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Full Length Research Paper

A fuzzy multi-criteria decision making approach for supplier selection in supply chain management

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In the recent years, supply chain management (SCM) has gained immense importance since enterprises are now competing on supply chain rather than manufacturing or service operations. One of the key strategic considerations in the supply chain is supplier selection problem. The supplier selection problem is a multi objective problem involving both qualitative and quantitative factors. These factors and their interdependencies make the problem highly complex one. From the managerial perspectives, it is always convenient to express the variables and weights through linguistic values. This paper uses a fuzzy approach to deal with the supplier selection problem in supply chain. The method is based on hierarchical multiple criteria decision making (MCDM) using fuzzy approach to select suitable supplier. In such type of decision making problems, all the decision makers are assumed to be equally important resulting in impractical aggregation of decision. Therefore, an analytic hierarchy process (AHP) like procedure based on Eigen value has been proposed to derive the weightages of decision makers. Then, weightages of decision makers are incorporated with fuzzy decision making paradigm to arrive at robust selection of suppliers in SCM. The methodology has been demonstrated with the help of a case study in a steel plant.

Key words: Supplier selection, multi criteria decision making, fuzzy numbers, supply chain management, distance measure.

INTRODUCTION

In the today's competitive corporate environment, all dimensions of product delivery viz., quality, flexibility, and response time need to be incorporated through effective design and operation of supply chain. Supplier evaluation and selection is one of the most important components of supply chain, which influence the long term commitments and performance of the company. Suppliers have varied strengths and weaknesses which require careful assessment by the purchasers before they are ranked based on some criteria. Therefore, every decision needs to be integrated by trading off performances of different suppliers at each supply chain stage (Liu and Hai, 2005).

The problem becomes more important in manufacturing units where lot of time and revenue is spent on purchase. Good suppliers allow enterprises to achieve good manufacturing performance and make the greatest benefits for practitioners. Supplier selection is viewed as a complex problem due to number of criteria and their interdependence (Chen et al., 2005). In general, the supplier selection problem in supply chain is a group decision making under multiple criteria (Chen et al., 2006). The group decision making process involves human judgment; crisp data are not adequate to model these judgments as it involves human preferences. The more pragmatic approach is to use linguistic values for assessment. So the ratings and weights of the criteria in the problem are assessed by means of linguistic variables (Bellman et al., 1970; Herrara et al., 1996; Herrara -Viedma, 2000).

LITERATURE REVIEW

Supply chain management has started playing an important role in the value chains of both industrial and service sectors. Recent data shows that company spends around 20% of the products cost in managing supply chain (http://mgtclass.com). As discussed, one of the most important

