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Full Length Research Paper

Use of nonlinear programming to determine the economically optimal energy density in laying hens diet during phase 1

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This study was designed to show the advantage of nonlinear programming in diet formulation. A nonlinear programming Excel workbook was developed that used the Excel solver to optimize energy density and bird performance. In this study 6 dietary treatments (include 2.515, 2.615, 2.715, 2.815, 2.915 and 3.015 Mcal of metabolizable energy per kilogram) were fed to Hy-line W-36 laying hens (n = 192) in phase 1 (from 24 to 32 weeks of age). Data were fitted to quadratic equations to express egg mass, feed consumption and objective function return over feed cost in terms of energy density. Nutrient: energy ratio constraints were transformed into equivalent linear constraints. To demonstrate the capabilities of the model, the prices for egg, corn and soybean meal were increased and decreased by 25% and the program solved for the maximum profit and optimized feed mix. Formulations were identical in all other respects. By increasing egg price, the model changed the optimal diet formulation and energy density in such a way as to improve performance and feed consumption and accepted a higher energy concentration. In order to make nutritional and economical decisions for a given feed formulation problem, the sensitivity analysis was performed. The sensitivity analysis for linear programming showed that if the protein level of the diet were to change from 16.046 to 15.046%, the cost of the diet would decrease by $0.0293 from $0.4089 to $0.3799 kg^{-1}. The sensitivity analysis for nonlinear programming showed that if the protein level of the diet were to change from 14.96 to 13.96%, the cost of the diet would decrease by $0.0272 kg^{-1} from $0.3547 to $0.3275 kg^{-1}. Results indicated that there are considerable savings to be made by egg producers from the use of the nonlinear programming model described here as opposed to a linear one with fixed minimums for energy and other nutrients. These savings result from the nonlinear programming models'' ability to determine the most profitable energy density that should be fed as energy and protein prices change.

Key words: Energy, feed formulation, laying hens, nonlinear programming, optimization.

INTRODUCTION

Animal nutritionists' main interests are in biology, not econometrics. As a result, the economic interpretation of feeding practices has been practically ignored in animal nutrition texts and in our teaching programs. Most animal nutrition texts have only a few paragraphs or pages on feed formulation, despite the fact that feed formulation is the primary objective of animal nutrition and the major cost in animal production. The development and implementation of easy, customized software models should facilitate the teaching of (and increase interest in) the economic ramifications of choices in animal nutrition and feeding (Pesti and Seila, 1999).

The concept of feeding economically optimal concentrations of nutrients based on diminishing returns functions is not new but has rarely been used in nutrition (Almquist, 1953). Linear programming is an effective method of finding the least possible cost of a unit of diet that satisfies a set of nutrient specifications. It selects

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proportionally the ingredients to be mixed to meet certain nutrient specifications that minimize cost per unit of weight of a diet. This technique of least cost feed mix implicity assumes constant marginal products and constant returns to scale of a production response. The commonly applied least-cost feed mix does not take productivity into account, nor does it use productivity information in setting energy and protein levels in response to their prices (Pesti et al., 1986). Static methods of diet formulation ignore the importance of economics and are not adequate to optimize the feeding of commercial laying hens. Reducing feed costs may make the cost side of the equation look attractive but the resulting loss in performance may have negative effects on profitability. Roland et al. (2000) demonstrated that nutrient requirements for maximum profits vary. Producers must let egg, protein and energy prices dictate nutrient requirements. Because egg and feed prices change, there can be no fixed nutrient requirements.

There is a wide range of dietary energy levels (2.684 - 2.992 Mkcal of ME/kg) currently being used by the egg industry. Part of the reason for this is that information is not available that would allow egg producers to know the ideal dietary energy level required for optimal performance and profits during phase one. Feed intake and egg weight can significantly affect cost of production and profits. With the sharp increase in dietary energy prices that can occur, it is even more important for egg producers to have information available that would allow them to continually optimize dietary energy use (Wu et al., 2005). Therefore, since it is widely accepted in poultry nutrition that nutrient requirements should be expressed as grams per megacalorie to take into account the effect of energy on feed intake (Scott et al., 1982; Waldroup et al., 1990; Leeson et al., 1996), a bird response function can be derived in terms of dietary energy density from either experimental or industry data to analyze profitability. Because the response of birds to energy density is a diminishing returns phenomenon, it should be evaluated economically to estimate an economic optimum level rather than a biological maximum. Guevara (2004) reported that non linear programming can be more useful than conventional linear programming to optimize performance response to energy density in broiler feed formulation because an energy level does not need to be set.

