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The jump-diffusion process for the VIX and the S&P 500 index

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This paper applies the CBP-GARCH model of Chan (2003) to analyze the discontinuous jump and the time-varying correlated jump intensity for the changes in the VIX and the S&P 500 returns over the period extending from January 15, 2001 to December 31, 2009. The empirical results provide evidence of the significant jump-diffusion process and the causal relationships in the bi-directions between the S&P 500 returns and the changes in the VIX. In addition, the relationships between the S&P 500 returns and the changes in the VIX exhibit joint jump behavior are not time varying.

Key words: VIX, CBP-GARCH model, jump-diffusion process.

INTRODUCTION

The CBOE market volatility index (hereafter, VIX) is a measure of the market's expectations regarding a 30 day implied volatility index.

In 1993, the Chicago board options exchange (hereafter, CBOE) introduced the CBOE Volatility Index, VIX, which comprises at-the-money S&P 100 index option prices. In 2003, the CBOE employed a new VIX based on the S&P 500 index that averages the weighted prices of the S&P 500 index puts and calls over a wide range of strike prices. On March 24, 2004, the CBOE introduced the first exchange-traded VIX futures contract on its new, all-electronic CBOE futures exchange. In February 2006, the CBOE launched VIX options and, as a result, in less than five years the combined trading activity in VIX options and futures has grown to more than 100,000 contracts per day (see the CBOE White Paper, 2009). The VIX also serves as an "investor fear gauge" that is a measurement index of the price variations and is widely discussed by both academics and practitioners (Whaley, 2000; Simon, 2003; Giot, 2005; Badshah, 2009; Whaley 2009). In addition, many studies have found evidence of a strong negative and asymmetric relationship between returns and the changes in the volatility index (Fleming et al., 1995; Whaley, 2000; Simon, 2003; Low, 2004; Giot, 2005; Selcuk, 2005; Dennis et al., 2006; Hibbert et al., 2008; Badshah, 2009). Accordingly, the observation and application of the VIX is a non-negligible topic in the financial field. Therefore, the need to speculate and hedge against the changes in volatility has motivated the rapid growth of the volatility derivatives markets in recent years. As regards applying the VIX to engage in arbitrage or hedging activities, we utilize the GARCH series model to analyze the short-term dynamic behavior between returns and changes in the volatility index.

Jorion (1988), Bakshi et al. (1997) and Das and Sundaram (1999) have pointed out that if models neglect discontinuous jump characteristics, they will give rise to mispricing. Hence, it is important to incorporate discontinuous jump characteristics into the models. As a result, Chan and Maheu (2002), Pan (2002), Eraker et al. (2003) and Maheu and McCurdy (2004) apply the jump model to analyze the stock or option markets; however, they fail to provide an in-depth discussion of the jumps in volatility markets. More recently, a number of studies have paid attention to the jump model to analyze the implied volatility index. Wagner and Szimayer (2004) are the first to investigate the jump characteristics in the implied volatility index by estimating the mean-reverting jump-diffusion process, and they conclude that there are significant positive jumps in implied volatilities. Dotsis et al. (2007) find that the jump-diffusion model performs best; in addition, they argue that the joint modeling of the implied volatility index and the corresponding stock index

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deserves to be a topic for future research. Becker et al. (2009) also discuss jumps in the VIX and their findings indicate that the VIX reflects past jump activity in the S&P 500 index and that the VIX forecast errors are indeed uncorrelated with information available in the past that is related to jump activity. Unfortunately, so far, the existing empirical literature only seems to consider the single jump-diffusion model but not joint jump-diffusion models, so that they can not together analyze the causal relationships in both directions between the implied volatility index and the stock index. In order to overcome this shortcoming, it is therefore necessary for future research to consider a joint modeling to investigate the relationships between the implied volatility index and the corresponding stock index.

