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The contribution of the African capital markets in the diversification of investment global portfolios

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This study aims to evaluate the contribution of the African capital markets in the diversification of investment global portfolios. The study used the methodology based on the application of optimization models like mean variance (MV), resample michaud (RM), semi variance (SV), mean absolute deviation (MAD), and filtered historical simulation (FHS). In-sample and out-of-sample approaches were used to analyze the data. The study results suggested the existence of a strong correlation between some African capital markets and global capital markets, that is, they tend to move in the same direction. The most important being the diversification of global portfolio with assets of African capital markets generate benefits for both types of investors, risk averse and taker investors; that is, it provides benefits in the return and reduce investment risk. Still, the study results suggested that the foreign investors should look for African capital markets with a chance to maximize their wealth and diversify the investment risk in their portfolios. In the same order, the study result went further to elaborate on the advantages of the international diversification and furthermore contributes to the literature through application of the FHS method in the optimization portfolio. This methodology in addition to producing good results, is more restrained in the composition of investment portfolios than the other methods.

Key words: African capital markets, diversification, investment global portfolios.

INTRODUCTION

Globalization phenomenon has provided funds transfers between financial markets, with special attention given to the capital markets through the investor, and fund managers that are seeking to invest in order to maximize wealth.

However, there are financiers that are willing to invest their assets in both domestic and international markets. This is done in order to minimize possible loss in the case of adverse events occurring in the domestic region that can negatively influence the expected result of their investments. The investors use diversification strategies to minimize risk and maximize return of portfolios in order to protect their investments.

Thus, this study attempts to tackle the issues of diversification in the international context, considering the fact that global investors hold domestic portfolios where

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through diversification strategy, they include the African assets in their portfolios in order to reduce the exposure of risk and maximize the return. This study aims to identify how the African capital markets will contribute in the diversification risk of the investment global portfolios. Moreover, it compares capital markets to the level of dependency and exposure with respect to events that occur in these large markets.

Based on the weekly data collected from the main Europe and Africa markets and the methodology used, and in particular the application of the optimization models (mean variance, resample Michaud, semivariance, mean absolute deviation, and filtered historical simulation), with both *in-sample* and *out-of-sample* approaches, the study results suggest that African markets have a significant relationship with some of the world markets included in the study. The diversification of global portfolios with African assets generates benefits for the investor, that is, provides benefits in return and reduces investment risk for both types of investors.

The study contribution to the literature is to test empirically the application of the filtered historical simulation (FHS) methodology in the portfolio optimization and contributes to the discussion on advantage of international diversification context. FHS methodology in addition of producing good results, reveals being more cautious in the constitution of investment portfolios than the other methods. However, this model presents lesser returns and higher risk than other models, however their results follow the trend of the other models.

LITERATURE REVIEW

Several studies have shown that diversification in the international context is an advantage for investors holding composite portfolios with domestic and foreign securities.

According to Mansourfar et al. (2010) and Dimitriou and Kenourgios (2012), the arguement that diversification of the international portfolio has been a feature of the global capital market and potential benefits encourage the investors to diversify their investments. According to them, these benefits came from the fact that prices of international assets are less correlated and are derived from different fundamental economic factors.

In addition, they point out the benefits of international diversification the investor’s bets in the emerging markets and consequently have huge gains in the short term. According to Baele and Inghelbrecht (2009) and Chiou (2009) based on strong empirical support, potential gains from international diversification are still sufficient to justify a global asset allocation strategy rather than industry/regional or local diversification.

Flavin and Panopoulou (2009) argue that diversification in the international context has long been advocated as an effective way to achieve a higher adjusted return on the investment risk in the domestic market, that is, facilitates risk sharing. Rezayat and Yavas (2006) examined short-term co-movements between the five major stock markets (USA, UK, France, Germany and Japan) to assess the benefits of International Portfolio Diversification (IPD) and concluded that despite the fact that there is still room for diversification, the benefits are minimal for American and European investors who would like to invest exclusively in these two major economic blocs (Europe and America).

Laopodis (2005) argued that analysts is of the opininon that financial integration among global capital markets has reduced IPD's benefits by increasing the correlation between equity markets. Coeurdacier and Guibaud (2011) argue that both theories and empirical evidence suggest that financial integration between countries has a positive impact on the correlation between equity markets, which tends to reduce IPD's benefits. The economic gains from international equity diversification are still substantial despite the growing markets correlation (Bouslama and Ouda, 2014).

The major focus of studies on IPD is focused on the portfolios of American, European and Asian investors which fixated on their diversification directed primarily at the assets of European and Asian capital markets, such as the studies[1] of Odier and Solnik (1993) on a global investment where they found that it was profitable for Japanese, British, German and American investors. Liljeblom et al. (1997) investigated the benefits of IPD from the point of view of Nordic investors; Ho et al. (1999) reported that reducing the risk of loss through IPD would be of substantial benefit to Canadian investors; Rowland and Tesar (2004) and Gerke et al. (2005) also examined the potential benefits of IPD from the perspective of the German investor; Dunis and Shannon (2005) who examined stock markets in Southeast Asia (Malaysia, Philippines and Indonesia) and Central Asia (China, Belize, Taiwan and India), found that IDP would be beneficial to investors in the USA; Kearney and Poti (2006) used two conditional and unconditional estimation methods and analyzed the dynamics of correlation in five leading European capital markets, and Égert and Kocenda (2007) analyzed the issue between Eastern European stock markets and Central Bank ,where they stated that there is no long-term bond between stock markets between these two blocs. Therefore, on the question of the International Portfolio Diversification in the African context, there are practically no studies done, except for the few references that however, did not have a great impact on the African capital markets.

The studies of Hassan et al. (2003), Bailey et al. (2005), Lagoarde-Segot and Lucey (2007), Yu and Hassan (2008) and Mansourfar et al. (2010) on the stock

[1] *Apud* Mansourfar *et al*. (2010).

markets in the Middle East and North Africa countries (MENA), concluded that there are many benefits to the portfolio diversification with titles of these regions which are both in dollars and local currency.

However, it was argued that these undervalued and under-investigated emerging markets could attract more value for portfolios in the future. According to Mansourfar et al. (2010), in the past years emerging equity markets have been subject of a large body of studies on international finance. Therefore, it makes sense to look at this issue as being relevant in the context of the financial markets and the major economic blocks, particularly for Africa given the dynamism of their capital markets combined with economic growth in recent years and due to the financial crisis, and confidence in other great world capital markets.

METHODOLOGY

Data

The sample consists of weekly data corresponding to the prices of the market index, collected in the Thomson Reuters Eikon. The database sample started 5th August, 2004 and ended on 7th July, 2016; making a total of 624 weekly observations collected of the forty two (42) major capital markets in the World, according to the classification given by MSCI World Index and thirteen (13) of the main African capital markets, as shown in Tables 1 to 3. Weekly returns measured in USA Dollar were considered. To measure the return, risk level, and composition of investment portfolios, we proposed the following optimization models: MV, RM, SV, MAD and FHS. To evaluate the relationship between capital markets, the correlation coefficient was used. Matlab was resorted to for the application of the optimization models and Excel to make the graphics of efficient portfolios and to estimate the performance indicators.

In-sample and out-of-sample approaches

In the first stage, the *in-sample* approach is used for the entire period T of returns observations, where the different investment strategies distribution is plotted and represented by curves of efficient frontiers. Then, to evaluate and measure portfolio performances and the contribution to diversification, we proposed Sharpe Ratio and Sortino Ratio according to the study of Lagoarde-Segot and Lucey (2007); and to measure the contribution of portfolio diversification, we proposed the measures suggested by Liang and McIntosh (1999).

In the second phase, in line with the works of DeMiguel et al. (2009), Daskalaki and Skiadopoulos (2011) and Bessler et al. (2014), we applied the rolling sample approach, in order to understand the contribution of African assets in the diversification of Europe´s portfolios. This rolling sample methodology consists of considering a window with M observations for a given sub-period. The next step is to add one (1) more observation to window M (we considered M=5 years, corresponding to the 260 observations), forgetting the first observation, and calculating tangential portfolios that maximizes performance.

The process is repeated by always adding one more observation in the window and dropping the oldest observation and so on, inorder to determine the optimum portfolios for each window bearing until it gets the total observation (the total of 363 portfolios weights for out-of-sample analyses). The following out-of-sample evaluation is based on the performance of the following statistics:

Excess return (ER), risk (R), sharpe ratio (SR) and Sortino ratio (S) in order to realize the contribution of African markets to the diversification of global portfolios.

However, before following this methodology, we need to divide the sample into two sub-periods of 5 years (first sub-period starting from 5th August, 2004 to 3rd July, 2009 and second sub-period from 3rd July, 2009 to 7th July, 2016). Therefore, for the out-of-sample analysis, we have *2* sub-periods to evaluate the performance of the investment distribution strategies.

To evaluate the contribution of African assets in the Global portfolio diversification, we defined some possible strategies that investors can follow. However, it is important to note that nothing assures us that foreign investors can adopt these strategies because as you know, each investor has his own profile when it comes to investment. We assume that a rational investor can choose these two strategies here presented:

Strategy 1: The investor makes an optimal distribution of 100 of his investment in global capital markets. We consider this portfolio composition such as domestic portfolios.
Strategy 2: The investor chooses to make an optimal distribution of 100 of his investment between global and African capital markets.

The in-sample analysis for each strategy are made of 50 optimal portfolios that include the efficient frontiers based on risk and return. To evaluate the performance of the strategies and test the statistical significance, we considered two (2) null hypotheses:

$$H_0: SR2 - SR1 = 0 \quad (1)$$

$$H_0: S2 - S1 = 0 \quad (2)$$

SR2 and S2 are the values of the Sharpe Ratio and Sortino Ratio index performances for strategy 2; SR1 and S1 are the values of the Sharpe Ratio and Sortino Ratio index performances for *Strategy 1*. The objective is to evaluate whether the differences between the performances of the strategies are statistically significant, considering a 1 significance level for both analyses. Therefore, we compare the diversification strategy with an undiversified strategy, that is *Strategy 1*.

Portfolio optimization models

Mean variance (MV)

The first work on portfolio optimization was developed by Markowitz (1952) known as Mean Variance model (MV). This model suggests that making decisions on portfolio composition risk and return must be a criteria. The risk measure is standard deviation and the return measure is given by the average value of assets returns. Although it is highly criticized, it is a model widely used in financial studies. The Markowitz paradigm expects return and volatility to be relevant aspects that investors take into consideration when making decisions about portfolio composition. Thus, for the risk adverse investors the expectation to minimize risk to a given return limit, according to Markowitz (1952), can be expressed as:

Table 1. African capital markets.

Country		Currency/code	Market index (Name)
Continent Africa	South Africa	Rand (R)	FTSE/JSE Africa top 40 index
	Egypt	Egyptian Pound (EGP)	Egyptian EGX30 index
	Morocco	Moroccan Dirham (MAD)	Moroccan All Share MASI
	Tunisia	Tunisian Dinar (TND)	Tunindex
	Botswana	Botswana Pula (BWP)	BSE Domestic Company DCIBT
	Malawi	Malawian Kwacha (MWK)	Malawi All share Index (MASI)
	Mauritius	Mauritian Rupi (MUR)	Semdex MDEX
	Namibia	Namibian Dollar (NAD)	Namibia Stock Exchange (NSX)
	Nigeria	Nigerian Naira (NGN)	NSE Index 30 (NSEINDX:IND)
	Kenya	Kenyan Shiling (KES)	Kenya NSE 20 (NSE20)
	Uganda	Ugandan Shiling (UGX)	Uganda All Share (ALSIUG)
	Zambia	Zambian Kwacha (ZMK)	LSE All Share (LASILZ)
	Rep Democratic of Congo	Congolese Franc (CDF)	All Share index
	Costa do Marfim/Cote D´ivoire	XOF	All Share index

This table shows all the African capital market included in this study. Therefore, the capital markets did not meet the requirements of the sample between periods of 5 August, 2004 to 7 July, 2016 they were excluded form the study. The first column shows the countries, second the local currency index quotation and the third column the main market index for each country.

Table 2. World developed markets.

Country		Currency/Code	Market index (Name)
Continent Europe and Middle East	Germany	Euro(€)	DAX INDEX
	United Kingdon	Euro(€)	FTSE 100 INDEX (FTSE)
	France	Euro(€)	CAC 40 INDEX
	Italy	Euro(€)	FTSE MIB INDEX
	Spain	Euro(€)	IBEX 35 INDEX
	Austria	Euro(€)	ATX (ATX)
	Switzerland	Swiss Franc (CHF)	SMI (SSMI
	Belgium	Euro(€)	BEL20 (BFX)
	Denmark	Danish Krone (DKK)	OMX COPENHAGEN 20 (OMXC20)
	Finland	Euro(€)	OMX Helsinki 25 (OMXH25)
	Ireland	Euro(€)	ISEQ Overall (ISEQ)
	Israel	Israeli Shekel (ILS)	Tel Aviv 25 Index (TA25)
	Netherlands	Euro(€)	AEX (AEX)
	Norway	Norwegian Krone (NOK)	Oslo Stock Exchange All Share Index (OSEAX:IND)
	Portugal	Euro(€)	PSI 20 (PSI20)
	Sweden	Swedish Krona (SEK)	OMX Stockholm 30 (0MXS30)
Continent American	Canada	Canadian Dolar (CAD)	S&P/ TSX (GSPTSE)
	United States	USA DOLAR (USD)	S&P 500 (SPX)
Continent Asia /Pacific	Australia	Australian Dolar (AUD)	S&P/ASX (AXJO)
	Hong Kong	Hong Kong Dolar (HKD)	Hang Seng (HSI)
	Japan	Japanese Yen (JPY)	Nikkei 225 (N225)
	New Zealand	New Zealand Dollar (NZD9	S&P/NZX 50 Index Gross (NZSE50Ffg:IND)
	Singapore	Singapore Dollar (SGD)	FTSE Singapore (FTWISGPL)

This table shows all the main global markets included in the study according to the MSCI Word Index classifed in the developed markets. Therefore, the capital markets did not meet the requirements of the sample between periods from 5 August, 2004 to 7 July, 2016 so they were excluded from study. The first column shows the countries, second the local currency index quotation and the third column the main market index for each country was found.

Table 3. World emergent markets.

