

Full Length Research Paper

# A study on reverse osmosis permeating treatment for yarn dyeing effluent using fuzzy linear regression model

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**This paper presents a fuzzy linear regression model for estimation of reverse osmosis permeating parameters conditions. The proposed model can effectively take on non-crisp, fuzzy and crisp data. This study model used for estimation of reverse osmosis permeating parameters data from Tirupur examines the variables that contribute to the deterioration of membrane. This result demonstrates the capability and effectiveness of the proposed model to assign membrane. The study system requires an accurate and efficient prediction model.**

**Key words:** Fuzzy regression, membership function, chlorine (Cl<sup>-</sup>), membrane, total dissolved solids (TDS), pH.

## INTRODUCTION

The textile industry uses valuable dyes, which are clearly visible if discharged into public water ways. However, these disposals create both an aesthetic and environmental wastewater problems. At the same time, the textile industry continually seeks to conserve water and economically benefits from dye recovered and reused. The textile dyeing industry is regarded as water intensive sector, because it uses water as the principal medium for application of dyes and chemicals as well as for removal of impurities (European Commission, 2002). Continuous effluent discharges have caused gross

damages to the nearby aquatic system receiving body Orathupalayam Dam, located downstream of River Noyyal, making the water quality unfit for irrigation. The TDS, chloride and sodium of the reservoir water were reported to be as high as 5054, 2869 and 1620 mg/L, respectively (Central Pollution Control Board, 2005). Also, the concentration of dissolved solids in the ground and river water is reported to be in the range of 5000 to 7000 mg/L, which is almost ten times higher than the desirable drinking water standard (Indian Standard 10500, 1991).

There are 57 textile processing units of large scale industries, 5156 cotton textiles units and 4799 hosiery and readymade garments units of small scale industries in Erode District, Tamil Nadu. Erode is therefore a well known textile centre in India, particularly for textile processing. The main water resources for these industries are Bhavani and Cauvery Rivers. Most of the textile industries situated in these regions are small and medium; they are unorganized in nature judging from the effluent treatment facilities. Most of the coloured effluents are discharged into river, particularly into River Cauvery, without any proper treatment. Hence, it is necessary to find low cost and affordable treatment for the coloured textile wastewater (Malathy, 2007).

Most of the effluents from different industrial sources used to be discharged directly into the soil or ground water, but due to stringent environmental restrictions,

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**Nomenclature:** FLR, Fuzzy linear regression; RO, reverse osmosis;  $\tilde{Y}$ , fuzzy observed data;  $\mu_{\tilde{Y},L}$ , left bound of  $\hat{Y}_i$  at membership  $\mu$  value;  $\mu_{\tilde{Y},R}$ , right bound of  $\hat{Y}_i$  at membership  $\mu$  value;  $e_i$ , left fuzzy width of  $\tilde{Y}_i$ ;  $a_0$ , centre value;  $\underline{a}$ , lower value;  $\bar{a}$ , upper value;  $\mu$ , membership value; TDS, total dissolved solids; PPM, parts per million.

Central Pollution Control Board (CPCB) has become stricter and has imposed very stringent laws for recovering pure water from such industrial effluents. However, for the treatment of an effluent by conventional methods like aerobic and non-aerobic digestion, the ratio of biological oxygen demand (BOD) to chemical oxygen demand (COD) should be  $>0.6$  (Chain and Dewalle, 1977). In membrane process, the increasing cost of water and its wasteful consumption have now induced a treatment process which is integrated into in-plant water circuits rather than as a subsequent treatment. From this standpoint, membrane filtration offers potential applications. Processes using membranes provide very interesting possibilities of separating hydrolyzed dyestuffs and dyeing auxiliaries, thus simultaneously reducing coloration and the BOD/COD of the wastewater (Machenbach, 1998).

Reverse osmosis membranes have a retention rate of 90% or more for most types of ionic compounds and produce high quality permeation (Tinghui, 1983). Decolouration and the elimination of chemical auxiliaries in dye house wastewater can be carried out using a single step. Reverse osmosis permeates the removal of all mineral salts, hydrolyzed reactive dyes and chemical auxiliaries. The problem involved is that the higher the concentration of salts, the more important the osmotic pressure becomes and therefore, the greater the energy required.