Accordingly, the purpose of this paper is to contribute to exploring the joint jump-diffusion activity both for the changes in the VIX and the S&P 500 returns and their special causal relationships. Based on a comparison of the models, we confirm that the best specification is a correlated bivariate Poisson jump model of Chan (2003, CBP-GARCH model hereafter), and then apply it to perform a further empirical analysis. It is an extension of a model with multivariate GARCH parameterization that includes a bivariate correlated jump process and a correlated bivariate Poisson function to examine the dynamic relationships between the changes in the VIX and S&P 500 returns. With respect to the relevant applications of CBP-GARCH model, Lee and Cheng (2007) employed the CBP-GARCH model to investigate the relationship between the volatility of crude oil and gasoline especially during the period of the Gulf War. In addition, Chiu and Hung (2007) investigated normal and abnormal information transmissions by using CBP-GARCH model to examine diffusion volatility and jump intensity spillovers in China's stock markets. Therefore, it has the following three advantages in this study. First, it can examine whether the changes in the VIX and S and P 500 returns exhibit significantly negative or positive return-volatility relationships regardless of the impact of changes in the VIX on S&P 500 returns or the impact of the S and P 500 returns on the changes in the VIX. This model efficiently incorporates both the additional lagged variables of the S&P 500 returns and changes in the VIX, respectively, so that this study can adequately describe how the current S&P 500 returns and the current changes in the VIX are affected by both of the lagged variables in this model. Bollerslev and Zhou (2006) have pointed out that the volatility feedback effect refers to the impact of the contemporaneous volatilities on the returns, and the leverage effect refers to the impact of the lagged returns on the current volatilities. In addition, Bollerslev et al. (2006) indicate that the causal relationships for the two effects are indistinguishable, are often inconclusive and sometimes lead to the conflicting results reported in the extant literature. Meanwhile, we also examines whether the CBP-GAECH model is consistent with the extrapolation bias behavior theory of Hibbert et al. (2008). Hibbert et al. (2008) mention that past changes in implied volatilities affecting current changes in implied volatility are consistent with the extrapolation bias behavioral theory, as investors would expect volatility changes to maintain a trend in the near future. In addition, Shefrin (2008) proposes an example to describe the extrapolation bias, namely, that U.S. home prices rose by about 85 from 1997 to 2006. Because of this extrapolation bias, the sentiment of many people was that housing prices would continue to increase by about 10 percent each year. Hence, it is fair to describe this as a bubble. When the prices did not continue to increase, the bubble burst, as evidenced by the decline in housing prices by more than 15% between June 2007 and June 2008. Consequently, this result led to the global financial crisis in 2007-2009. This study thus argues that, if changes in implied volatility really represent “mean reverting” behavior but not investors’ cognition, then this is so-called extrapolation bias in volatility markets. Secondly, CBP-GAECH model investigates whether both the changes in the VIX and S&P 500 returns exhibit jump-diffusion activity and co-movement jump behavior. Finally, CBP-GAECH model further confirms whether the S&P 500 returns lead to the changes in the VIX or whether the changes in the VIX lead the S&P 500 returns.

The remainder of this article is organized as follows. Section II describes the data and the econometric model. The empirical results and conclusions are reported in section III and section IV, respectively. The final section contains our discussion.

DATA AND THE CBP-GARCH MODEL

The data used are the daily S&P 500 index and the VIX of the CBOE, and cover the period from January 15, 2001 to December 31, 2009. All daily data are obtained from Cmoney. The S&P 500 returns and the changes in the VIX are defined as $r_{sp,t} = (\ln p_t - \ln p_{t-1}) \times 100$ and $r_{vix,t} = (\ln VIX_t - \ln VIX_{t-1}) \times 100$, respectively, where p_t is the S&P 500 index and VIX_t is the VIX index. This paper applies the CBP-GARCH model of Chan (2003) to investigate the relationships between the S&P 500 returns and the changes in the VIX, which both postulate that the jump intensity obeys an ARMA process and incorporates the GARCH effect of the series. This model can adequately capture the diffusion volatility and jump intensity spillover effects between the S&P 500 returns and the changes in the VIX. For the purpose of examining whether there are jump-diffusion processes as well as the causal relationships in the bi-directions between the S and P 500 returns and the changes in the VIX, the CBP-GARCH model is described as follows:

$$r_{sp,t} = \beta_{10} + \beta_{11} r_{sp,t-1} + \beta_{12} r_{sp,t-2} + \beta_{13} r_{vix,t-1} + \beta_{14} r_{vix,t-2} + \varepsilon_{sp,t} + J_{sp,t} \quad (1)$$

$$r_{vix,t} = \beta_{20} + \beta_{21} r_{vix,t-1} + \beta_{22} r_{vix,t-2} + \beta_{23} r_{sp,t-1} + \beta_{24} r_{sp,t-2} + \varepsilon_{vix,t} + J_{vix,t} \quad (2)$$