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area in the supply chain is supplier evaluation and selection. Dickson (1996) identified twenty three criteria for supplier selection based on the extensive survey, the result shows that quality is the most important parameter followed by delivery and performance history. A number of quantitative techniques have been used to supplier selection problem such as weighing method, statistical methods, Analytic Hierarchy Process (AHP), Data Envelopment Analysis etc. Kagnicioglu (2006) has used fuzzy multi-objective model with capacity, demand and budget constraint for supplier selection problem. They used two models to solve the problem. First Zimmerman's approach of symmetric model is used followed by Tiwari, Dharmar and Rao's weighted additive model as asymmetric model. Gnanasekaran et al. (2006) has applied Analytical Hierarchy Process (AHP) for effective supplier selection in a leading automobile component manufacturing company. The study shows that application of AHP enhances the decision making process and reduces the time taken to select the supplier. The paper uses Additive Normalisation Method and Eigen vector Method to find priority vector. Muralidharan et al. (2002) uses a novel model based on aggregation technique for combining group members' preferences into one consensus ranking for multi-criteria group decision making for supplier rating. Ibrahim and Ugur (2003) have used activity based costing (ABC) approach under the fuzzy variables by considering multi period of supplier-purchaser relationship for vendor selection. Bhutta and Huq (2002) use total cost of ownership and analytical hierarchy process for supplier selection problem and a comparison is made among different approaches. Taqi (2006) uses a non parametric technique called Data Envelopment Analysis for identifying and selecting vendors. The paper makes a comparison between DEA approach with the current practices for vendor selection and superiority of DEA is illustrated. Pearson and Ellram (1995) examine the supplier selection and evaluation criterion in small and large electronic firms. The results confirm the importance of the quality criteria in the supplier selection and evaluation. The other criteria found to be relatively important are speed to market, design capability and technology. The result shows that the nature of industry and its competitive environment may have a greater influence on selection criteria in comparison to the size of the firm. Singpurwalla (1999) has used probabilistic hierarchical classification model for rating suppliers in context of software development. In this approach, a positive probability is assigned to a supplier belonging to each class of supplier then this probabilistic classification is employed as an input to any decision making procedure that is used for actual supplier selection. Liu and Hai (2005) have used voting analytic hierarchy process (VAHP) for supplier selection. The method uses "vote ranking" rather than "paired comparison" for quantifying and measuring consistence. The study uses the vote ranking to determine the weights in the selected rank in place of the paired comparison method. In such a situation, fuzzy set theory can be very useful for supplier selection. Kumar et al. (2004) has used fuzzy goal programming for supplier selection. Kumar et al. (2006) used fuzzy multi-objective mathematical programming for supplier selection with three goals: cost minimization, quality maximization and on-time delivery maximization with constraints as demand, capacity, and quota flexibility.

CONCEPT OF FUZZY NUMBERS

In this section, we discuss some basic definitions related with fuzzy sets, fuzzy numbers and their operations. As we know the conventional models of operations research are dichotomous, deterministic and precise in character, but real situations are often uncertain or vague. To supersede these situations, research works present fuzzy approach to solve the many models of operations research. Zadeh (1965) developed the concept of fuzzy set theory in contrast of fuzziness of the problems. The applications of fuzzy set theory in mathematical programming have been discussed by many authors, (Zadeh, 1983; Zimmermann, 1983; Zimmermann, 1996).

Zadeh (1965) introduced fuzzy system to deal with the issue of uncertainty in systems modeling. Zadeh defined fuzzy sets as sets with boundaries that are not precise. "The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of degree." The concept of fuzzy set theory challenged conventional two-valued logic.

Fuzzy set

By a fuzzy set $\tilde{A}$ in a set X we mean the set of ordered pairs

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) \mid x \in X\}$ When membership space $M = [0,1]$ the set $\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$ becomes a characteristic function of $\tilde{A}$. Hence a fuzzy set is a generalization of classical set and the membership function is a generalization of the characteristic function.

Membership function

A membership function is a function which assigns to each element *x* of *X* a number, $\mu_{\tilde{A}}(x)$, in the closed unit interval [0, 1] that characterizes the degree of member-ship of *x* in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to one, the greater the membership of *x* in $\tilde{A}$. Thus, a fuzzy set $\tilde{A}$ can be defined precisely by associating with each element *x*, a number between 0 and 1, which represents its grade of membership in $\tilde{A}$. The membership function of a fuzzy set *A* can also be represented as $\tilde{A}$ (*x*).

Fuzzy numbers

Fuzzy sets that are defined on the set R of real numbers are

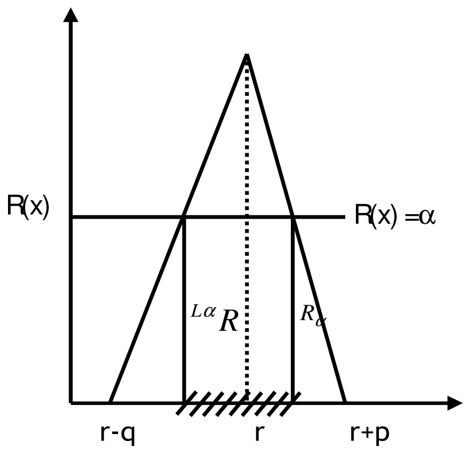

Figure 1. Triangular Fuzzy Number R "close to crisp number " r.