## MATERIALS AND METHODS

A fuzzy regression model is a non parametric model that can be used to explain the variation of a dependent variable  $Y$  in terms of the variations of the independent variable,  $X$  as  $Y = f(x)$ , where  $f(x)$  is a linear function (Wang and Tsaur, 2000). Fuzzy regression provides means for handling regression problem where there is lack of significant amount of data and where there is vague relationship between the explanatory and response variables (Xue et al., 2005). It was first introduced by Tanaka et al. (1982). The first linear regression analysis with a fuzzy model used fuzzy numbers that can be expressed as intervals, having membership values as the regression co-efficient (Tanaka et al., 1982). Since the regression co-efficient is the set of fuzzy numbers, then the estimated dependent variable  $Y$  is also a fuzzy number.

Chang and Ayyub (1996) had described the three approaches to fuzzy regression namely; the possible regression, which is based on minimizing fuzziness as an optimal criterion; the second is based on the least squares of errors as a fitting criterion and the third approach is described as interval regression analysis. Tanaka's possibility regression describes the response variable  $Y$  as:

$$\tilde{Y} = A_0 + A_1 X_1 + A_2 X_2 + \dots + A_j X_j + \dots + A_k X_k \dots \quad (1)$$

Where,  $Y$  is the fuzzy output,  $X = [X_1, X_2, \dots, X_k]^T$  is the real valued input vector of independent variables and each regression co efficient  $A_j$ ,  $j = 0, 1, \dots, k$  is assumed to be asymmetric triangular fuzzy number with centre  $a_j$  and half width  $c_j$ ,  $c_j > 0$ .

In fuzzy regression, deviation between observed and estimated values is assumed to be due to the fuzziness of the system or fuzziness of regression co-efficient values. In other words, according to fuzzy regression theory, the residuals between the estimators and observations are produced by the uncertainty of the

parameter in the model, and not by measurement errors (Tseng and Lin, 2005). Fuzzy regression models can also be classified by the conditions of the dependent and independent variables as follows:

- (1) Both input and output numbers are non- fuzzy numbers
- (2) Both input and output numbers are fuzzy numbers.
- (3) Input data are non- fuzzy numbers but output data are fuzzy numbers (Buckley and Choi, 2005).

Generally, fuzzy regression model could be estimated using several methods. The method developed by Tanaka et al., (1982) was based on numerical methods that minimized the fuzziness of the response variables. Another researcher, Diamond (1988) used statistical methods that minimized the difference between estimated and observed outputs.

## Preliminaries

A fuzzy number  $\tilde{A}$  convex normalized fuzzy subset of the real line  $R$  with an upper semi-continuous membership function of bounded support (Dubois and Prade, 1980).

## Definition

**A symmetric fuzzy number  $\tilde{A}$ , denoted by:**

$\tilde{A} = (a, c)_L$ , is defined as

$$\tilde{A}(x) = L((x - a) / c), c > 0,$$

Where,  $a, c$  is the centre and spread of  $\tilde{A}$  and  $L(x)$  is a shape function of fuzzy numbers such that:

- (i)  $L(x) = L(-x)$ ,
- (ii)  $L(0) = 1, L(1) = 0$ ,
- (iii)  $L$  is strictly decreasing on  $[0, \infty]$ ,
- (iv)  $L$  is invertible on  $[0, 1]$ .

The set of symmetric fuzzy numbers is denoted by  $F_L(R)$ . If  $L(x) = 1 - |x|$ , then the fuzzy number is a symmetric triangular fuzzy number.

## Definition

Suppose  $\tilde{A} = (a, c)_L$  is a symmetric fuzzy number and  $\lambda \in R$ , then  $\lambda \tilde{A} = (\lambda a, |\lambda| c)_L$ .

## Fuzzy linear regression analysis

The fuzzy regression equation is considered as,

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 X$$

$$(\underline{y}, \bar{y}) = (a_0, \underline{a}, \bar{a}) + (a_1, \underline{a}, \bar{a})x_1 + (a_2, \underline{a}_2, \bar{a}_2)x_2 + (a_3, \underline{a}_3, \bar{a}_3)x_3$$

