African Journal of Agricultural Research Vol. 6(18), pp. 4385-4391, 12 September, 2011
Available online at http://www.academicjournals.org/AJAR
DOI: 10.5897/AJAR10.993
ISSN 1991-637X ©2011 Academic Journals

Full Length Research Paper

Differential equation model for durability of the tractor´s engine with application to the model Massey Ferguson 8160

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Accepted 2 August, 2011

A novel mathematical model for the tractor engine durability, based on the appropriate differential equation and conditions, is formulated and presented in this paper. Its practical applicability is experimentally verified for the tractor type Massey-Ferguson 8160. The probability density function of the engine's lifespan, as long as the overhaul is needed, is established and approximated by normal Gaussian function. Fitting coefficients are evaluated through the non-linear fitting of empirical data by Levenberg-Marquardt numerical algorithm.

Key words: Mathematical model, partial differential equation, tractor, engine lifespan distribution, non-linear fitting.

INTRODUCTION

Agricultural systems demand detailed planning and control of relevant biological, technical, technological and other processes (Mileusnić et al., 2010). Besides fuel consumption (Sümer et al., 2010), machinery assets (Keskin et al., 2010), ergonomic properties of the tractors (Melemez and Tunay, 2010) etc. Machinery statistics represents crucial information that influences the agricultural machinery management (Qi et al., 2009; Ozturk, 2010). The adequate database containing especially, data related to renewing and maintenance of the tractor fleet is an initial point for appropriate decision-making (Mileusnic et al., 1995). Fast development of digital computers in the past few decades, together with appropriate software and hardware accessories, has enabled the application of advanced technologies based on mathematical modeling and simulations (Dyer and Desjardins, 2003; Petrovic et al., 2007). Sørensen and Bochtis (2010) analyze the preventive technical maintenace of agricultural machines. The main goal is to minimise the machine malfunctions. Alvarez and Huet (2008) have applied rule-based electronic test system (Scorpio) which compares the present performance of the tractor to the reference values specified by the appropriate standard. The system analyzes the differences between real (measured) and optimal values of all tractor relevant parameters, and suggests which parts may be responsible for the lack of performance.

Equipment and repair and maintenance expenses are important cost elements for agricultural concerns (Rotz and Bowers, 1991). Therefore, keeping tractors in good operational condition is financially important and it also reduces the risk of major failures when the tractor is needed. It is also essential to keep the tractor engine operating at its optimum performance level, in order to correctly perform work that requires power and torque reserves (Siemens and Bowers, 1999). This paper presents the results of a tractor engine durability examination, performed on Serbian farms. Initial results of less comprehensive investigation of such kind have been previously reported by Petrović et al. (2010). Specifically, the engine durability of the popular heavy-tractor model, Massey-Ferguson 8160, is analyzed. On the basis of questionnaire, that is data acquired on various Serbian farms, the frequency distribution and probability density function of the engine lifespan (up to

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Figure 1. The tractor Massey-Ferguson 8160.

the overhaul is done) has been established. An original mathematical model, which includes the differential equation with adequate conditions, has been developed for this purpose. A solution of this equation is the normal Gaussian function, which is used for analytical approximation of the lifespan probability distribution. Coefficients of the fitting model function are evaluated through the non-linear fitting of empirical data (Seber and Wild, 2003).

MATERIALS AND METHODS

Tractor model

The present study complies with the well–known and widely used standards ASAE D230.2 (1971), ASAE D497.4 (2003) and ASAE D497.6 (2009). The focus of this work is to formulate a mathematical model and establish the analytical approximation of the frequency distribution function of the tractor engine lifespan, as long as the overhaul is needed and performed. The tractor model Massey Ferguson 8160, produced in Beauvais, France, (Figure 1) has been monitored and analyzed on various larger Serbian farms, such as "PKB - Brazda" near Belgrade, etc. The fairly old tractor model Massey Ferguson 8160 was chosen for analysis, because it represents one of the most popular models (in the category over 140 kW), imported from western countries to Serbia (about 200 in total). In addition, the built-in engine (Figure 2) was of respectable quality and design at that time. Basic technical characteristics of the tractor, as well as of the engine, are given in Table 1.