Country		Currency/code	Market index (Name)
Continent Europe and Middle East	Czech Republic	Czech Koruna (CZK)	PX (PX)
	Greece	Euro(€)	Athens General (ATG)
	Hungary	Hungarian Forint (HUF)	Budapest SE (BUX)
	Poland	Polish Zloty (PLN)	WIG 20 (WIG20)
	Qatar	Qatari Riyal (QAR)	Stock Market DOHA (QSI)
	Russia	Russian Ruble (RUB)	MICEX (MCX)
	Turkey	Turkish Lira (TRY)	BIST 100 (XU100)
	United Arab Emirates	AED	ADX General (ADI)
Continent American	Brasil	Brasilian Real (BRL)	Ibovespa Brasil Sao Paulo SE Index (IBOV:iND)
	Chile	Chilean Peso (CLP)	IPSA (IPSA)
	Peru	Peruvian Sol (PEN)	S&P Lima General (SPBLPGPT)
	Mexico	Mexican Peso (MXN)	IPC (MXX)
	Colombia	Colombian Peso (COP)	Colombian COLCAP Index (COLCAP:IND)
Continent Asia /Pacific	China	Chinese Yuan Renminbi (CNY)	Shanghai SE Composite Index (SHCOMP:IND)
	India	Indian Rupee (INR)	BSE Sensex 30 (BSESN)
	Indonesia	Indonesian Rupiah (IDR)	IDX Composite (JKSE)
	Korea	South Korean Won (KRW)	KOSPI (KS11)
	Malaysia	Malaysian Ringgit (MYR)	FTSE Malaysia KLCI (KLSE)
	Philippines	Philippine Peso (PHP)	PSEI Composite (PSI)
	Taiwan	Taiwan Dollar (TWD)	Taiwn Weighted (TWII)
	Thailand	Thai Baht (THB)	FTSE SET All-Share (FTFSTHA)

This table shows all the emergent markets included in study according to the MSCI Word Index. Therefore, the capital markets did not meet the requirements of the sample between periods starting from 05 August, 2004 to 07July, 2016 they were excluded from the study. The first column shows the countries, second the local currency index quotation and the third column the main market index for each country.

$$Minimize\ portfolio\ risk = \sqrt{\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(x_i\, x_j\, \rho_{ij}\sigma_i\sigma_j\ )} \tag{3}$$

subject to a minimum expected return is given by:

$$\sum_{i=1}^{N} x_i\, \overline{r_i} \geq r_C \tag{4}$$

total investment in the portfolio is given by:

$$\sum_{i=1}^{N} x_i = 1 \tag{5}$$

and to ensure that there are no negative investment is given by:

$$x_i \geq 0\ \forall i \tag{6}$$

N is the number of assets; x_i and x_j are the weights of the assets in the portfolio; σ_i and σ_j are the standard deviations of the assets i and j; ρ_{ij} is the correlation between assets i and j; $\overline{r_i}$ corresponds to the average return of the asset and r_c corresponds to the minimum desired portfolio return.

Resample michaud (RM)

This method was developed by Michaud (1998) and according to Becker et al. (2015), the basic concept of Michaud (1998) comprises of three aspects:

(1) A generation of sequence of returns, which are statistically equivalent to the actual time series of returns, through a Monte Carlo Simulation.
(2) The subsequent determination of portfolio weights for every resample.
(3) The averaging over the obtained portfolio weights to obtain the optimal portfolio weights.

This method can be considered as a "sophistication" of the MV model but based on the simulation method.

The algorithm that explains how to implement this method is described as follows:

(1) From the original database, two parameters are estimated, the vector of expected excess returns (μ) and the variance-covariance matrix (∑).
(2) Resample applying multivariate normal distribution with mean μ and covariance ∑ considering T draws. For each resample that is generated, there is a new mean μ and covariance ∑ to estimate optimal portfolio weights over T draws; and
(3) Choosing the optimal portfolio weights depends on the required portfolio number. The portfolio risks and returns that make up the Efficient Frontiers by Michaud are then estimated.

Semivariance **(SV)**

This model has emerged as an alternative to the mean-variance model (MV) which aims to remedy its shortcomings raised by scholars and researchers in the field of finance. Thus, Markowitz (1959) recognized the shortcomings of the MV model and proposed the SV model as the most appropriate measure of risk for investment portfolios. In general, according to Markowitz (1959), cited by Bond and Satchell (2002), the SV model for an individual asset is defined as follows:

$$SV = \frac{\sum_{j=1}^{T}\{\min[0, (r_i - \bar{r}_i)]\}^2}{T} \quad (7)$$

The standard deviation of the semi-variance of an asset is given by:

$$SV = \sqrt{\frac{\sum_{j=1}^{T}\{\min[0, (r_{it} - \bar{r}_i)]\}^2}{T}} \quad (8)$$

The semi-variance of an investment portfolio (SV_C) is given as:

$$SV_C = \frac{\sum_{j=1}^{T}\{\min[0, (r_{Ct} - \bar{r}_C)]\}^2}{T} \quad (9)$$

However, there are authors (Estrada, 2008) that suggest the estimation portfolio semi-variance approach by the expression:

$$SV_C \approx \sum_{i=1}^{N}\sum_{j=1}^{N}(x_i\, x_j\, SC_{ij}) \quad (10)$$

According to Estrada (2008) and Cumova and Nawrocki (2011), semi-covariance (SC) between the assets of the portfolios is estimated as:

$$SC_{ij} = \frac{1}{T}\sum_{t=1}^{T}\left[Min(r_{it} - \bar{r}_i, 0).\, Min\left(r_{jt} - \bar{r}_j, 0\right)\right] \quad (11)$$

The expected return of an investment portfolio is obtained from the following expression:

$$E(R_C) = \sum_{i=1}^{N} x_i\, \bar{r}_i \quad (12)$$

The mathematical formulation of the portfolio optimization problem using this model has as objective function to minimize the SV subject to certain restrictions as:

Minimize

$$SV_C \approx \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}(x_i\, x_j\, SC_{ij})} \quad (13)$$

subject to a minimum expected return is given by:

$$\sum_{i=1}^{N} x_i\, \bar{r}_i \geq r_C$$

total investment in the portfolio is given by:

$$\sum_{i=1}^{N} x_i = 1$$

and to ensure that there are no negative investment is given by:

$$x_i \geq 0\ \forall i$$

where, T is the size of the observation period; t is the sample period over T; r_{it}, r_{jt} and r_{Ct} are the observed returns of assets i, j and portfolio c in the period t; $\bar{r}_i$, $\bar{r}_j$ and $\bar{r}_C$ are the observed mean returns of the assets and portfolio. In the maximization problem, the objective function is that portfolio returns subjected to restrictions.

Mean absolute deviation (MAD)

To overcome the shortcomings of the model mean variance, Konno and Yamasaki (1991) suggested the model MAD as linear programming or linear optimization of portfolios, where the risk measure is the designed Average Deviation Absolute.

According to these authors, the MAD is based on dividing the distribution of a variable randomized into two groups, those afroementioned and below the average, and giving estimates for the absolute deviations of observations in each group from the average. MAD is preferred over standard deviation because of its propertises, especially when the distribution is not normal. It can still be designated as a model used to measure risk in the portfolio optimization (Miller and Ruszczynski, 2008), taking into consideration that the relevance for investors is to minimize the risks and maximize returns for their portfolios. It is a general measure of risk and can be used in other risk management practices (Xue and Titterington, 2011). The linear formulation takes advantage of a less computational effort (unlike quadratic formulation) and more applicability in practical terms (Moon and Yao, 2011). The authors formulated it as follows:

$$MAD_C = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt} - \bar{r}_j)x_j\right| \quad (14)$$

The mathematical formulation of the portfolio optimization problem posed by this model suggested by Konno and Yamazaki (1991) can be summarized by the following expressions:

$$Minimize\ MAD_C = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt} - r_j)x_j\right|$$

subject to a minimum expected return is given as:

$$\sum_{i=1}^{N} x_i\,\overline{r_i} \geq r_C$$

total investment in the portfolio is given as:

$$\sum_{i=1}^{N} x_i = 1$$

and to ensure there are no negative investments is given as:

$x_i \geq 0\ \forall i.$

Filtered historical simulation (FHS)

This method is quite credible and acceptable among scholars and researchers. Some articles have addressed this method and it is use in the estimation of portfolio risk, but yet, unknown articles have used the FHS in portfolio optimization, and this is one of the important contributions of this study. Thus, through a simple clear language all steps for implementing the FHS method can be shown (Giannopoulos and Tunaru, 2005). The FHS is one of the methods of Value-at-Risk (VaR) that combines the traditional method Historical Simulation (HS) with volatility models (Garch or EGARCH). The algorithm to implementation in determining the level of risk and portfolio optimization requires some steps:

(1) Application of the historical simulation method.
(2) Estimation of volatilities of returns series of the portfolios through the GARCH (1.1) model.
(3) Estimation of residual returns standardized, obtained by dividing the residual value of returns by the respective variance.
(4) Application bootstrapping method where each standardized return period t randomly multiplies the variance of the period t + 1; and finally
(5) Estimates the VaR through the percentile of returns, considering a certain confidence interval, significance level, and period of portfolio tenure.

Historical simulation (HS)

The application of VaR method is quite simple and requires some steps:

(1) The estimation of periodic returns of the assets that makes up the initial portfolio
(2) Periodic portfolios, adding the products of periodicals returns of each asset at its initial weight is estimated to be 1 / N, where N is the total number of assets.
(3) Considering a certain significance level and period detention portfolios, estimated VaR, which is given by the expression:

$$VaR\_HS = -Percentil\left\{\left\{\sum_{i=1}^{N} x_i\ r_i\right\}^m, \alpha\%\right\} \quad (15)$$

Where, r_i is the periodic return of the asset i and *m* refers to the observation period (m only illustrates the period that corresponds to summation, which does not have any mathematical effect on the formula) and α corresponds to the specified significance level.

The GARCH volatility model

It is assumed that the GARCH (1.1) model is to estimate periodic variances of portfolios. However, nothing ensures the possibility of the historical returns of the assets assuming a normal distribution or t-student. Considering the simple GARCH model, standardized residual returns are estimated by the expression:

$$z_{t+1} = \frac{R_{t+1}}{\sigma_{t+1}} \quad (16)$$

Where the variance is given as:

$$\sigma_{t+1}^2 = \omega + \varphi {R_t}^2 + \beta \sigma_t^2 \tag{17}$$

and, ω, φ and β are model parameters whose estimation can be by maximizing the sum of the function Maximum Likelihood Estimation (MLE) which is given by the expression similar to that of Aldrich (1997):

$$MLE_{t+1} = LN\left(\frac{1}{\sqrt{2\pi\sigma_{t+1}^2)}} * \exp\left(-0{,}5 * \frac{R_{t+1}^2}{\sigma_{t+1}^2}\right)\right) \tag{18}$$

Where R_{t+1} is the residual value of the return; ${R_t}^2$ is the residual value squared and σ_t^2 is the unconditional variance in period t.

Bootstrapping method

This method, given a certain period of detention portfolios from observations of standardized residual returns, randomly generates return for period t to be multiplied by the variance in period t + 1. Random returns of portfolios will be estimated with the FHS VaR, which can be given by the expression:

$$VaR\_FHS = -Percentil\{\{random\ returns\}^m, \alpha\%\} \tag{19}$$

The use of this method in portfolio optimization requires some care because the process is a little different from other methods, although apparently it has an almost similar mathematical formulation. There are two (2) objectives function to consider:

$$Minimizar\ VaR\_HS = -Percentil\left\{\left\{\sum_{i=1}^{N} x_i\ r_i\right\}^m, \alpha\%\right\}$$

$$VaR\_FHS = -Percentil\{\{random\ returns\}^m, \alpha\%\}$$

subject to a minimum expected return is given by:

$$\sum_{i=1}^{N} x_i\, \overline{r_i} \geq r_C$$

total investment in the portfolio is given by:

$$\sum_{i=1}^{N} x_i = 1$$

and to ensure there are no negative investment is given by:

$$x_i \geq 0\ \forall i$$

Performance measures and contribution of portfolio diversification strategies

***Sharpe ratio* (SR)**

The SR index of a particular investment strategy is measured by the ratio between the risk premiums or excess return, and risk of strategy i as the expression (Sharpe, 1994):

$$IS_i = \frac{\hat{\mu}_i}{\sigma_i} \tag{20}$$

In that, μ_i corresponds to the risk premium (risk-free rate asset[2]) and σ_i is the risk of strategy i. This indicator shows how much the investor receives the strategy i defined for each unit of risk associated with the strategy i. The higher value for this measure indicate higher quality of the investment in the strategy i. Assuming a normal distribution, to determine whether SR, S, and PT between the strategies are statistically significant, we propose two-sample t-test according with the Matlab code in the appendices to test the null hypotheses.

Sortino ratio (S)

Just as Sharpe ratio, the Sortino ratio is also an important statistical indicator used to measure the investment portfolio performance. Dr. Frank Sortino proposed it in the 80s. However, it is different from Sharpe Ratio because it uses the standard deviation of negative returns; while Sharpe Ratio uses the standard deviation of positive and negative returns. This is one of the reasons appointed as insufficient of MV model. The Sortino ratio is a modification of the Sharpe ratio, and can be expressed by:

$$S = \frac{\hat{\mu}_i}{Downside\ risk^1} \tag{21}$$

Contribution measures of portfolio diversification

To measure the contribution of African capital markets assets in global portfolio, we propose three measures according to the study of Liang and McIntosh (1999):

[2] We propose as the benchmark risk-free rate asset, the average weekly interest rate of Treasury bills to monthly of USA bills during the data observation period.

Overall benefit (OB_i)

This indicator measures the general benefit of the investment diversification effect, that is, it measures benefits in reducing risk and return. It is given by the following expression:

$$OB_i = (R_i - R_f) - (\rho\sigma_i/\sigma_m)(R_m - R_f) \tag{22}$$

Diversification benefit (DB_i)

This indicator measures only the benefits of investment diversification in the risk reduction. It is given as follows:

$$DB_i = (R_m - R_f)(1 - \rho\sigma_i/\sigma_m) \tag{23}$$

Return benefit (RB_i)

This indicator measures only the benefits of investment diversification in the return. It is represented by the expression:

$$RB_i = R_i - R_m \tag{24}$$

Where, R_m= existing portfolio return m; σ_m= volatility of portfolio m; R_i= Return i proposed investment; σ_i = Volatility i proposed investment; ρ = correlation coefficient between portfolio m and investment i; and R_f = risk-free rate.

RESULTS AND DISCUSSION

In- sample analysis

In analyzing Table 4, strong positive and negative correlation between African and World capital markets can be found. However, African capital markets in general shows significant positive correlation with world capital markets.

The study results show that there are African capital markets with the tendencies to follow the behavior of World capital markets but also we can find African capital market with behaviors that are contrary to the global markets, such as Rep. Democratic Gongo, Cote D´ivoire, Nigerian, Morocco, Tunisia and Mauritius. The most important African capital markets like South African, Namibia, Egypt, Kenya, Botswana, Uganda and Zambia presents significant and positive correlation with World capital markets between periods of data analysis from 5th August, 2004 to 7July, 2016 as seen in the test p-value results correlation shown in Table 5 considering the significance level of 1.

For in-sample analysis, Tables 5 to 10 shows the results for each strategy based on trade-off risk and return, where we can find global portfolio optimization before and after diversification with their respective performances based on optimization models. The results shows that the global portfolio diversification with African assets contributes in reducing the risk and maximizing the return.

As Figures 1 and 2 shows, we can see different efficient frontiers for each of the optimization models used in this study that represents the two (2) investment strategies. To all optimization models, the strategy of the global portfolio diversification with assets of the African capital market show higher return than global portfolio without diversification as illustrated in Table 11.

On the other hand, on the same table for MV model, the global portfolio diversification with assets of African capital market increase the risk level but for SV, RM, MAD and FHS models, it does not increase. In other words, for these models, global portfolio diversification with assets of African capital markets reduces the risk level. However, the diversification strategy of global portfolio with assets of African capital markets presents better performance than global portfolio without diversification, according to the results of the Sharpe Ratio and Sortino Ratio performance in Figures 3 to 5 where the African capital markets in the diversification global portfolio was observed. These results are statistically significant for all models included in the study, since it rejects all null hypotheses according to the results on Table 12.

Therefore, the investment strategy 2 shows better performance than strategy 1. The real contribution of the diversification of global portfolio with assets of African capital markets is illustrated in Table 13. For all models, this strategy generates benefit in diversification and return benefit as shown in Table 13.