Sterling et al. (2005) used quadratic programming model to determine economically optimal dietary crude protein and lysine levels for broiler chicks and demonstrated that maximum profit model should only give the same or better, never worse, formulations compared with current least cost formulation models. The NRC (1994) committee concluded, “It would be desirable to have mathematical models available that would facilitate the selection of the most economical combinations of dietary concentrations of protein/amino acids (and other nutrients) and energy to achieve poultry production goals.” At present, there is no reliable dynamic computer method of diet formulation to determine how changing prices of egg and feed ingredients affect performance and dietary energy density that maximizes margin over feed cost. A study examining the effects of different energy density on the difference between returns from egg production and feed cost may provide a rationale for formulating diets based on market situations that maximize profits rather than minimizing feed costs alone.

The objective of our study was to evaluate the efficiency of a nonlinear programming optimization model to determine the impact of variation in ingredients and egg prices on the optimal energy density and feed mix and margin over feed cost.

MATERIALS AND METHODS

In this experiment, Hy-line W-36 hens (n=192) in phase 1 (24 to 32 weeks of age) were randomly divided into 6 treatments (4 replicates of 8 birds per treatment). Replicates were equally distributed into upper and lower cages to minimize cage level effect. All hens were housed in an environmentally-controlled house with temperature maintained at approximately 24°C. The house had controlled ventilation and lighting (16 h/day). All hens were supplied with feed and water *ad libitum*. Dietary treatments included the following energy densities: 2.515, 2.615, 2.715, 2.815, 2.915, and 3.015 Mcal of metabolizable energy per kilogram. The 2.915 energy level was computer-formulated to meet the requirements of the Hy-line commercial management guide (2008). Nutrient levels were kept in a constant ratio to energy level.

Egg production and egg weight were recorded daily and feed consumption was recorded weekly. Egg mass and feed conversion (g feed /g egg) were calculated from egg production, egg weight and feed consumption. Data were fitted to quadratic equations by using Excel polynomial regression to ex-press egg mass and feed consumption in terms of energy density. The optimization model and conventional linear programming was solved using the solver which is the default solver of Excel (Frontline System, Inc., 1999). It uses the generalized reduced gradient method to solve nonlinear problems. The options, which are specified by the user, were set as follows: iterations = 1,000, precision = 0.00001, convergence = 0.001, estimates = tangent, derivatives = forward and search = Newton.

The feed formulation model used in this study has been detailed by Guevara (2004). It will be described here only briefly. This program takes the technically derived equation for egg mass, adds economic data on the cost of feed ingredients and the value of egg and calculates the density of energy that maximize profit (return over feed cost). The quadratic production function and feed consumption were derived from the response of the birds to various dietary energy densities as:

$$EM = a + bE - cE^2$$
$$F = d - eE + fE^2$$

Where EM = egg mass (kg), E = energy density (Mcal/kg), F = feed intake (kg) and a, b, c, d, e and f are constants. These functions were used to create windows for economic modeling in a Microsoft Excel worksheet (2003). The nonlinear programming model is listed in Table 1.

The model identifies the combination of feed ingredients that maximize return over feed cost. The linear programming matrix of Pesti et al. (1986) was adapted to take energy density expressed as a ratio and bird response into account. Nutrient concentrations

Table 1. Nonlinear programming model.