Basic statistics

Basic statistical parameters of the relative frequency f (%) distribution of the tractor engine lifespan t (h) are calculated following the common statistic procedure (Marques de Sá, 2007). The own computer program, coded using "C" compiler, is used for calculations. Source data (which originate from questionnaire), that is engine lifespan, are sorted at intervals of Δt = 1000 (*h*) in width, starting from 3500 (*h*) and up to 14500 (*h*). This way, each of *m* temporal size-classes is represented by the interval midpoint t_i (*h*) and the absolute frequency n_i - number of tractor engines characterized by lifespan $t_i \pm (\Delta t/2)$. Consequently, the sum of absolute frequencies equals the total number of tractor engines under consideration $n = \sum_{i=1}^{m} n_i$. Starting from the relative frequencies:

$$f_i = \frac{n_i}{n}; \; (i = 1, 2, \ldots, m), \tag{1}$$

The basic statistical parameters are calculated as follows:

1) The arithmetic mean:

$$\bar{t} = \sum_{i=1}^{m} f_i \cdot t_i \tag{2}$$

2) Standard deviation and coefficient of variation, respectively:

$$\sigma_t = \sqrt{\sum_{i=1}^{m} f_i \cdot \left(t_i - \bar{t}\right)^2}, \; C_{V_t} = \frac{\sigma_t}{\bar{t}}, \tag{3-4}$$

3) Skewness and flatness factor:

$$S_t = \frac{\sum_{i=1}^{m} f_i \cdot \left(t_i - \bar{t}\right)^3}{\sigma_t^3}, \; F_t = \frac{\sum_{i=1}^{m} f_i \cdot \left(t_i - \bar{t}\right)^4}{\sigma_t^4}. \tag{5-6}$$

In addition, maximum, minimum, range, mode, median, lower and upper quartile, quartile range, 10 and 90% percentiles are also evaluated.

Data fitting

Characterization of some distribution by different statistical description parameters provides useful information. However, additional approach based on data fitting is also used in the present paper. The software realization (Press et al., 2002) of the numerical procedure of Levenberg-Marquardt (Seber and Wild, 2003), coded using "C" compiler, is applied for this purpose. The tested model functions are given in Table 1. Lastly, the probability density function of the engine lifespan:

$$pdf\left(t\right) = \frac{f}{\Delta t} \; \frac{\%}{h} \tag{7}$$

is calculated, where Δt = 1000 (*h*).

Experimental *pdf's* are approximated by the normal Gaussian function:

$$y = y_0 + \frac{A}{w \cdot \sqrt{\pi / 2}} \cdot e^{-2 \cdot \frac{\left(t - t_c\right)^2}{w^2}}, \tag{8}$$

Figure 2. Diesel engine of the tractor Massey-Ferguson 8160.

Table 1. Technical characteristics of the tractor Massey-Ferguson 8160 and the built-in engine.

Technical characteristics	**Massey-Ferguson 8160**
Tractor type	4WD
Engine model	Valmet 637 DS
Power engine[kW]	147
Nominal rated engine speed (min^{-1})	2200
Rated engine speed at max. power (min^{-1})	2000
$Torque_{max.}$ (Nm)	844
Rotation rate at max. torque (min^{-1})	1200
Spec. fuel consumption (g/kWh)	267
No off speed front/rear	32/32
Traction systems	wheels
- front	480/70-30
- rear	620/70-42
Tractor dimension: (mm)	
- length	5610
- width	2480
Energy supply in reference at konst. mass [kW/t]	18.56
Specific mass without ballast [kg/kW]	53.87
Specific mass with ballast [kg/kW]	76.19
Noise level in cab [dB] (A)	79.5

where t represents the tractor engine lifespan (up to the overhaul), while y corresponds to the $pdf(t)$ defined in (7). Fitting parameters, offset y_0, center (or mean) t_C, width w and area A (between the function curve and abscissa) are evaluated numerically by the Levenberg-Marquardt non-linear fitting method. Fitting accuracy is estimated by the mean square error (or standard error):