We can see weak contribution of the World capital markets in the diversification of global portfolio, being outweighed by large contributions of the African capital markets. According Tables 14 to 19, the African capital markets with great performance in the composition of the Europe portfolio diversifying with higher weights are; Cote D´ivoire, Republic Democratic Gongo, Zambia and Tunisia. Already with less weight, we found the following markets; Botswana, Mauritius, Egypt, Uganda, Nigerian, Egypt, Namibia and South Africa. According to MV, RM, SV and MAD models, the study data analyses shows that in general, the African capital markets are efficient in the global portfolio composition as we can see their weights in the first portfolios.

In summary, the study in-sample analysis of the database in the period considered allows the realization that the diversification of global portfolio with assets of African markets contributes in reducing risk and maximize the return of the portfolio, where investor prefer high level of risk at the expense of a high return.

Table 4. Correlation coefficients between African and global capital markets.

Country	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa
Germany	0.39	-0.17	0.74	0.64	0.41	0.28	0.47	0.58	0.74	0.33	0.86	0.73	0.82
UK	0.67	0.44	-0.04	-0.17	0.58	0.18	-0.2	0.5	0.2	0.81	0.54	0.38	0.51
France	0.61	0.71	-0.33	-0.37	0.73	0.37	-0.3	0.56	0.06	0.86	0.41	0.28	0.28
Italy	0.42	0.82	-0.69	-0.7	0.56	0.19	-0.59	0.29	-0.33	0.75	0.03	-0.08	-0.11
Spain	0.63	0.75	-0.41	-0.33	0.8	0.62	-0.13	0.63	0.13	0.79	0/27	0.32	0/24
Austrian	0.65	0.8	-0.52	-0.54	0.71	0.46	-0.32	0.49	-0.04	0.85	0.2	0.2	0.17
SWISS	0.31	-0.27	0.74	0.62	0.29	0.08	0.38	0.45	0.63	0.29	0.84	0.62	0.76
Belgium	0.53	0.7	-0.35	-0.44	0.68	0.24	-0.39	0.47	-0.04	0.84	0.38	0.19	0.21
Denmark	0.08	-0.3	0.75	0.57	0.21	-0.02	0.34	0.35	0.53	0.12	0.76	0.48	0.6
Finland	0.62	0.29	0.25	0.14	0.68	0.39	0.12	0.68	0.48	0.71	0.75	0.58	0.67
Ireland	0.26	0.66	-0.47	-0.62	0.41	-0.1	-0.66	0.19	-0.36	0.68	0.16	-0.12	-0.09
Israel	0.3	-0.43	0.86	0.78	0.27	0.31	0.73	0.5	0.84	0.07	0.75	0.69	0.82
Netherlands	0.62	0.64	-0.21	-0.31	0.72	0.33	-0.25	0.56	0.13	0.85	0.49	0.35	0.37
Norway	0.86	0.26	0.25	0.19	067	0.58	0.31	0.69	0.64	0.72	0.66	0.73	0.83
Portugal	0.58	0.83	-0.62	-0.55	0.66	0.52	-0.3	0.5	-0.08	0.74	0.02	0.12	0.04
Sweden	0.55	-0.23	0.68	0.57	0.32	0.2	0.44	0.55	0.7	0.4	0.79	0.64	0.88
Czech Republic	0.75	0.74	-0.42	-0.41	0.77	0.7	-0.05	0.6	0.18	0.78	0.2	0.35	0.29
Greece	0.38	0.9	-0.79	-0.75	0.57	0.3	-0.57	0.27	-0.36	0.69	-0.11	-0.1	-0.22
Hungary	0.69	0.62	-0.4	-0.42	0.68	0.55	-0.06	0.58	0.12	0.7	0.17	0.21	0.28
Poland	0.82	0.69	-0.32	-0.32	0.75	0.65	-0.05	0.66	0.24	0.85	0.32	0.43	0.41
Qatar	0.06	-0.13	0.42	0.35	0.31	-0.09	0.05	0.02	0.38	0.18	0.63	0.47	0.42
Russian	0.9	0.47	-0.09	-0.14	0.65	0.73	0.23	0.67	0.44	0.71	0.32	0.52	0.59
Turkey	0.78	-0.12	0.41	0.41	0.43	0.48	0.59	0.62	0.69	0.41	0.55	0.61	0.85
UAE	-0.1	0.22	-0.01	-0.05	0.39	-0.2	-0.37	-0.07	-0.02	0.32	0.45	0.17	0.03
Brasil	0.73	0.06	0.16	0.26	0.48	0.83	0.69	0.59	067	0.27	0.2	0.57	0.63
Chile	0.6	-0.44	0.63	0.61	0.09	0.47	0.85	0.45	0.8	0.01	0.34	0.57	0.81
Peru	0.72	-0.18	0.54	0.49	0.25	0.61	0.76	0.62	0.8	0.19	0.37	0.67	0.83
Mexico	0.62	-0.27	0.76	0.67	0.35	0.47	0.73	0.63	0.88	0.31	0.74	0.8	0.95
Canada	0.83	0.01	0.47	0.43	058	0.61	0.56	0.68	0.82	0.52	0.69	0.81	0.95
EUA	0.05	-0.3	0.67	0.47	0.12	-0.21	0.16	0.22	04	0.14	0.72	0.38	0.53
Australia	0.85	0.09	0.39	0.33	0.57	0.54	0.44	0.72	0.73	0.6	0.66	0.73	0.9
Hong Kong	0.6	-0.09	0.61	0.55	0.55	0.5	0.61	0.73	0.8	0.4	0.77	0.76	0.89
Japan	0.28	-0.05	0.41	0.23	0.33	-0.09	0.03	0.34	0.29	0.42	0.72	0.35	0.54
Newzealand	0.33	0.14	0.22	0.05	0.31	-0.13	-0.2	0.31	0.2	0.51	0.58	0.29	0.44
Singapore	0.7	-0.21	0.69	0.62	0.4	0.49	0.7	0.68	0.87	0.36	0.72	0.79	0.97
China	0.41	0.1	0.42	0.33	0.53	0.56	0.51	0.8	0.6	0.27	0.52	0.58	0.59
India	0.57	-0.15	0.63	0.57	0.55	0.54	0.7	0.72	0.82	0.31	0.71	0.72	0.86
Indonesia	0.57	-0.15	0.63	0.57	0.55	0.54	0.7	0.72	0.82	0.31	0.71	0.72	0.86
South Korea	0.7	-0.14	0.62	0.5	0.45	0.4	0.54	0.67	0.78	0.41	0.73	0.73	0.95
Malasya	0.46	-0.48	0.85	0.81	0.11	0.3	0.75	0.45	0.88	0.07	0.63	0.72	0.89
Philipine	0.12	-0.56	0.93	0.78	-0.02	-0.03	0.57	0.29	0.67	-0.07	0.68	0.54	0.71
Taiwan	0.6	-0.28	0.67	0.61	0.37	0.34	0.58	0.54	0.8	0.29	0.72	0.69	0.92

The correlation level between returns of African and European capital markets considering the significance level was presented. We recall that the returns was measured in dollar. We can find strong positive correlation between African capital market and European capital markets. However, in general , some African capital markets such as Nigerian, Democratic Republic Gongo, Mauritius, Tunisia and Cote D´ivoire show inverse correlation with global markets, particularly with European capital markets but they show strong and positive correlation with some American and Asia-pacific markets. The study results show that African capital markets have tendencies to follow the behavior of the global markets, in the same direction and opposite, as seen in negative values of correlation. The main African capital markets, such as South Africa, Namibia, Egypt, Morocco, Tunisia and Kenya presents high and positive correlation with European capital markets between periods of data analysis.

Table 5. P-value test of the correlation coefficients between African and global capital markets.

Country	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa
Germany	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UK	0.000	0.000	0.271	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
France	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.161	0.000	0.000	0.000	0.000
Italy	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.498	0.049	0.006
Spain	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000
Austrian	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.366	0.000	0.000	0.000	0.000
SWISS	0.000	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Belgium	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.355	0.000	0.000	0.000	0.000
Denmark	0.039	0.000	0.000	0.000	0.000	0.664	0.000	0.000	0.000	0.003	0.000	0.000	0.000
Finland	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000
Ireland	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.000	0.002	0.025
Israel	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.086	0.000	0.000	0.000
Netherlands	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
Norway	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Portugal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.046	0.000	0.664	0.004	0.302
Sweden	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Czech Republic	0.000	0.000	0.000	0.000	0.000	0.000	0.207	0.000	0.000	0.000	0.000	0.000	0.000
Greece	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.017	0.000
Hungary	0.000	0.000	0.000	0.000	0.000	0.000	0.113	0.000	0.002	0.000	0.000	0.000	0.000
Poland	0.000	0.000	0.000	0.000	0.000	0.000	0.257	0.000	0.000	0.000	0.000	0.000	0.000
Qatar	0.113	0.001	0.000	0.000	0.000	0.032	0.174	0.658	0.000	0.000	0.000	0.000	0.000
Russian	0.000	0.000	0.022	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Turkey	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UAE	0.009	0.000	0.883	0.253	0.000	0.000	0.000	0.092	0.588	0.000	0.000	0.000	0.431
Brasil	0.000	0.149	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Chile	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.750	0.000	0.000	0.000
Peru	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mexico	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Canada	0.000	0.845	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EUA	0.180	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Australia	0.000	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Hong Kong	0.000	0.032	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Japan	0.000	0.209	0.000	0.000	0.000	0.023	0.498	0.000	0.000	0.000	0.000	0.000	0.000
Newzealand	0.000	0.001	0.000	0.184	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Singapore	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
China	0.000	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
India	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Indonesia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
South Korea	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Malasya	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.090	0.000	0.000	0.000
Philipine	0.003	0.000	0.000	0.000	0.670	0.481	0.000	0.000	0.000	0.093	0.000	0.000	0.000
Taiwan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The result of p-value test of the correlation coefficients between the returns of the capital markets, considering significance level of 1 was illustrated. In general, there are significant correlation between markets in the world. The efficient frontiers of the investment strategies for each optimization model was presented. The Global Market portfolio without diversification considered as a strategy 1 and the Global Market portfolio diversified with African asset as strategy 2 was also seen. The following tables provide risk and return of 50 portfolios for each strategy and the performance measure through Sharpe Ratio (SR) and Sortino Ratio (S) for each optimization model and for each investment strategy was shown. The risk-free rate used in this study correspond with the monthly US treasury bills with a weekly rate of 0.0675%. In general, we can see in the next five (5) tables that diversification strategy of global investment portfolios with African assets show better performance than global investment portfolio for all optimization models.

Table 6. Efficient portfolios based in the Mean Variance model (MV).

Mean variance model										
Portfolio	Risk global market (%)	Return global market (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)	Risk global market + Africa (%)	Return global market + Africa (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)
MVP	1.718	0.083	0.911	1.169	1.338	1.062	0.122	5.109	0.685	7.921
P2	1.718	0.087	1.148	1.168	1.689	1.063	0.128	5.672	0.681	8.855
P3	1.720	0.091	1.385	1.168	2.038	1.067	0.134	6.222	0.681	9.742
P4	1.722	0.095	1.620	1.169	2.386	1.072	0.140	6.756	0.683	10.605
P5	1.726	0.099	1.853	1.171	2.732	1.079	0.146	7.272	0.686	11.438
P6	1.730	0.104	2.085	1.174	3.074	1.089	0.152	7.760	0.691	12.226
P7	1.736	0.108	2.314	1.177	3.413	1.103	0.158	8.215	0.699	12.967
P8	1.742	0.112	2.541	1.180	3.749	1.119	0.164	8.633	0.708	13.652
P9	1.748	0.116	2.765	1.185	4.080	1.140	0.170	9.007	0.720	14.271
P10	1.756	0.120	2.985	1.191	4.402	1.165	0.176	9.335	0.734	14.825
P11	1.765	0.124	3.202	1.198	4.718	1.193	0.182	9.620	0.750	15.314
P12	1.775	0.128	3.414	1.205	5.028	1.225	0.188	9.865	0.768	15.733
P13	1.786	0.132	3.621	1.213	5.330	1.261	0.194	10.062	0.790	16.068
P14	1.798	0.136	3.824	1.223	5.625	1.302	0.200	10.214	0.814	16.340
P15	1.811	0.140	4.022	1.232	5.913	1.346	0.207	10.327	0.840	16.554
P16	1.826	0.144	4.214	1.242	6.194	1.394	0.213	10.405	0.867	16.723
P17	1.841	0.149	4.402	1.253	6.468	1.446	0.219	10.450	0.897	16.845
P18	1.857	0.153	4.583	1.264	6.736	1.502	0.225	10.465	0.930	16.893
P19	1.874	0.157	4.760	1.275	6.997	1.561	0.231	10.454	0.966	16.904
P20	1.892	0.161	4.929	1.286	7.251	1.625	0.237	10.420	1.003	16.873
P21	1.912	0.165	5.093	1.299	7.497	1.691	0.243	10.369	1.043	16.815
P22	1.932	0.169	5.251	1.312	7.734	1.760	0.249	10.304	1.084	16.736
P23	1.954	0.173	5.402	1.325	7.964	1.833	0.255	10.229	1.127	16.639
P24	1.976	0.177	5.548	1.339	8.185	1.907	0.261	10.146	1.170	16.532
P25	1.999	0.181	5.687	1.354	8.398	1.984	0.267	10.059	1.215	16.418
P26	2.024	0.185	5.821	1.369	8.604	2.064	0.273	9.963	1.263	16.279
P27	2.049	0.189	5.949	1.385	8.801	2.148	0.279	9.855	1.312	16.128
P28	2.075	0.193	6.071	1.400	8.995	2.235	0.285	9.739	1.363	15.970
P29	2.102	0.198	6.187	1.413	9.206	2.326	0.291	9.619	1.416	15.807
P30	2.131	0.202	6.295	1.426	9.407	2.420	0.297	9.495	1.469	15.644
P31	2.161	0.206	6.397	1.440	9.598	2.517	0.303	9.372	1.522	15.496
P32	2.192	0.210	6.491	1.455	9.780	2.623	0.309	9.224	1.555	15.558

Table 6. Contd.

P33	2.225	0.214	6.579	1.470	9.958	2.743	0.315	9.041	1.594	15.561
P34	2.261	0.218	6.657	1.486	10.128	2.875	0.322	8.837	1.638	15.512
P35	2.298	0.222	6.726	1.503	10.284	3.017	0.328	8.620	1.687	15.421
P36	2.338	0.226	6.785	1.522	10.424	3.169	0.334	8.399	1.740	15.296
P37	2.381	0.230	6.836	1.543	10.547	3.334	0.340	8.166	1.782	15.276
P38	2.426	0.234	6.877	1.566	10.655	3.517	0.346	7.912	1.832	15.188
P39	2.474	0.238	6.910	1.590	10.751	3.717	0.352	7.649	1.884	15.090
P40	2.524	0.242	6.935	1.616	10.832	3.936	0.358	7.378	1.941	14.956
P41	2.576	0.247	6.952	1.643	10.899	4.170	0.364	7.108	2.008	14.759
P42	2.630	0.251	6.964	1.672	10.954	4.418	0.370	6.846	2.084	14.514
P43	2.687	0.255	6.969	1.703	10.998	4.678	0.376	6.595	2.167	14.235
P44	2.745	0.259	6.969	1.735	11.031	4.947	0.382	6.359	2.258	13.934
P45	2.806	0.263	6.965	1.768	11.055	5.224	0.388	6.138	2.354	13.620
P46	2.868	0.267	6.957	1.802	11.069	5.508	0.394	5.931	2.456	13.302
P47	2.931	0.271	6.946	1.839	11.068	5.798	0.400	5.739	2.562	12.985
P48	2.998	0.275	6.928	1.882	11.036	6.093	0.406	5.561	2.673	12.673
P49	3.079	0.279	6.878	1.930	10.971	6.392	0.412	5.395	2.788	12.370
P50	3.293	0.283	6.556	2.052	10.522	6.695	0.418	5.241	2.905	12.077

Source: Author.

