTS AND DISCUSSIONS

Real-world example

The numerical example of water network design is Yeosunetwork (Jang, 1968) located in South Korea as shown in Figure 2, which has one reservoir (pressure head = 65 m), 19 nodes, 29 links, and 11 loops. Detailed

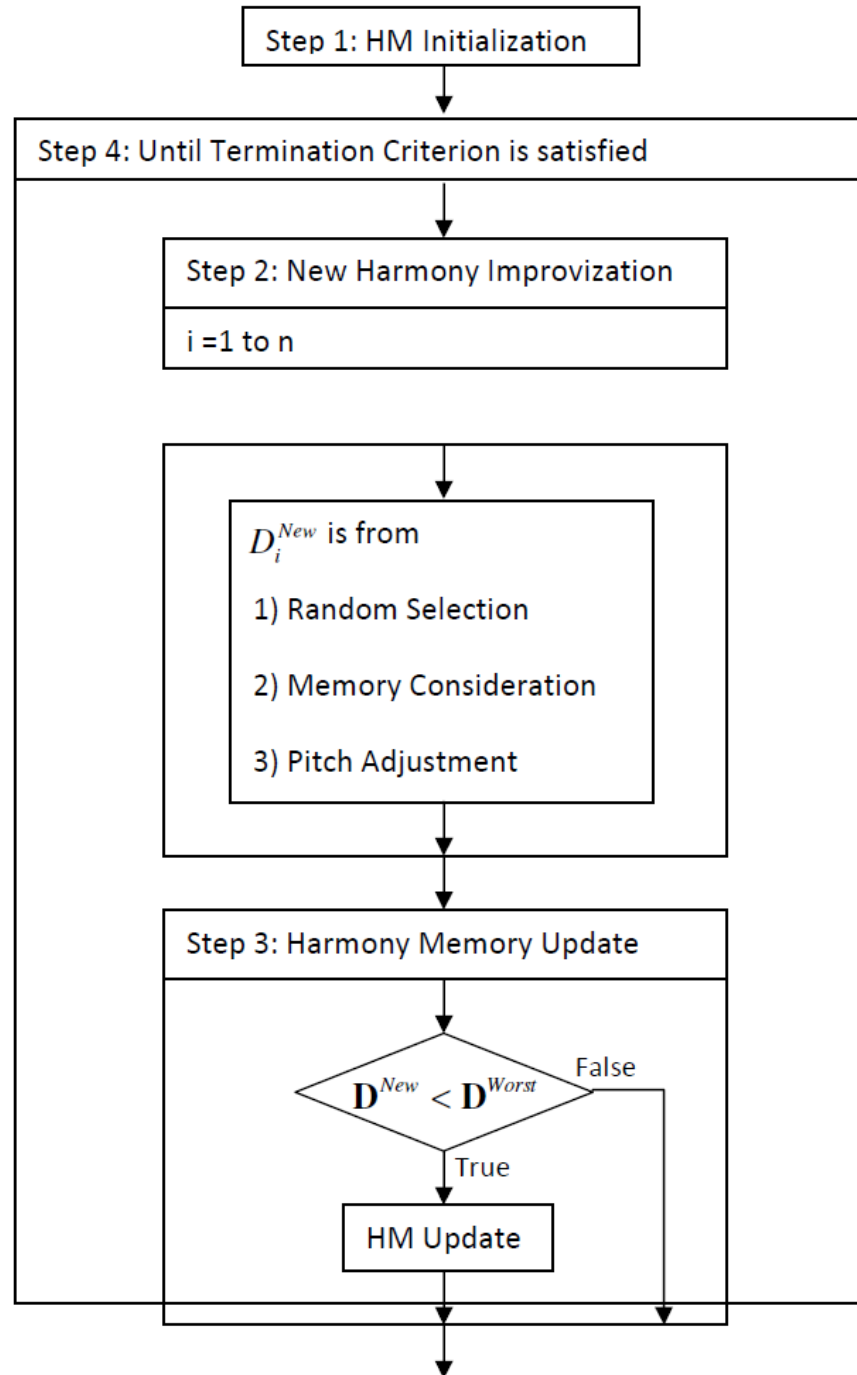


Figure 1. Flowchart of harmony search algorithm.

information, such as ground elevation, nodal water demand, pipe length, and pipe cost, can be found in Tables 1 to 3.