Where $r_{sp,t-i}$ ($r_{vix,t-i}$) denotes the S&P 500 returns (the changes in VIX) on day *t-i*, i=0, 1, 2. $\varepsilon_{sp,t}$ ($\varepsilon_{vix,t}$) is a error term and $J_{sp,t}$ ($J_{vix,t}$) is a jump component for $r_{sp,t}$ ($r_{vix,t}$). The error term and the jump component are assumed to be independent, that is, $E(\varepsilon_{sp,t}, J_{sp,t}) = 0$ and $E(\varepsilon_{vix,t}, J_{vix,t}) = 0$. The error term $\varepsilon_{sp,t}$ ($\varepsilon_{vix,t}$) has a bivariate normal distribution with zero mean and conditional covariance matrix $\tilde{\mathrm{H}}_t$; besides, the jump component $J_{sp,t}$ ($J_{sp,t}$) also has a bivariate normal distribution with zero mean and conditional covariance matrix Δ_t. In a bivariate framework, the jump component is defined as:

$$J_t = \begin{bmatrix} \sum_{i=1}^{n_{1t}} Y_{1t,i} - E_{t-1}(\sum_{i=1}^{n_{1t}} Y_{1t,i}) \\ \sum_{j=1}^{n_{2t}} Y_{2t,j} - E_{t-1}(\sum_{j=1}^{n_{2t}} Y_{2t,j}) \end{bmatrix} \tag{3}$$

Where $\sum_{i=1}^{n_{1t}} Y_{1t,i}$ ($\sum_{j=1}^{n_{2t}} Y_{2t,i}$) denotes the summation of n jumps or the jump intensity for $r_{sp,t}$ ($r_{vix,t}$) over any period t. In addition, each stochastic variable Y_{it} follows a normal distribution with mean θ for its intercept term and variance δ_i^2; in other words, the bivariate jump intensities can be described as:

$$Y_{1t,i} \sim N(\theta_1, \delta_1^2) \text{ and } Y_{2t,j} \sim N(\theta_2, \delta_2^2) \tag{4}$$

In equation (3), the variables n_{1t} and n_{2t} both denote individual counting variables of jump intensity in that the two variables are constructed by the independent Poisson variables, namely, n_{1t}^*, n_{2t}^* and n_{3t}^*. Each one of these variables has a probability density function given by:

$$\mathrm{P}(\mathrm{n}_{\mathrm{it}}^* = \mathrm{j} \mid \Phi_{\mathrm{t-1}}) = \mathrm{e}^{-\lambda_\mathrm{i}} \lambda_\mathrm{i}^\mathrm{j} / \mathrm{j}!. \tag{5}$$

The expected values and variances of $\mathrm{n}_{\mathrm{it}}^*$ are each equal to λ_i, which is also referred to as the expected number of jumps or the jump intensity. The correlated jump intensity counters (M'Kendrick, 1926; Campbell, 1934) are defined as:

$$n_{1t} = n_{1t}^* + n_{3t}^* \text{ and } n_{2t} = n_{2t}^* + n_{3t}^* \tag{6}$$

By construction, each of these counting variables, n_{1t} or n_{2t}, is capable of generating independent jumps as well as correlated jumps. The independent jumps are initiated by n_{1t}^* and n_{2t}^* in time period *t*. The correlated jumps are produced by the additional Poisson variable, n_{3t}^*, which contributes jumps to both series. Using the change of variables method and integrating out n_{3t}^* yields the joint probability density for n_{1t} and n_{2t} as:

$$P(n_{1t} = i, n_{2t} = j \mid \Phi_{t-1}) = \sum_{k=0}^{\min(i,j)} e^{-(\lambda_1+\lambda_2+\lambda_3)} \frac{\lambda_1^{i-k} \lambda_2^{j-k} \lambda_3^k}{(i-k)!(j-k)!k!} \tag{7}$$

And the expected number of jumps is equal to

$$E(n_{it}) = \lambda_i + \lambda_3 \tag{8}$$

According to Chan (2003), the jump intensity parameter λ_{it} is the time-varying jump intensity, i = 1, 2, 3, and it is defined as:

$$\lambda_{1t} = \lambda_1 + \eta_1^2 r_{sp,t-1}^2 \tag{9}$$

$$\lambda_{2t} = \lambda_2 + \eta_2^2 r_{vix,t-1}^2 \tag{10}$$

$$\lambda_{3t} = \lambda_3 + \eta_3^2 r_{sp,t-1}^2 + \eta_4^2 r_{vix,t-1}^2 \tag{11}$$

Where $r_{sp,t-1}$ and $r_{vix,t-1}$ denote the rates of return for the S&P 500 index and the changes in the VIX at time $t-1$, respectively. The individual jump intensities λ_{1t} (λ_{2t}) are assumed to be related to market conditions which are reflected in $r_{sp,t-1}^2$ ($r_{vix,t-1}^2$) as an approximation of last period's volatility. Similarly, the covariance λ_{3t} is governed by the variations in the last period's volatilities from both series.