Activity	Ingredient			Energy (E)	RHS	Ranges	
	X1	X2	X3	X4		Minimum	Maximum
Cost	c1	c2	c3		=Px		
Weight	w1	w2	w3		=1		
Energy	e1	e2	e3	-1	=0		
Protein (P)	p1	p2	p3	-(P/E)	≥0		
Amino acid (A)	a1	a2	a3	-(A/E)	≥0		
				1	=E	E1	E2

Objective function Maximize: $Py(a + bE - cE^2) - Px(d - eE + fE^2)$.

Table 2. Partial nonlinear programming matrix and constraint set used in optimization.

Component	Corn	Wheat	Soybean meal	Fish meal	Soy oil	Canola meal	Oyster	ME	RHS
Cost	0.3	0.275	0.51	1.2		0.39	0.04		0.353
Weight	1.0	1.0	1.0	1.0	1.0	1.0	1.0		1.000
Protein (%)	8.5	11.5	44	60		34.8		-5.48	0.000
ME (Mcal/kg)	3.35	3.120	2.230	2.82	8.5	2		-1.00	0.000
Calcium (%)	0.04	0.05	0.29	5.11		0.68	38.0	-1.33	0.000
Av.phosph (%)	0.10	0.10	0.27	2.88		0.30		-0.164	0.000
Sodium (%)	0.06	0.06	0.01	0.65		0.05	0.05	-0.058	0.000
Lysine (%)	0.26	0.31	2.69	4.51		1.94		-0.301	0.000
Met (%)	0.18	0.15	0.62	1.63		0.71		-0.147	0.000
Met+Cys (%)	0.36	0.37	1.28	2.20		1.58		-0.246	0.006
Threonine (%)	0.29	0.32	1.72	2.46		1.53		-0.2332	0.095
Tryptophan (%)	0.06	0.12	0.74	0.49		0.44		-0.0617	0.020
								1.000	2.915
Minimum					0				
Maximum					0.03				

were kept in a constant ratio with energy level, and each nutrient constraint expressed as a ratio was transformed into equivalent linear constraints before using nonlinear programming. Energy density was entered as an extra ingredient on the left-hand side of the model.

Since nutrient: energy ratio is equal to the requirement divided by energy level, then the requirement in the right-hand side of the model is equivalent to ratio times energy level and could pass to the left-hand side of the model as a linear function. The effects of changes in different variables on the optimum energy density, diet formulation, performance and profitability were performed using nonlinear programming model and were compared with conventional linear programming. To demonstrate the capabilities of the model, the prices for egg, corn and soybean meal were increased and decreased by 25% and the program solved for the maximum profit and optimized feed mix. Formulations were identical in all other respects. The nonlinear programming matrix and constraint set used in optimization are shown in Table 2. In order to make nutritional and economical decisions for a given feed formulation problem, the sensitivity analysis was performed for both linear programming and nonlinear programming feed formulation (Roush et al., 2009).

RESULTS AND DISCUSSION

Egg mass and feed consumption response functions are shown in Figures 1 and 2, respectively. The objective function obtained was:

$$\text{Margin} = \text{Egg price} * (-0.3214*E^2 + 2.2318*E - 1.1391) - \text{feed cost} * (2.1786*E^2 - 13.208*E + 24.366)$$

The effect of changing prices of egg, corn and soybean meal on optimal performance and energy density are shown in Table 3. The profit margin was much higher when the prices of ingredients were low. By increasing egg price, the model changed the optimal diet formulation and energy density in such a way as to improve performance and feed consumption and accepted a higher energy concentration. As the price of soybean meal increased, its dietary usage decreased and the model increased the optimal energy density, as well as