Here  $\tilde{Y} = (\underline{y}, \bar{y})$

$$\tilde{A}_0 = (a_0, \underline{a}_0, \bar{a}_0)$$

**Table 1.** Reverse osmosis permeating parameters.

DAY	$\bar{y}$	$y$	pH	TDS (ppm)	Cl <sup>-</sup> (ppm)
1	0.4	0.3	6.13	190	85
2	0.5	0.4	6.12	160	79
3	0.5	0.4	6.17	130	80
4	0.4	0.3	6.14	130	71
5	0.6	0.5	6.30	110	92
6	0.5	0.4	6.03	110	72
7	0.6	0.5	6.20	115	74
8	0.4	0.3	6.02	90	55
9	0.3	0.2	6.00	92	61
10	0.5	0.4	6.42	150	90
11	0.8	0.7	6.52	130	86
12	0.7	0.6	6.40	130	78
13	0.6	0.5	6.39	110	92
14	0.9	0.8	6.52	130	84
15	0.7	0.6	6.48	122	88

The three crisp independent variables are  $X_1, X_2$  and  $X_3$  and fuzzy dependent variable  $\check{Y}$ . The linear regression can be expressed as:  
 $(\underline{y}, \bar{y}) = (a_0, \underline{a}, \bar{a}) + (a_1, \underline{a}, \bar{a})x_1 + (a_2, \underline{a}, \bar{a})x_2 + (a_3, \underline{a}, \bar{a})x_3$ .

The lower bound is;

$$\underline{y} = \mu_{\check{y},L} = y_i - (1 - \mu)(\underline{a}_0 + \underline{a}_1x_1 + \underline{a}_2x_2)$$

The upper bound is;

$$\bar{y} = \mu_{\check{y},R} = y_i - (1 - \mu)(\bar{a}_0 + \bar{a}_1x_1 + \bar{a}_2x_2)$$

The normal equations are:

$$\sum y_i = na + b \sum x_i$$

$$\sum x_i y_i = a \sum x_i + b \sum x_i^2$$

$$na_0 + a_1 \sum x_1 + a_2 \sum x_2 + a_3 \sum x_3 = \sum y_i \tag{1}$$

$$a_0 \sum x_1 + a_1 \sum x_1^2 + a_2 \sum x_1 x_2 + a_3 \sum x_1 x_3 = \sum x_1 y_i \tag{2}$$

$$a_0 \sum x_2 + a_1 \sum x_1 x_2 + a_2 \sum x_2^2 + a_3 \sum x_2 x_3 = \sum x_2 y_i \tag{3}$$

$$a_0 \sum x_3 + a_1 \sum x_1 x_3 + a_2 \sum x_2 x_3 + a_3 \sum x_3^2 = \sum x_3 y_i \tag{4}$$

$$na_0 + a_1 \sum x_1 + a_2 \sum x_2 + a_3 \sum x_3 = \sum e_i \tag{5}$$

$$a_0 \sum x_1 + a_1 \sum x_1^2 + a_2 \sum x_1 x_2 + a_3 \sum x_1 x_3 = \sum e_i x_1 \tag{6}$$

$$a_0 \sum x_2 + a_1 \sum x_1 x_2 + a_2 \sum x_2^2 + a_3 \sum x_2 x_3 = \sum e_i x_2 \tag{7}$$

$$a_0 \sum x_3 + a_1 \sum x_1 x_3 + a_2 \sum x_2 x_3 + a_3 \sum x_3^2 = \sum e_i x_3 \tag{8}$$

$$\bar{a}_0 + \bar{a}_1 \sum x + \bar{a}_2 \sum x_2 + \bar{a}_3 \sum x_3 = \sum \bar{e}_i \tag{9}$$

$$\bar{a}_0 \sum x_1 + \bar{a}_1 \sum x_1^2 + \bar{a}_2 \sum x_1 x_2 + \bar{a}_3 \sum x_1 x_3 = \sum \bar{e}_i x_1 \tag{10}$$

After the regression coefficients are derived, it is valuable to measure how well the regression explains the relationship between the dependent and independent variables. The data are collected for 15 days and represented in Table 1. The RO process variable using FLR model is regarded as fuzzy dependent variable ( $\check{Y}$ ). The corresponding two crisp predictor variables included pH ( $X_1$ ), TDS ( $X_2$ ), and Cl ( $X_3$ ). The data are mixed numerical and fuzzy values. The proposed fuzzy linear regression model is used here.