Figure 2. crisp number r.

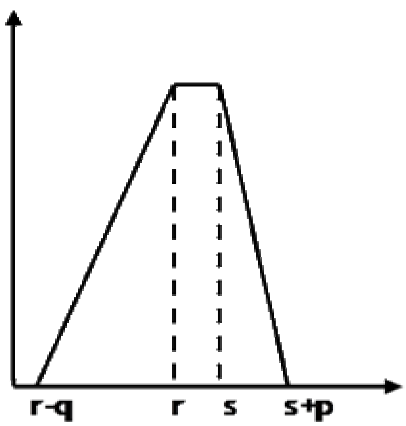

Figure 3. Fuzzy Interval r - s.

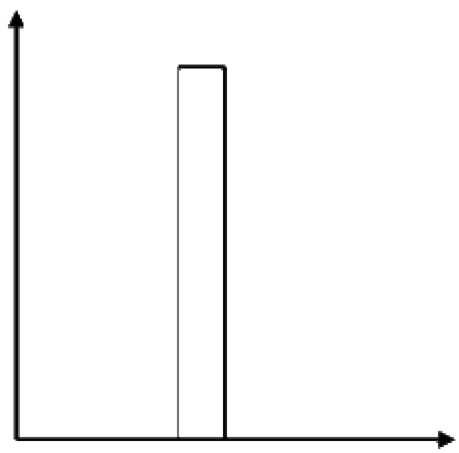

Figure 4. Crisp Interval r-s.

called fuzzy numbers (Klir and Yuan, 1995). Membership functions of these sets have a quantitative meaning and are represented as: $\tilde{A} : \mathbf{R} \longrightarrow [0, 1]$

Normalised fuzzy set

A fuzzy set $\tilde{A}$ in the universe X is said to be normalized if the height which is the largest membership grade attained by any element in the set is equal to unity.

Fuzzy matrix

A matrix $\tilde{D}$ is called a fuzzy matrix if at least one element of the matrix is fuzzy number.

Linguistic variable

A linguistic variable is a variable whose values are expressed in linguistic terms. The graphical representation of fuzzy numbers is given in Figures 1 – 4.

Operations on fuzzy numbers

Let $\tilde{m}$ *and* $\tilde{n}$ be two triangular fuzzy numbers given by $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ respectively and p be a positive real number, then the basic arithmetic expressions are given as follows:

$$\tilde{m} \oplus \tilde{n} = (m_1 + n_1, m_2 + n_2, m_3 + n_3)$$

$$\tilde{m} \Theta \tilde{n} = (m_1 - n_1, m_2 - n_2, m_3 - n_3)$$

$$\tilde{m} \otimes \tilde{n} \cong (m_1 n_1, m_2 n_2, m_3 n_3)$$

$$\tilde{m} \otimes p = (m_1 p, m_2 p, m_3 p)$$

It shall be noted here that fuzzy addition and subtraction of two triangular fuzzy numbers is a triangular fuzzy number whereas multiplication of two triangular fuzzy numbers is only approximately triangular fuzzy number.

Distance between two fuzzy numbers

The distance between two triangular fuzzy numbers can be calculated using the vertex method (Chen, 2000) as follows;

$$d(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{3}\left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2\right]}$$

A NUMERICAL PROBLEM AND COMPUTATIONAL PROCEDURE FOR SELECTING THE VENDOR

A case study

XYZ is the largest public owned steel manufacturing company in India. It has got five major plants at various places in India; one among them is situated at 'A' called 'ABC' in the mineral rich state of India. Raw Material Division (R. M. D.) is a part of ABC with its captive mines at 'B' and 'C' around 100 km from the steel city of ABC. These mining unit supplies raw materials mainly iron ore and lime stone to the steel plant and have its head office at Kolkata. The mines at B known as B. I. M. supplies iron ores to ABC mainly in two forms - fines (2 - 8 mm) and lumps (8 - 40 mm). One of the primary units of B. I. M. is Jigging plant which improves quality of iron ore. Through jigging the circuit alumina and silica are separated from the iron ore.