Combining the GARCH model with the CBP function, the probability density function both for $r_{sp,t-1}$ and $r_{vix,t-1}$ given i jumps in currency 1 and j jumps in currency 2 is defined by:

$$f(X_t \mid n_{1t} = i, n_{2t} = j, \Phi_{t-1}) = \frac{1}{2\pi^{N/2}} \left|H_{ij,t}\right|^{-1/2} \exp\left[-u'_{ij,t} H_{ij,t}^{-1} u_{ij,t}\right] \tag{12}$$

$$u_{ij,t} = \begin{bmatrix} r_{sp,t} - E_{t-1}(r_{sp,t}) - i\theta_1 + (\lambda_1 + \lambda_3)\theta_1 \\ r_{vix,t} - E_{t-1}(r_{vix,t}) - i\theta_2 + (\lambda_2 + \lambda_3)\theta_2 \end{bmatrix} \tag{13}$$

Where X_t denotes $r_{sp,t}$ and $r_{vix,t}$; and $u_{ij,t}$ is the usual error term with the jump component $J_{ij,t}$. $\mathrm{H}_{ij,t}$ is the covariance matrix of $r_{sp,t}$ and $r_{vix,t}$. Under the normal disturbance, ε_t, is independent of the jump component, and $\mathrm{H}_{ij,t}$, is the summation of the covariance matrix for the normal disturbance $\tilde{\mathrm{H}}_t$ and the jump component $\Delta_{ij,t}$.

The covariance matrix for the normal disturbance $\tilde{H}_t$ is defined as:

$$\tilde{H}_t = \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 \end{bmatrix} \quad (14)$$

Where $\sigma_{1,t}^2$, $\sigma_{2,t}^2$ and $\sigma_{12,t}$ denote $\omega_1 + \alpha_1\varepsilon_{1,t-1}^2 + \beta_1\sigma_{1,t-1}^2$, $\omega_2 + \alpha_2\varepsilon_{2,t-1}^2 + \beta_2\sigma_{2,t-1}^2$ and $\omega_{12}\sqrt{\sigma_{1,t}^2\sigma_{2,t}^2}$, respectively; notably, ω_{12} denotes the diffusion correlation coefficient.

The covariance matrix for the jump component $\Delta_{ij,t}$ is derived from the assumption that the correlation between the jump sizes is constant across contemporaneous equations and zero across time:

$$Corr(Y_{1t}, Y_{2t}) = \rho_{12} \text{ and } Corr(Y_{1t}, Y_{2s}) = 0 \text{ where } t \neq s \quad (15)$$

Therefore, the covariance matrix for the jump component $\Delta_{ij,t}$ can be presented as:

$$\Delta_{ij,t} = \begin{bmatrix} i\delta_1^2 & \rho_{12}\sqrt{ij}\,\delta_1\delta_2 \\ \rho_{12}\sqrt{ij}\,\delta_1\delta_2 & j\delta_2^2 \end{bmatrix} \quad (16)$$

Where parameter ρ_{12} denotes the jump correlation coefficient of Y_1 and Y_2. The covariance matrix of the CBP-GARCH model is indicated by the summation of $\tilde{H}_t$ and $\Delta_{ij,t}$. Finally, to complete the specification, the conditional density function is defined as:

$$P(X_t|\Phi_{t-1}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} f(X_t|n_{1t}=i, n_{2t}=j, \Phi_{t-1}) P(n_{1t}=i, n_{2t}=j, \Phi_{t-1}) \quad (17)$$

The log likelihood function is simply the sum of the log conditional densities:

$$\ln L = \sum_{t=1}^{N} \ln P(X_t|\Phi_{t-1}) \quad (18)$$

Since the information matrix is not a block diagonal matrix, evaluation of the full likelihood is required. To estimate the CBP-GARCH model, a truncation point must be selected for the probability function in equation (17). We choose a truncation point, which is sufficiently large so that the likelihood function and parameter estimates stabilize as a set of converged values. Of course, this paper will try to compare the models to confirm that the CBP-GARCH model fits. Thus, when not taking jump variables in account (setting $\theta_1 = \theta_2 = \delta_1 = \delta_2 = \rho_{12} = \lambda_1 = \lambda_2 = \lambda_3 = \eta_1 = \eta_2 = \eta_3 = \eta_4 = 0$), the CBP-GARCH model will reduce to a bivariate GARCH model (hereafter, the BGARCH model).