$$(15)a_0 + (93.84)a_1 + (1899)a_2 + (1887)a_3 = (14.6) \tag{1}$$

$$(93.84)a_0 + (586.62)a_1 + (11890.9)a_2 + (7667.1)a_3 = (1370.06) \tag{2}$$

$$(1899)a_0 + (11890.9)a_1 + (249673)a_2 + (152128)a_3 = (27725.4) \tag{3}$$

$$(1887)a_0 + (7667.1)a_1 + (152128)a_2 + (95621)a_3 = (173302) \tag{4}$$

$$(15)\underline{a}_0(93.84)\underline{a}_1 + (1899)\underline{a}_2 + (1887)\underline{a}_3 = (6.4) \quad (5)$$

$$(93.84)\underline{a}_0 + (586.62)\underline{a}_1 + (11890.9)\underline{a}_2 + (7667.1)\underline{a}_3 = (600.576) \quad (6)$$

$$(1899)\underline{a}_0 + (11890.9)\underline{a}_1 + (249673)\underline{a}_2 + (152128)\underline{a}_3 = (12153.6) \quad (7)$$

$$(1887)\underline{a}_0 + (7667.1)\underline{a}_1 + (152128)\underline{a}_2 + (95621)\underline{a}_3 = (7596.8) \quad (8)$$

$$(15)\overline{a}_0(93.84)\overline{a}_1 + (1899)\overline{a}_2 + (1887)\overline{a}_3 = (8.2) \quad (9)$$

$$(93.84)\overline{a}_0 + (586.62)\overline{a}_1 + (11890.9)\overline{a}_2 + (7667.1)\overline{a}_3 = (769.48) \quad (10)$$

$$(1899)\overline{a}_0 + (11890.9)\overline{a}_1 + (249673)\overline{a}_2 + (152128)\overline{a}_3 = (15571.8) \quad (11)$$

$$(1887)\overline{a}_0 + (7667.1)\overline{a}_1 + (152128)\overline{a}_2 + (95621)\overline{a}_3 = (344.4) \quad (12)$$

Results for Equations 1 are  $\underline{a}_0 = -739.3550$ ;  $\underline{a}_1 = 17.0013$ ;  $\underline{a}_2 = 1.7227$  and  $\underline{a}_3 = 5.2553$ . Solving Equations 5 to 8 yields the lower bound values as,  $\underline{a}_0 = -324.0812$ ;  $\underline{a}_1 = 7.4580$ ;  $\underline{a}_2 = 0.7554$  and  $\underline{a}_3 = 2.3027$ . While Equations 9 to 12 yield the upper bound values as,  $\overline{a}_0 = -173.8299$ ;  $\overline{a}_1 = -29.3085$ ;  $\overline{a}_2 = 1.0278$  and  $\overline{a}_3 = 2.8763$ .

The fuzzy linear equation is,

$$(\underline{y}, \overline{y}) = (\underline{a}_0, \overline{a}_0) + (\underline{a}_1, \overline{a}_1)x_1 + (\underline{a}_2, \overline{a}_2)x_2 + (\underline{a}_3, \overline{a}_3)x_3$$

$$\check{Y} = (-739.3550, -324.0812, -173.8299) + (17.0013, 7.4580, -29.3085) X_1 + (1.7227, 0.7554, 1.0278) X_2 + (5.2553, 2.3027, 2.8763) X_3.$$

## RESULTS AND DISCUSSION

The slope of  $X_1$  is (17.0013, 7.4580, -29.3085)  $X_1$ . This means that in spite of the average pH ( $X_1$ ) and TDS( $X_2$ ), TDS( $X_2$ ) and Cl area( $X_3$ ), had an increase of 17.0013 in the reverse osmosis parameter ( $\check{Y}$ ), decrease of 7.4580 in the lower bound, decrease of -29.3085 in the upper bound for one day pH( $X_1$ ), and then (1.7227, 0.7554, 1.0278). This shows that  $X_2$  decreases by 1.7227 with decrease of 0.7554 and increase of 1.0278 in the lower and upper bound values. For the coefficient of  $X_3$  (5.2553, 2.3027, 2.8763), the values of 5.2553 increase; 2.3027 and 2.8763 also increase in the lower and upper bound values. Then the above values are substituted for

the fuzzy estimated equations.

## Conclusion

This paper presents a multivariate fuzzy regression model, an extension of Chang's model for predicting RO process conditions. The proposed model utilizes different membership values to adequately fit fuzzy regression models on for RO process data. In addition, this model employs two different types of two membership function in order to overcome the limitation of Chang's approach, which can only be applied to normalized triangular fuzzy numbers. The proposed model has been demonstrated as an effective tool for dealing with mixture of crisp and fuzzy data. The result also shows that the model has potential to be used with similar type of industry process problems. The prediction outcomes given by the proposed model depend on the RO process data. This model identifies function of RO process variables. The result can provide and reduce the variables of pH, TDS and Cl.

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