Originally, Yeosu network was designed using a mathematical approach (linear programming module named ALLEGRO on CDC 3600 computer) (Jang, 1968). The third column in Table 2 shows optimized diameters for 29 links (two values in parenthesis mean starting and

ending node numbers). However, because those diameters are continuous rather than discrete, round-off process is added in order to obtain commercial discrete diameters as described in the fourth column in Table 2. The optimal cost obtained by linear programming module is \$241,772.

The HS algorithm also tackled the problem, obtaining a better result in terms of diameter as shown in the sixth

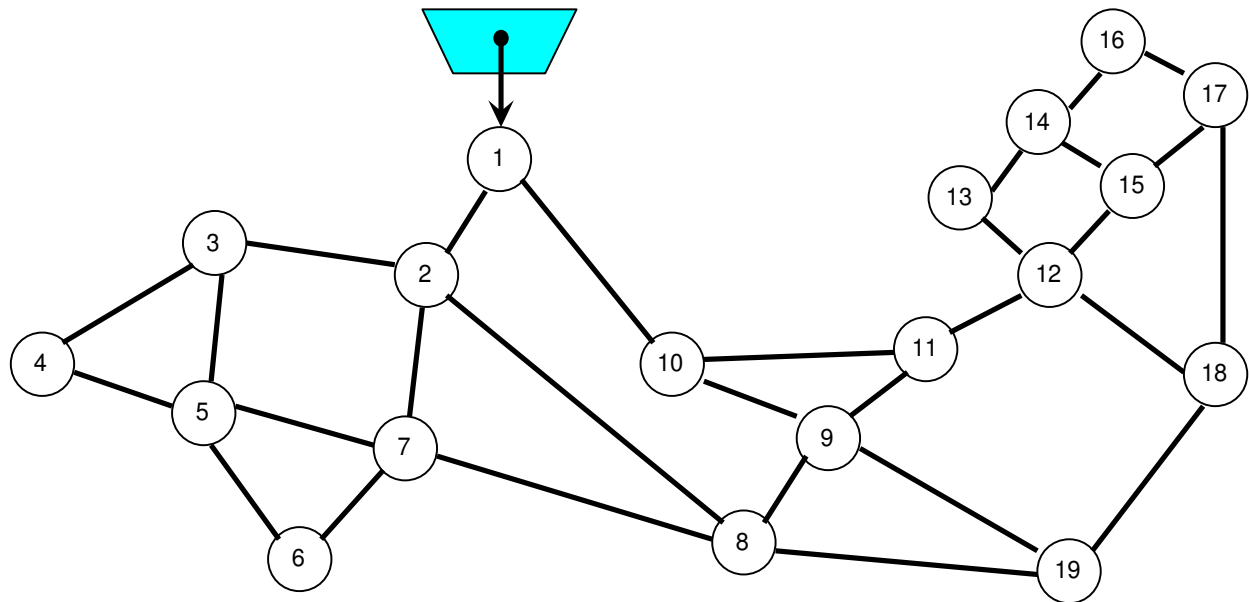


Figure 2. Schematic of Yeosu water network.

Table 1. Nodal information.

Node number	Elevation (m)	Demand (liter/s)	Pressure head (m)			
			LP ¹	LP ²	HS ¹	HS ²
1	0	13.0	65.00	65.00	65.00	65.00
2	0	10.7	60.04	64.08	63.02	63.11
3	0	17.2	55.69	60.04	58.26	58.95
4	40	45.0	14.77	19.20	16.55	17.34
5	0	19.5	55.04	59.47	57.88	58.74
6	0	14.8	55.57	60.28	58.51	59.51
7	0	6.5	57.17	62.19	60.21	61.39
8	0	19.0	56.41	62.28	59.42	58.51
9	0	8.6	56.26	62.11	58.24	57.57
10	0	12.6	57.16	62.51	60.67	60.29
11	0	21.8	56.00	61.85	57.01	56.34
12	0	15.0	55.82	61.70	56.19	55.49
13	0	8.3	55.62	61.54	55.64	55.32
14	0	11.7	55.58	61.51	55.20	55.12
15	0	13.9	55.72	61.63	55.21	55.08
16	45	3.9	10.56	16.51	10.16	10.06
17	0	12.2	55.57	61.52	55.16	55.05
18	0	11.0	55.54	61.49	55.66	55.23
19	40	17.3	15.65	21.65	16.91	16.27

column in Table 2. Because the algorithm proposed the design cost of \$192,383, it saved 20.4% when compared with linear programming technique. Tables 1 and 2 provide additional information, such as flow velocity and pressure head.