Table 7. Efficient portfolios based in the resample Michaud model (RM).

	Resample michaud model									
Portfolio	Risk global market (%)	Return global market (%)	sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)	Risk global market + Africa (%)	Return global market + Africa (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)
MVP	1.773	0.107	2.238	0.926	4.288	1.005	0.155	8.661	0.547	15.930
P2	1.774	0.113	2.593	0.925	4.975	1.006	0.160	9.152	0.547	16.841
P3	1.776	0.120	2.945	0.924	5.658	1.008	0.165	9.628	0.548	17.729
P4	1.779	0.126	3.294	0.925	6.335	1.012	0.170	10.088	0.549	18.595
P5	1.783	0.132	3.640	0.927	7.004	1.017	0.175	10.532	0.551	19.440
P6	1.789	0.139	3.980	0.928	7.675	1.023	0.180	10.955	0.553	20.268
P7	1.796	0.145	4.315	0.930	8.337	1.033	0.185	11.342	0.556	21.064
P8	1.805	0.151	4.645	0.932	8.990	1.045	0.190	11.683	0.561	21.775
P9	1.814	0.158	4.969	0.936	9.632	1.061	0.195	11.982	0.567	22.405
P10	1.824	0.164	5.286	0.940	10.262	1.080	0.200	12.239	0.576	22.950
P11	1.835	0.170	5.598	0.944	10.879	1.101	0.205	12.456	0.586	23.424
P12	1.847	0.177	5.902	0.950	11.482	1.125	0.210	12.641	0.596	23.848

Table 7. Contd.

P13	1.861	0.183	6.199	0.956	12.071	1.150	0.215	12.797	0.608	24.208
P14	1.874	0.189	6.489	0.962	12.643	1.177	0.220	12.926	0.621	24.513
P15	1.889	0.195	6.772	0.969	13.200	1.206	0.225	13.033	0.635	24.768
P16	1.905	0.202	7.047	0.977	13.740	1.236	0.230	13.119	0.649	24.975
P17	1.922	0.208	7.314	0.985	14.264	1.268	0.235	13.187	0.665	25.140
P18	1.939	0.214	7.573	0.994	14.772	1.301	0.240	13.238	0.682	25.265
P19	1.957	0.221	7.825	1.004	15.256	1.335	0.245	13.276	0.699	25.357
P20	1.977	0.227	8.067	1.015	15.715	1.370	0.250	13.300	0.717	25.432
P21	1.998	0.233	8.298	1.026	16.160	1.407	0.255	13.312	0.735	25.483
P22	2.020	0.240	8.519	1.038	16.579	1.445	0.260	13.303	0.754	25.509
P23	2.044	0.246	8.729	1.051	16.972	1.487	0.265	13.270	0.775	25.461
P24	2.069	0.252	8.928	1.065	17.339	1.531	0.270	13.211	0.798	25.349
P25	2.095	0.258	9.117	1.080	17.683	1.579	0.275	13.130	0.823	25.181
P26	2.122	0.265	9.296	1.095	18.013	1.629	0.280	13.029	0.849	24.999
P27	2.151	0.271	9.467	1.111	18.322	1.684	0.285	12.901	0.877	24.766
P28	2.180	0.277	9.628	1.128	18.601	1.743	0.290	12.753	0.908	24.482
P29	2.211	0.284	9.778	1.147	18.845	1.805	0.295	12.591	0.941	24.165
P30	2.244	0.290	9.917	1.167	19.063	1.871	0.300	12.418	0.975	23.818
P31	2.278	0.296	10.045	1.189	19.243	1.939	0.305	12.240	1.012	23.451
P32	2.314	0.303	10.161	1.213	19.390	2.010	0.310	12.059	1.050	23.073
P33	2.352	0.309	10.266	1.237	19.511	2.082	0.315	11.878	1.090	22.690
P34	2.391	0.315	10.361	1.263	19.609	2.160	0.320	11.682	1.133	22.273
P35	2.432	0.322	10.446	1.290	19.686	2.245	0.325	11.465	1.180	21.803
P36	2.474	0.328	10.521	1.319	19.745	2.335	0.330	11.235	1.232	21.300
P37	2.518	0.334	10.588	1.348	19.787	2.432	0.335	10.995	1.286	20.795
P38	2.564	0.340	10.647	1.378	19.814	2.534	0.340	10.751	1.343	20.286
P39	2.610	0.347	10.698	1.407	19.843	2.640	0.345	10.506	1.402	19.783
P40	2.660	0.353	10.736	1.438	19.863	2.752	0.350	10.262	1.465	19.283
P41	2.712	0.359	10.761	1.470	19.859	2.870	0.355	10.016	1.530	18.781
P42	2.767	0.366	10.774	1.503	19.833	2.992	0.360	9.773	1.599	18.285
P43	2.825	0.372	10.777	1.539	19.788	3.121	0.365	9.530	1.670	17.809
P44	2.886	0.378	10.770	1.575	19.737	3.270	0.370	9.248	1.758	17.206
P45	2.949	0.385	10.753	1.612	19.669	3.458	0.375	8.892	1.872	16.427
P46	3.014	0.391	10.728	1.651	19.586	3.677	0.380	8.497	2.007	15.569
P47	3.082	0.397	10.696	1.692	19.490	3.927	0.385	8.084	2.157	14.720
P48	3.153	0.403	10.657	1.733	19.385	4.235	0.390	7.615	2.336	13.805
P49	3.225	0.410	10.614	1.776	19.271	4.598	0.395	7.123	2.551	12.840
P50	3.300	0.416	10.565	1.822	19.134	5.066	0.400	6.564	2.825	11.768

Source: Author.

Table 8. Efficient portfolios based in the SEMIVARIANCE model (SV).

	Semi variance model									
Portfolio	Risk global market (%)	Return global market (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)	Risk global market + Africa (%)	Return global market + Africa (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)
MVP	1.146	0.089	1.917	1.146	1.917	0.647	0.132	9.930	0.647	9.930
P2	1.146	0.093	2.261	1.146	2.261	0.647	0.138	10.822	0.647	10.822
P3	1.147	0.097	2.605	1.147	2.605	0.650	0.143	11.672	0.650	11.672
P4	1.149	0.101	2.945	1.149	2.945	0.656	0.149	12.469	0.656	12.469
P5	1.151	0.105	3.283	1.151	3.283	0.663	0.155	13.224	0.663	13.224
P6	1.154	0.109	3.618	1.154	3.618	0.671	0.161	13.933	0.671	13.933
P7	1.158	0.113	3.946	1.158	3.946	0.681	0.167	14.591	0.681	14.591
P8	1.164	0.117	4.268	1.164	4.268	0.692	0.173	15.194	0.692	15.194
P9	1.169	0.121	4.585	1.169	4.585	0.705	0.179	15.736	0.705	15.736
P10	1.176	0.125	4.896	1.176	4.896	0.721	0.184	16.211	0.721	16.211
P11	1.183	0.129	5.200	1.183	5.200	0.739	0.190	16.606	0.739	16.606
P12	1.191	0.133	5.498	1.191	5.498	0.760	0.196	16.927	0.760	16.927
P13	1.200	0.137	5.789	1.200	5.789	0.782	0.202	17.185	0.782	17.185
P14	1.209	0.141	6.072	1.209	6.072	0.807	0.208	17.387	0.807	17.387
P15	1.218	0.145	6.349	1.218	6.349	0.833	0.214	17.539	0.833	17.539
P16	1.228	0.149	6.619	1.228	6.619	0.862	0.219	17.639	0.862	17.639
P17	1.239	0.153	6.882	1.239	6.882	0.892	0.225	17.693	0.892	17.693
P18	1.250	0.157	7.139	1.250	7.139	0.924	0.231	17.709	0.924	17.709
P19	1.261	0.161	7.388	1.261	7.388	0.958	0.237	17.692	0.958	17.692
P20	1.273	0.165	7.630	1.273	7.630	0.994	0.243	17.648	0.994	17.648
P21	1.286	0.169	7.865	1.286	7.865	1.031	0.249	17.582	1.031	17.582
P22	1.298	0.173	8.092	1.298	8.092	1.069	0.255	17.500	1.069	17.500
P23	1.312	0.177	8.312	1.312	8.312	1.108	0.260	17.405	1.108	17.405
P24	1.325	0.180	8.525	1.325	8.525	1.149	0.266	17.302	1.149	17.302
P25	1.339	0.184	8.731	1.339	8.731	1.190	0.272	17.191	1.190	17.191
P26	1.354	0.188	8.929	1.354	8.929	1.233	0.278	17.067	1.233	17.067
P27	1.369	0.192	9.120	1.369	9.120	1.278	0.284	16.930	1.278	16.930
P28	1.384	0.196	9.305	1.384	9.305	1.324	0.290	16.786	1.324	16.786
P29	1.400	0.200	9.482	1.400	9.482	1.371	0.296	16.637	1.371	16.637
P30	1.416	0.204	9.653	1.416	9.653	1.419	0.301	16.484	1.419	16.484
P31	1.433	0.208	9.818	1.433	9.818	1.468	0.307	16.331	1.468	16.331
P32	1.450	0.212	9.976	1.450	9.976	1.518	0.313	16.178	1.518	16.178
P33	1.467	0.216	10.126	1.467	10.126	1.569	0.319	16.027	1.569	16.027
P34	1.485	0.220	10.269	1.485	10.269	1.620	0.325	15.878	1.620	15.878

Table 8. Contd.

P35	1.504	0.224	10.406	1.504	10.406	1.673	0.331	15.731	1.673	15.731
P36	1.523	0.228	10.535	1.523	10.535	1.726	0.336	15.589	1.726	15.589
P37	1.544	0.232	10.651	1.544	10.651	1.779	0.342	15.449	1.779	15.449
P38	1.566	0.236	10.752	1.566	10.752	1.835	0.348	15.297	1.835	15.297
P39	1.590	0.240	10.840	1.590	10.840	1.894	0.354	15.130	1.894	15.130
P40	1.615	0.244	10.916	1.615	10.916	1.958	0.360	14.933	1.958	14.933
P41	1.642	0.248	10.980	1.642	10.980	2.029	0.366	14.700	2.029	14.700
P42	1.670	0.252	11.032	1.670	11.032	2.106	0.372	14.443	2.106	14.443
P43	1.699	0.256	11.072	1.699	11.072	2.188	0.377	14.166	2.188	14.166
P44	1.731	0.260	11.096	1.731	11.096	2.277	0.383	13.872	2.277	13.872
P45	1.765	0.264	11.107	1.765	11.107	2.371	0.389	13.567	2.371	13.567
P46	1.802	0.268	11.100	1.802	11.100	2.470	0.395	13.259	2.470	13.259
P47	1.842	0.271	11.076	1.842	11.076	2.573	0.401	12.953	2.573	12.953
P48	1.884	0.275	11.037	1.884	11.037	2.681	0.407	12.653	2.681	12.653
P49	1.933	0.279	10.964	1.933	10.964	2.792	0.413	12.360	2.792	12.360
P50	2.052	0.283	10.522	2.052	10.522	2.905	0.418	12.077	2.905	12.077

Source: Author.

Table 9. Efficient portfolios based on mean absolute deviation (MAD).

	Mean absolute deviation model									
Portfolio	Risk global market (%)	Return global market (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)	Risk global market + Africa (%)	Return global market + Africa (%)	Sharpe ratio (%)	Downside deviation (%)	Sortino ratio (%)
MVP	1.232	0.091	1.931	0.771	3.085	0.427	0.143	17.669	0.175	43.037
P2	1.233	0.095	2.248	0.772	3.591	0.429	0.149	18.910	0.172	47.226
P3	1.233	0.099	2.565	0.772	4.099	0.439	0.154	19.750	0.179	48.503
P4	1.234	0.103	2.881	0.773	4.597	0.458	0.160	20.150	0.190	48.608
P5	1.235	0.107	3.195	0.776	5.089	0.482	0.165	20.307	0.205	47.755
P6	1.237	0.111	3.507	0.776	5.589	0.509	0.171	20.340	0.222	46.626
P7	1.240	0.115	3.817	0.778	6.083	0.537	0.177	20.309	0.240	45.409
P8	1.243	0.119	4.123	0.781	6.560	0.567	0.182	20.245	0.260	44.070
P9	1.247	0.123	4.424	0.785	7.028	0.597	0.188	20.162	0.281	42.883
P10	1.251	0.127	4.721	0.787	7.506	0.628	0.194	20.083	0.301	41.887
P11	1.256	0.130	5.014	0.789	7.981	0.658	0.199	20.003	0.321	41.072
P12	1.263	0.134	5.298	0.794	8.425	0.689	0.205	19.911	0.341	40.296
P13	1.271	0.138	5.571	0.799	8.865	0.721	0.210	19.813	0.361	39.609
P14	1.281	0.142	5.834	0.804	9.293	0.753	0.216	19.719	0.380	39.088

Table 9. Cont'd.

P15	1.292	0.146	6.091	0.810	9.708	0.785	0.222	19.631	0.400	38.555
P16	1.302	0.150	6.342	0.816	10.126	0.817	0.227	19.546	0.422	37.886
P17	1.314	0.154	6.584	0.823	10.512	0.850	0.233	19.465	0.442	37.426
P18	1.326	0.158	6.817	0.831	10.886	0.882	0.238	19.389	0.462	36.987
P19	1.340	0.162	7.042	0.838	11.256	0.914	0.244	19.317	0.483	36.558
P20	1.354	0.166	7.258	0.846	11.618	0.947	0.250	19.248	0.504	36.182
P21	1.368	0.170	7.467	0.854	11.964	0.979	0.255	19.182	0.523	35.900
P22	1.384	0.174	7.668	0.862	12.314	1.012	0.261	19.116	0.544	35.565
P23	1.400	0.178	7.859	0.871	12.636	1.045	0.267	19.054	0.565	35.256
P24	1.417	0.181	8.043	0.881	12.940	1.078	0.272	18.995	0.585	34.980
P25	1.434	0.185	8.219	0.889	13.252	1.111	0.278	18.938	0.606	34.713
P26	1.452	0.189	8.385	0.898	13.562	1.144	0.283	18.884	0.626	34.478
P27	1.472	0.193	8.539	0.908	13.848	1.177	0.289	18.833	0.647	34.252
P28	1.493	0.197	8.685	0.918	14.123	1.210	0.295	18.779	0.666	34.123
P29	1.513	0.201	8.825	0.929	14.380	1.244	0.300	18.710	0.687	33.916
P30	1.535	0.205	8.956	0.940	14.629	1.279	0.306	18.645	0.707	33.721
P31	1.558	0.209	9.077	0.950	14.884	1.314	0.312	18.582	0.727	33.569
P32	1.582	0.213	9.184	0.962	15.097	1.349	0.317	18.512	0.748	33.402
P33	1.608	0.217	9.279	0.974	15.327	1.384	0.323	18.445	0.768	33.244
P34	1.635	0.221	9.367	0.984	15.557	1.420	0.328	18.378	0.787	33.140
P35	1.663	0.225	9.445	0.997	15.758	1.456	0.334	18.310	0.808	32.996
P36	1.692	0.228	9.513	1.012	15.909	1.492	0.340	18.242	0.828	32.863
P37	1.723	0.232	9.569	1.025	16.095	1.528	0.345	18.177	0.849	32.742
P38	1.756	0.236	9.616	1.040	16.227	1.565	0.351	18.110	0.868	32.644
P39	1.790	0.240	9.652	1.058	16.331	1.602	0.357	18.043	0.888	32.550
P40	1.824	0.244	9.684	1.076	16.425	1.640	0.362	17.970	0.909	32.409
P41	1.861	0.248	9.706	1.096	16.471	1.679	0.368	17.887	0.930	32.298
P42	1.899	0.252	9.717	1.116	16.537	1.718	0.373	17.803	0.950	32.201
P43	1.939	0.256	9.719	1.135	16.603	1.759	0.379	17.712	0.972	32.055
P44	1.981	0.260	9.710	1.156	16.635	1.800	0.385	17.624	0.995	31.881
P45	2.024	0.264	9.695	1.178	16.657	1.842	0.390	17.525	1.014	31.845
P46	2.068	0.268	9.678	1.202	16.652	1.888	0.396	17.395	1.034	31.760
P47	2.115	0.272	9.650	1.227	16.630	1.939	0.402	17.231	1.057	31.617
P48	2.164	0.276	9.611	1.254	16.587	1.994	0.407	17.036	1.080	31.457
P49	2.241	0.279	9.459	1.295	16.365	2.062	0.413	16.747	1.120	30.827
P50	2.422	0.283	8.914	1.409	15.325	2.141	0.418	16.391	1.164	30.159

Source: Author.