The entire movement of materials in the plant, that is, from mining to dispatch (loading point) is done using the conveyor belt. The belt rolls on idlers and is driven by electric motors through pulley. The idlers are of various types, troughing idlers, troughing training idlers, return idlers and return training idlers. The idlers require frequent replacement as it gets damaged due to jamming. The jamming occurs due to materials falling on it. The material spillage occurs for the following reasons; excessive feeding, slurry materials, imperfect belt alignment e.t.c. The cost of idler is approximately two to four thousand Indian Rupees and due to its high demand, a high level stock is maintained. The idler used are of non-greasing type, that is, once it gets damaged it is replaced rather than repaired (use and throw). There are number manufacturer producing idlers, and selection of vendor for idlers is an important problem for the management of B. I. M. The selection of vendor is done by a committee which evaluates the vendors on number of parameters to arrive at decision. B. I. M. being public sector company lots of emphasis is given on procedure and transparency in the process.

In the following section, we suggest a solution to the above mentioned problem by considering a situation involving four suppliers S_1, S_2, S_3 and S_4 evaluated by a committee of three members D_1, D_2, and D_3, on four criteria's C_1, C_2, C_3 and C_4.

Computational procedure

In the discussed problem there are four suppliers S_1, S_2, S_3 and S_4, the decisions are taken by a committee of three members D_1, D_2, and D_3; the weightage of these committee members/decision makers varies ba-sed on various criteria like years of experience, technical expertise e.t.c.

Table 1. Rating of criteria by decision makers.

Criteria	Decision maker		
	D_1	D_2	D_3
C_1	H	H	H
C_2	VH	H	VH
C_3	VH	VH	VH
C_4	H	H	VH

Step 1: In this step, the weightage of each decision maker is computed. To find the weightage of each decision maker many methods are employed. Keeny and Kirkwood (1975) and Keeny (1976) have suggested the use of interpersonal comparison to obtain the values of scaling constants in the weighted additive social choice function. Bash (1980) has used a Nash bargaining based approach to estimate the weights intrinsically. Mirkin (1979) has developed an Eigen vector method for deriving weightage of group members. In our problem, we ask the decision maker D_1, D_2, and D_3 to rank each member in a scale of 1 - 3 based on their judgment to get the following matrix A.

$$A = \begin{array}{c|ccc} & D_1 & D_2 & D_3 \\ D_1 & 1 & 3 & 2 \\ D_2 & \frac{1}{3} & 1 & 2 \\ D_3 & \frac{1}{2} & \frac{1}{2} & 1 \end{array}$$

The above matrix is solved for its Eigen value using MATLAB. The principal Eigen vector is found to be 1.000, 0.4799 and 0.3468 respectively with principal Eigen value being 3.1333. The normalized Eigen vector is calculated to be 0.5474, 0.2627 and 0.1899 respectively with consistency index value 0.0667 and consistency ratio 0.1149, the result is obtained in three comparisons. So the weightage of three decision makers D_1, D_2, and D_3 is taken as 0.5474, 0.2627 and 0.1899 respectively.

Step 2: In this step each of the three decision makers D_1, D_2, and D_3 is asked to rate the four criteria C_1, C_2, C_3 and C_4 in a linguistic scale- low (L), medium low (ML), medium (M), medium high (MH), high (H) and very high (VH). The response is recorded in Table 1.

Table 2. Rating of suppliers by decision makers under various criteria.

