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Sensitivity analysis of Doorenbos and Kassam (1979) crop water production function

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Doorenbos and Kassam's (1979) equation and the multiplicative and additive forms of it that were developed by Rao et al. (1988) were extensively used in optimization models and deficit irrigation planning. Although these equations were validated and successfully used by several investigators to predict crop yield at several locations, however, some signs of anomalies exist. This paper present the results of sensitivity analysis of these equations for potato crop in relation to three parameters namely maximum evapotranspiration (PET), actual evapotranspiration (AET) and crop yield response factor (K_y). Results show that plus error of PET and K_y and minus error of AET results in over predicting of relative yield (Y_r). For a specific error value of K_y, when water shortage increases, error percentage of estimated Y_r by Doorenbos and Kassam (1979) equation and by multiplicative and additive forms of Rao et al. (1988) equation increases. For a specific error value of PET or AET, however, when water shortage increases, error percentage of estimated Y_r by Doorenbos and Kassam (1979) equation and by multiplicative form of Rao et al. (1988) equation decreases while error percentage of estimated Y_r by additive form of Rao et al. (1988) equation increases. Sensitivity of Doorenbos and Kassam (1979) equation and additive form of Rao et al. (1988) equation is equal for plus or minus error of K_y and AET but their sensitivities are greater for minus error of PET than plus error. However, sensitivity of multiplicative form of Rao et al. (1988) equation is greater for minus error of K_y and PET and plus error of AET. It was shown that error percentage on estimation of Y_r by multiplicative form of Rao et al. (1988) equation arising from error of PET, AET or K_y is less than additive form. In addition, calculated Y_r by multiplicative equation is higher than additive form and the difference between two forms of this equation increases severely when water shortage increases. According to the results, it is recommended that multiplicative form of Rao et al. (1988) equation instead of additive form be used in optimization models and deficit irrigation planning.

Key words: Sensitivity analysis, Doorenbos and Kassam (1979) equation, multiplicative and additive equations, Rao et al. (1988) equation

INTRODUCTION

Doorenbos and Kassam (1979) according to Stewart et al. (1977) presented the following linear relationship between relative yield and relative evapotranspiration:

$$\left(1-\frac{Y_a}{Y_m}\right)=K_y\left(1-\frac{AET}{PET}\right) \qquad (1)$$

Where Y_a and Y_m are actual and maximum crop yields, corresponding to AET and PET, actual and maximum evapotranspiration, respectively; and K_y is crop yield response factor. Yield response factor varies depending on species, variety, irrigation method and management, and growth stage when deficit evapotranspiration is imposed (Kirda, 2002).

Rao et al. (1988) proposed multiplicative form of Equation (1) which covers all growth stages simultaneously and is an extended form of one growth stage water production function developed by Doorenbos and Kassam (1979)

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$$Y_r = \frac{Y_a}{Y_m} = \prod_{i=1}^{n} \left[1 - K_{yi} \left(1 - \frac{AET_i}{PET_i} \right) \right] \qquad (2)$$

Where i is an index for growth stage and n is total number of crop growth stages.

In addition to the multiplicative form of equation (1), Rao et al. (1988) have also addressed an additive form as follow:

$$Y_r = \frac{Y_a}{Y_m} = 1 - \sum_{i=1}^{n} K_{yi} \left(1 - \frac{AET_i}{PET_i} \right) \qquad (3)$$

They supported both equations (2) and (3) with similar results. Allen et al. (1998) in FAO Irrigation and Drainage Paper 56 (entitled Crop Evapotranspiration; Guidelines for Computing Crop Water Requirements) published calculation procedure for maximum evapotranspiration (PET). Calculation procedures for actual evapotranspiration (AET) and K_y values presented by Doorenbos and Kassam (1979) in FAO Irrigation and Drainage Paper 33 (entitled Yield Response to Water). The Joint FAO/IAEA Division of Nuclear Techniques in Food and Agriculture coordinated a research project between 1990 and 1995 entitled "The use of nuclear and related techniques in assessment of irrigation schedules of field crops to increase effective use of water in Irrigation projects". Some results of this project, such as yield response factor (Ky), were published in FAO Water Report 22 (entitled Deficit Irrigation Practices, 2002).

Although performance of Doorenbos and Kassam (1979) equation and similar equations such as Jensen (1968) were considered fairly adequate, however, such empirical models can rarely simulate field condition properly due to some inherent variability in field data that these models may not be able to capture (Igbadun et al., 2007). Despite the above shortcomings, these relationships particularly Doorenbos and Kassam (1979) and Rao et al. (1988) equations played a central role in mathematical programming models aiming at optimizing the water allocation and cropping pattern (Mannocchi and Mecarelli, 1994; Wardlaw and Barnes, 1999; Kipkorir et al., 2002a; Ghahraman and Sepaskhah, 2004; Montazar and Rahimikob, 2008). Therefore, optimal solutions of these optimization models are very dependent on the ability of Doorenbos and Kassam's (1979) equation in estimating the relative yield. It is also true that accurate estimation of relative yield by these equations is more dependent on accurate estimation of their parameters (AET, PET and K_y). Although enough information and valid methodologies have been presented for estimation of these parameters, however, some uncertainties exists which may cause inaccurate estimation of relative yield by Doorenbos and Kassam (1979) equation. These uncertainties can be grouped into factors related to three parameters (AET, PET and K_y).

Factors related to K_y

K_y values reported by Doorenbos and Kassam (1979) in FAO 33 are commonly used in calculating relative yield. However, according to the following reasons, their application may cause some errors:

- The two groups of K_y presented by FAO 33 and FAO/IAEA, showed wide ranges of variations of this parameter: $0.20 < K_y < 1.15$ (FAO), and $0.08 < K_y < 1.75$ (FAO/IAEA). The two data sets, while showing the same trends, gave neither identical average values for K_y nor similar ranges of variations. For example, Table 1 shows wide ranges of variations of K_y for potato crop.
- The yield response factors (K_y) of Doorenbos and Kassam (1979) were validated and successfully used by several investigators to predict crop yield at several locations in the USA, India, China, Korea, etc. (Ghahraman and Sepaskhah, 2004). However, some signs of anomalies have been seen in using them. For example, Igbadun et al. (2007) obtained yield response factor for the vegetative, flowering and grainfilling growth stages of maize as equal to 0.21, 0.86, and 0.49, respectively while Doorenbos and Kassam (1979) presented these values as 0.4, 1.5 and 0.5, respectively. Kipkorir et al. (2002b) found the seasonal K_y of the onion was 1.28 while Doorenbos and Kassam (1979) gave this parameter as 1.1. Prieto and Angueira (1999) obtained a value of 0.48 K_y for cotton in flowering stage while Ananc et al. (1999) gave it as equal to 0.67 (Kirda, 2002).
- The yield response factors (K_y) is dependent on locations. For example, Dehghanisanij et al. (2009) presented the seasonal K_y for winter wheat in Mashhad and Karaj (both in Iran) as 1.03 and 1.23, respectively while Doorenbos and Kassam (1979) presented it as one. There were similar results for the maize in Orumieh and Mashhad (both in Iran). - The seasonal K_y of the maize in Orumieh and Mashhad were 1.03 and 1.46, respectively while it was given as 1.25 by Doorenbos and Kassam (1979). Moutonnet (2002) presented K_y for the vegetative growth stage of cotton in Argentina and Pakistan as equal to 0.75 and 0.80 respectively while it was given as equal to 0.2 by Doorenbos and Kassam (1979). He reported similar results for the flowering stage and total growth season in Argentina, Pakistan and Turkey. In flowering stage, K_y of cotton in Argentina, Pakistan and Turkey were reported as 0.48, 0.60, and 0.76, respectively while Doorenbos and Kassam (1979) value was 0.5. In addition, for total growth season of the cotton, K_y values in Argentina, Pakistan and Turkey were reported as 1.02, 0.71, and 0.99, respectively while Doorenbos and Kassam (1979) gave it as equal to 0.85.
- It seems that yield response factor depends on irrigation method (Kirda, 2002). Madanoglu (1977) reported the seasonal K_y for the wheat under sprinkler and basin irrigaion as equal to 0.76 and 0.93, respectively (after Kirda, 2002) while it was reported as 1 and 1.5 for the winter and spring wheat, respectively by Doorenbos and

Table 1. Wide ranges of variations of K_y for potato crop.

Vegetative stage			Flowering stage	Yield formation stage	Ripening stage	Total season	Reference
Early	Late	Total					
0.45	0.8	-	-	0.7	0.2	1.1	FAO 33
	0.4		0.33	0.46	-	0.83	FAO 22

Kassam (1979) irrespective of irrigation method.
- In using K_y presented for other locations, it is necessary to pay attention to the number and the definition of growth stages. For example, Igbadun et al. (2007) divided maize total growing season to 3 stages while Doorenbos and Kassam (1979) divided it to 4 stages. Therefore, application of this K_y results in error when it is used for estimation of relative yield in other locations with different number and length of growth stages.
- High-yielding varieties are more sensitive to water stress than low-yielding varieties. For example, deficit irrigation had more adverse effects on the yield of new maize varieties than traditional ones (Kirda, 2002). Therefore, updating of different crops K_y is necessary because of the progress in biotechnology in production of new drought tolerant varieties.

Factors related to maximum evapotranspiration (PET)

FAO-Penman-Monteith (FPM) method is a standard method for calculation of reference evapotranspiration (ET_o). The length of four development stages (initial, development, mid and late season), K_c values and crop height reported in FAO 56 are valid for calculation of maximum evapotranspiration (PET), particularly where calibrated and validated data is not available. If so, according to the following reasons, there maybe some errors in the calculation of PET.

- The length of development stages, K_c values and crop height may be different from the data given by FAO 56 due to diversity of crop varieties. Consequently, there may be some errors in calculated PET.
- In some localities in the world, weather stations are located outside of agricultural land (such as airport or areas close to industrial or residential region). In such cases, even when the weather data is standardized, some errors may exist in calculated PET.
- In the old weather stations, where instruments and the methods of obtaining data are old fashioned, the possibility of finding errors is very high. Therefore, misleading estimation of PET may occur.
- In some localities in the world, the number of weather stations in an extensive region may be insufficient. In such cases, calculated PET should be extended from a few point stations to regional scale by interpolating between the available weather stations in time and space. As a result , there may be some errors in calculated PET in some areas.
- In some locatlities in the world, the length of the collection period of weather data is not sufficient. In such cases, the calculated PET cannot be applied with sufficient certainty for planning of irrigation scheme, particularly in deficit irrigation strategies.
- In some weather stations, calculation of ET_o according to FPM method is not possible. In such cases, calculation of PET should be taken by other methods. Jensen et al. (1990) compare 20 different ET_o estimation methods. Results of this comparative study show that estimation error of ET_o is between -18 to 35 and -37 to +21% in humid and arid region, respectively.
- Due to the stochastical nature of climatic parameters, PET prediction of crops even with precise and lengthy weather data is not possible. Therefore, there exist some uncertainties in the calculation of PET. For example, Reca et al. (2001a, b) show the variability of the production functions due to stochastical nature of climatic parameters.