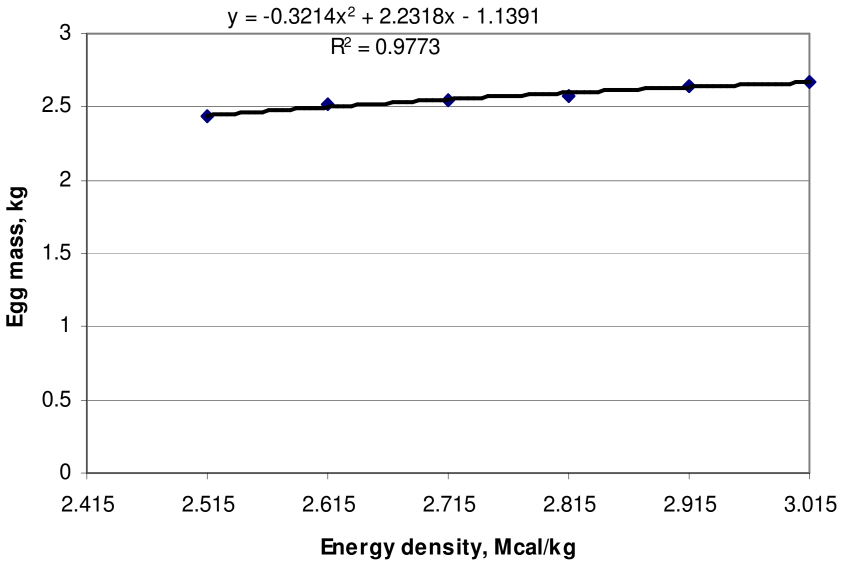

Figure 1. Egg mass response to energy density during 24 to 32 weeks of age.

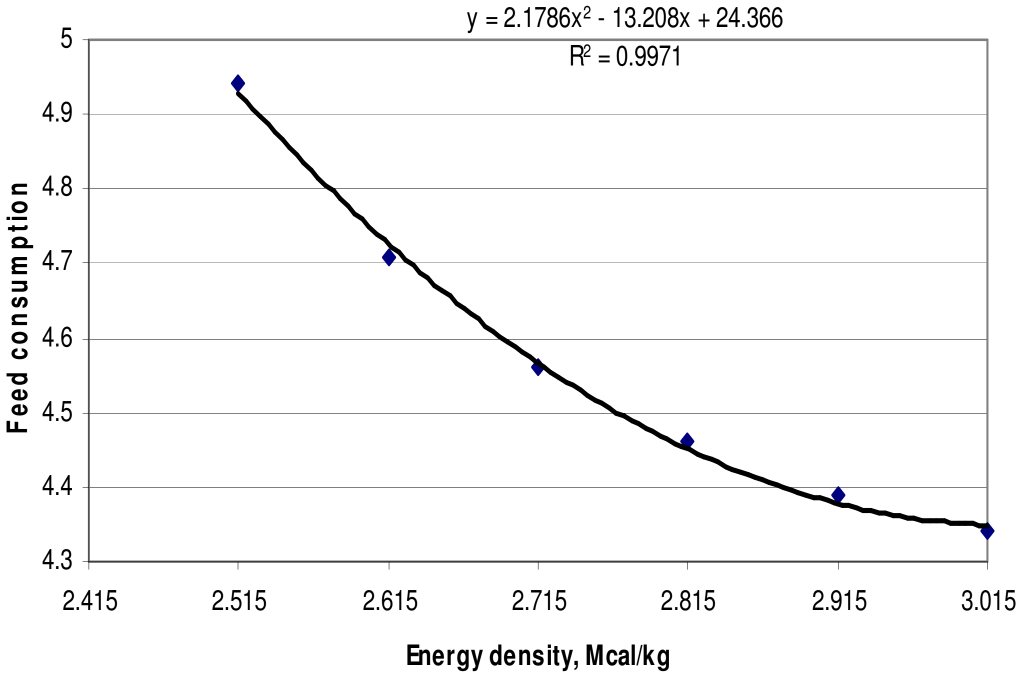

Figure 2. Laying hens feed consumption response to energy density during 24 to 32 weeks of age.

optimizes the diet composition that leads to improved feed conversion and bird performance. Different profitabilities for similar energy densities reflected changes in diet formulation.

In Table 4 least cost feed formulation (linear programming method) is compared with the maximum profit, using the nonlinear programming model. Margin and diet cost changed compared with conventional linear programming. The diet formulated using nonlinear programming was more profitable. If the maximum profit formulation model is used, the economically optimal energy density may change depending on the prices of energy and the value of the egg.

In Tables 5 and 6 the sensitivity analysis for linear and nonlinear programming solutions are shown respectively. The terms shadow price and reduced cost are familiar

Table 3. Effect of changing the price of egg, corn and soybean meal on optimum performance and energy density.