EMPIRICAL RESULTS

Table 1 presents the descriptive statistics for $r_{sp,t}$ and $r_{vix,t}$ during the research period. The sample mean and standard error of $r_{sp,t}$ ($r_{vix,t}$) are -0.0088 (-0.0019) and 1.4065 (6.0118), respectively, implying that the volatility of $r_{vix,t}$ is larger than that of $r_{sp,t}$. In addition, the statistics show that $r_{sp,t}$ and $r_{vix,t}$ both have leptokurtic distributions and fat tails because their skewness, kurtosis (excess) and JB-test measures are highly significant at the 5% level. As to the Ljung–Box Q (Q^2) statistics for $r_{sp,t}$ and $r_{vix,t}$, both have five lags and are significant at the 1% level. These indicate that both $r_{sp,t}$ and $r_{vix,t}$ exhibit autocorrelation, linear dependence and strong ARCH effects.

Table 2 presents the estimated results of the BGARCH model and the CBP-GARCH model. The CBP-GARCH model of Table 2 reveals that the statistical significance of the coefficients is rather similar to that of the BGARCH model. By testing the Q(5) and $Q^2(5)$ statistics, the BGARCH and CBP-GARCH models are both statistically insignificant, indicating that neither of the two models exhibits autocorrelation of the residuals nor conditional heteroskedasticity. In addition, this paper employs a likelihood ratio test to compare the CBP-GARCH model with the BGARCH model. We find that *LR* = 304.2394 which is significant at the 1% level, thus revealing that the CBP-GARCH model is better than the BGARCH model in terms of its statistics. Therefore, this paper will conduct further empirical analyses by using the CBP-GARCH model.

The parameters β_{11} = -0.1014 and β_{13} = -0.0094 are significant, indicating that the current S&P 500 returns is negatively influenced by both the lagged-one S&P 500 returns and changes in the VIX, but not by both the lagged-two S&P 500 returns and changes in the VIX. These findings not only demonstrate the significantly negative relationships of lagged return-volatility but also reveal the negative impact of the lagged-one returns on the current returns. This paper provides support of the negative lagged return-volatility relationships, that is, the current returns, are negatively correlated with the lagged-one changes in the VIX. The empirical result is consistent with the views of negative return-volatility relationships in most of the literature.

The parameter β_{21} = -0.0504 is significant, indicating that the current changes in the VIX are negatively influenced by the lagged-one changes in the VIX, this finding implying consistency with the extrapolation bias behavior theory of Hibbert et al. (2008). In addition, the parameters β_{23} = 0.1703 and β_{24} = 0.2204 are also

Table 1. Descriptive statistics.

Index	$r_{sp,t}$	$r_{vix,t}$
Sample mean	-0.0088	-0.0019
Standard error	1.4065	6.0118
Skewness	-0.1240**	0.6194***
Kurtosis (excess)	8.3266***	4.4771***
Jarque-Bera	6447.8557***	2005.0846***
Q(5)	47.5150***	44.1200***
$Q^2(5)$	111.2850***	111.2850***

Note ** and *** denote significance at the 5% and 1% levels. The Jarque-Bera statistic is used to determine whether the data come from a normal distribution. Q and Q^2 are Ljung-Box Q test statistics for serial correlation in the standardized residuals and squared standardized residuals, respectively.

significant, indicating that the current changes in the VIX is positively influenced by both the lagged-one and lagged-two S&P 500 returns. Hence, these findings seem to be consistent with the views of Ghysels et al. (2005), Bali and Peng (2006) and Lundblad (2007). They consistently suggest a significant positive risk-return trade-off relationship.