Factors related to actual evapotranspiration (AET)

Doorenbos and Kassam (1979) equation is used extensively in optimization models for estimation of relative yield (Mannocchi and Mecarelli, 1994; Wardlaw and Barnes, 1999; Kipkorir et al., 2002a; Ghahraman and Sepaskhah, 2004; Montazar and Rahimikob, 2008). In these models, AET is calculated by water balance equation throughout crop root zone. According to following reasons, calculated AET may face with several errors:

- Many parameters in water balance equation are approximate and their precise values are not available. Effective rainfall, time dependency of rooting depth, soil moisture variations at variuos depths and groundwater share in crop water requirements are some instances of uncertainties.
- Due to the stochastical nature of climatic parameters, predicting of crops AET even with precise data is not possible. Therefore, calculated AET is usually faced with some uncertainties.

Generally, over or under estimation of AET or PET can cause over or under estimation of relative yield. Similarly, the calculated relative yield may be non exact because of the application of non exact values of K_y into Doorenbos

Table 2. Gradient of linear relationship between error percentage of K_y and error percentage of estimated Y_r.

AET_i/PET_i	Growth stage			
	1	2	3	4
0.95	-0.023	-0.041	-0.036	-0.010
0.9	-0.047	-0.087	-0.075	-0.020
0.8	-0.098	-0.190	-0.162	-0.041
0.7	-0.156	-0.315	-0.265	-0.063
0.6	-0.219	-0.470	-0.388	-0.087
0.5	-0.290	-0.667	-0.538	-0.111

and Kassam (1979) equation. For example, suppose that in deficit irrigation planning for potato (seasonal K_y equal to 1.1), PET and AET are calculated based on current methods and the seasonal $AET/PET = 0.7$ is proposed as an excellent option for increasing of water productivity. Relative yield under this situation will be equal to 0.67 based on Doorenbos and Kassam (1979) equation. If AET calculation is correct but PET under predicted only by 10%, the seasonal AET/PET will not be equal 0.7 but it will be equal to 0.64. Consequently, relative yield based on Doorenbos and Kassam (1979) equation is 0.60. In other words, this discrepancy in PET (10%) will cause an over prediction of 11.7% in relative yield [(0.67 - 0.60)/0.60 x 100 = 11.7%]. Although, this amount of error seems to be small but in a high yielding crop such as potato (according to FAO 33, maximum yield is equal to 15 to 35 tons per hectare), it may reach several tons (in this case, this error is equal to 1.75 to 4.1 tons per hectare). Thus, this level of deficit irrigation may not be an excellent option for increasing of water productivity based on this relative yield (0.60). Therefore, before implementing a deficit irrigation programme, it is necessary to know the accuracy of crop production functions, particularly Doorenbos and Kassam (1979) equation that is the most common one, for estimation of relative yield. Since in many locations, it is not possible to calibrate and to validate the Doorenbos and Kassam (1979) equation and its parameters. This brings to mind the question, how much deviation in relative yield can the amount of error in the named parameters (AET, PET and K_y) cause? In this work, the above question is answered by sensitivity analysis of Doorenbos and Kassam (1979) equation.

MATERIALS AND METHODS

This paper presents results of sensitivity analysis of Doorenbos and Kassam (1979) equation and two developed forms of it by Rao et al. (1988) (multiplicative and additive forms) for potato. the three discussed paremeters are : 1- yield response factor (K_y), 2- maximum evapotranspiration (PET) and 3- actual evapotranspiration (AET). The results were made dimensionless in order to make possible generalization. Therefore, error percentage in estimating of relative yield resulting from error of each parameters was calculated. Sensitivity analysis of the equation was conducted for a wide range of water shortage levels from no shortage ($AET/PET = 1$) to 50% water shortage ($AET/PET = 0.5$) on each growth stages of potato crop. Basic values of K_y were adopted from Doorenbos and Kassam (1979) in FAO 33 these are presented in Table 1. Sensitivity analysis of this equation was conducted in two scenarios. In scenario 1, it considered the effect of each parameter error only in one growth stage on the estimation of relative yield while it supposed that values of parameters were correct in other stages. Therefore, in this scenario, sensitivity analysis of equation is conducted for three parameters in each growth stage independent of other stages. It is evident that equations 1, 2 and 3 have similar responses in this scenario. In scenario two it was assumed that error of each parameter occurred in more than one growth stage (two, three or four stages) while it had correct values in other stages. Therefore, in this scenario, sensitivity analysis of equation was conducted for each parameter in two or more growth stages simultaneously. In this scenario, diverse situations occur from the viewpoint of combinations of different growth stages and amount of parameters error (plus or minus). In order to facilitate results comparison, it considered that identical error percentage of each parameter at the first two stages, three stages form 1 to 3 and four stages from one to four caused how many percentages of error for estimation of relative yield. It is evident that, in this scenario, equation 2 (multiplicative form of equation 1) and three (additive form of equation 1) do not have a similar responses. Finally, it presents the relationship between error percentage of each parameter and error percentage of estimated relative yield.

RESULTS

Effect of error in one growth stage

Effect of yield response factor (K_y) error

A linear relationship ($y = mx$) was found between error percentage of K_y (x) and error percentage of estimated Y_r (y). Table 2 shows the gradient (m) of this relationship. In addition, the effect of yield response factor error on estimating of relative yield (Y_r) by Doorenbos and Kassam (1979) equation in late vegetative stage of potato is shown in Figure 1. It is observed that plus error of K_y results in under prediction of Y_r and minus error of K_y results in over prediction of Y_r. In addition, plus and

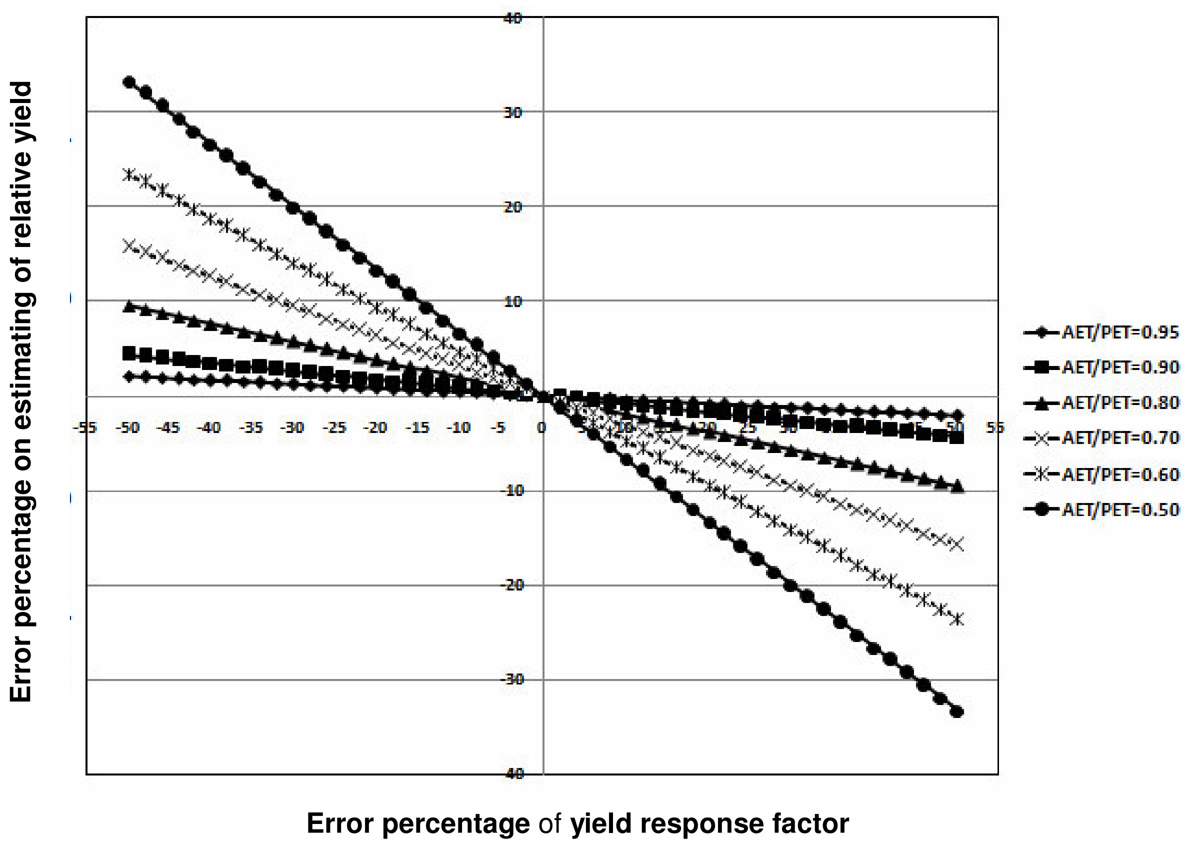

Figure 1. Effect of K_y error in late vegetative stage of potato on estimating of Y_r.

minus error of K_y have a similar error on prediction of Y_r. As shown, with decreasing of AET_i/PET_i (increasing of water shortage) and with increasing of K_y (for different growth stages), the gradient of this relationship increases. In other words, for specific error of K_y, when water shortage increases, error percentage of Y_r increases. This error is particularly higher in stages where crop is more sensitive to water shortage (K_y is greater) than they are in late vegetative stage (K_y = 0.8), yield formation stage (K_y = 0.7), early vegetative stage (K_y = 0.45) and ripening stage (K_y = 0.2), respectively for potato. For example, a 10% plus error of K_y in early vegetative stage of potato (K_y = 0.495 instead of K_y = 0.45) under $AET_i/PET_i = 0.5$ results in 2.9% minus error on estimating of Y_r (0.7525 instead of 0.775), while the same situation in late vegetative stage (K_y = 0.88 instead of K_y = 0.80) results in 6.7% minus error on estimating of Y_r (0.56 instead of 0.60). Therefore, when water shortage and K_y increase, sensitivity of Doorenbos and Kassam (1979) equation related to error of K_y parameter increases.

Effect of calculated PET error

The effect of calculated PET error on estimating of relative yield (Y_r) by Doorenbos and Kassam (1979) equation in the first, second, third and fourth growth stage of potato is shown in Figures 2 to 5, respectively. In these figures, PET1 is the true value of PET in each stage. Since, PET is the denominator of Doorenbos and Kassam (1979) equation, relationship between PET error and Y_r error is nonlinear. As shown, overestimation of PET (plus error) results in underestimation of Y_r and underestimation of PET (minus error) results in overestimation of Y_r. In addition, percentage error on estimating of Y_r decreases when water shortage increases and it increases when crop sensitivity to water shortage increases. It is remarkable that underestimation or over--estimation of PET has different effects on estimating of Y_r so that to underestimate of PET causes higher error of estimated Y_r than to overestimate of PET. For example, a 10% plus error in estimation of PET in late vegetative stage of potato under 10% water shortage ($AET_i/PET_i = 0.9$) results in 7.11% minus error of Y_r while a 10% minus error on estimating of PET under the same stage results in 8.7% plus error of Y_r. Overall, Doorenbos and Kassam (1979) equation is more sensitive to underestimation of PET than overestimation of it. In addition, sensitivity decreases when water shortage increases and it increases when K_y is higher. Therefore, when deficit irrigation, particularly low water