Price	ME (Mcal/kg)	Egg mass (kg)	Feed consumption (kg)	FCR[1] (g/g)	Margin (US $/hen)
LP[2]					1.64
NLP[3]					
Normal	2.731	2.559	4.547	1.78	1.71
Corn (%)					
+25	2.734	2.560	4.543	1.77	1.51
-25	2.737	2.562	4.538	1.77	1.94
Soybean meal (%)					
+25	2.783	2.583	4.485	1.74	1.62
-25	2.745	2.565	4.529	1.77	1.83
Egg[4] (%)					
+25	2.734	2.560	4.543	1.77	2.55
-25	2.727	2.557	4.552	1.78	0.88

[1]FCR= feed conversion ratio; [2]LP= linear programming; [3]NLP= nonlinear programming; [4]Egg price assumed= 1.3 US $/kg.

Table 4. Effect of changing prices on diet formulations.

Ingredient	Price ($/kg)	Normal	Corn+	Corn-	SBM+	SBM-	Egg+	Egg-	LP[1]
Corn	0.3	59.2	58.63	67.93	60.6	68.26	59.37	59.01	61.62
Wheat	0.275	10	10	0	10	0	10	10	0
SBM[2]	0.51	18.70	18	18.92	12.91	20.53	19.44	17.84	20
Fish meal	1.2	0	0	0	4	0	0	0	1.90
Canola meal	0.39	0.92	2	2	2	0	0	2	2
Soybean oil	1.2	0	0.19	0	0	0	0	0	3.66
Limestone	0.04	4	4	4	4	4	4	4	4
O.S.M.[3]	0.04	4.28	4.27	4.29	4.25	4.33	4.29	4.25	4.33
D.C.P.[4]	1	1.81	1.81	1.8	1.28	1.81	1.81	1.8	1.69
DL-Met	9.5	0.16	0.16	0.15	0.13	0.16	0.16	0.16	0.17
Lys-HCL	3.8	0.15	0.16	0.13	0.11	0.13	0.15	0.16	0.08
Salt	0.04	0.28	0.28	0.28	0.22	0.28	0.28	0.28	0.05
Mineral premix	1.6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Vitamin premix	1.6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	Cost, $/kg	0.3547	0.3997	0.3060	0.3879	0.3326	0.3556	0.3537	0.4089
	Margin, $/kg	1.71	1.51	1.94	1.62	1.83	2.55	0.88	1.64
Calculated analysis									
ME (Mcal/kg)		2.731	2.734	2.737	2.783	2.745	2.734	2.727	2.915
Protein (%)		14.96	14.98	15	15.25	15.04	14.98	14.94	16.04
Lysine (%)		0.82	0.82	0.82	0.84	0.83	0.82	0.82	0.88
Methionine (%)		0.4	0.4	0.4	0.41	0.4	0.4	0.4	0.448
Methionine-cystine (%)		0.66	0.67	0.67	0.67	0.66	0.66	0.66	0.72
Calcium (%)		3.63	3.64	3.64	3.7	3.65	3.64	3.63	3.9
Available phosphorus (%)		0.45	0.45	0.45	0.46	0.45	0.45	0.45	0.48
Sodium (%)		0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.17

[1]LP = linear programming; [2]SBM = Soybean meal; [3]OSM = oyster shell meal; [4]DCP = Dicalcium phosphate.

Table 5. Sensitivity report for the linear solution of diet.