Table 10. Efficient portfolios based on filtered historical simulation (FHS).

Filtered historical simulation model												
Portfolio	Risk Europe HS (%)	Risk Europe FHS (%)	Return Europe (%)	Sharpe ratio (%)	Downside deviation FHS (%)	Sortino ratio (%)	Risk Europe + Africa HS (%)	Risk Europe + Africa FHS (%)	Return Europe+ Africa (%)	Sharpe ratio (%)	Downside deviation FHS (%)	Sortino ratio (%)
MVP	4.099	9.436	0.088	0.220	4.264	0.486	3.466	6.607	0.091	0.357	3.707	0.636
P2	4.111	8.637	0.088	0.232	4.281	0.468	3.514	7.728	0.101	0.439	3.890	0.872
P3	4.125	9.724	0.091	0.238	4.289	0.541	3.564	7.277	0.093	0.354	3.894	0.662
P4	4.137	7.905	0.086	0.229	4.307	0.420	3.576	7.487	0.090	0.305	3.895	0.586
P5	4.145	8.279	0.094	0.319	4.308	0.613	3.610	6.991	0.098	0.442	3.900	0.792
P6	4.148	8.476	0.089	0.248	4.311	0.489	3.614	7.763	0.091	0.306	3.901	0.608
P7	4.157	8.906	0.085	0.196	4.316	0.405	3.615	7.268	0.090	0.306	3.905	0.570
P8	4.159	8.562	0.088	0.240	4.319	0.476	3.619	8.081	0.097	0.359	3.912	0.742
P9	4.166	7.370	0.080	0.166	4.329	0.282	3.621	6.884	0.093	0.375	3.913	0.660
P10	4.167	9.377	0.087	0.211	4.332	0.456	3.624	7.194	0.091	0.321	3.918	0.589
P11	4.170	8.435	0.086	0.217	4.333	0.422	3.625	6.916	0.092	0.348	3.919	0.613
P12	4.172	8.519	0.087	0.231	4.334	0.455	3.626	5.524	0.090	0.411	3.923	0.579
P13	4.173	8.736	0.088	0.238	4.335	0.480	3.627	7.217	0.091	0.325	3.931	0.596
P14	4.175	8.208	0.082	0.180	4.345	0.340	3.630	8.093	0.100	0.402	3.932	0.827
P15	4.177	8.325	0.087	0.233	4.351	0.446	3.631	7.271	0.092	0.336	3.933	0.620
P16	4.180	8.283	0.082	0.176	4.357	0.334	3.632	7.173	0.091	0.322	3.936	0.586
P17	4.189	8.215	0.088	0.253	4.367	0.477	3.632	7.387	0.092	0.331	3.937	0.621
P18	4.189	8.754	0.089	0.248	4.367	0.498	3.633	7.368	0.093	0.351	3.938	0.656
P19	4.191	8.245	0.086	0.218	4.368	0.412	3.633	7.973	0.091	0.300	3.940	0.606
P20	4.195	7.559	0.079	0.152	4.369	0.263	3.634	7.212	0.091	0.332	3.940	0.607
P21	4.196	8.355	0.085	0.210	4.371	0.401	3.635	6.957	0.087	0.274	3.944	0.483
P22	4.196	8.285	0.091	0.280	4.373	0.530	3.637	6.470	0.081	0.214	3.944	0.351
P23	4.197	7.159	0.081	0.191	4.376	0.312	3.639	7.154	0.093	0.351	3.949	0.635
P24	4.201	8.992	0.083	0.172	4.377	0.353	3.640	5.215	0.084	0.310	3.959	0.408
P25	4.201	8.330	0.086	0.219	4.378	0.416	3.641	7.236	0.091	0.331	3.962	0.604
P26	4.202	8.265	0.086	0.220	4.381	0.416	3.645	7.023	0.089	0.311	3.963	0.551
P27	4.204	7.433	0.080	0.168	4.383	0.285	3.652	5.316	0.092	0.459	3.964	0.616
P28	4.205	8.361	0.086	0.217	4.385	0.414	3.652	9.100	0.097	0.329	3.964	0.755
P29	4.210	7.300	0.085	0.241	4.386	0.401	3.654	6.957	0.089	0.305	3.967	0.536
P30	4.211	7.586	0.082	0.192	4.387	0.331	3.655	7.283	0.092	0.336	3.969	0.617
P31	4.213	9.128	0.089	0.241	4.388	0.501	3.658	6.547	0.092	0.380	3.973	0.626
P32	4.213	6.684	0.081	0.202	4.388	0.307	3.660	7.406	0.090	0.307	3.982	0.571
P33	4.216	8.309	0.086	0.221	4.391	0.419	3.662	7.242	0.090	0.315	3.983	0.573
P34	4.223	7.437	0.083	0.212	4.393	0.358	3.663	7.589	0.092	0.323	3.984	0.615

Table 10. Contd.

P35	4.223	7.864	0.083	0.196	4.396	0.350	3.665	7.541	0.095	0.368	3.988	0.696
P36	4.225	7.904	0.085	0.221	4.399	0.397	3.665	6.576	0.083	0.232	3.988	0.383
P37	4.226	8.024	0.087	0.244	4.412	0.444	3.667	5.228	0.089	0.406	3.991	0.532
P38	4.228	8.114	0.088	0.248	4.419	0.456	3.672	7.254	0.090	0.307	3.992	0.558
P39	4.229	7.388	0.082	0.193	4.421	0.323	3.675	7.526	0.093	0.332	3.994	0.627
P40	4.231	8.492	0.085	0.200	4.424	0.384	3.677	5.873	0.087	0.324	3.996	0.476
P41	4.238	8.291	0.081	0.167	4.431	0.313	3.678	5.505	0.087	0.356	4.000	0.489
P42	4.245	7.821	0.081	0.173	4.433	0.305	3.678	6.477	0.087	0.302	4.004	0.488
P43	4.246	7.776	0.085	0.222	4.442	0.388	3.689	7.345	0.090	0.306	4.010	0.561
P44	4.250	8.333	0.086	0.217	4.447	0.406	3.690	6.257	0.089	0.336	4.015	0.524
P45	4.279	7.128	0.080	0.174	4.451	0.279	3.691	6.496	0.082	0.219	4.017	0.353
P46	4.284	10.122	0.093	0.249	4.455	0.567	3.695	7.335	0.090	0.311	4.039	0.565
P47	4.286	8.857	0.087	0.216	4.459	0.429	3.706	7.522	0.085	0.231	4.052	0.429
P48	4.296	7.098	0.085	0.247	4.462	0.393	3.708	7.215	0.086	0.254	4.071	0.451
P49	4.299	7.049	0.086	0.263	4.477	0.414	3.709	7.301	0.087	0.263	4.080	0.471
P50	4.344	8.518	0.078	0.127	4.580	0.236	3.770	5.893	0.087	0.331	4.093	0.477

Source: Author.

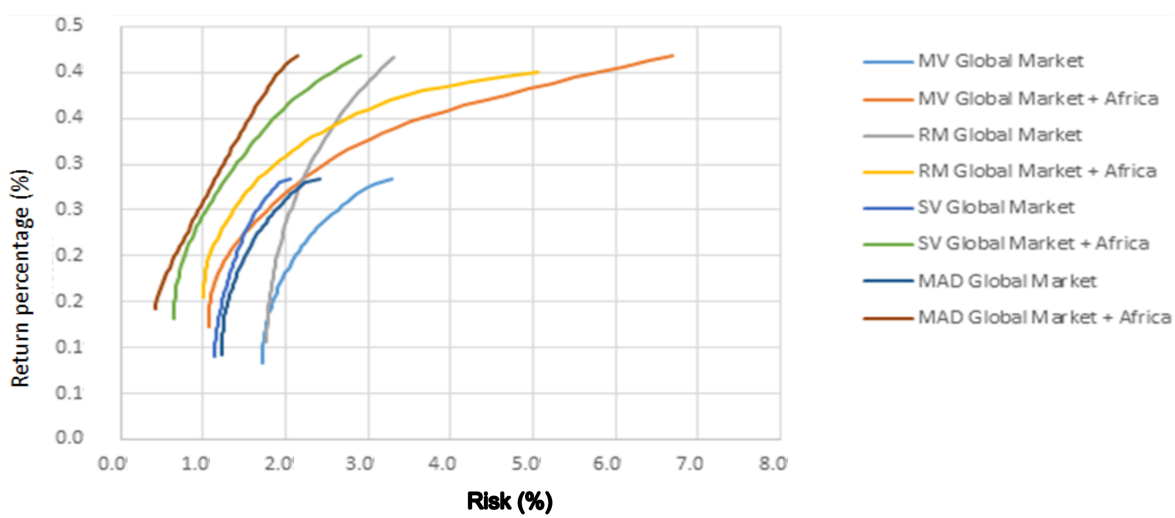

Figure 1. Efficient frontiers of the investment strategies. This figure shows us the efficient frontiers of the investment strategies for each optimization model form period 05/08/2004 to 07/07/2016 based in the criteria Risk and Return. Thus, we have the following models: Mean Variance (MV), Resample Michaud (RM), SemiVariance, Mean Absolute and Deviation (MAD)).

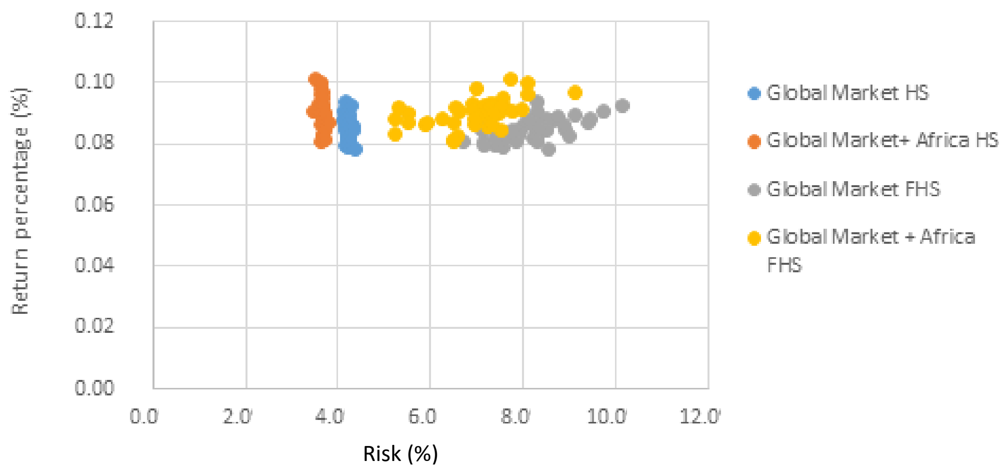

Figure 2. Efficient portfolios of the investment strategies using FHS model. This figure shows us the contribution to literature, application of the FHS methodology combine with the Historical Simulation method (HS) and Garch volatility model. We represent efficient portfolios through the point graphics instead of line because it shows better presentation due to convex properties of the model).

Table 11. The average performance of investment strategies.

Model	Mean variance		Resample Michaud		Semi variance		Mean absolute devition		Filtered historical simulation	
	Global market	Global market + Africa	Global market	Global market + Africa	Global market	Global market + Africa	Global market	Global market + Africa	Global market	Global market + Africa
Risk	2.152	2.611	2.251	1.982	1.410	1.378	1.548	1.159	8.207	7.005
Return	0.183	0.270	0.262	0.277	0.186	0.275	0.19	0.281	0.085	0.090
Sharpe ratio	5.083	8.431	8.199	11.291	8.024	15.39	7.36	18.785	0.216	0.329
Downside risk	1.420	1.390	1.188	1.059	1.410	1.38	0.94	0.620	4.381	3.960
Sortino ratio	7.730	14.631	15.540	21.302	8.024	15.39	12.15	36.645	0.406	0.581

The mean contribution in terms of risk and return of the investment strategies and their performances through Sharpe Ratio and Sortino Ratio was illustrated. With MV model, the diversification of the global investment portfolios with African assets is riskier than global investment portfolios but presents better return and performance. With RM, SV, MAD and FHS models, the diversification of global investment portfolios with African assets is seen to be more efficient than global investment portfolio. To all optimization models, the diversification strategy of the global investment portfolios with African assets is seen to have better performances than strategy not diversified.

However, the study results show that Tables 5 to 10 for all models even for investors that prefer Minimum Portfolio Variance (MPV), the diversification of global portfolio with African assets, reduce risk and maximize return. Even if the diversification of global portfolio with African assets increased risk, the benefits in return compensate for the increased risk.

Out-of-sample analysis

In this analysis, we also analyzed the contribution of the African capital market in the global portfolio

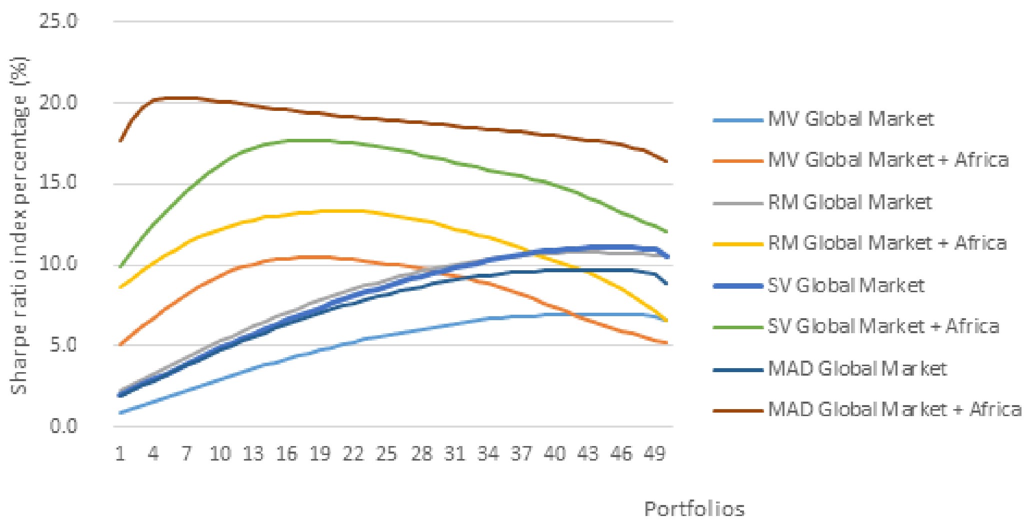

Figure 3. Portfolio performances of the investment strategies for each optimization models. This figure shows the portfolio performances of the investment strategies for each optimization model measured by Sharpe Ratio. However, investment strategy with higher value of Sharpe Ratio show better performance).