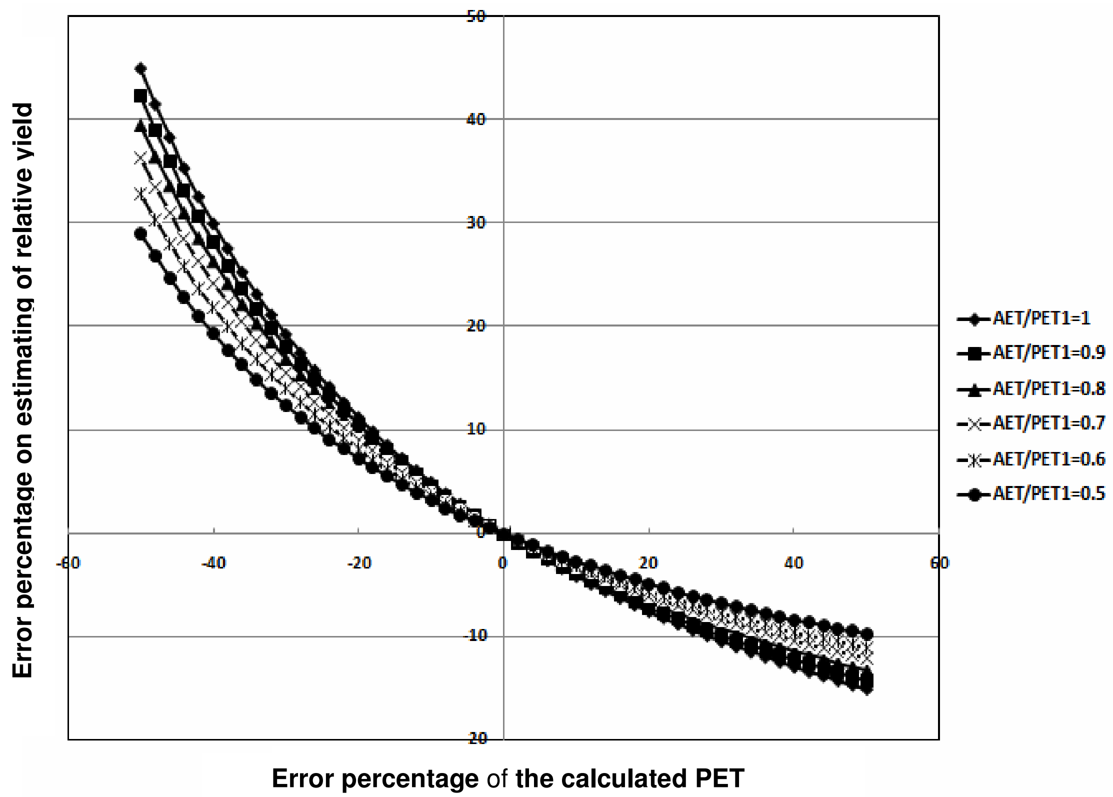

Figure 2. Effect of PET error in early vegetative stage of potato on estimating Y_r.

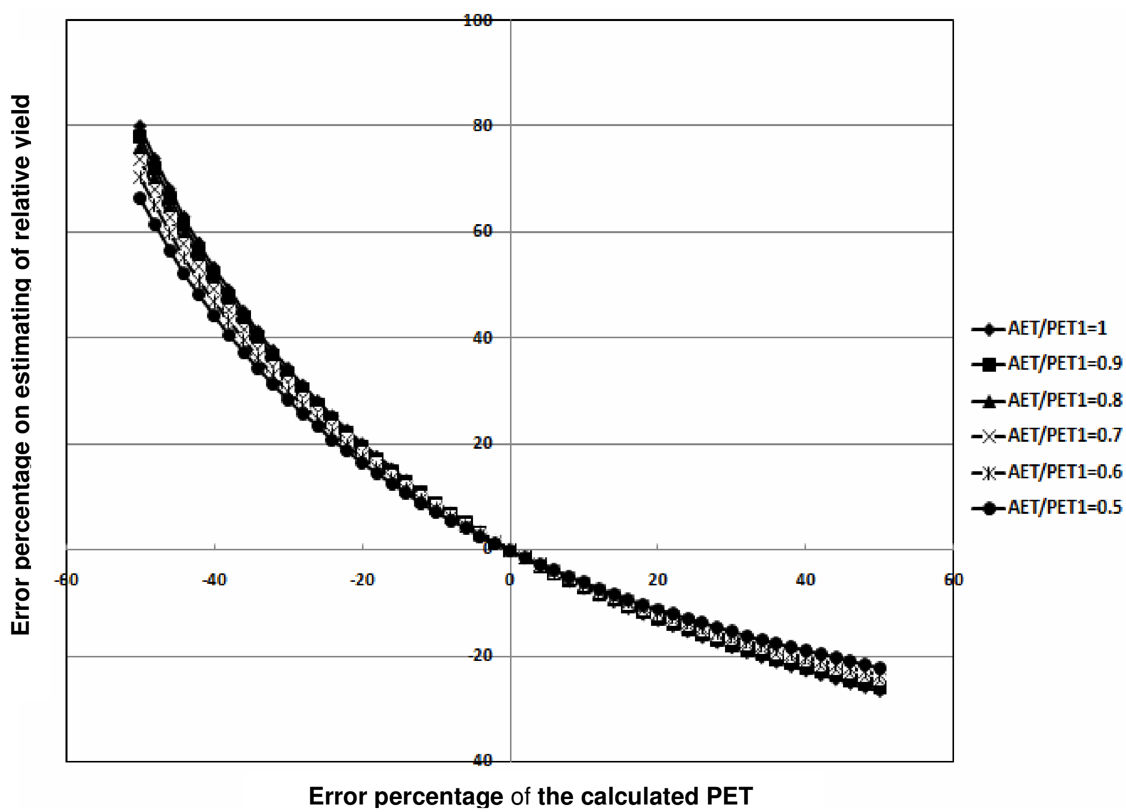

Figure 3. Effect of PET error in late vegetative stage of potato on estimating of Y_r.

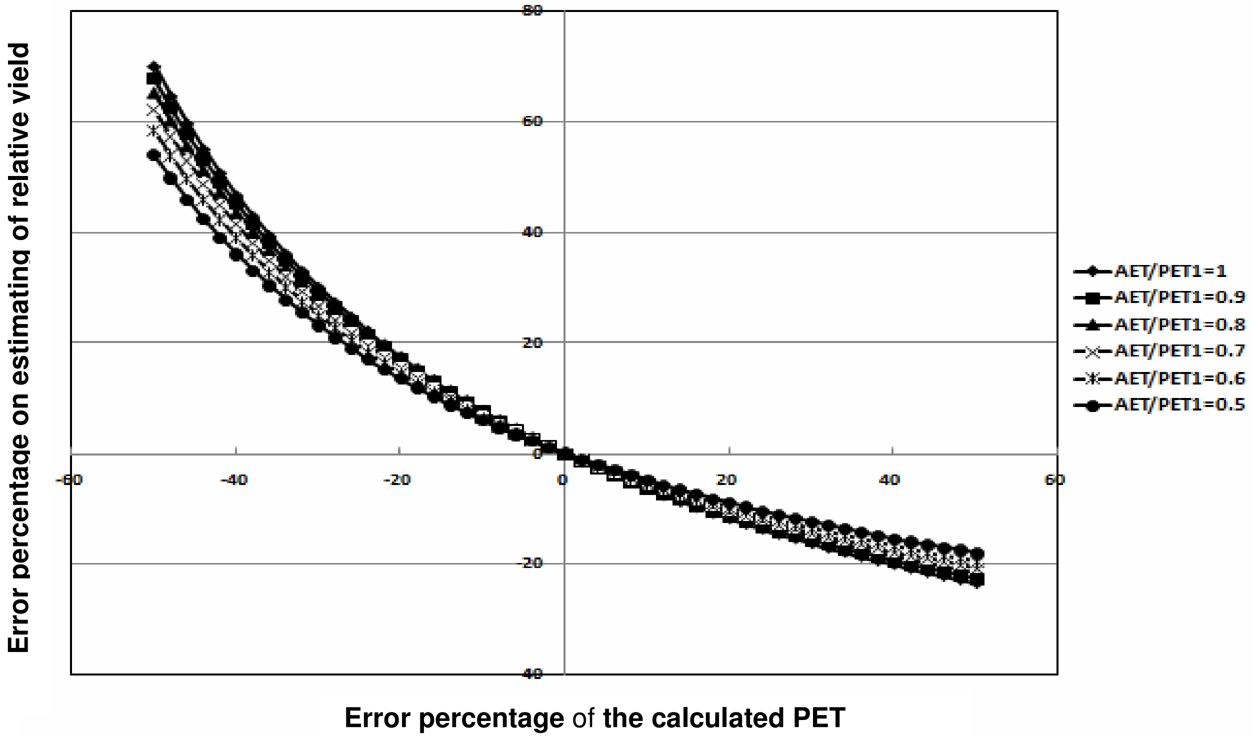

Figure 4. Effect of PET error in yield formation stage of potato on estimating of Y_r.

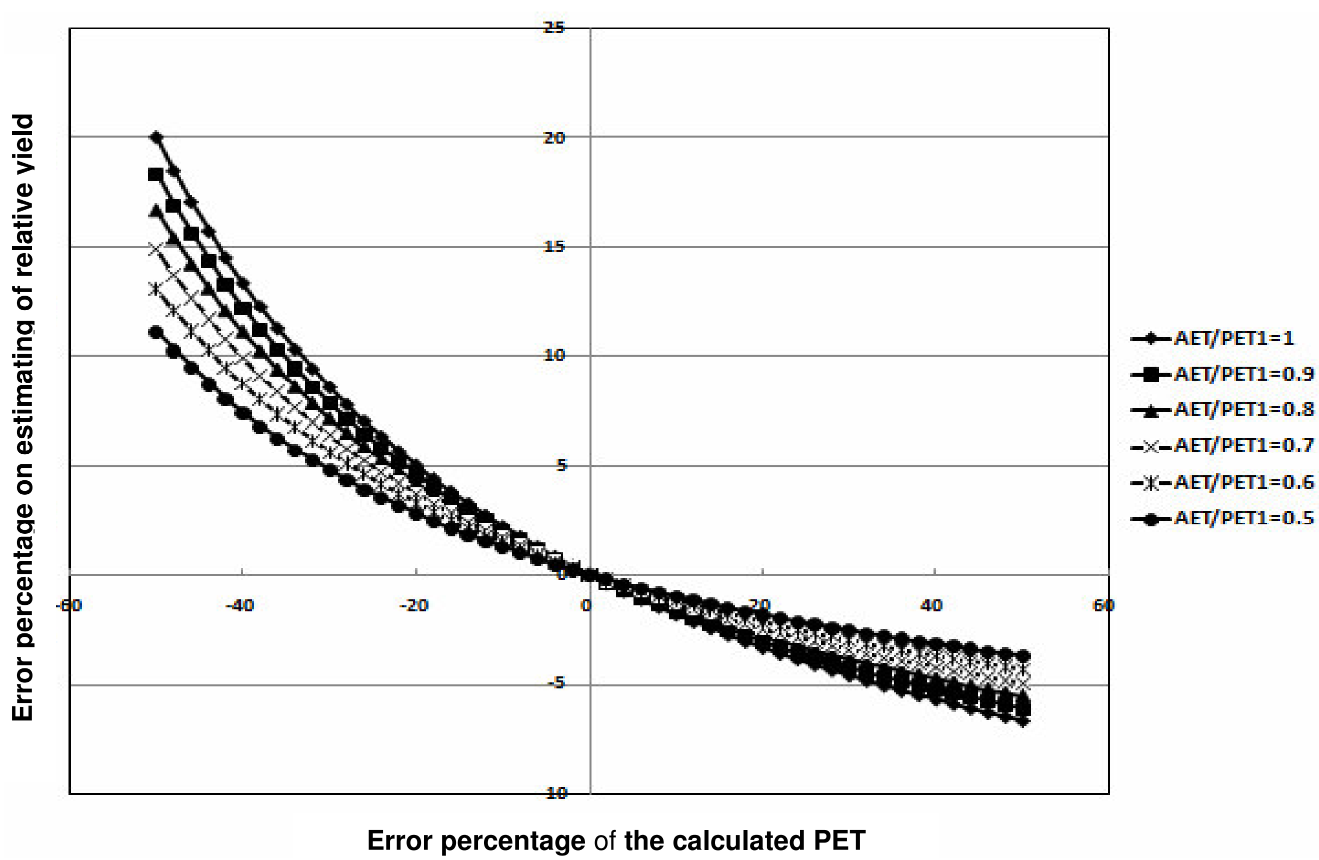

Figure 5. Effect of PET error in ripening stage of potato on estimating of Y_r.