In terms of the conditional variance equations, the $\alpha_1 + \beta_1$ ($\alpha_2 + \beta_2$) is 0.9876 (0.9393), indicating a strong GARCH effect and persistence of conditional variance for $r_{sp,t}$ ($r_{vix,t}$). This implies that any exogenous fluctuation will together impact the S&P 500 returns and the changes in the VIX, thus giving rise to volatility impact effects both for $r_{sp,t}$ and $r_{vix,t}$. In addition, both the jump intensity and size parameters are significant, that is, the $r_{sp,t}$ and $r_{vix,t}$ exhibit jump behavior. Therefore, the Poisson jump components play a critical role in modeling the S&P 500 returns and the changes in the VIX.

Moreover, the diffusion correlation coefficient $\omega_{12} = -0.8354$ and jump correlation coefficient $\rho_{12} = -0.7790$, respectively. They both present a significant correlation and negative interactions with S&P 500 returns and the changes in the VIX.

The parameters θ_1 and θ_2 (δ_1 and δ_2) are the means and standard errors of the jumps, respectively, and most of them are statistically significant, except for θ_1. These exhibit abnormal information which will cause the average of the jumps to be about -0.1431 and 3.52898 and the standard error of the instantaneous jumps to be about 0.8691 and 5.7124 for the S&P 500 returns and the changes in the VIX, respectively. Owing to $\delta_1 < \delta_2$, this result thus reveals that the changes in the VIX give rise to more risk of jumps.

The individual jump intensity parameters λ_1 and λ_2 are statistically insignificant, except for the joint jump intensity parameter λ_3. These findings imply that there only exists joint jump behavior for the S&P 500 returns and the changes in the VIX, and no single jump behavior is found to exist. In addition, the total parameters η_1, η_2, η_3 and η_4 are statistically insignificant, revealing that the joint jump intensity parameter λ_3 is not related to market conditions and is not reflected in $r^2_{sp,t-1}$ ($r^2_{vix,t-1}$). In other words, these findings imply that only joint jump behavior exists, but it does not follow that the S&P 500 returns and the changes in the VIX are times varying.

Finally, we employ the Granger causality test to examine whether the S&P 500 returns and the changes in the VIX have lead or lag causal relationships with each other. If the impact of changes in the VIX on S&P 500 returns is statistically significant, this indicates that the lagged changes in the VIX will affect the current S&P 500 returns. Likewise, if the impact of S&P 500 returns on the changes in the VIX is statistically significant, this indicates that the lagged S&P 500 returns will affect the current changes in the VIX. Consequently, the test statistics are 5.0968 and 6.9445, respectively. Thus, the empirical results indicate that they are both statistically significant regardless of whether the changes in the VIX has an impact on the S&P 500 returns or the S&P 500 returns has an impact on the changes in the VIX. These findings imply that there are bi-directional causal relationships between the S&P 500 returns and the changes in the VIX; thus, the empirical findings are similar to those of Bollerslev et al. (2006) regarding leads or lags in the returns and the implied volatilities.

Conclusions

This study utilizes the CBP-GARCH model to investigate whether the S&P 500 returns and the changes in the VIX are characterized by negative or positive correlations, jump-diffusion process or special causal relationships. In addition, this analysis also compares the BGARCH model

Table 2. The estimation results for the BGARCH and CBP-GARCH models.

BGARCH model				CBP-GARCH model			
Parameter	**coefficient**		**Standard error**	**Parameter**	**Coefficient**		**Standard error**
β_{10}	0.0534	***	0.0295	β_{10}	0.0491	***	0.0158
β_{11}	-0.1055	***	0.0017	β_{11}	-0.1014	***	0.0262
β_{12}	-0.0407	***	0.0016	β_{12}	-0.0271		0.0246
β_{13}	-0.0092	***	0.0017	β_{13}	-0.0094	**	0.0042
β_{14}	-0.0036		0.3868	β_{14}	-0.0016		0.0039
ω_1	0.0059	***	0.5346	ω_1	0.0017		0.0015
α_1	0.0637	***	0.0020	α_1	0.0669	***	0.0065
β_1	0.9334	***	0.0118	β_1	0.9207	***	0.0069
β_{20}	-0.2226	***	0.0292	β_{20}	-0.1877	*	0.0971
β_{21}	-0.0724	**	0.2771	β_{21}	-0.0504	**	0.0243
β_{22}	-0.0058		0.0195	β_{22}	-0.0201		0.0244
β_{23}	0.1533		0.0066	β_{23}	0.1703	*	0.1020
β_{24}	0.2875	***	0.0128	β_{24}	0.2204	**	0.1051
ω_2	1.7183	***	0.0133	ω_2	1.0092	***	0.2195
α_2	0.0573	***	0.0114	α_2	0.0543	***	0.0078
β_2	0.8923	***	0.0134	β_2	0.8850	***	0.0160
ω_{12}	-0.8156	***	0.0064	ω_{12}	-0.8354	***	0.0091
				θ_1	-0.1431		0.0934
				θ_2	3.2898	***	0.6566
				δ_1	0.8691	***	0.0977
				δ_2	5.7124	***	0.5477
				ρ_{12}	-0.7790	***	0.0438
				λ_1	0.0000		0.1906
				λ_2	0.0000		0.0884
				λ_3	-0.4263	***	0.0367
				η_1	$1.1*10^{-7}$		0.0434
				η_2	$-4.7*10^{-7}$		0.0255
				η_3	$-3.2*10^{-7}$		0.0378
				η_4	$-1.5*10^{-7}$		0.0152
Log-likelihood value			-9108.5024				-8956.3827
Granger causality test							
VIX causes S and P 500			4.8576*	VIX causes S&P 500			5.0968*
S and P 500 causes VIX			5.8081*	S&P 500 causes VIX			6.9455*