Furthermore, in order to perform a more practical design, the constraint of flow velocity range in Equation 7

is considered in this study, while the constraint was not included in the original design (Jang, 1968). Actually, the flow velocity constraint has been rarely adopted in previous researches (Samani and Mottaghi, 2006), and there is no fixed rule for velocity range. However, pressures start to drop off when velocity reaches 3.0 m/s (Walski, 2003). Water hammer (surge), due to fast valve

Table 2. Link information.

Link number	Length (m)	Diameter (mm) and Velocity (m/s)						
		LP ¹	LP ²	HS ¹	HS ²			
1 (1,2)	140	315	450	1.42	350	1.84	350	1.79
2 (2,3)	270	199	200	1.33	200	1.45	200	1.35
3 (3,4)	480	199	200	0.42	200	0.61	200	0.59
4 (3,5)	420	200	200	0.36	200	0.29	200	0.21
5 (5,4)	220	299	300	0.45	200	0.82	200	0.84
6 (2,7)	210	252	250	1.16	250	1.44	300	1.24
7 (7,5)	450	216	200	0.82	200	0.75	200	0.80
8 (6,5)	390	199	200	0.46	200	0.40	200	0.45
9 (7,6)	250	199	200	0.93	200	0.87	200	0.92
10 (1,10)	480	209	250	0.86	300	1.31	300	1.37
11 (2,8)	520	317	400	0.93	250	1.01	200	1.00
12 (7,8)	430	199	200	0.13	200	0.43	200	0.86
13 (10,11)	600	227	250	0.38	200	0.82	200	0.85
14 (10,9)	300	199	200	0.36	250	1.10	250	1.17
15 (8,9)	160	449	450	0.54	200	0.91	200	0.80
16 (9,11)	300	450	450	0.47	300	0.85	300	0.85
17 (8,19)	1,070	200	200	0.23	200	0.49	200	0.46
18 (9,19)	700	225	250	0.28	200	0.44	200	0.43
19 (19,18)	870	199	200	0.12	200	0.38	200	0.34
20 (11,12)	180	430	450	0.45	300	0.91	300	0.92
21 (12,18)	700	202	200	0.16	200	0.26	200	0.18
22 (17,18)	570	199	200	<u>0.07</u>	200	0.29	200	0.17
23 (12,13)	130	199	200	0.36	200	0.67	300	0.46
24 (12,15)	260	421	450	0.26	200	0.63	200	0.39
25 (13,14)	260	198	200	<u>0.09</u>	200	0.41	300	0.34
26 (15,14)	130	199	200	0.30	200	<u>0.07</u>	200	0.16
27 (15,17)	260	273	300	0.25	200	0.12	200	0.11
28 (14,16)	300	199	200	<u>0.02</u>	200	0.10	250	0.15
29 (17,16)	130	199	200	0.10	200	<u>0.02</u>	200	0.11
Total Cost (\$)		N/A	241,772		192,383		195,853	

Table 3. Pipe cost.

Diameter (mm)	Unit cost (\$/m)
200	15.7652
250	20.2867
300	24.7882
350	35.8312
400	44.5225
450	51.5675

operation or power failure, also requires maximum velocity limit to alleviate any structural damage. Minimum velocity is desirable in order to prevent fine material (such as soil particle) sedimentation, which chokes pipes eventually.

If minimum velocity is set to 0.1 m/s and maximum velocity is set to 3.0 m/s, LP² (links 22, 25, and 28) and

HS¹ (links 26 and 29) violate the minimum velocity constraint as shown in Table 2. However, the result HS², which considered the velocity constraint, did not violate any minimum velocity limit, obtaining a design cost of \$195,853. The new design cost is slightly higher than HS¹, but still saves 19.0% of the original cost obtained in LP².

Thus, HS² can be a cost-effective design solution, as well as a structure-effective design solution. As seen in Table 2, LP² has too slow velocities in Pipe 22 (0.07 m/s between Nodes 17 and 18), Pipe 25 (0.09 m/s between Nodes 13 and 14), and Pipe 28 (0.02 m/s between Nodes 14 and 16); and HS¹ has too slow velocities in Pipe 26 (0.07 m/s between Nodes 15 and 14), and Pipe 29 (0.02 m/s between Nodes 17 and 16). However, every flow velocity in HS² is more than 0.10 m/s, which prevents the low-velocity sedimentation problem.