Table 3. Gradient of linear relationship between error percentage of AET and error percentage of estimated Y_r.

AET_i/PET_i	Growth stage			
	1	2	3	4
1	0.45	0.800	0.700	0.200
0.9	0.424	0.782	0.677	0.183
0.8	0.395	0.761	0.651	0.166
0.7	0.364	0.736	0.620	0.148
0.6	0.329	0.705	0.583	0.130
0.5	0.290	0.667	0.538	0.111

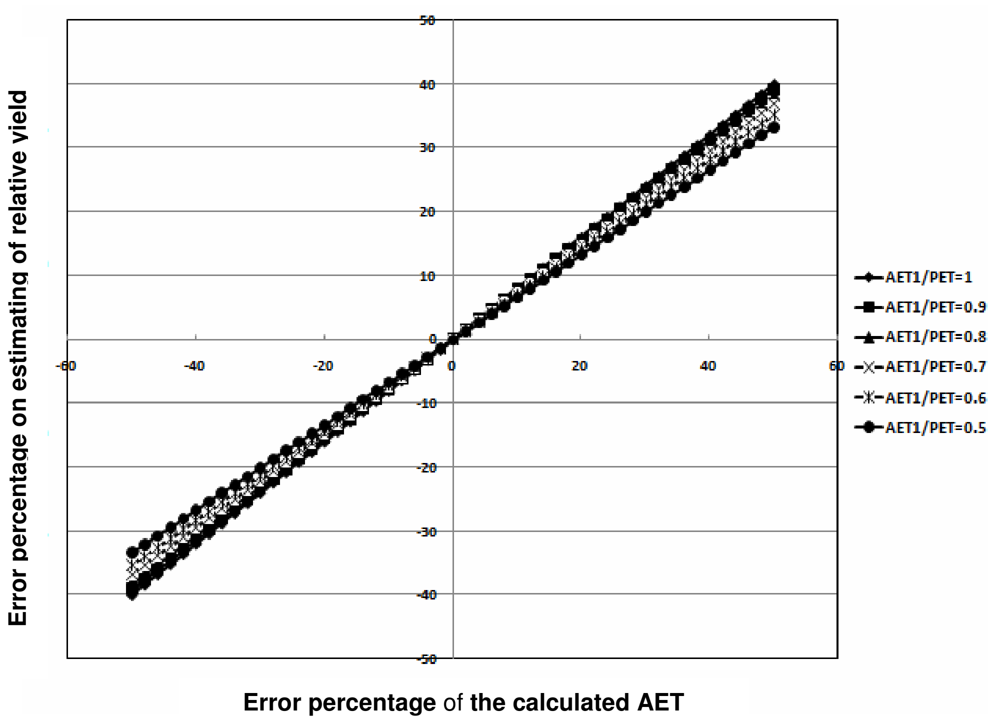

Figure 6. Effect of AET error in late vegetative stage of potato on estimating of Y_r.

shortage, is implemented in only one of the crop growth stages in which value of K_y is high, PET should be estimated carefully.

Effect of estimated AET error

The linear relationship ($y = mx$) was found between error percentage of AET (x) and error percentage of estimated Y_r (y). Table 3 shows the gradient (m) of this relationship. Also, the effect of estimated AET error on estimating of relative yield (Y_r) by Doorenbos and Kassam (1979) equation in late vegetative stage of potato is shown in Figures 6. In this figure, AET1 is the true value of AET in that stage. It is observed that overestimation of AET (plus error) results in over prediction of Y_r and underestimation of AET (minus error) results in under prediction of Y_r. In addition, over and under estimation of AET have a similar error on prediction of Y_r. As seen, with increasing of AET_i/PET_i (decreasing of water shortage) and with increasing of K_y (in different growth stages), the gradient of this relationship increases. In other words, for specific error of AET, when water shortage decreases, error percentage of estimated Y_r increases. This error is particularly higher in stages which crop is more sensitive to water shortage (K_y is greater) that they are late vegetative stage (K_y=0.8), yield formation stage (K_y=0.7), early vegetative stage (K_y=0.45) and ripening stage (K_y=0.2), respectively for potato. For example, a 10% error on estimating of AET in early and late vegetative, yield formation and ripening stages of potato under $AET_i/PET_i = 0.9$ result in 4.24, 7.82, 6.77 and 1.83

Table 4. Gradient of linear relationship between error percentage of K_y in two or more growth stages and error percentage of estimated Y_r.

AET_i/PET_i	Growth stages			
	1 and 2	**1 to 3**	**1 to 4**	**Total season**
0.95	-0.066	-0.108	-0.120	-0.058
0.9	-0.142	-0.242	-0.273	-0.123
0.8	-0.333	-0.639	-0.754	-0.282
0.7	-0.600	-1.409	-1.816	-0.492
0.6	-1	-3.545	-6.142	-0.785
0.5	-1.667	-39	+14.33	-1.222

percentage error on estimating of Y_r, respectively.

Overall, sensitivity of Doorenbos and Kassam (1979) equation related to AET parameter decreases when water shortage increases. In addition, sensitivity increases when K_y is higher. Therefore, when deficit irrigation, particularly low water shortage, is implemented in only one of the crop growth stages in which the value of K_y is high, PET should be estimated carefully. By comparing Tables 2 and 3, it can be seen that the sensitivity of Doorenbos and Kassam (1979) equation in relation to error of AET parameter is more than K_y error, particularly in low water shortage, but this sensitivity is equal in $AET_i/PET_i = 0.5$.

Effect of error in two or more growth stages

If error of estimation of Doorenbos and Kassam (1979) equation parameters occurs in two or more growth stages, evidently, equation two (multiplicative form of Equation 1) and two (additive form of equation 1) do not have similar responses. So, they are considered separately.

Additive form

Effect of yield response factor (K_y) error

Table 4 shows the gradient of linear relationship between error percentage of K_y in two or more growth stages (x) and error percentage of estimated Y_r (y) by additive form of Rao et al. (1988) equation. Moreover, the gradient concerned total growth season is presented in this table. Similar to section 1-1, when AET_i/PET_i decreases (increasing of water shortage), the gradient of this relationship increases. In other words, for specific error of K_y in two or more growth stages, when water shortage increases, error percentage of Y_r increases too. It is necessary to mention when 50% water shortage in four growth stages is imposed on potato, Y_r by additive form of Rao et al. (1988) equation is estimated less than zero. Obviously, it does not have any physical interpretations. Because of this, its gradient in Table 4 is dissimilar.

Clearly, in additive form of Rao et al. (1988) equation, yield reduction is equal to sum of calculated yield reductions by the right side of Doorenbos and Kassam (1979) equation for separate stages. By comparison of Tables 2 and 4, however, it can be seen that error percentage of Y_r (estimated by additive equation) arising from error of K_y in two or more growth stages, is greater than the sum of errors percentage of Y_r (estimated by Doorenbos and Kassam equation) for separate stages. For example, under $AET_i/PET_i = 0.7$, a 10% error of K_y in the growth stages 1 to 4 results in 18.16% error in estimating of Y_r by additive form of Rao et al. (1988) equation while the sum of error percentage of estimated Y_r by Doorenbos and Kassam (1979) equation, under same condition, is equal to 7.99%. In addition, as shown in Table 4, error percentage of Y_r arising from error in K_y when Y_r is estimated by Doorenbos and Kassam (1979) equation (according the seasonal K_y) is less than when it is estimated by additive equation of Rao et al. (1988). For example, under $AET_i/PET_i = 0.7$, a 10% error of K_y in growth stages 1 and 2, 1 to 3 and 1 to 4 results in 6, 14.09 and 18.16% error respectively on estimating of Y_r by additive form of Rao et al. (1988) equation while this error, under same water shortage for total growth season, is equal to 4.92% when Y_r is estimated by Doorenbos and Kassam (1979) equation.

Effect of calculated PET error

The effect of PET error in two or more growth stages on estimating of relative yield (Y_r) by additive form of Rao et al. (1988) equation is presented in Figures 7 to 9. In addition, Figure 10 shows the effect of PET error in total growth season on estimating of Y_r by Doorenbos and Kassam (1979) equation for potato. As shown in Figures 7 to 10, contrary to section 1 - 2, when AET_i/PET_i decreases (increasing of water shortage), error percentage of estimated Y_r increases. In other words, for specific error of PET in one stage of growth season, as presented in Figures 2 to 5, with decreasing of AET_i/PET_i, error percentage of estimated Y_r equation

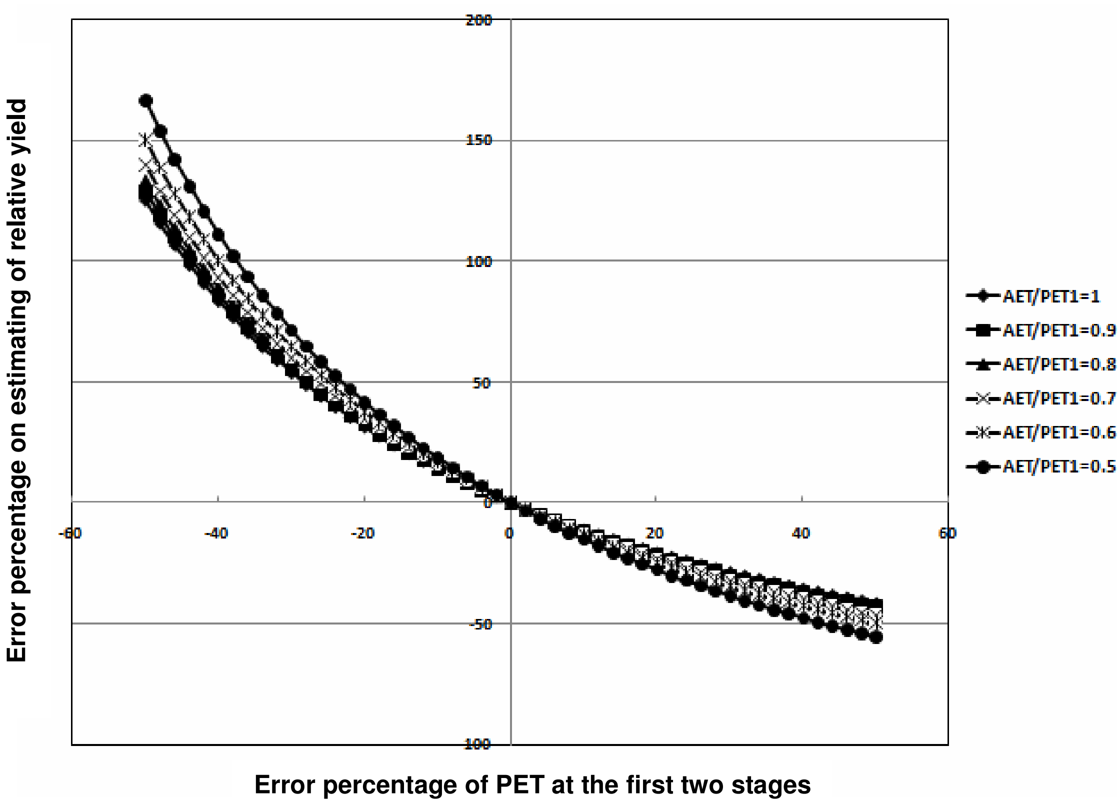

Figure 7. Effect of PET error at the first two stages on estimating of Y_r by additive equation of Rao et al. (1988).