$$MSE = \frac{\sum_{i=1}^{m}\left(y_i - \hat{y}_i\right)^2}{m-k} = \frac{\sum_{i=1}^{m}\varepsilon_i^2}{m-k}\ , \qquad (9)$$

that is the sum of squares of differences between the measured values y_i and fitted values $\hat{y}_i$, divided by "degree of freedom" (difference between the number *m* of fitted data "points" and the number *k* of constants in a model function). The root mean square error, or standard error of estimate:

$$RMSE = \sqrt{MSE}\,, \tag{10}$$

is a parameter also used for this purpose. The smaller values of parameters, defined by expressions (9) and (10), result in better fitting accuracy. An additional and very popular "measure" of fitting accuracy is the well-known coefficient of determination or R-square factor:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}\,. \tag{11}$$

The closer the value of R^2 to 1, means a better fitting.

RESULTS AND DISCUSSION

Theoretical results - formulation of the mathematical model

Developing the novel mathematical model complies with the following nomenclature. Dependent variable $y = y(t,\tau)$ denotes *pdf* of tractor engine lifespan, defined according to Equation (7). However, in this case, this is a function of two independent variables: *t* (lifespan – the number of working hours before the overhaul of the engine) and the actual year of tractor manufacturing $\tau>0$ (Marques de Sá, 2007), that is:

$$y = y(t,\tau) = pdf(t)_\tau\,. \tag{12}$$

Then, *y* satisfies partial differential equation (Rothe, 1984; Grindrod, 1996; Cerrai, 2001; Buerger and Haller, 2005):

$$\frac{\partial y}{\partial \tau} = \frac{w^2}{8}\cdot\frac{\partial^2 y}{\partial t^2}\,, \tag{13}$$

where *w* is a known constant. Let us seek solutions of the form:

$$y = \tau^{-\alpha}\cdot f\left(t\cdot\tau^{-\alpha}\right), \tag{14}$$

with

$$f(t) = B\cdot e^{-g(t)} \tag{15}$$

where α and *B* are unknown constants which should be determined. For the functions $g(t)$ that satisfy $g(0)=0$, there exists $\alpha > 0$ that satisfies the scaling condition:

$$g\left(t\cdot\tau^{-\alpha}\right) = g(t)\cdot\tau^{-1}\,. \tag{16}$$

The constant B has to be determined by the normalization condition:

$$\int_{-\infty}^{+\infty} f(t)\cdot dt = B\cdot\int_{-\infty}^{+\infty} e^{-g(t)}\cdot dt = 1\,. \tag{17}$$

Using (14), (15) and (16) we can calculate a particular solution of (13) in the form:

$$y = B\cdot\tau^{-\alpha}\cdot e^{-g(t)\cdot\tau^{-1}}\,. \tag{18}$$

Since (13) has constant coefficients, we set:

$$y(t,\tau) = e^{b\cdot t + c\cdot\tau} \tag{19}$$

where *b* and *c* are arbitrary constants. Substituting (19) into Equation (13), we find that:

$$c = \frac{w^2 b^2}{8}\,. \tag{20}$$

Replacing *b* by $i\cdot b$, with the imaginary unit *i,* we get the complex solution:

$$y(t,\tau) = e^{-\frac{w^2}{8}\cdot b^2\cdot\tau}\cdot cos(b\cdot t) + i\cdot e^{-\frac{w^2}{8}\cdot b^2\cdot\tau}\cdot sin(b\cdot t)\,. \tag{21}$$

The real and imaginary parts of (21) are solutions of (13). This implies that a solution of (13) is also given by:

$$\int_0^{\infty} e^{-\frac{w^2}{8}\cdot b^2\cdot\tau}\cdot\cos(b\cdot t)\cdot db = \frac{\sqrt{2\pi}}{w}\cdot\frac{1}{\sqrt{\tau}}\cdot e^{-\frac{2\cdot t^2}{w^2\cdot\tau}}\,, \tag{22}$$

because *b* represents any arbitrary constant. If we compare the function on the right hand side of (21) with the sleeked solution (18), we obtain:

$$g(t) = \frac{2\cdot t^2}{w^2};\ \alpha = \frac{1}{2}\,. \tag{23}$$

After substituting $g(t)$ from the previous expression (23)