Item	Amount	Reduced cost ($)	Objective coefficient ($)	Allowable increase ($)	Allowable decrease ($)
Corn	0.6162	0.0000	0.3	0.054	0.083
Wheat	0.0000	0.0761	0.275	-	0.076
Wheat middling	0.0000	0.1913	0.25	-	0.191
Soybean meal	0.2000	0.0000	0.51	0.054	0.045
Soybean oil	0.0366	0.0000	1.2	0.261	0.177
Canola meal	0.0200	-0.1724	0.39	0.172	-
Oyster shell meal	0.0433	0.0000	0.04	0.010	58.750
Fish meal	0.0190	0.0000	1.2	0.069	0.088
Wheat bran	0.0000	0.1848	0.17	-	0.184
Limestone	0.0400	0.0000	0.05	-	0.010
Dicalcium phosphate	0.0169	0.0000	1	0.625	0.492
Lys	0.0008	0.0000	3.8	10.475	8.593
DL Met	0.0017	0.0000	9.5	14.585	11.459
Vitamin premix	0.0025	1.9676	1.6	-	1.967
Mineral premix	0.0025	1.9676	1.6	-	1.967
Salt	0.0005	0.0000	0.04	6.713	0.407
		Constraints			
Item	**Amount**	**Shadow price ($)**	**Constraint R.H. side**	**Allowable increase**	**Allowable decrease**
Weight	1.0000	-0.3676	1	0.021	-
ME (Mcal/kg)	2.9150	0.1844	2.915	-	0.149
Protein (%)	16.0406	-0.0293	16.0406	-	0.053
Calcium (%)	3.9000	0.0107	3.9	0.00019	-
Available phosphorus (%)	0.4800	0.0537	0.48	0.005	0.302
Sodium (%)	0.1700	0.0105	0.17	-	0.02
Lysine (%)	0.8800	0.0907	0.88	0.046	-
Methionine (%)	0.4489	0.0000	0.43	0.018	-
Met+Cys (%)	0.7200	0.1179	0.72	0.09	-
Threonine (%)	0.6000	0.7811	0.6	0.002	-
Tryptophan (%)	0.2031	0.0000	0.18	0.023	-

Table 6. Sensitivity report for the nonlinear solution of diet.

Item	Amount	Reduced gradient ($)
Corn	0.5920	0.0000
Wheat	0.1000	0.0320
Wheat middling	0.0000	-0.3485
Soybean meal	0.1871	0.0000
Soybean oil	0.0000	-0.8753
Canola meal	0.0092	0.0000
Oyster shell meal	0.0428	0.0000
Fish meal	0.0000	-0.4977
Wheat bran	0.0000	-0.5691
Limestone	0.0400	0.0000
Dicalcium phosphate	0.0181	0.0000
Lys	0.0015	0.0000
DL Met	0.0016	0.0000

Table 6. Cont'd.

Vitamin premix	0.0025	0.0000
Mineral premix	0.0025	0.0000
Salt	0.0028	0.0000
	Constraints	
Item	**Amount**	**Lagrange multiplier**
Weight	1.0000	1.3867
ME (Mcal/kg)	2.731	0.000
Protein (%)	14.96	-0.0272
Calcium (%)	3.63	-0.0412
Available phosphorus (%)	0.45	-0.2791
Sodium (%)	0.16	-0.0402
Lysine (%)	0.82	-0.2162
Methionine (%)	0.40	-0.4390
Met+Cys (%)	0.66	0.00
Threonine (%)	0.54	0.0000
Tryptophan (%)	0.19	0.0000

linear programming terms to feed formulators. Different terminology is used to define shadow prices and reduced costs for sensitivity analysis of the nonlinear programming solution. The Lagrange multiplier is used instead of shadow price to describe marginal value of nutrients. Reduced gradient is used instead of reduced cost to describe the price at which ingredients not used in the formulation would be included in the solution (Roush et al., 2009). The sensitivity analysis for linear programming showed that if the price of corn by $0.054/kg from $0.3 to $0.354/kg or decreases by $0.083/kg from $0.3 to $0.217/kg, it will stay at the same level of 61.62% of the diet. If the protein level of the diet were to change from 16.046 to 15.046%, the cost of the diet would decrease by $0.0293 from $0.4089 to $0.3799 kg^{-1}. The sensitivity analysis for nonlinear programming showed that if the protein level of the diet were to change from 14.96 to 13.96%, the cost of the diet would decrease by $0.0272 kg^{-1} from $0.3547 to $0.3275 kg^{-1}. A caveat is that, sensitivity analysis for nonlinear program is valid only for the single point of the optimal solution.