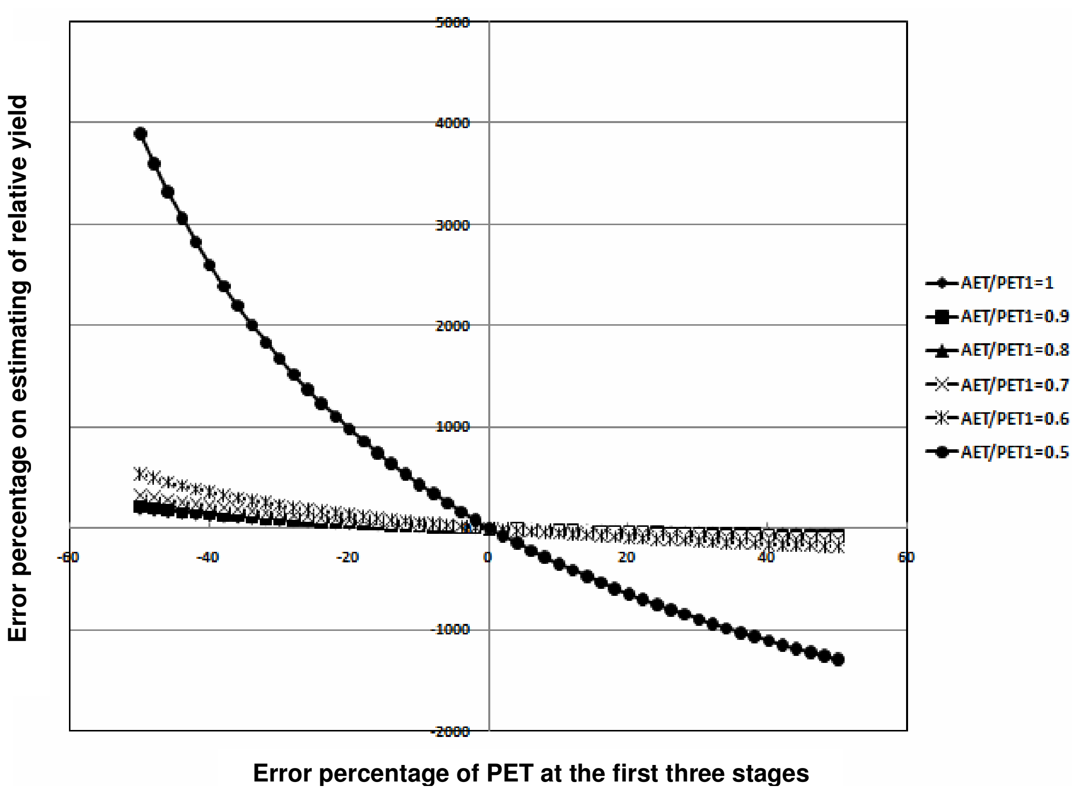

Figure 8. Effect of PET error at the first three stages on estimating of Y_r by additive equation of Rao et al. (1988).

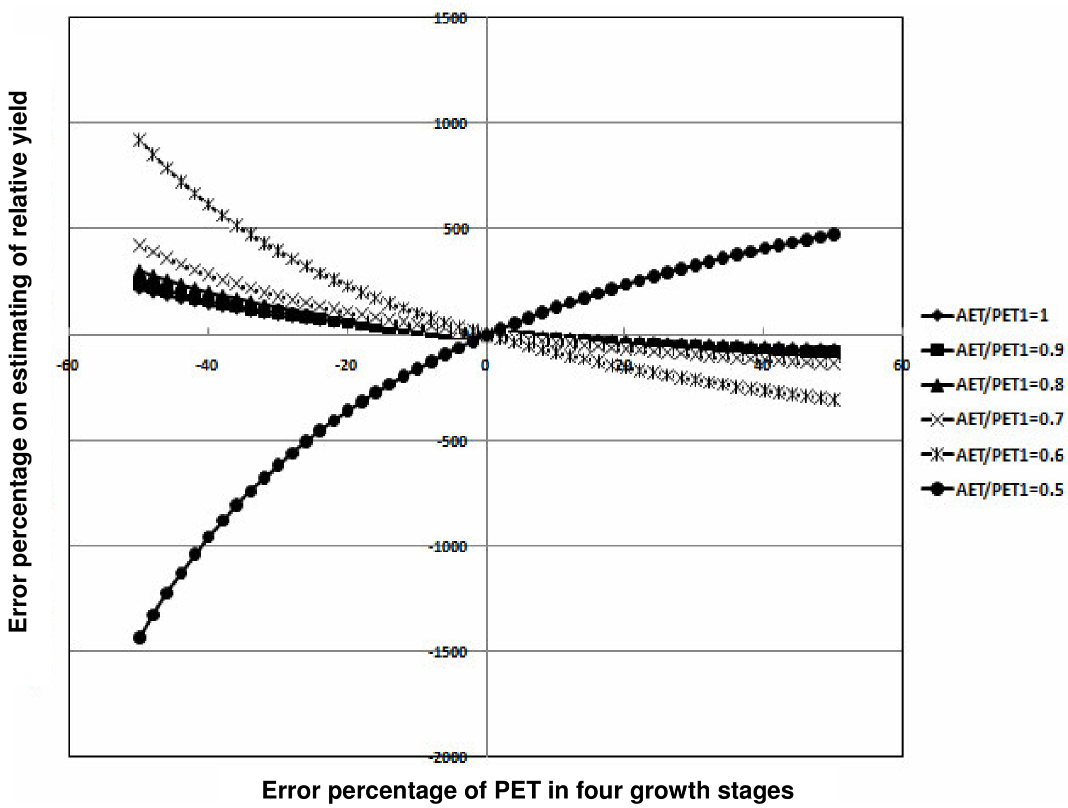

Figure 9. Effect of PET error in four stages on estimating of Y_r by additive equation of Rao et al. (1988).

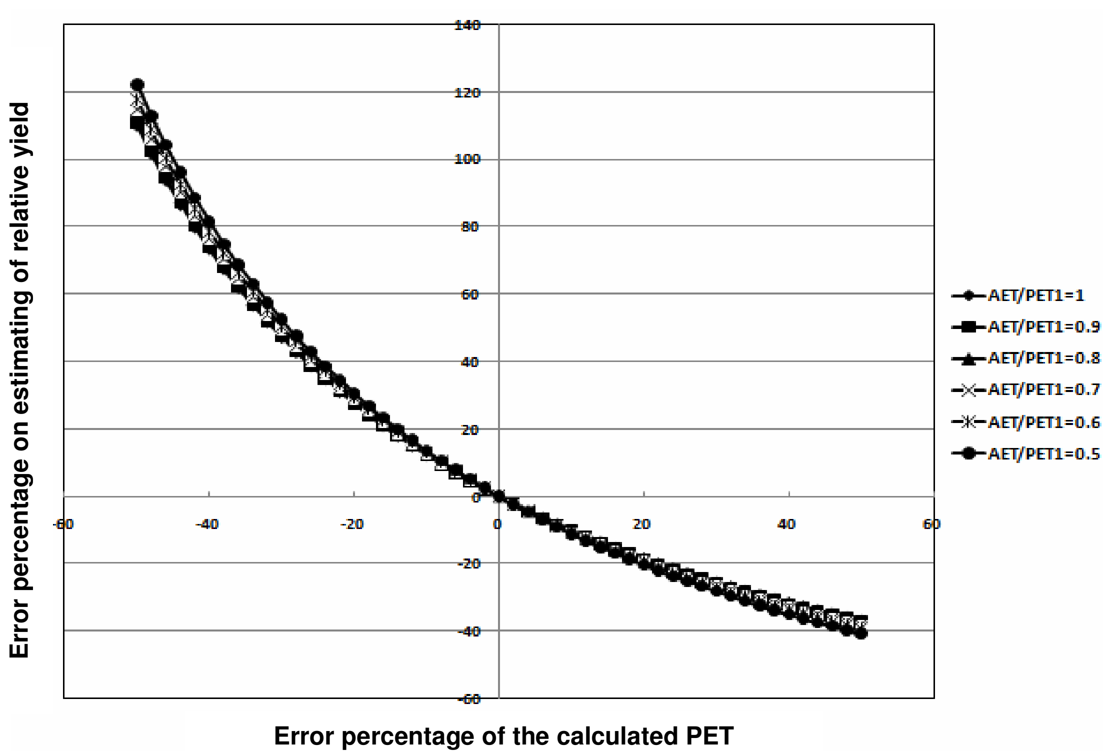

Figure 10. Effect of PET error in total growth season on estimating of Y_r by Doorenbos and Kassam (1979) equation.

Table 5. Gradient of linear relationship between error percentage of AET in two or more growth stages and error percentage of estimated Y_r.

AET_i/PET_i	growth stages			
	1 and 2	1 to 3	1 to 4	Total season
1	1.250	1.950	2.150	1.1
0.9	1.285	2.180	2.465	1.112
0.8	1.333	2.557	3.017	1.128
0.7	1.400	3.289	4.239	1.149
0.6	1.500	5.318	9.214	1.178
0.5	1.667	39	-14.330	1.222

(according the seasonal K_y) is less than when it is estimated by additive equation of Rao et al. (1988). For example, under $AET_i/PET_i = 0.7$, a 10% error of PET in growth stages 1 and 2, 1 to 3 and 1 to 4 results 15.56, 36.55 and 47.11 percentage error respectively in estimating of Y_r by additive form of Rao et al. (1988) equation while this error, under same water shortage for total growth season, is equal to 12.77% when Y_r is estimated by Doorenbos and Kassam (1979) equation.

Effect of estimated AET error

Table 5 shows the gradient of linear relationship between error percentage of AET in two or more growth stages (x) and error percentage of estimated Y_r (y) by additive form of Rao et al. (1988) equation. Moreover, the gradient concerned total growth season is presented in this table. As shown in Table 5, contrary to section 1 - 3, when AET_i/PET_i decreases (increasing of water shortage), gradient of this relationship increases. In other words, for specific error of AET in two or more growth stages when water shortage increases, error percentage of Y_r increases. It must be mentioned that when 50% water shortage in four growth stages is imposed on potato, the Y_r estimated by additive form of Rao et al. (1988) equation less than zero.

As mentioned in sections 2-1-1 and 2-1-2, by comparison of Tables 3 and 5, it can be seen that error percentage of Y_r (estimated by additive equation) arising from error of AET in two or more growth stages is greater than the sum of errors percentage of Y_r (estimated by Doorenbos and Kassam equation) for separate stages. For example, under $AET_i/PET_i = 0.7$, a 10% error of AET in growth stages 1 to 4 results 42.39% error in estimating of Y_r by additive form of Rao et al. (1988) equation while the sum of errors percentage of estimated Y_r by Doorenbos and Kassam (1979) equation under same condition, according to Table 3, is equal to 18.68%. In addition, as seen in Table 5, error percentage of Y_r arising from error of AET when Y_r estimated by Doorenbos and Kassam (1979) equation (according the seasonal K_y) is less than when it is estimated by additive quation of Rao et al. (1988). For example, under $AET_i/PET_i = 0.7$, a 10% error of AET in growth stages 1 and 2, 1 to 3 and 1 to 4 results 14, 32.89 and 42.39 percentage error respectively in estimating of Y_r by additive form of Rao et al. (1988) equation while this error, under same water shortage for total growth season, is equal to 11.49% when Y_r is estimated by Doorenbos and Kassam (1979) equation.