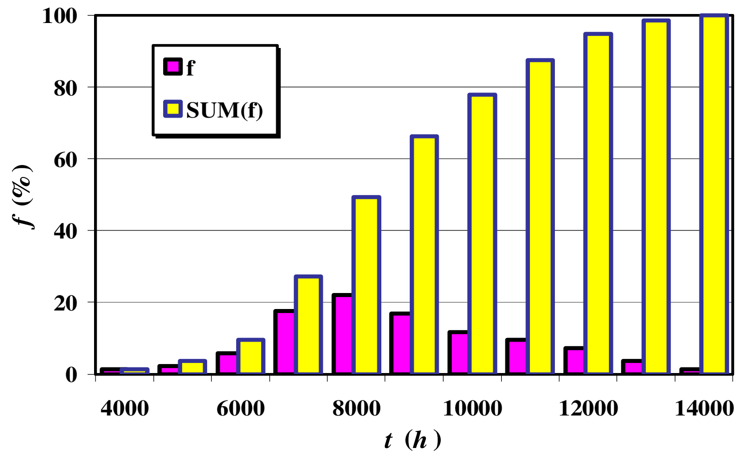

Figure 3. Frequency distribution and the cumulative function (ascending) of the tractor Massey-Ferguson 8160 engine lifespan.

Table 2. Descriptive statistics of the engine lifespan frequency distribution.

Parameter	Value	Parameter	Value
Mean (*h*)	8850.00	Median (*h*)	9000.00
Mode (*h*)	8000.00	Frequency of Mode (%)	22.00
Minimum (*h*)	4000.00	Maximum (*h*)	14000.00
Lower 25% Quartile (*h*)	7000.00	Upper 75% Quartile (*h*)	10000.00
Percentile 10% (*h*)	7000.00	Percentile 90% (*h*)	12000.00
Range (*h*)	10000.00	Quartile range (*h*)	3000.00
Standard deviation (*h*)	2026.96	Coeff. of variation (-)	22.90
Skewness (-)	0.33	Flatness (-)	2.72

into (17), B can be determined as:

$$B = \frac{1}{w}\sqrt{\frac{2}{\pi}}\,. \tag{24}$$

After introducing (24) in the formula (18), the desired solution (18) arises in the form:

$$y(t,\tau) = \frac{1}{w}\cdot\sqrt{\frac{2}{\pi\cdot\tau}}\cdot e^{-\frac{2\cdot t^2}{w^2\cdot\tau}}\,. \tag{25}$$

Since $y(t-t_c,\tau)$ also represents a solution of the problem for any constant t_c, it follows:

$$y(t,\tau) = \frac{1}{w}\cdot\sqrt{\frac{2}{\pi\cdot\tau}}\cdot e^{-\frac{2\cdot(t-t_c)^2}{w^2\cdot\tau}}\,. \tag{26}$$

Taking $\tau=1$ and replacing $\frac{y-y_0}{A}$ by y in (26), the fitting function (8) arises.

EXPERIMENTAL RESULTS – VERIFICATION OF THE MATHEMATICAL MODEL

Frequency distribution of the tractor, Massey-Ferguson 8160 engine, lifespan is presented in Figure 3, together with the ascending cumulative function. It is fairly close to the normal Gaussian distribution, but still slightly asymmetrical. This is evident from Table 2, which presents the basic statistical parameters: flatness factor is nearly 3 (the normal Gaussian distribution value), while the skewness has a value of 0.46 that is not too far away from the Gaussian value, which is zero. The fitting of experimental data by normal Gaussian model function is presented in Figure 4, while its constants are presented in Table 3, and the parameters describing the fit quality are given in Table 4. The approximation model function, namely "normal Gaussian function", has been successfully fitted. The mean square error MSE = 7.4431 and root mean square error RMSE = 2.7282, while the R-square or determination factor is 0.8990 and adjusted R-square is 0.8558 – fairly close to 1 for this function.