Criteria	Supplier	Decision maker		
		D_1	D_2	D_3
C_1	S_1	MG	MG	MG
	S_2	VG	VG	VG
	S_3	MG	MG	G
	S_4	G	G	G
C_2	S_1	MG	MG	MG
	S_2	G	G	G
	S_3	VG	VG	VG
	S_4	G	G	MG
C_3	S_1	G	G	G
	S_2	G	VG	VG
	S_3	MG	MG	MG
	S_4	G	G	G
C_4	S_1	G	G	G
	S_2	G	VG	G
	S_3	G	G	G
	S_4	G	G	VG

Next the decision makers are asked to rate the four sup-pliers S_1, S_2, S_3 *and* S_4 based on four criteria C_1, C_2, C_3 and C_4 using the linguistic scale - poor (P), medium poor (MP), fair (F), medium good (MG), good (G) and ve-ry good (VG). The results of this rating are shown in Table 2.

Now as the linguistic assessments simply approximate the subjective judgment of decision-makers, we consider the linear triangular membership functions to capture the vagueness of linguistic assessment. The linguistic varia-bles are expressed as positive triangular fuzzy numbers and are shown in Figures 5 and 6. The decision makers are asked to use the linguistic variables as shown in Fi-gures 5 and 6 to evaluate the importance of the criteria and the ratings of alternatives with respect to qualitative criteria.

Now the Figure 5 and 6 is used for rating the criteria by the decision-makers and the rating of suppliers on va-rious criteria. The ratings are represented using fuzzy va-lues in Tables 3 and 4.

Step 3: In this step the weights of the decision-maker as calculated in the step-1 is incorporated in the ratings. The weightage of three decision makers D_1, D_2, *and* D_3 is 0.5474, 0.2627 and 0.1899 respectively. The fuzzy ra-tings of criteria and the suppliers on various criteria with weightage of decision-makers are shown in Tables 5 and 6.

Table 3. Fuzzy rating of criteria by decision makers.

Criteria	Decision maker		
	D_1	D_2	D_3
C_1	(0.8, 0.9, 1.0)	(0.8, 0.9, 1.0)	(0.8, 0.9, 1.0)
C_2	(0.9, 1.0, 1.0)	(0.8, 0.9, 1.0)	(0.9, 1.0, 1.0)
C_3	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)
C_4	(0.8, 0.9, 1.0)	(0.8, 0.9, 1.0)	(0.9, 1.0, 1.0)

Table 4. Rating of suppliers by decision makers under various criterias.

Criteria	Supplier	Decision maker		
		D_1	D_2	D_3
C_1	S_1	(6, 7, 8)	(6, 7, 8)	(6, 7, 8)
	S_2	(9, 10, 10)	(9, 10, 10)	(9, 10, 10)
	S_3	(6, 7, 8)	(6, 7, 8)	(8, 9, 10)
	S_4	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
C_2	S_1	(6, 7, 8)	(6, 7, 8)	(6, 7, 8)
	S_2	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
	S_3	(9, 10, 10)	(9, 10, 10)	(9, 10, 10)
	S_4	(8, 9, 10)	(8, 9, 10)	(6, 7, 8)
C_3	S_1	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
	S_2	(8, 9, 10)	(9, 10, 10)	(9, 10, 10)
	S_3	(6, 7, 8)	(6, 7, 8)	(6, 7, 8)
	S_4	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
C_4	S_1	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
	S_2	(8, 9, 10)	(9, 10, 10)	(8, 9, 10)
	S_3	(8, 9, 10)	(8, 9, 10)	(8, 9, 10)
	S_4	(8, 9, 10)	(8, 9, 10)	(9, 10, 10)

Step 4: Now, the aggregate fuzzy weights $\tilde{w}_j$ of each cri-terion can be calculated as follows;

$$\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$$

where;

$$w_{j1} = \underset{k}{Min}\{w_{jk1}\}$$

$$w_{j2} = \frac{1}{k}\sum_{k=1}^{k} w_{jk2}$$

$$w_{j3} = \underset{k}{Max}\{w_{jk3}\}$$

k = number of decision makers = 3

For first criteria C_1

$$w_{j1} = \underset{k}{Min}\{w_{jk1}\}$$

$$= Min([0.4379, 0.2102, 0.1519])$$

$$= 0.1519$$

Table 5. Fuzzy rating of criteria with weight of decision makers.