Multiplicative form

Effect of yield response factor (K_y) error

The effect of yield response factor (K_y) error in two or more growth stages on estimating of relative yield (Y_r) by multiplicative form of Rao et al. (1988) equation is shown in Figures 11 to 13. As shown, there is a nonlinear relationship between error percentage of K_y and error percentage of estimated Y_r that it is because of nonlinear relationship (multiplicative) between Y_r and K_y. When water shortage or number of growth stages increases, linearity of this relationship decreases. Similar to additive form of Rao et al. (1988) equation (section 2-1-1) when AET_i/PET_i decreases (increasing of water shortage), error percentage of K_y increases. Contrary to additive form, however, plus or minus error of K_y has different effect on estimating of Y_r so that minus error of K_y produces higher error on estimating of Y_r than plus error. Contrary to additive form, the estimated Y_r by multiplicative form of Rao et al. (1988) equation is not less than zero when 50% water shortage in four growth stages is imposed on potato. As shown in Table 6, calculated Y_r by multiplicative form of Rao et al. (1988) equation is higher than the additive form. In addition, the difference between two forms of this equation increased severely when water shortage is increased. For example, under $AET_i/PET_i = 0.7$ in stages 1 to 4, percentage of difference between two forms of Rao et al. (1988) equation is -37.52 and 27.28 relations to additive and multiplicative forms, respectively (relative yield is 0.3550 and 0.4882, respectively). This result is opposed to Rao et al. (1988) findings that assert both Equations 2 and 3

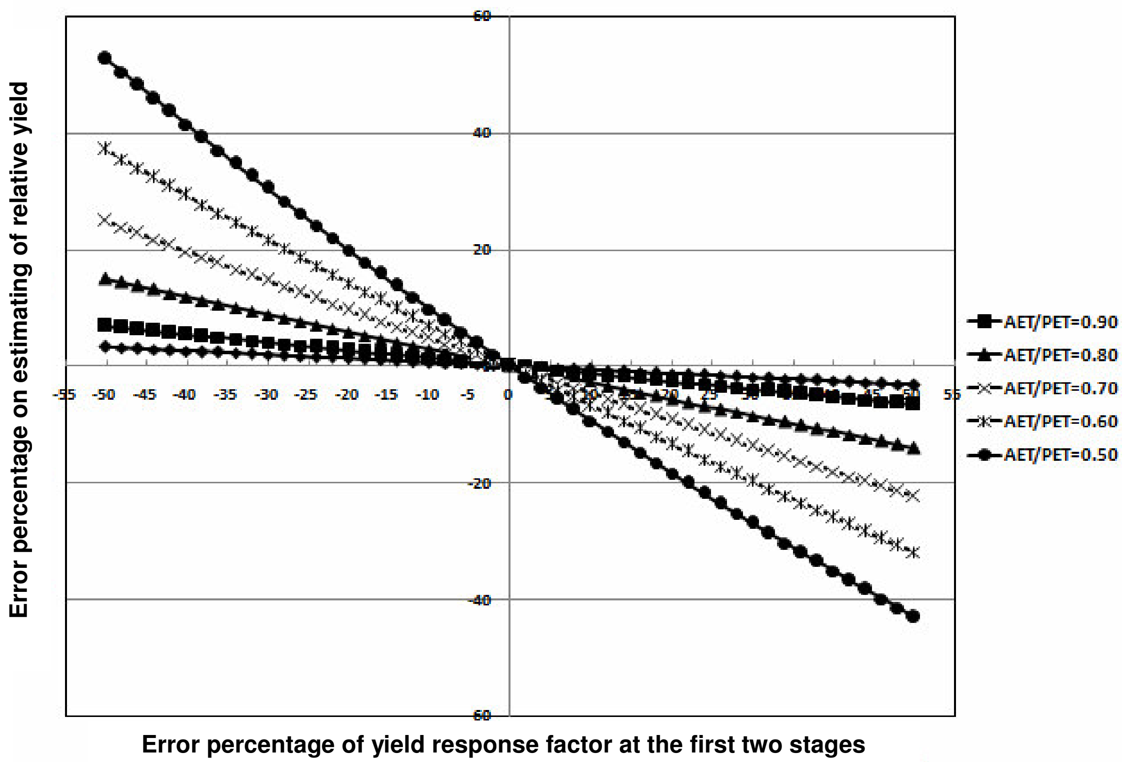

Figure 11. Effect of K_y error at the first two stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

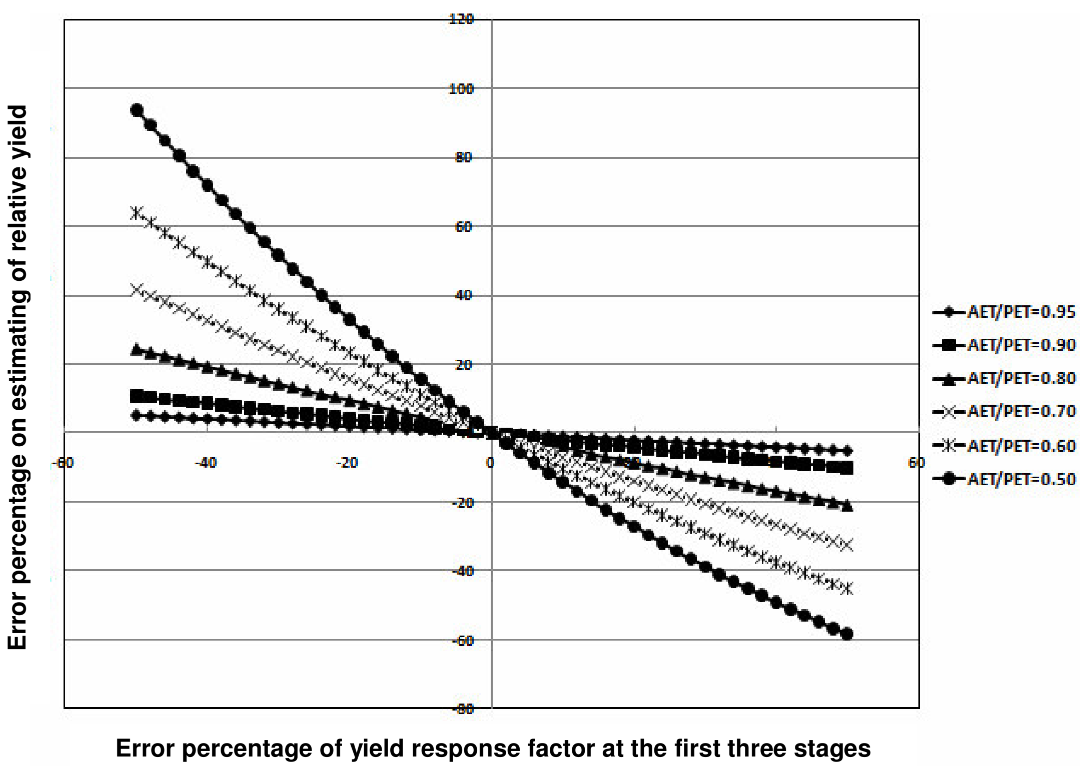

Figure 12. Effect of K_y error at the first three stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

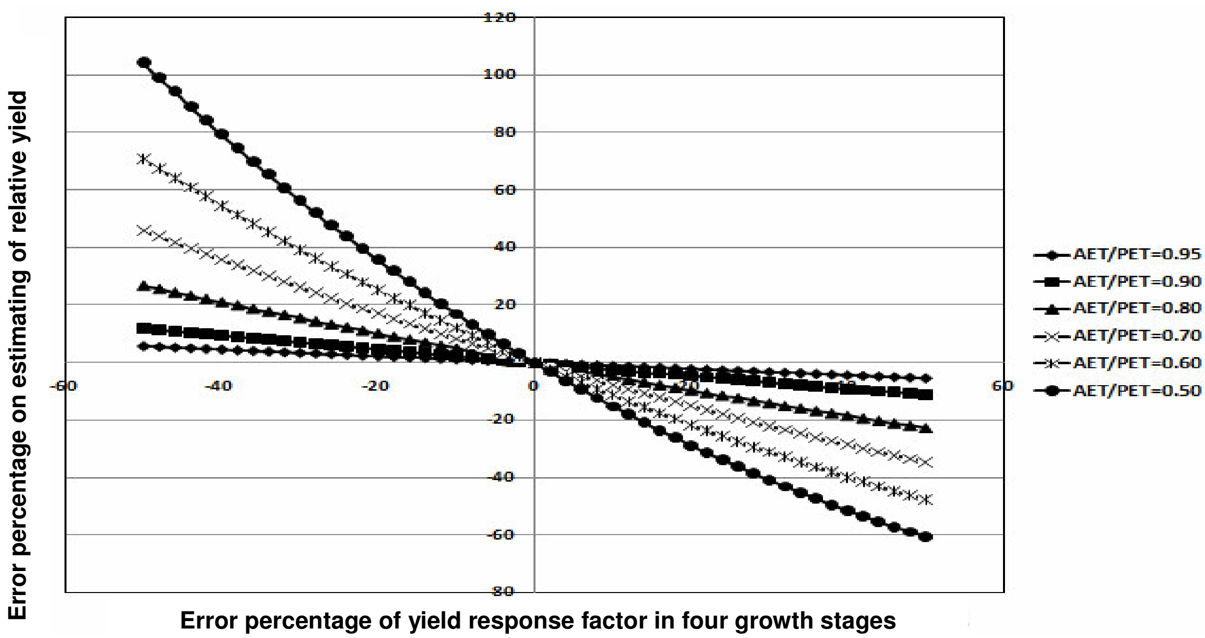

Figure 13. Effect of K_y error in four stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

Table 6- Difference between calculated Y_r by additive and multiplicative forms of Rao et al. (1988) equation.

Occurring stages	AET_i/PET_i	0.95	0.9	0.8	0.7	0.6	0.5
The first two stages	additive	0.9375	0.8750	0.7500	0.6250	0.5000	0.3750
	multiplicative	0.9384	0.8786	0.7644	0.6574	0.5576	0.4650
	% diff. relation to add.	-0.10	-0.41	-1.92	-5.184	-11.52	-24
	% diff. relation to mult.	0.10	0.41	1.88	4.93	10.33	19.35
The first three stages	additive	0.9025	0.8050	0.6100	0.4150	0.2200	0.0250
	multiplicative	0.9056	0.8171	0.6574	0.5193	0.4015	0.3023
	% diff. relation to add.	-0.34	-1.50	-7.77	-25.14	-82.49	-1109
	% diff. relation to mult.	0.34	1.48	7.21	20.09	45.20	91.73
Stages 1 to 4	additive	0.8925	0.7850	0.5700	0.3550	0.1400	-0.0750
	multiplicative	0.8965	0.8008	0.6311	0.4882	0.3694	0.2720
	% diff. relation to add.	-0.45	-2.01	-10.72	-37.52	-163.82	462.7
	% diff. relation to mult.	0.45	1.97	9.68	27.28	62.10	127.57

give similar results. However, Ghahraman and Sepaskhah (2004) reported the same findings as this paper for corn. Therefore, with respect to severe differences between two forms of Rao et al. (1988) equation, it is necessary that before implementing of Rao et al. (1988) equation in optimization models or deficit irrigation programmes, it be determinated which forms of this equation is suitable.