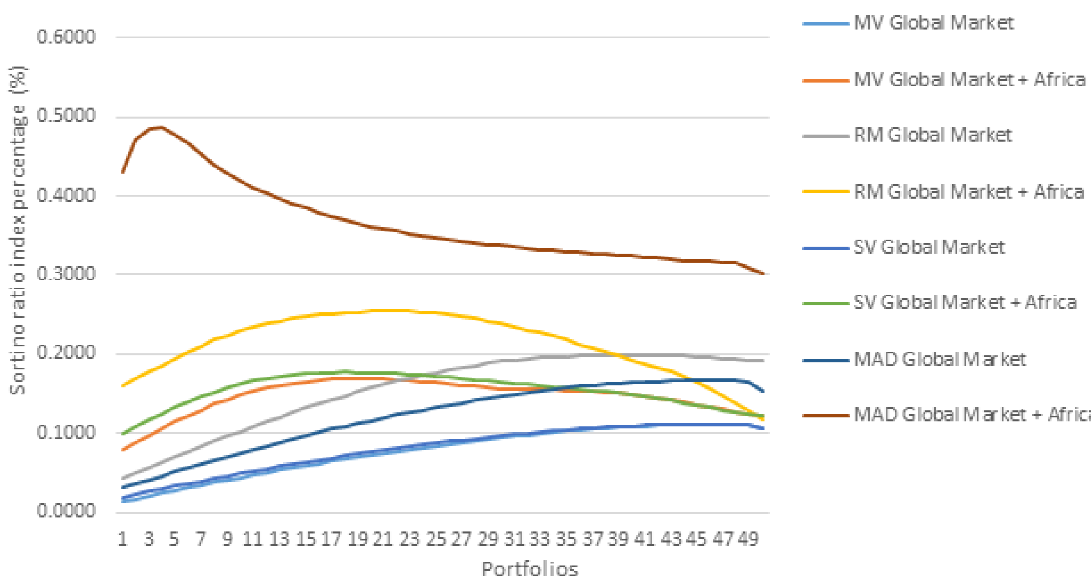

Figure 4. Portfolio performances of the investment strategies for each optimization models (This figure shows us the portfolio performances of the investment strategies for each optimization model measured by Sortino Ratio. However, investment strategy with higher value of Sharpe Ratio show better performance).

diversification using out-of-sample analysis. Furthermore, the objective of this methodology is to analyze the portfolio performance measured by ER, risk (R), SR and S over the period where it is applied by the rolling sample approach. The study out-of-sample results show that the strategy of diversification of global portfolio with assets of African market present better performance measured by ER, Risk, SR and Sortino Ratio than global portfolio according to the models as shown in Table 20 and Figures 6 to 9. To test statistically, the study investment performances was measured by SR *and* Sortino ratio, Table 21 provided the test results. For all optimization models, MV, RM, SV, MAD and FHS shows the rejection of the null hypotheses. The result shows high positive

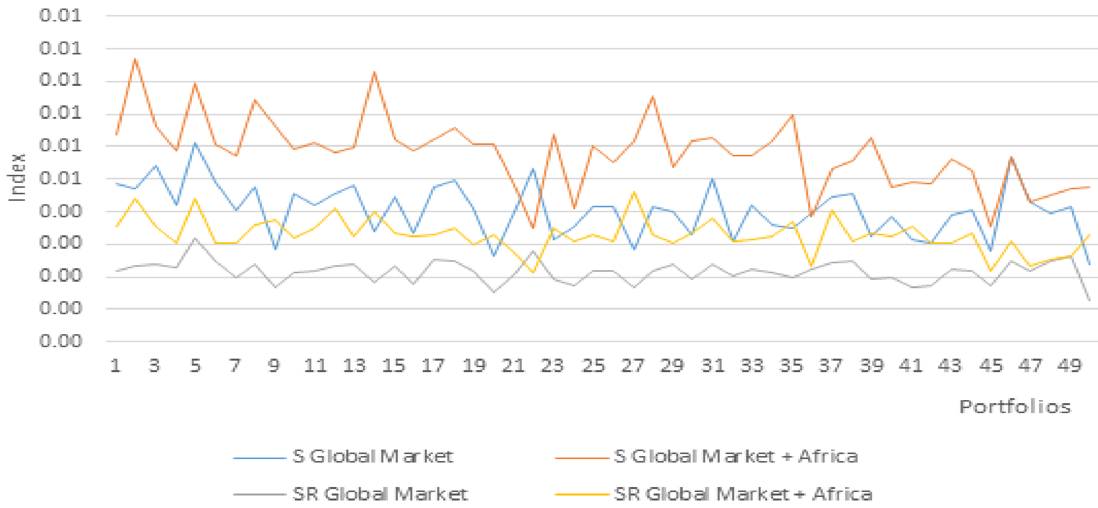

Figure 5. Portfolio performances using FHS model. This figure shows Sharpe Ratio (SR) and Sortino Ratio (S) like a portfolio performance through the FHS model. They were represented separately because the application of this model is the study contribution to the literature. Individual result were other models).

Table 12. Statistical test results of performances investment strategies.

Model	Test result of sharpe ratio portfolio performances			
	Null hypotheses	**t-statistic**	**P-value**	**Reject or No reject**
Mean variance (MV)	SR2-SR1=0	9.12	0.000	Reject
Resample Michaud (RM)	SR2-SR1=0	6.69	0.000	Reject
Semi variance (SV)	SR2-SR1=0	14.75	0.000	Reject
Mean absolute deviation (MAD)	SR2-SR1=0	30.52	0.000	Reject
Filtered historical simulation (FHS)	SR2-SR1=0	12.72	0.000	Reject
Models	**Null hypotheses**	**t-statistic**	**P-value**	**Reject or No reject**
Mean variance (MV)	S2-S1=0	13.00	0.0000	Reject
Resample Michaud (RM)	S2-S1=0	6.57	0.0000	Reject
Semi variance (SV)	S2-S1=0	14.75	0.0000	Reject
Mean absolute deviation (MAD)	S2-S1=0	25.60	0.0000	Reject
Filtered historical simulation (FHS)	S2-S1=0	9.08	0.0000	Reject

The statistical test result of performance investment strategies was presented, where SR2 corresponding to the *strategy 2* performance and SR1 is *strategy 1* performance. Thus, 1 was considered to have significance level. As seen, all null hypotheses have been rejected, this means that the higher performance of strategy 2 over strategy1 is statistically significant because high value of the t-statistic and p-value is lesser than 0.01.

Table 13. Contribution of global portfolio diversification with African capital market assets.

Contribuition measure	MV	RM	SV	MAD	FHS
Overall benefit	0.163	0.164	0.176	0.189	0.019
Diversification benefit	0.076	0.148	0.087	0.096	0.014
Return benefit	0.087	0.016	0.089	0.093	0.005

The real contribution of the Europe portfolio diversification with African capital market assets, based on equation 22, 23 and 24 considering all optimization models used in this study was presented. To all optimization models, the diversification of the global investment portfolios with African assets generates benefits in the returns and diversification that correspond with the overall benefits.

Table 14. Global portfolio weights diversified with African capital markets by mean variance model.

Portfolio	Namibia	Nigeria	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa	Global markets	Total portfolio weight
MPV	0	1	26	2	0	0	23	13	10	3	1	3	0	18	100
P2	0	1	26	3	0	0	23	12	10	2	2	4	0	18	100
P3	0	1	26	3	0	0	24	12	10	1	2	4	0	17	100
P4	0	0	26	3	0	0	24	12	10	0	3	4	0	17	100
P5	0	0	26	3	0	0	24	11	10	0	3	5	0	17	100
P6	0	0	26	4	0	0	25	10	10	0	3	6	0	17	100
P7	0	0	27	4	0	0	25	9	9	0	3	6	0	17	100
P8	0	0	27	5	0	0	25	7	9	0	3	7	0	18	100
P9	0	0	27	5	0	0	26	5	8	0	3	8	0	19	100
P10	0	0	27	6	0	0	26	3	7	0	2	8	0	20	100
P11	0	0	27	7	0	0	26	1	6	0	2	9	0	22	100
P12	0	0	27	7	0	0	26	0	5	0	2	10	0	23	100
P13	0	0	27	8	0	0	25	0	3	0	2	11	0	24	100
P14	0	0	27	9	0	0	24	0	2	0	2	11	0	25	100
P15	0	0	27	10	0	0	23	0	0	0	2	12	0	26	100
P16	0	0	26	10	0	0	22	0	0	0	1	13	0	27	100
P17	0	0	26	11	0	0	20	0	0	0	1	14	0	28	100
P18	0	0	25	12	0	0	18	0	0	0	1	14	0	29	100
P19	0	0	25	13	0	0	16	0	0	0	0	15	0	31	100
P20	0	0	24	14	0	0	14	0	0	0	0	16	0	33	100
P21	0	0	23	15	0	0	11	0	0	0	0	16	0	35	100
P22	0	0	23	16	0	0	8	0	0	0	0	17	0	37	100
P23	0	0	22	17	0	0	6	0	0	0	0	17	0	39	100
P24	0	0	21	18	0	0	3	0	0	0	0	18	0	41	100
P25	0	0	20	19	0	0	0	0	0	0	0	18	0	43	100
P26	0	0	17	20	0	0	0	0	0	0	0	18	0	45	100
P27	0	0	13	21	0	0	0	0	0	0	0	19	0	47	100
P28	0	0	10	23	0	0	0	0	0	0	0	19	0	49	100
P29	0	0	7	24	0	0	0	0	0	0	0	19	0	51	100
P30	0	0	3	25	0	0	0	0	0	0	0	19	0	53	100
P31	0	0	0	26	0	0	0	0	0	0	0	19	0	54	100
P32	0	0	0	29	0	0	0	0	0	0	0	15	0	56	100
P33	0	0	0	31	0	0	0	0	0	0	0	12	0	57	100
P34	0	0	0	34	0	0	0	0	0	0	0	8	0	58	100

Table 14. Contd.

P35	0	0	0	36	0	0	0	0	0	0	0	4	0	60	100
P36	0	0	0	39	0	0	0	0	0	0	0	0	0	61	100
P37	0	0	0	43	0	0	0	0	0	0	0	0	0	57	100
P38	0	0	0	46	0	0	0	0	0	0	0	0	0	54	100
P39	0	0	0	51	0	0	0	0	0	0	0	0	0	49	100
P40	0	0	0	55	0	0	0	0	0	0	0	0	0	45	100
P41	0	0	0	60	0	0	0	0	0	0	0	0	0	40	100
P42	0	0	0	64	0	0	0	0	0	0	0	0	0	36	100
P43	0	0	0	69	0	0	0	0	0	0	0	0	0	31	100
P44	0	0	0	73	0	0	0	0	0	0	0	0	0	27	100
P45	0	0	0	78	0	0	0	0	0	0	0	0	0	22	100
P46	0	0	0	82	0	0	0	0	0	0	0	0	0	18	100
P47	0	0	0	87	0	0	0	0	0	0	0	0	0	13	100
P48	0	0	0	91	0	0	0	0	0	0	0	0	0	9	100
P49	0	0	0	96	0	0	0	0	0	0	0	0	0	4	100
P50	0	0	0	100	0	0	0	0	0	0	0	0	0	0	100

Table 15. Global portfolio weights diversified with African capital markets by resample Michaud model.

Portfolio	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa	Global markets	Total Port folio weight
MPV	0	2	26	1	0	0	24	12	10	0	4	4	0	17	100
P2	0	1	25	2	0	0	24	13	10	0	5	4	0	17	100
P3	0	1	24	2	0	0	25	13	10	0	5	4	0	16	100
P4	0	1	24	2	0	0	25	13	10	0	5	4	0	16	100
P5	0	0	23	2	0	0	25	13	11	0	5	4	0	16	100
P6	0	0	22	2	0	0	26	13	11	0	5	4	0	16	100
P7	0	0	20	3	0	0	27	12	11	0	6	5	0	17	100
P8	0	0	18	3	0	0	27	11	10	0	6	5	0	19	100
P9	0	0	16	3	0	1	28	10	10	0	6	6	0	19	100
P10	0	0	15	4	0	1	28	9	10	0	7	6	0	20	100
P11	0	0	13	4	0	2	28	8	10	0	7	6	0	21	100
P12	0	0	12	4	0	2	29	7	10	0	7	7	0	22	100
P13	0	0	10	4	0	3	29	6	9	0	8	7	0	23	100
P14	0	0	9	5	0	4	30	5	9	0	8	8	0	23	100
P15	0	0	8	5	0	4	30	4	9	0	8	8	0	24	100
P16	0	0	7	5	1	5	30	3	9	0	8	8	0	24	100

Table 15. Contd.

P17	0	0	5	5	1	5	31	2	8	0	8	9	0	25	100
P18	0	0	4	5	1	6	31	1	8	0	9	9	0	26	100
P19	0	0	3	6	1	6	31	0	8	0	9	9	0	26	100
P20	0	0	1	6	2	7	32	0	7	0	9	10	0	26	100
P21	0	0	0	6	2	7	32	0	7	0	9	10	0	27	100
P22	0	0	0	6	3	8	31	0	6	0	10	10	0	26	100
P23	0	0	0	7	4	9	30	0	4	0	10	11	0	26	100
P24	0	0	0	7	4	10	28	0	2	0	10	11	0	27	100
P25	0	0	0	7	5	11	27	0	1	0	10	11	0	27	100
P26	0	0	0	8	6	12	24	0	0	0	11	11	0	28	100
P27	0	0	0	8	7	13	21	0	0	0	11	11	0	30	100
P28	0	0	0	9	8	14	17	0	0	0	11	11	0	31	100
P29	0	0	0	9	9	14	14	0	0	0	11	11	0	32	100
P30	0	0	0	9	10	15	10	0	0	0	11	11	0	34	100
P31	0	0	0	10	10	16	7	0	0	0	11	11	0	36	100
P32	0	0	0	10	11	17	3	0	0	0	11	10	0	37	100
P33	0	0	0	11	12	17	0	0	0	0	11	10	0	39	100
P34	0	0	0	11	13	17	0	0	0	0	10	9	0	40	100
P35	0	0	0	12	15	16	0	0	0	0	10	7	0	41	100
P36	0	0	0	12	16	15	0	0	0	0	9	6	0	42	100
P37	0	0	0	13	17	14	0	0	0	0	8	4	0	44	100
P38	0	0	0	13	18	13	0	0	0	0	7	2	0	46	100
P39	0	0	0	14	20	12	0	0	0	0	6	1	0	48	100
P40	0	0	0	15	21	10	0	0	0	0	4	0	0	50	100
P41	0	0	0	15	22	8	0	0	0	0	2	0	0	53	100
P42	0	0	0	16	23	5	0	0	0	0	0	0	0	55	100
P43	0	0	0	17	24	1	0	0	0	0	0	0	0	58	100
P44	0	0	0	17	24	0	0	0	0	0	0	0	0	59	100
P45	0	0	0	17	23	0	0	0	0	0	0	0	0	60	100
P46	0	0	0	17	22	0	0	0	0	0	0	0	0	61	100
P47	0	0	0	16	18	0	0	0	0	0	0	0	0	65	100
P48	0	0	0	14	9	0	0	0	0	0	0	0	0	77	100
P49	0	0	0	12	0	0	0	0	0	0	0	0	0	88	100
P50	0	0	0	0	0	0	0	0	0	0	0	0	0	100	100

Table 16. Global portfolio weights diversified with African capital markets by semi variance model.