Criteria	Decision maker		
	D_1	D_2	D_3
C_1	(0.4379, 0.4927, 0.5474)	(0.2102, 0.2634, 0.2627)	(0.1519, 0.1709, 0.1899)
C_2	(0. 4927, 0.5474, 0.5474)	(0. 2102, 0. 2634, 0.2627)	(0. 1709, 0.1899, 0.1899)
C_3	(0. 4927, 0.5474, 0.5474)	(0. 2634, 0.2627, 0.2627)	(0. 1709, 0.1899, 0.1899)
C_4	(0. 4379, 0. 4927, 0.5474)	(0. 2102, 0. 2634, 0.2627)	(0. 1709, 0.1899, 0.1899)

Table 6. Rating of suppliers under various criteria with weight of decision makers.

Criteria	Supplier	Decision maker		
		D_1	D_2	D_3
C_1	S_1	(3.2844, 3.8318, 4.3792)	(1.5762, 1.8389, 2.1016)	(1.1394, 1.3293, 1.5192)
	S_2	(4.9266, 5.474, 5.474)	(2.3643, 2.627, 2.627)	(1.7091, 1.899, 1.899)
	S_3	(3.2844, 3.8318, 4.3792)	(1.5762, 1.8389, 2.1016)	(1.5192, 1.7091, 1.899)
	S_4	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
C_2	S_1	(3.2844, 3.8318, 4.3792)	(1.5762, 1.8389, 2.1016)	(1.1394, 1.3293, 1.5192)
	S_2	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
	S_3	(4.9266, 5.474, 5.474)	(2.3643, 2.627, 2.627)	(1.7091, 1.899, 1.899)
	S_4	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.1394, 1.3293, 1.5192)
C_3	S_1	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
	S_2	(4.3792, 4.9266, 5.474)	(2.3643, 2.627, 2.627)	(1.7091, 1.899, 1.899)
	S_3	(3.2844, 3.8318, 4.3792)	(1.5762, 1.8389, 2.1016)	(1.1394, 1.3293, 1.5192)
	S_4	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
C_4	S_1	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
	S_2	(4.3792, 4.9266, 5.474)	(2.3643, 2.627, 2.627)	(1.5192, 1.7091, 1.899)
	S_3	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.5192, 1.7091, 1.899)
	S_4	(4.3792, 4.9266, 5.474)	(2.1016, 2.3643, 2.627)	(1.7091, 1.899, 1.899)

Table 7. Weights of the criteria.

	Criteria			
	C_1	C_2	C_3	C_4
Weights	(0.1519,0.309,0.5474)	(0.1709,0.3336,0.5474)	(0.1709,0.3333,0.5474)	(0.1709,0.3153,0.5474)

$$w_{j2} = \frac{1}{k}\sum_{k=1}^{k} w_{jk2}$$
$$= \frac{1}{3}(0.4927 + 0.2634 + 0.1709)$$
$$= 0.309$$

$$w_{j3} = \underset{k}{Max}\{w_{jk3}\}$$
$$= Max(0.5474, 0.2627, 0.1899)$$
$$= 0.5474$$

So, weights of the criteria can be given as shown in Table 7.

The aggregate fuzzy rating of suppliers on various criteria can be defined as,

$$\tilde{R} = (a, b, c)$$

where

$$a = \underset{k}{Min}\{a_k\}$$
$$b = \frac{1}{k}\sum_{k=1}^{k} b_k$$
$$c = \underset{k}{Max}\{c_k\}$$

Table 8. Fuzzy decision matrix and fuzzy weight of suppliers.