By comparison, of Table 4 and Figures 11 to 13, it can be found that error percentage on estimating of Y_r by multiplicative form of Rao et al. (1988) equation arising from error of K_y in two or more growth stages is less than additive one. For example, it supposed a 10% error of K_y in 1 and 2, 1 to 3 and 1 to 4 growth stages of potato under $AET_i/PET_i = 0.7$. Under this condition, additive form of Rao et al. (1988) equation results respectively in 6, 14 and 18 percentage error on estimating of Y_r while multiplicative form of Rao et al. (1988) equation results in 4.7, 7.2 and 7.8 percentage error respectively on estimating of Y_r. Therefore, where validated or calibrated values of K_y is not available, it is recommended that multiplicative form of Rao et al. (1988) equation instead of additive form be used in optimization or deficit irrigation models.

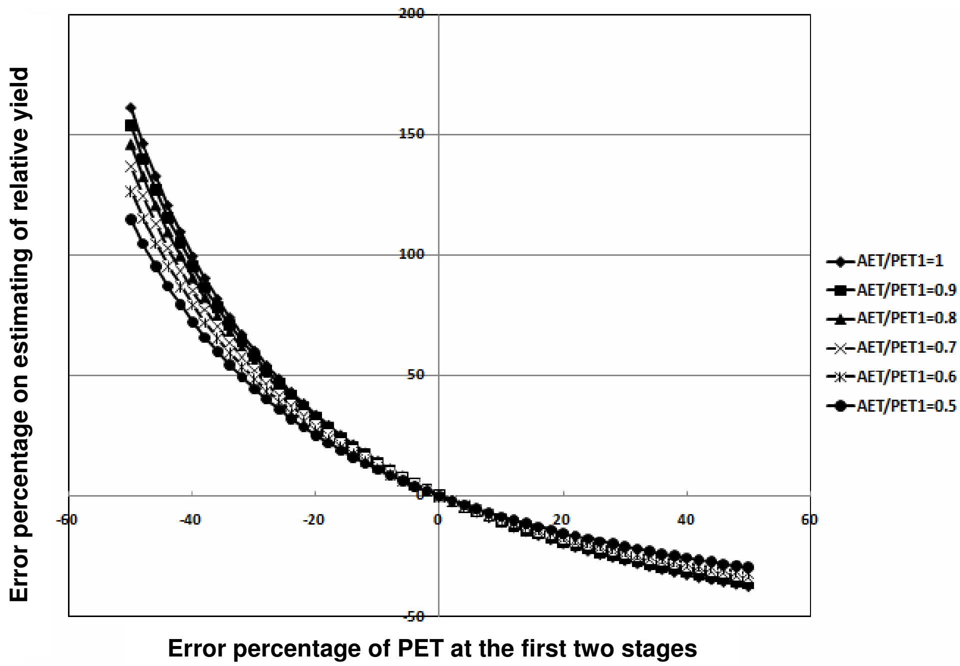

Figure 14. Effect of PET error at the first two stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

Effect of calculated PET error

The effect of PET error in two or more stages on estimating of relative yield (Y_r) by multiplicative form of Rao et al. (1988) equation is presented in Figures 14 to 16. As shown, similar to sections 1-2 and 2-1-2, plus or minus error of PET has different effects on estimating of Y_r so that minus error of PET produces higher error on estimating of Y_r than plus error. Contrary to additive form of Rao et al. (1988) equation (section 2-1-2) but similar to section 1-2, when AET_i/PET_i decreases (increasing of water shortage), error percentage on estimating of Y_r decreases. In other words, for specific error of PET in two or more growth stages, as shown in Figures 8 to 10, when AET_i/PET_i decreases, error percentage on estimating of Y_r by additive form of Rao et al. (1988) equation increases while error percentage on estimating of Y_r by multiplicative form of Rao et al. (1988) equation, as shown in Figures 14 to 16, decreases. With respect to Figures 8 to 10 and 14 to 16, it can be seen that when error on estimating of PET occurs in two or more growth stages, error percentage on estimating of Y_r by additive form of Rao et al. (1988) equation is greater than when Y_r is estimated by multiplicative one. For example, under $AET_i/PET_i = 0.7$, a 10% error of PET in stages 1 and 2, 1 to 3 and 1 to 4 results in 15.56, 36.55 and 47.11 percentage error respectively on estimating of Y_r by additive form of Rao et al. (1988) equation while this condition results in 9.79, 14.87 and 16.03 percentage error respectively on estimating of Y_r by multiplicative form of Rao et al. (1988) equation. Therefore, where precise values of PET are not available or there are some uncertainties in calculated PET, it is recommended to use multiplicative form of Rao et al. (1988) equation instead of additive form in optimization or deficit irrigation models.

Effect of estimated AET error

The effect of AET error in two or more growth stages on estimating of relative yield (Y_r) by multiplicative form of Rao et al. (1988) equation is presented in Figures 17 to 19. There is a nonlinear relationship between error percentage of K_y and error percentage of estimated Y_r that it is because of nonlinear relationship (multiplicative) between Y_r and K_y. As shown, with increasing the number of growth stages, linearity of this relationship decreases. Contrary to additive form, however, plus or minus error of AET have different effect on estimating of Y_r so that minus error of AET produces lesser error on estimating of Y_r than plus error.

Contrary to additive form of Rao et al. (1988) equation (section 2-1-3) but similar to section 1-3, when AET_i/PET_i decreases (increasing of water shortage), error percentage on estimating of Y_r decreases. In other words, for specific error of AET in two or more growth stages, as shown in table 5, when AET_i/PET_i decreases, error percentage on estimating of Y_r by additive form of Rao et al. (1988) equation increases while error percentage on estimating of Y_r by

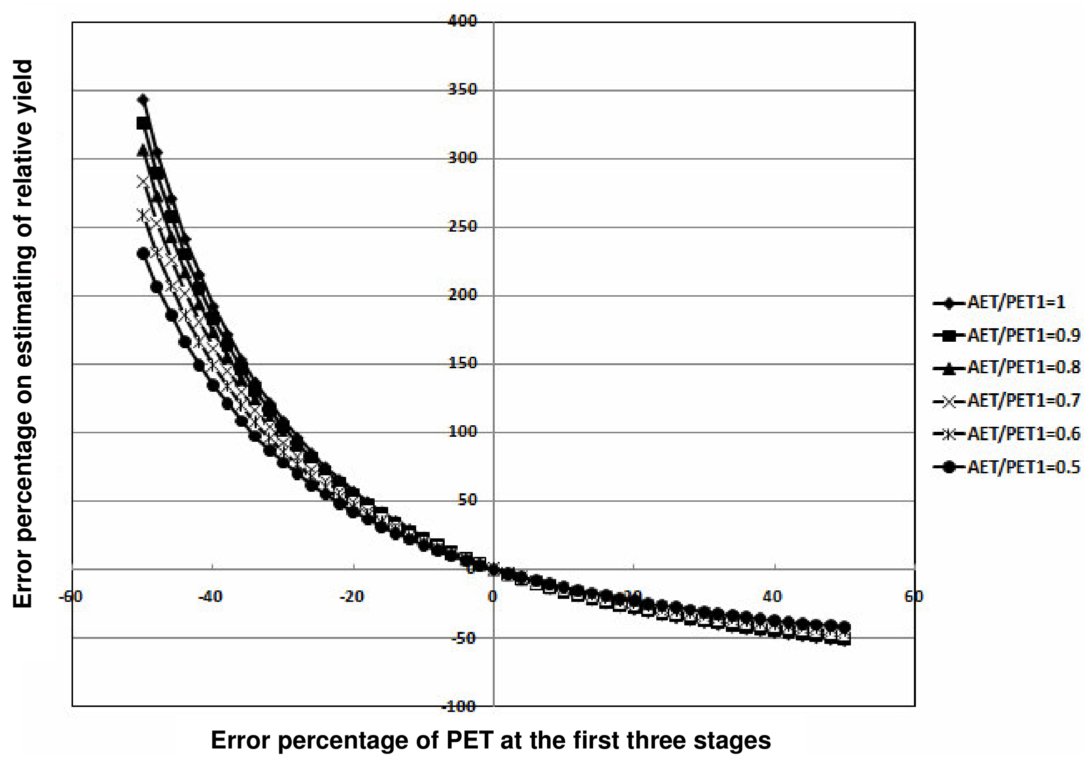

Figure 15. Effect of PET error at the first three stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

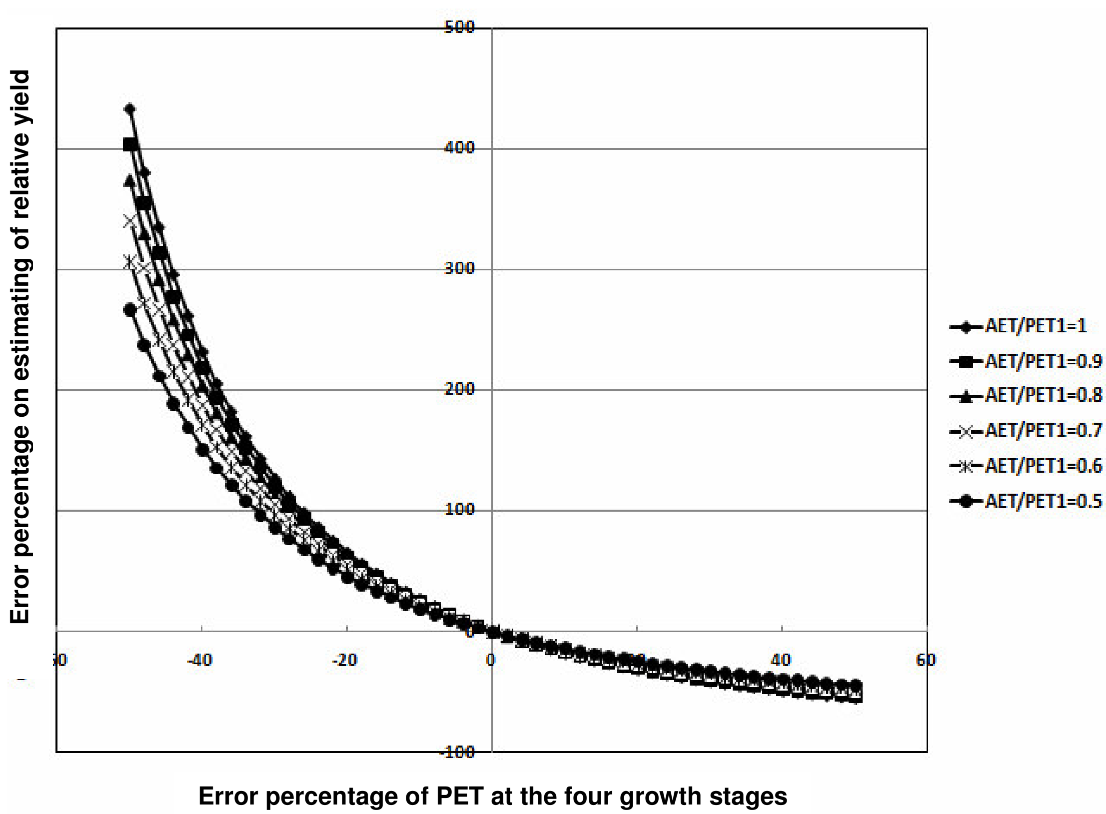

Figure 16. Effect of PET error in four stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