Table 2. Cont'd.

Diagnostics on standardized residuals			
$Q_{sp}(5)$	2.8210	$Q_{sp}(5)$	3.3790
$Q_{sp}^2(5)$	7.8210	$Q_{sp}^2(5)$	5.8470
$Q_{vix}(5)$	6.2470	$Q_{vix}(5)$	7.4660
$Q_{vix}^2(5)$	0.3540	$Q_{vix}^2(5)$	0.1530

Note: *, ** and *** denote significance at the 10, 5 and 1% levels. Q_{sp} (Q_{vix}) and Q_{sp}^2 (Q_{vix}^2) are Ljung-Box Q tests for the serial correlation of $r_{sp,t}$ ($r_{vix,t}$) in the standardized residuals and squared standardized residuals, respectively.

with the CBP-GARCH model.

The empirical results reveal several findings regarding the relationships between the S&P 500 returns and changes in the VIX. First, our empirical results confirm that the CBP-GARCH model is better than the other models and that it can adequately describe whether the current S&P 500 returns and changes in the VIX are affected by both the lagged changes in the VIX and S&P 500 returns. For instance, these results have provided evidence of the negative relationships for the impacts of both the lagged-one returns and changes in the VIX on the current returns, implying consistency with the negative return-volatility relationships in most of the literature and describing the negative impact of the lagged-one returns on the current returns. In addition, the negative relationships for the impact of the lagged-one changes in the VIX on the current changes in the VIX imply consistency with the extrapolation bias behavior theory of Hibbert et al. (2008).

Furthermore, the positive relationships for the lagged-one and lagged-two returns also impact the current changes in the VIX, implying consistency with the positive risk-return relationships (Ghysels et al. 2005; Bali and Peng, 2006; Lundblad, 2007). Second, the diffusion and jump processes exhibit significant correlations and negative interactions with the S&P 500 returns and the changes in the VIX. This implies that there is a strong GARCH effect and persistence in the conditional variance between the S&P 500 returns and the changes in the VIX. Moreover, this study finds that the changes in the VIX lead to more risk of jumps than the S&P 500 returns.

In addition, the relationships between the S&P 500 returns and the changes in the VIX only exhibit joint jump behavior and are not time varying. Finally, this paper finds that there are bi-directional causal relationships between the S&P 500 returns and the changes in the VIX, similar to the viewpoint of Bollerslev et al. (2006) regarding the leads or lags in returns and their implied volatility. Therefore, this paper argues that the volatility transmissions are influenced by both the lagged S&P 500 returns and changes in the VIX.

DISCUSSION

According to the empirical finding that it only exits joint jump behavior, this paper hence argues that if the market gives rise to fluctuations, then it will simultaneously impact the S&P 500 returns and the changes in the VIX. In addition, owing to the evidence of bi-directional causal relationships, then the current S&P 500 returns and the current changes in the VIX will be related to the lagged counterpart factor for the S&P 500 returns (the changes in the VIX).

Therefore, according to above descriptions, this study provides an advice that, when the institutions or investors apply the VIX to engage in arbitrage or hedging activities, ought to monitor the simultaneous informational impact on spot market and option market in order to arrange the efficient strategies of investment. As to the future research issue, this study suggests that examining the informational content of forecast errors regarding jump characteristics will be a subject of future research and discussion.

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