Portfolio	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa	Global Markets	Total Port folio weight
MPV	0	0	36	2	0	0	26	10	10	0	0	1	0	14	100
P2	0	0	37	3	0	0	26	9	10	0	0	2	0	13	100
P3	0	0	37	4	0	0	26	8	10	0	0	3	0	12	100
P4	0	0	37	4	0	0	26	7	10	0	0	3	0	12	100
P5	0	0	38	5	0	0	26	6	10	0	0	4	0	12	100
P6	0	0	38	6	0	0	26	5	10	0	0	4	0	11	100
P7	0	0	38	6	0	0	27	4	9	0	0	5	0	11	100
P8	0	0	39	7	0	0	27	2	9	0	0	5	0	12	100
P9	0	0	39	8	0	0	27	0	8	0	0	6	0	13	100
P10	0	0	39	9	0	0	26	0	7	0	0	6	0	13	100
P11	0	0	39	10	0	0	25	0	5	0	0	7	0	14	100
P12	0	0	39	11	0	0	24	0	4	0	0	7	0	16	100
P13	0	0	38	12	0	0	23	0	2	0	0	7	0	17	100
P14	0	0	38	13	0	0	22	0	1	0	0	8	0	18	100
P15	0	0	38	15	0	0	20	0	0	0	0	8	0	19	100
P16	0	0	38	16	0	0	18	0	0	0	0	8	0	20	100
P17	0	0	37	17	0	0	16	0	0	0	0	8	0	21	100
P18	0	0	36	19	0	0	14	0	0	0	0	8	0	22	100
P19	0	0	36	20	0	0	12	0	0	0	0	9	0	24	100
P20	0	0	35	21	0	0	10	0	0	0	0	9	0	25	100
P21	0	0	34	23	0	0	8	0	0	0	0	9	0	27	100
P22	0	0	33	24	0	0	6	0	0	0	0	9	0	28	100
P23	0	0	33	25	0	0	3	0	0	0	0	9	0	29	100
P24	0	0	32	27	0	0	1	0	0	0	0	9	0	31	100
P25	0	0	31	28	0	0	0	0	0	0	0	9	0	32	100
P26	0	0	28	30	0	0	0	0	0	0	0	9	0	33	100
P27	0	0	25	31	0	0	0	0	0	0	0	9	0	35	100
P28	0	0	23	33	0	0	0	0	0	0	0	8	0	36	100
P29	0	0	20	35	0	0	0	0	0	0	0	8	0	37	100
P30	0	0	18	36	0	0	0	0	0	0	0	8	0	38	100
P31	0	0	15	38	0	0	0	0	0	0	0	8	0	39	100
P32	0	0	12	39	0	0	0	0	0	0	0	7	0	41	100
P33	0	0	10	41	0	0	0	0	0	0	0	7	0	42	100
P34	0	0	7	43	0	0	0	0	0	0	0	7	0	43	100
P35	0	0	5	44	0	0	0	0	0	0	0	7	0	44	100
P36	0	0	2	46	0	0	0	0	0	0	0	6	0	45	100

Table 16. Contd.

P37	0	0	0	48	0	0	0	0	0	0	0	6	0	46	100
P38	0	0	0	51	0	0	0	0	0	0	0	3	0	47	100
P39	0	0	0	54	0	0	0	0	0	0	0	0	0	46	100
P40	0	0	0	58	0	0	0	0	0	0	0	0	0	42	100
P41	0	0	0	61	0	0	0	0	0	0	0	0	0	39	100
P42	0	0	0	65	0	0	0	0	0	0	0	0	0	35	100
P43	0	0	0	70	0	0	0	0	0	0	0	0	0	30	100
P44	0	0	0	74	0	0	0	0	0	0	0	0	0	26	100
P45	0	0	0	78	0	0	0	0	0	0	0	0	0	22	100
P46	0	0	0	83	0	0	0	0	0	0	0	0	0	17	100
P47	0	0	0	87	0	0	0	0	0	0	0	0	0	13	100
P48	0	0	0	91	0	0	0	0	0	0	0	0	0	9	100
P49	0	0	0	96	0	0	0	0	0	0	0	0	0	4	100
P50	0	0	0	100	0	0	0	0	0	0	0	0	0	0	100

Table 17. Global portfolio weights diversified with African capital markets by mean absolute deviation model.

Portfolio	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa	Global markets	Total portfolio weight
MPV	0	1	92	0	0	0	2	0	2	0	1	0	0	3	100
P2	0	0	93	0	0	0	2	0	1	0	1	1	0	3	100
P3	0	0	92	1	0	0	1	0	0	0	0	1	0	3	100
P4	0	0	91	3	0	0	1	0	0	0	0	2	0	3	100
P5	0	0	90	5	0	0	0	0	0	0	0	2	0	3	100
P6	0	0	88	6	0	0	0	0	0	0	0	2	0	4	100
P7	0	0	85	8	0	0	0	0	0	0	0	2	0	4	100
P8	0	0	83	10	0	0	0	0	0	0	0	2	0	4	100
P9	0	0	80	12	0	0	0	0	0	0	0	3	0	5	100
P10	0	0	77	14	0	0	0	0	0	0	0	3	0	5	100
P11	0	0	75	16	0	0	0	0	0	0	0	3	0	5	100
P12	0	0	73	18	0	0	0	0	0	0	0	4	0	5	100
P13	0	0	70	19	0	0	0	0	0	0	0	4	0	6	100
P14	0	0	68	21	0	0	0	0	0	0	0	4	0	6	100
P15	0	0	66	23	0	0	0	0	0	0	0	4	0	6	100
P16	0	0	63	25	0	0	0	0	0	0	0	5	0	7	100
P17	0	0	61	27	0	0	0	0	0	0	0	5	0	7	100
P18	0	0	58	29	0	0	0	0	0	0	0	5	0	8	100

Table 17. Contd.

P19	0	0	56	30	0	0	0	0	0	0	0	5	0	9	100
P20	0	0	53	32	0	0	0	0	0	0	0	5	0	9	100
P21	0	0	51	34	0	0	0	0	0	0	0	5	0	9	100
P22	0	0	49	36	0	0	0	0	0	0	0	6	0	10	100
P23	0	0	46	38	0	0	0	0	0	0	0	6	0	10	100
P24	0	0	44	40	0	0	0	0	0	0	0	6	0	11	100
P25	0	0	41	41	0	0	0	0	0	0	0	6	0	11	100
P26	0	0	38	43	0	0	0	0	0	0	0	6	0	12	100
P27	0	0	36	45	0	0	0	0	0	0	0	7	0	12	100
P28	0	0	34	47	0	0	0	0	0	0	0	7	0	12	100
P29	0	0	32	49	0	0	0	0	0	0	0	7	0	13	100
P30	0	0	30	51	0	0	0	0	0	0	0	7	0	13	100
P31	0	0	28	53	0	0	0	0	0	0	0	7	0	13	100
P32	0	0	25	55	0	0	0	0	0	0	0	7	0	13	100
P33	0	0	23	56	0	0	0	0	0	0	0	7	0	14	100
P34	0	0	21	59	0	0	0	0	0	0	0	7	0	13	100
P35	0	0	18	60	0	0	0	0	0	0	0	7	0	14	100
P36	0	0	16	62	0	0	0	0	0	0	0	7	0	14	100
P37	0	0	14	64	0	0	0	0	0	0	0	7	0	14	100
P38	0	0	12	66	0	0	0	0	0	0	0	7	0	14	100
P39	0	0	10	68	0	0	0	0	0	0	0	7	0	15	100
P40	0	0	8	71	0	0	0	0	0	0	0	7	0	14	100
P41	0	0	6	73	0	0	0	0	0	0	0	7	0	14	100
P42	0	0	5	75	0	0	0	0	0	0	0	6	0	14	100
P43	0	0	3	77	0	0	0	0	0	0	0	6	0	14	100
P44	0	0	0	79	0	0	0	0	0	0	0	6	0	14	100
P45	0	0	0	82	0	0	0	0	0	0	0	4	0	13	100
P46	0	0	0	85	0	0	0	0	0	0	0	3	0	12	100
P47	0	0	0	88	0	0	0	0	0	0	0	1	0	10	100
P48	0	0	0	92	0	0	0	0	0	0	0	0	0	8	100
P49	0	0	0	96	0	0	0	0	0	0	0	0	0	4	100
P50	0	0	0	100	0	0	0	0	0	0	0	0	0	0	100

value of t-statistic and p-value lesser than 1. It means that the superiority of the investment performances of the strategy of diversification of global portfolio with assets of African capital markets is statically significant. Finally, such in-sample analysis shows the out-of-sample

Table 18. Global portfolio weights diversified with African capital markets by filtered historical simulation model.

Portfolio	Namibia	Nigerian	Gongo	Cote D´Ivoire	Egypt	Morrocco	Tunisia	Botswana	Mauritius	Kenya	Uganda	Zambia	South Africa	Global markets	Total portfolio weight
MPV	2	2	1	2	1	2	2	2	2	2	2	2	2	75	100
P2	2	1	2	3	2	2	2	2	2	2	2	2	2	75	100
P3	2	2	2	3	2	2	2	2	2	2	2	2	2	74	100
P4	2	2	2	2	2	2	2	2	3	2	2	2	2	75	100
P5	2	1	3	1	2	2	2	0	2	2	2	2	2	76	100
P6	2	2	2	4	2	2	1	2	2	2	1	2	2	75	100
P7	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P8	2	2	2	2	1	2	1	0	2	3	2	4	2	74	100
P9	1	2	1	2	2	4	2	2	2	2	2	2	2	75	100
P10	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P11	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P12	2	1	2	2	2	2	2	2	2	2	2	2	2	77	100
P13	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P14	2	1	2	2	2	2	1	2	2	2	2	2	0	78	100
P15	2	1	2	2	2	2	2	2	2	2	2	2	2	76	100
P16	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P17	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P18	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P19	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P20	2	2	2	2	2	2	2	2	2	2	2	2	2	77	100
P21	2	2	4	1	2	2	2	2	2	1	2	2	2	75	100
P22	1	3	2	2	1	2	2	4	1	2	2	1	1	73	100
P23	2	2	2	2	2	2	2	2	2	1	2	2	2	76	100
P24	2	2	1	2	2	5	3	2	1	2	2	2	1	73	100
P25	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P26	2	2	2	2	2	2	2	2	2	2	2	2	2	77	100
P27	2	1	2	2	2	2	2	2	2	2	2	2	2	76	100
P28	3	2	3	2	2	0	2	2	0	0	2	2	1	79	100
P29	2	1	2	2	2	2	2	2	2	2	2	2	2	77	100
P30	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P31	2	2	2	2	2	2	2	2	2	2	2	2	2	77	100
P32	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P33	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P34	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100

Table 18. Contd.

P35	2	1	2	2	2	2	2	2	2	2	2	2	2	76	100
P36	2	2	2	1	2	2	2	2	2	2	2	2	2	75	100
P37	2	2	2	2	2	2	2	2	2	2	2	2	2	77	100
P38	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P39	2	2	2	2	2	2	2	2	2	2	2	2	2	77	100
P40	3	2	2	3	2	3	1	3	3	1	2	2	2	71	100
P41	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P42	2	2	2	1	2	2	2	2	2	2	2	2	2	77	100
P43	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P44	2	2	2	3	2	2	2	2	1	2	2	2	2	75	100
P45	1	1	2	1	2	2	2	2	2	1	2	1	1	80	100
P46	2	1	2	2	2	2	2	2	2	2	2	2	2	77	100
P47	2	2	2	0	2	2	2	2	2	2	2	2	2	77	100
P48	2	2	2	2	2	2	2	2	2	2	2	2	2	76	100
P49	1	2	1	3	2	1	3	2	2	2	1	2	1	77	100
P50	1	1	2	2	1	2	4	2	2	1	1	2	2	77	100

Table 19. The average weights of the global market portfolio diversified.

Market	MV	RM	SV	MAD	FHS	Market	MV	RM	SV	MAD	FHS	Market	MV	RM	SV	MAD	FHS
Namibia	0.00	0.00	0.00	0.00	1.77	Spain	0.00	0.00	0.00	0.00	1.84	Russian	0.00	12.63	0.00	0.00	1.79
Nigerian	0.06	0.10	0.01	0.01	1.67	Austrian	0.00	0.00	0.00	0.00	1.72	Turkey	0.00	0.00	0.00	0.00	1.84
Gongo	13.67	5.71	21.58	42.09	1.95	Swiss	0.00	0.00	0.00	0.00	1.87	UAE	1.38	1.90	0.21	0.15	1.77
Cote D´Ivoire	30.50	8.27	35.32	43.73	2.03	Belgium	0.00	0.00	0.00	0.00	1.79	Brasil	0.00	0.00	0.00	0.00	1.84
Egypt	0.00	7.68	0.00	0.01	1.78	Denmark	0.00	1.37	0.00	0.00	1.86	Chile	0.00	0.00	0.00	0.00	1.78
Morrocco	0.00	6.38	0.00	0.00	1.94	Finland	0.00	0.00	0.00	0.00	1.83	Peru	0.51	2.32	0.25	0.00	1.82
Tunisia	9.71	16.06	9.30	0.11	1.94	Ireland	0.00	0.00	0.00	0.00	1.74	Mexico	0.00	0.00	0.00	0.00	1.87
Botswana	1.89	3.12	1.03	0.00	1.83	Israel	0.44	1.91	0.33	0.01	1.90	Canada	0.00	0.00	0.00	0.00	1.78
Mauritius	2.23	4.23	2.13	0.06	1.85	Netherlands	0.00	0.00	0.00	0.00	1.74	Australia	1.05	0.65	0.81	0.06	1.90
Kenya	0.12	0.00	0.00	0.01	1.74	Norway	0.00	0.00	0.00	0.00	1.74	Hong Kong	0.00	0.00	0.00	0.00	1.81
Uganda	0.77	6.50	0.00	0.10	1.87	Portugal	0.00	0.00	0.00	0.00	1.76	Japan	0.00	0.00	0.00	0.00	1.86
Zambia	8.34	5.94	5.10	4.65	1.89	Sweden	0.00	0.00	0.00	0.00	1.78	Newzealand	0.00	10.66	0.00	0.00	1.75
South Africa	0.00	0.00	0.00	0.00	1.74	Czech Republic	0.00	0.00	0.00	0.00	1.81	Singapore	0.06	0.01	0.50	0.00	1.90
Germany	0.00	0.00	0.00	0.00	1.84	Greece	0.00	0.00	0.00	0.00	1.65	China	25.03	0.00	19.95	6.51	1.87
UK	0.00	0.00	0.00	0.00	1.75	Hungary	0.00	0.51	0.00	0.00	1.73	India	0.00	0.08	0.00	0.00	1.82
France	0.00	0.00	0.00	0.00	1.77	Poland	0.00	0.00	0.00	0.00	1.74	Indonesia	0.00	2.59	0.00	0.00	1.91
Italy	0.00	0.00	0.00	0.00	1.74	Qatar	0.00	1.24	0.00	0.00	1.82	South Korea	0.00	0.00	0.00	0.00	1.85

Table 19. Contd.