	C_1	C_2	C_3	C_4
S_1	(1.1394, 2.3333, 4.3792)	(1.1394, 2.3333, 4.3792)	(1.5192, 3, 5.474)	(1.5192, 3, 5.474)
S_2	(1.7091, 3.3333, 5.474)	(1.5192, 3, 5.474)	(1.7091, 3.1508, 5.474)	(1.5194, 3.0876, 5.474)
S_3	(1.5192, 2.4599, 4.3792)	(1.7091, 3.3333, 5.474)	(1.1394, 2.3333, 4.3792)	(1.5192, 3, 5.474)
S_4	(1.5192, 3, 5.474)	(1.1394, 2.8734, 5.474)	(1.5192, 3, 5.474)	(1.7091, 3.0633, 5.474)
Weights	(0.1519,0.309,0.5474)	(0.1709,0.3336,0.5474)	(0.1709,0.3333,0.5474)	(0.1709,0.3153,0.5474)

Table 9. Normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4
S_1	(0.2081, 0.4263, 0.8)	(0.2081, 0.4263, 0.8)	(0. 2775, 0.548, 1.0)	(0. 2775, 0.548, 1.0)
S_2	(0.3122, 0.6089, 1.0)	(0. 2775, 0.548, 1.0)	(0. 3122, 0.5756, 1.0)	(0.2776, 0.564, 1.0)
S_3	(0.2775, 0.4494, 0.8)	(0.3122, 0.6089, 1.0)	(0. 2081, 04263, 0.8)	(0.2775, 0.548, 1.0)
S_4	(0. 2775, 0.548, 1.0)	(0. 2081, 0.5249, 1.0)	(0.8, 0.9, 1.0)	(0.3122, 0.5596, 1.0)
Weights	(0.1519,0.309,0.5474)	(0.1709,0.3336,0.5474)	(0.1709,0.3333,0.5474)	(0.1709,0.3153,0.5474)

So, aggregation of first supplier S_1 on criteria C_1 can be represented as follows;

(3.2844, 3.8318, 4.3792) (1.5762, 1.8389, 2.1016) (1.1394, 1.3293, 1.5192)

$$a = \underset{k}{Min}\{a_k\}$$

= Min (3.2844, 1.5762, 1.1394)
= 1.1394

$$b = \frac{1}{k}\sum_{k=1}^{k} b_k$$

= 1/3 (3.8318 + 1.8389 + 1.3293) = 2.3333

$$c = \underset{k}{Max}\{c_k\}$$

= Max (4.3792, 2.1016, 1.5192) = 4.3792

So, fuzzy representation of aggregation of first supplier S_1 on criteria C_1 can be represented as (1.1394, 2.3333, 4.3792). Similarly all the elements can be calculated and is shown in Table 5 in step-5.

Step 5: The triangular fuzzy numbers obtained through the linguistics evaluations is used to construct the fuzzy decision matrix as in Table 8. Mathematically, the supplier selection problem is now can be expressed in matrix format as follows;

$$\tilde{D} = \begin{pmatrix} \hat{x}_{11} & \hat{x}_{12} & \dots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \dots & \hat{x}_{2n} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ \hat{x}_{m1} & \hat{x}_{m2} & \dots & \hat{x}_{mn} \end{pmatrix}$$

$$\widehat{W} = [\widehat{w}_1 \quad \widehat{w}_2 \quad \dots \quad \widehat{w}_n]$$

where;

$\hat{x}_{ij} = (a_{ij}, \; b_{ij}, \; c_{ij})$ and $w_j = (w_{j1}, \; w_{j2}, \; w_{j3})$; $i = 1,2,\dots,m, j = 1,2,\dots,n$ can be approximated by positive triangular fuzzy numbers.