Our results follow the law of diminishing returns and basic principles of production economics: If the price of the inputs increases, their use tends to decrease; if the value of the product increases, the level of output tends to increase, and conversely. Results indicated that there are considerable savings to be made for egg producers from the use of the nonlinear programming model described here as opposed to a linear one with fixed minimums for energy and other nutrients. These savings result from the nonlinear programming models ability to determine the most profitable energy density that should be fed as energy and protein prices change. Linear programming models contain no features that allow them to make this determination (Pesti et al., 1986). The nonlinear model workbook gives producers a working tool to demonstrate the interdependencies of costs and technical response functions and the value of egg.

One of nonlinear programming model's advantage is its flexibility. Parameters that can be expressed in terms of energy density of the diet can be included in the model (Arraes, 1983). Egg weight is an example of these parameters that depends on diet energy density (Wu et al., 2007). Constraints can then be set and the diets chosen that will produce eggs with their average weight above or below a specified weight.

The nonlinear model may be used not only for optimization but also for evaluation of current feeding situation as well. In this way, bird performance estimated by the model can be compared with the actual performance observed in the farm to evaluate the efficiency of current feeding practices.

Wu et al. (2005) demonstrated that strain of hen has a significant effect on egg mass. It is important that producers develop their production functions with the genetic stocks and under the conditions they will be using commercially.

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REFERENCES

Almquist HJ (1953). Application of the law of diminishing returns to estimation of B-vitamins requirements for growth. Poult. Sci., 32: 1001.

Arraes RA (1983). Alternative evaluations of economically optimal rations for broilers. Ph.D. dissertation. University of Georgia Athens, GA.

Frontline Systems Inc. (1999). Solver User's Guide. Incline Village, NV.

Guevara VR (2004). Use of nonlinear programming to optimize performance response to energy density in broiler feed formulation. Poult. Sci., 83: 147-151.

Hy-line variety W-36 commercial management guide. 2008. http://www.hyline.com/

Lesson S, Caston L, Summers JD (1996). Broiler response to diet energy. Poult. Sci., 75: 529-535.

National Research Council (1994). Nutrient Requirements of Poultry. 9th rev. ed. National Academy Press, Washington, DC.

Pesti GM, Seila AF (1999). The use of an electronic spreadsheet to solve linear and non-linear "stochastic" feed formulation problems. J. Appl. Poult. Res., 8: 110-121.

Pesti GM, Arraes RA, Miller BR (1986). Use of the quadratic growth response to dietary protein and energy concentrations in least-cost feed formulation. Poult. Sci., 64: 1040-1051.

Roland DA, Bryant MM, Zhang JX, Roland DA, Self J (2000). Econometric feeding and management of commercial Leghorns: Optimizing profits using new technology. In Egg Nutrition and Biotechnology (Sim JS, Nakai S, and Guenter W eds.). pp 463-472. CABI Publishing, CAB Int. allingford, UK.

Roush WB, Purswell J, Branton SL (2009). Microsoft Excel sensitivity analysis for linear and stochastic program feed formulation. J. Appl. Poult. Res., 18: 85-89.

Scott ML, Nesheim MC, Young RJ (1982). Nutrition of the chicken. M. L. Scott and Associates, Ithaca, NY.

Sterling KG, Vedenov DV, Pesti GM, Bakalli RI (2005). Economically optimal dietary crude protein and lysine levels for starting broiler chicks. Poult. Sci., 84: 29-36.

Waldroup PW, Tidwell NM, Izat AL (1990). The effects of energy and amino acid levels on performance and carcass quality of male and female broilers grown separately. Poult. Sci., 69: 1513-1521.

Wu G, Bryant MM, Gunawardana P, Roland A (2007). Effect of nutrient density on performance, egg components, egg solids, egg quality and profits in eight commercial leghorn strains during phase one. Poult. Sci., 86: 691-697.

Wu G, Bryant MM, Voitle RA, Roland DA (2005). Effect of dietary energy on performance and egg composition of Bovans White and Dekalb White hens during phase I. Poult. Sci., 84: 1610-1615.