Malasya	4.00	0.00	3.49	2.45	1.87	Philipine	0.00	0.00	0.00	0.00	1.93	Taiwan	0.00	0.00	0.00	0.00	1.77
EUA	0.25	0.15	0.00	0.04	1.86												

Source: Author.

Table 20. The average out-of-sample performance.

Model	Global market					Global market + Africa				
	Excess return	Risk	Sharpe ratio	Downside risk	Sortino ratio	Excess return	Risk	Sharpe ratio	Downside risk	Sortino ratio
MV	0.038	2.804	1.215	1.809	1.779	0.047	2.531	1.707	1.642	2.460
RM	0.028	2.606	1.079	1.388	2.002	0.036	2.363	1.523	1.269	2.796
SV	0.038	1.809	1.779	1.809	1.779	0.047	1.642	2.460	1.642	2.460
MAD	0.038	2.068	1.720	1.229	2.728	0.047	1.878	2.399	1.123	3.762
FHS	0.039	5.356	0.686	4.507	0.753	0.047	4.872	0.947	4.159	1.018

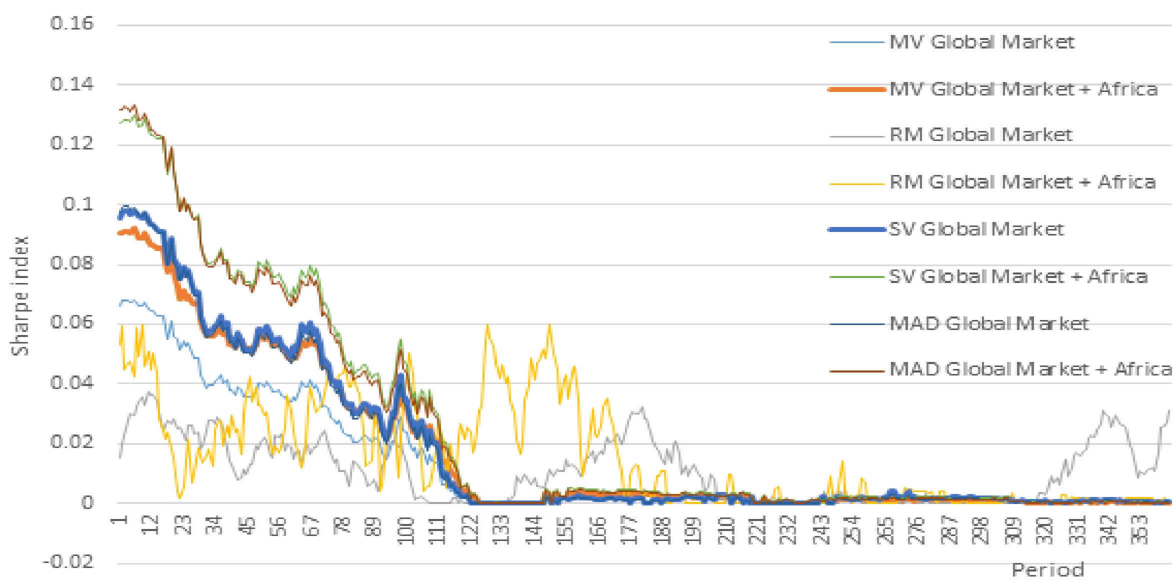

Figure 6. Out-of-sample sharpe ratio performance. This figure shows the out-of-sample portfolio performances of the investment strategies for each optimization model measured by Sharpe Ratio weekly. However, investment strategy with higher value of Sharpe Ratio, show better performance. For this analysis, we divide the database into two sub-period, being the first sub-period which started from 05/08/2004 to 23/07/2009 and the second sub-period started from 30/07/2009 to 07/07/2016).

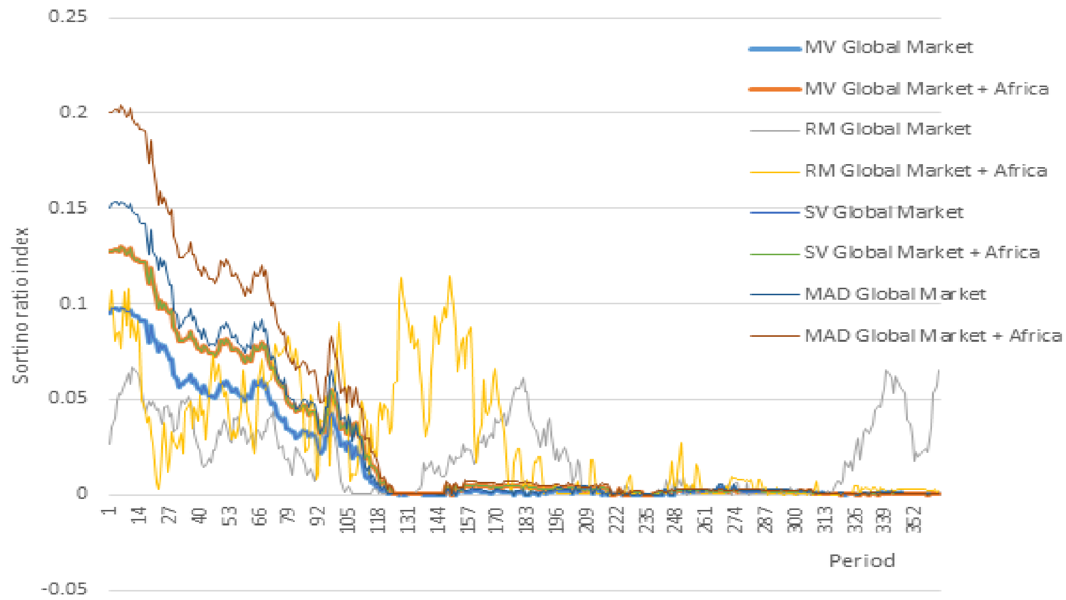

Figure 7. Out-of-sample sortino ratio performance. This figure shows the out-of-sample portfolio performances of the investment strategies for each optimization model measured by Sortino Ratio weekly. However, investment strategy with higher value of Sharpe Ratio show better performance. For this analysis, the database were divided into two sub-period, the first sub-period started 05/08/2004 to 23/07/2009 and the second sub-period started 30/07/2009 to 07/07/2016).

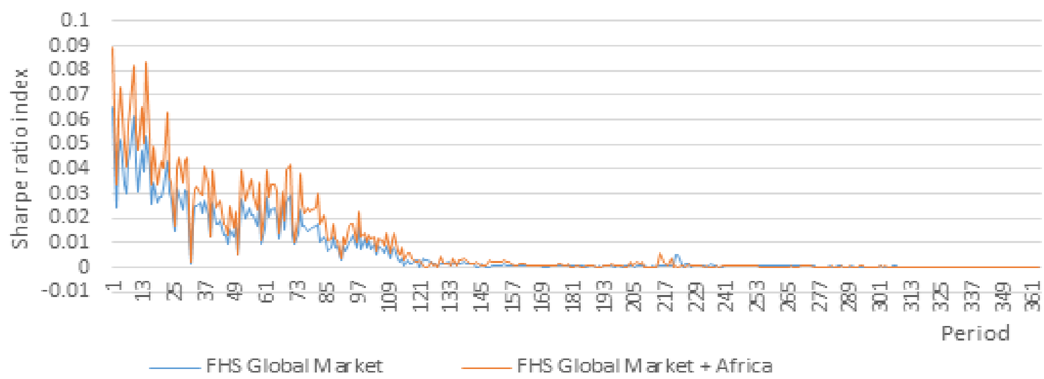

Figure 8. Out-of-sample sharpe ratio performance by FHS method. This figure show the out-of-sample portfolio performances of the investment strategies for each optimization model measured by Sharpe Ratio weekly.

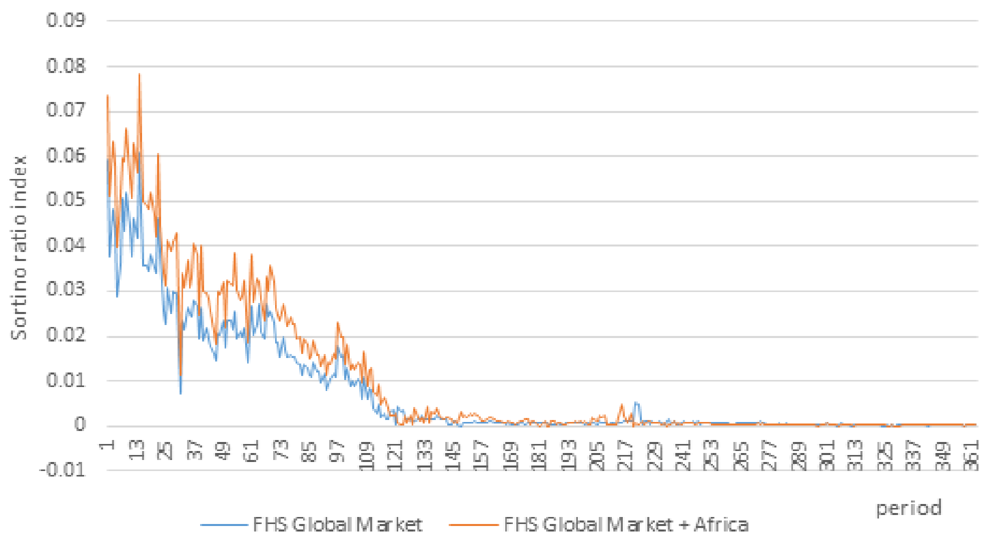

Figure 9. Out-of-sample sortino ratio performance by FHS method. This figure shows the out-of-sample portfolio performances of the investment strategies for each optimization model measured by Sortino Ratio weekly.

Table 21. Statistics test result for out-of-sample performance.

Model	Test result sharpe ratio performance			
	Null hypothesis	**t-statistic**	**P-value**	**Reject or No reject**
Mean variance (MV)	SR2-SR1=0	2.89	0.004	Reject
Resample Michaud (RM)	SR2-SR1=0	4.24	0.000	Reject
SemiVariance (SV)	SR2-SR1=0	2.78	0.006	Reject
Mean absolute deviation (MAD)	SR2-SR1=0	2.79	0.005	Reject
Filtered historical simulation (FHS)	SR2-SR1=0	2.49	0.013	Reject
Test result sortino ratio performance				
Mean variance (MV)	S2-S1=0	2.78	0.0056	Reject
Resample Michaud (RM)	S2-S1=0	4.10	0.0000	Reject
Semi variance (SV)	S2-S1=0	2.78	0.0056	Reject
Mean absolute deviation (MAD)	S2-S1=0	2.72	0.0066	Reject
Filtered historical simulation (FHS)	S2-S1=0	2.46	0.0141	Reject

analysis which is also a great contribution to the African capital market in the global portfolio composition as seen in Table 22.

Conclusion

The study data analysis from the period of 5th August, 2004 to 7th July, 2016 using the optimization models MV, RM, SV, MAD and FHS allowed the study to conclude that the diversification of global portfolio with assets of African capital market contributes in minimizing the risk and maximizing the return of the portfolio for the risk averse investors.

On the other hand, for risk loving investors, the diversification of global portfolio with assets of African capital markets increase the level of risk; but the benefit returns compensate for the risk increase. The study results are also in line with other studies (Lagoarde-Segot and Lucey, 2007; Yu and Hassan, 2008; Mansourfar et al., 2010) in the context of the international diversification.

The study results suggested that the foreign investors should look for an African capital market for an opportunity to maximize their wealth and diversify the investment risk. In the same order, the study result contributes to the discussion on the advantage of international diversification, even if it took place in the African context; and it further contributes to the literature

Table 22. The average weight of global market portfolio diversified with assets of African capital markets.

Markets	**MV**	**RM**	**SV**	**MAD**	**FHS**
Namibia	1	2	1	1	1
Nigerian	1	1	1	1	1
Gongo	2	2	2	2	2
Cote D´Ivoire	3	4	3	3	3
Egypt	1	2	1	1	1
Morrocco	1	2	1	1	1
Tunisia	1	2	1	1	1
Botswana	1	2	1	1	1
Mauritius	1	2	1	1	2
Kenya	1	2	1	1	1
Uganda	2	2	2	2	3
Zambia	2	3	2	2	2
South Africa	1	2	1	1	1
Germany	2	2	2	2	1
UK	1	2	1	1	1
France	1	2	1	1	1
Italy	1	2	1	1	1
Spain	1	2	1	1	1
Austrian	1	2	1	1	1
SWISS	2	2	2	2	2
Belgium	1	2	1	1	1
Denmark	7	3	7	7	7
Finland	1	2	1	1	1
Ireland	6	1	6	6	5
Israel	2	2	2	2	2
Netherlands	1	2	1	1	1
Norway	1	2	1	1	1
Portugal	1	2	1	1	1
Sweden	2	2	2	2	1
Czech Republic	1	2	1	1	1
Greece	1	2	1	1	1
Hungary	1	2	1	1	1
Poland	1	2	1	1	1
Qatar	2	2	2	2	2
Russian	1	3	1	1	1
Turkey	1	2	1	1	1
UAE	3	2	3	3	4
Brasil	1	2	1	1	1
Chile	1	2	1	1	1
Peru	1	2	1	1	1
Mexico	2	2	2	2	2
Canada	1	2	1	1	1
Australia	1	2	1	1	1
Hong Kong	1	2	1	1	1
Japan	1	2	1	1	1
Newzealand	1	2	1	1	1
Singapore	2	2	2	2	2
China	12	2	12	12	12
India	1	2	1	1	1
Indonesia	1	2	1	1	1

Table 22. Contd.

South Korea	2	2	2	2	1
Malasya	2	2	2	2	2
Philipine	2	2	2	2	1
Taiwan	1	2	1	1	1
EUA	3	2	3	3	3

through application of the FHS in the optimization portfolio. This methodology in addition of producing good results, reveals being more cautious in the constitution of investment portfolios than the other methods. However, this model presents lesser returns than others models.

The result of this study is important for Africa because it encourage the European, American, and Asia-Pacific investors to transfer part of their financial wealth to Africa by buying assets of African companies. These companies can help with the financial resources to develop new project which will pave way in improving the quality of lives of Africans.

On the other hand, these African companies can also use these financial resources to create new jobs that will encourage people to stay in their country, which will also reduce illegal immigration. For example, like the tragedy of deaths in the seas of the Mediterranean as it has been happening where thousands of people lose their lives trying to cross seas in small boat in the hope to find better quality of life in Europe and help their family that are in Africa. Many of these people could not get to Europe due to the bad traveling conditions, ruining their lives and dreams in the seas.

The result of this study encourages global investors to look at this problem and help Africa to solve it by buying African assets that can increase the value of their investment portfolios. The result of this study can contribute in the same way to provide transfer of knowledge or idea to Africa through canalization of these investments; this is because sometimes where there are money transfer new ideas are also shared. We can say that the result of this study can indirectly contribute to eliminate the inequality between other continents and Africa, through their investors that are looking for means to diversify their portfolios with African assets. On the other hand, this attitude on the part of global investors with the idea of buying African assets can contribute to poverty eradication in Africa.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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