Actually, this new formulation with velocity constraint has been applied to popular bench-mark problems, such as two-loop and Hanoi networks (Geem, 2009). However, both problems were not suitable for this approach. For the two-loop network, the velocity constraint was not working because the original result without the velocity constraint lies within the velocity range (0.32 m/s to 1.90 m/s). For Hanoi network, this approach was not properly working either because the original result without the velocity constraint is much greater than velocity range. While the maximum allowable velocity is 3.0 m/s in this approach, Hanoi network has the maximum velocity of 6.83 m/s for a pipe although the pipe has a maximum diameter (40 inch).

Conclusions

This study briefly explained the reason why engineers prefer meta-heuristic algorithms to mathematical techniques in water network design. Then, it applied one of the popular meta-heuristic algorithms (HS) to a real-world network with a more practical constraint (flow velocity range). Results showed that the meta-heuristic algorithm found better solutions than the mathematical method (LP) in terms of design cost and practical viewpoint.

ACKNOWLEDGEMENT

This research was supported by the Chung-Ang University Research Scholarship Grants in 2008.

REFERENCES

- BBC News, Sanitation 'Best Medical Advance' (2007). <http://news.bbc.co.uk/2/hi/health/6275001.stm> (accessed Oct. 6, 2010).
- Mays LW, Tung YK (1992). *Hydrosystems Engineering and Management*. McGraw Hill, New York.
- Goulter IC (1992). "Systems Analysis in Water-Distribution Network Design: From Theory to Practice." *J. Water Resour. Plann. Manage.*, 118(3): 238-248.
- Geem ZW (2006). "Optimal Cost Design of Water Distribution Networks Using Harmony Search." *Eng. Optim.*, 38(3): 259-280.
- Geem ZW (2008). "Novel Derivative of Harmony Search Algorithm for Discrete Design Variables." *Appl. Math. Comput.*, 199(1): 223-230.
- Geem ZW (2006). "Parameter Estimation for the Nonlinear Muskingum Model Using BFGS Technique." *J. Irrig. Drain. Eng. ASCE*, 132(5): 474-478.
- Broad DR, Dandy GC, Maier HR (2005). "Water Distribution System Optimization Using Metamodels." *J. Water Resour. Plann. Manage. ASCE*, 131(3): 172-180.
- Cunha MC, Ribeiro L (2004). *Tabu Search Algorithms for Water Network Optimization*. *Eur. J. Oper. Res.*, 157: 746-758.
- Maier HR, Simpson AR, Zecchin AC, Foong WK, Phang KY, Seah HY, Tan CL (2003). "Ant Colony Optimization for Design of Water Distribution Systems." *J. Water Resour. Plann. Manage. ASCE*, 129(3): 200-209.
- Eusuff M, Lansey KE (2003). "Optimization of Water Distribution Network Design Using the Shuffled Frog Leaping Algorithm." *J. Water Res. Plann. Manage. ASCE*, 129(3): 210-225.
- Geem ZW (2009). "Particle-Swarm Harmony Search for Water Network Design." *Eng. Optim.*, 41(4): 297-311.
- Schaake J, Lai D (1969). *Linear programming and dynamic programming – application of water distribution network design*. Report 116 (MIT Press: Cambridge, MA).
- Geem ZW (2007). "Optimal Scheduling of Multiple Dam System Using Harmony Search Algorithm." *Lect. Notes Comput. Sci.*, 4507: 316-323.
- Geem ZW (2005). "Harmony Search in Water Pump Switching Problem." *Lect. Notes Comput. Sci.*, 3612: 751-760.
- Rossman LA (2000). *EPANET 2 Users Manual*. US Environmental Protection Agency, Cincinnati.
- Geem ZW (2009). *Music-Inspired Harmony Search Algorithms: Theory and Applications*, Springer, Berlin.
- Geem ZW, Kim JH, Loganathan GV (2001). "A New Heuristic Optimization Algorithm: Harmony Search." *Simulation*, 76(2): 60-68.
- Jang HS (1968). "Rational Design Method of Water Network Using Computer." *J. Korean Prof. Eng. Assoc.*, 1(4): 3-8.
- Samani HM, Mottaghi A (2006). "Optimization of Water Distribution Networks Using Integer Linear Programming." *J. Hydraulic Eng.*, 132(5): 501-509.
- Walski, T, Chase DV, Savic D, Grayman WM, Beckwith S, Koelle E (2003). *Advanced Water Distribution Modeling and Management*. Haestad Press, Waterbury.