The average lifespan of analyzed tractors was 8850 (h), while the minimum and maximum were 4000 (h) and 14000 (h). This data dispersion originates from different

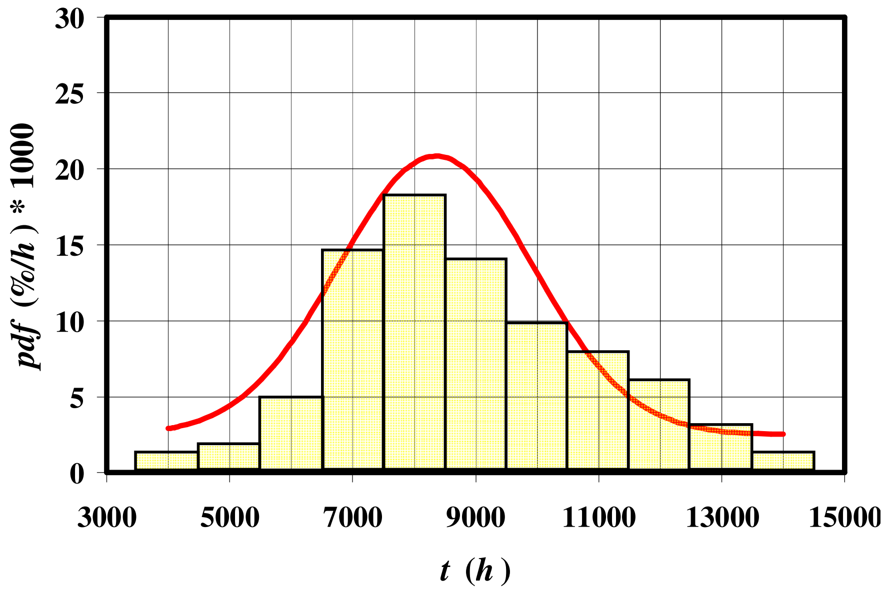

Figure 4. Fitting the probability density function of the engine lifespan by the normal (Gaussian) function - the tractor Massey-Ferguson 8160.

Table 3. Fitting parameters of the engine lifespan probability density function, modeled according to the normal Gaussian function.

Parameter	Value
y_0	2.492318
x_c	8351.0744
w	3158.1928
A	72652.174

Table 4. Parameters describing the fitting quality.

Parameter	Value
Number of points	11
Degrees of freedom	7
Reduced Chi-Square – Mean square error (MSE)	7.4431
Residual sum of squares	52.1018
R value	0.9482
R-square(COD)	0.8990
Adj. R-square	0.8558
Root-mean square error (RMSE)	2.7282

operational and maintenance conditions at various farms in Serbia, which represent evident problem nowadays. Our results are in fairly good agreement with standard ASAE D497.4 (2003), which assumes lifespan of 10000(h) under technical conditions specified by manufacturer. Furman et al. (1998) analysed a similar number of tractors and reported that engine failure after average working time of 615.4 (h). This gives 14.4 repairments before the general repairment of the engine.

Conclusions

It is recognized worldwide that mathematical and statistical modeling are useful tools that can facilitate and improve decision making processes in the area of farm machinery management. This way, the engine durability of more than a hundred tractors of popular Massey-Ferguson 8160 model, imported within a year in Serbia, has been carefully monitored. The presented analysis verifies that frequency distribution of the engine lifespan, as long as the overhaul is needed, is fairly close to the normal Gaussian distribution.

With respect to this basically statistical information, an adequate mathematical model has been formulated. It is based on differential equation and corresponding additional conditions, whose solution is just the normal Gaussian function. This way, a basis for future modeling of the process of tractor maintenance, repair and operation is partially established. A similar approach can be used for other tractor models and farm machinery components in general.

ACKNOWLEDGMENTS

This research was supported by the Ministry of Science and Technological Development, Republic of Serbia – projects ''Improvement of biotechnological procedures as a function of rational utilization of energy, agricultural products productivity and quality increase'' (Project no. TR 31051) and ''Dynamical stability and instabillity of mechanical systems exposed to stochastical disturbances'' (Project no. OI 174011).

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