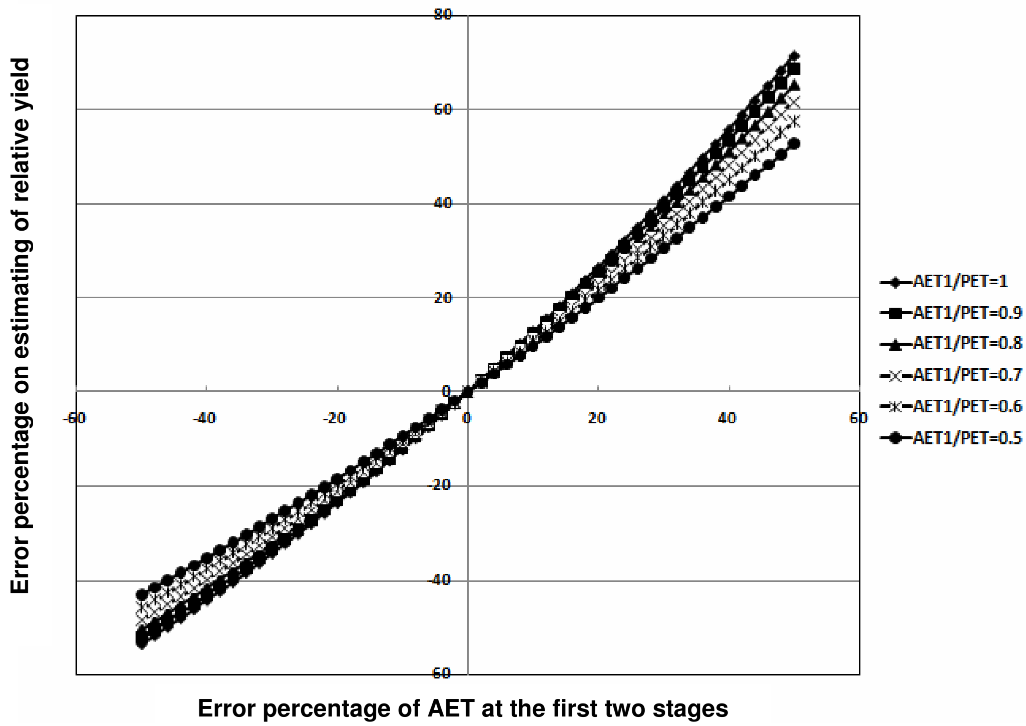

Figure 17. Effect of AET error at the first two stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

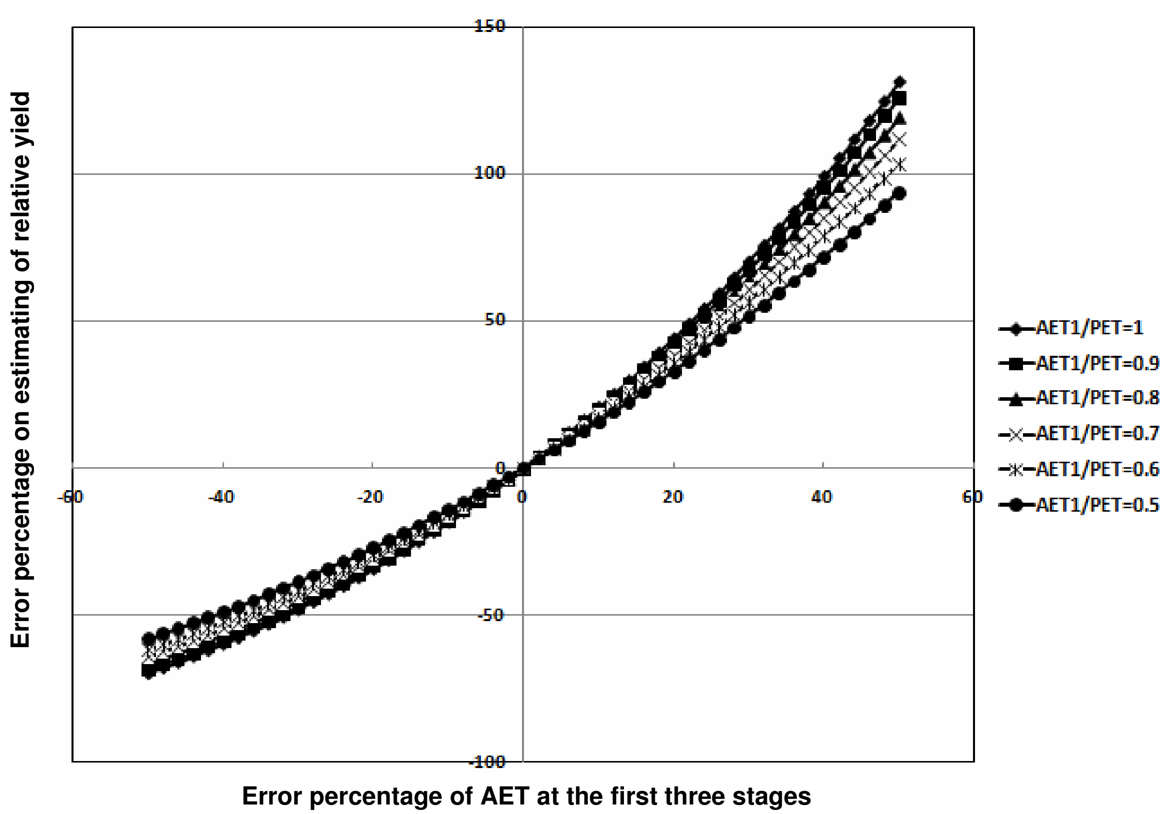

Figure 18. Effect of AET error at the first three stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

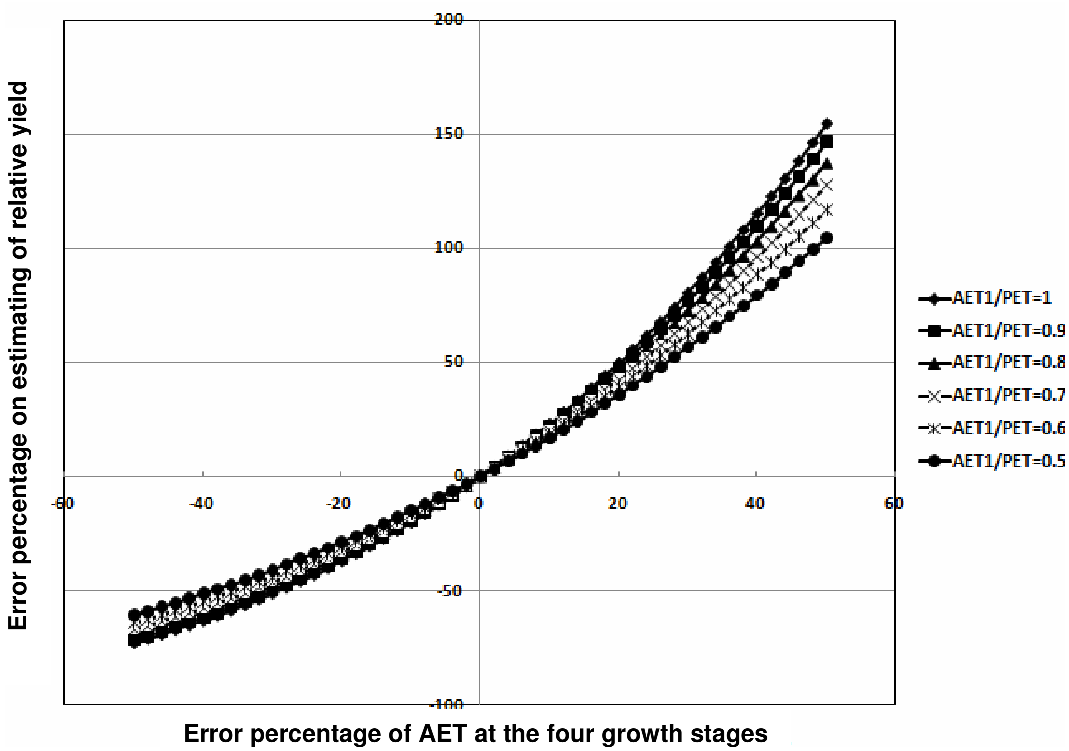

Figure 19. Effect of AET error in four stages on estimating of Y_r by multiplicative equation of Rao et al. (1988).

multiplicative form of Rao et al. (1988) equation, as shown in Figures 17 to 19, decreases. With respect to Table 5 and Figures 17 to 19, it can be found when error on estimating of AET occurs in two or more growth stages; error percentage on estimating of Y_r by additive form of Rao et al. (1988) equation is greater than when Y_r is estimated by multiplicative form. For example, under $AET_i / PET_i = 0.7$, a 10% error of AET in stages 1 and 2, 1 to 3 and 1 to 4 results in 14, 32.89 and 42.39% error respectively on estimating of Y_r by additive form of Rao et al. (1988) equation while this condition result in 11.28, 18.18 and 19.94 percentage error respectively on estimating of Y_r by multiplicative form of Rao et al. (1988) equation. Therefore, where non-reliable methods or non-precise values of AET are used, it is recommended to use multiplicative form of Rao et al. (1988) equation instead of additive form in optimization or deficit irrigation models.

DISCUSSION

This paper presents results of sensitivity analysis of Doorenbos and Kassam (1979) equation and its modified forms that developed by Rao et al. (1988) namely multiplicative and additive forms. Analysis was carried for potato in relation to 3 paremeters of these equations (PET, AET and K_y). According to this research, plus error of PET and K_y and minus error of AET results in over predicting of relative yield (Y_r). Error values of estimating Y_r by these equations arising from PET or AET error is depended on the value of K_y so that this error is higher in growth stages which have higher K_y. For a specific error value of K_y, when water shortage increases, error percentage of estimated Y_r by Doorenbos and Kassam (1979) equation and by multiplicative and additive forms of Rao et al. (1988) equation increases. For a specific error value of PET or AET, however, when water shortage increases, error percentage of estimated Y_r by Doorenbos and Kassam (1979) equation and by multiplicative form of Rao et al. (1988) equation decreases while error percentage of estimated Y_r by additive form of Rao et al. (1988) equation increases. Sensitivity of Doorenbos and Kassam (1979) equation and additive form of Rao et al. (1988) equation is equal for plus or minus error of K_y and AET but their sensitivities are greater for minus error of PET than plus error. However, sensitivity of multiplicative form of Rao et al. (1988) equation is greater for minus error of K_y and PET and plus error of AET. It is shown that error percentage on estimating of Y_r by multiplicative form of Rao et al. (1988) equation arising from error of PET, AET or K_y is less than additive form. In addition, calculated Y_r by multiplicative equation is higher than additive form and the difference between two forms of this equation increases severely when water shortage increases. Error percentage on estimating of Y_r by additive equation of Rao et al. (1988) arising from error of each one of parameters (PET, AET and K_y) in two or more growth stages is greater than the sum of errors percentage of estimated Y_r by Doorenbos and Kassam equation for separate stages while yield reduction of additive form of Rao et al. (1988) equation is equal to the sum of calculated yield reductions by the right side of Doorenbos and Kassam (1979) equation for separate stages. According to the results obtained in this research, it is recommended that multiplicative form of Rao et al. (1988) equation instead of additive form be used in optimization or deficit irrigation models.

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