The normalized fuzzy decision matrix can be representted as;

$$\hat{R} = [\hat{r}_{ij}]_{m \times n}$$

$$\hat{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \; \frac{b_{ij}}{d_j^*}, \; \frac{c_{ij}}{d_j^*}\right); \text{ where } d_j^* = \max_i d_{ij}$$

The normalization process so described preserves the property in which the elements $\hat{r}_{ij} \; \forall \; i, j$ are normalized triangular fuzzy number.

Next the normalized fuzzy decision matrix is constructed as shown in Table 9. Now, considering the importance of each criterion the weighted normalized fuzzy decision matrix is constructed as,

$$\hat{V} = [\hat{v}_{ij}]_{m \times n}, \; i = 1,2,\dots,m, \quad j = 1,2,\dots,n$$

Where, $\hat{v}_{ij} = \hat{r}_{ij} (.) \widehat{w}_j$

The weighted normalized fuzzy decision matrix is shown Table 10.

Step 6: In this step, the fuzzy positive –ideal solution (FPIS) S* and fuzzy negative –ideal solution (FNIS) S^- is calculated. The fuzzy positive –ideal solution (FPIS) S* and fuzzy negative –ideal solution (FNIS) S^- is defined as,

$$S^* = (\hat{v}_1^*, \; \hat{v}_2^*, \; \dots, \; \hat{v}_n^*)$$

Table 10. Weighted normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4
S_1	(0.0316, 0.1317, 0.4379)	(0.0355, 0.1422,0.4379)	(0.0474,0.1826,0.5474)	(0.0474,0.1728,0.5474)
S_2	(0.0474,0.1881,0.5474)	(0.0474,0.1828,0.5474)	(0.0533,0.1918, 0.5474)	(0.0474,0.1778,0.5474)
S_3	(0.0422,0.1388,0.43792)	(0.0533,0.2031,0.5474)	(0.0355,0.1420,0.4379)	(0.0474,0.1728,0.5474)
S_4	(0.0422,0.1693,0.5474)	(0.0355, 0.1751, 0.5474)	(0.0474,0.1826,0.5474)	(0.0533,0.1764,0.5474)

Table 11. Distance between S_i (I = 1, 2, 3, 4) and S^* with respect to each criterion.

	C_1	C_2	C_3	C_4	Sum
$d_1^* = d(S_1, S^*)$	0.3876	0.3822	0.3573	0.3607	1.4878
$d_2^* = d(S_2, S^*)$	0.3555	0.3573	0.3514	0.3589	1.4231
$d_3^* = d(S_3, S^*)$	0.3804	0.3477	0.3823	0.3607	1.471
$d_4^* = d(S_4, S^*)$	0.3643	0.3654	0.3573	0.3567	1.4437

Table 12. Distance between S_i (I = 1, 2, 3, 4) and S^- with respect to each criterion.

	C_1	C_2	C_3	C_4	Sum
$d_1^- = d(S_1, S^-)$	0.2415	0.2431	0.3075	0.2976	1.0897
$d_2^- = d(S_2, S^-)$	0.3113	0.3076	0.3091	0.3049	1.2329
$d_3^- = d(S_3, S^-)$	0.2426	0.3111	0.2403	0.2976	1.0916
$d_4^- = d(S_4, S^-)$	0.3082	0.3063	0.3075	0.2981	1.2201

and, $S^- = (\hat{v}_1^-, \hat{v}_2^-, \ldots, \hat{v}_n^-)$
where, $\hat{v}_j^* = \max_i\{v_{ij3}\}$ and $\hat{v}_j^- = \min_i\{v_{ij1}\}$; $i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n$

S* = [(0.5474, 0.5474, 0.5474), (0.5474, 0.5474, 0.5474), (0.5474, 0.5474, 0.5474), (0.5474, 0.5474, 0.5474)]
S^- = [(0.0316, 0.0316, 0.0316), (0.0355, 0.0355, 0.0355), (0.0355, 0.0355, 0.0355), (0.0474, 0.0474, 0.0474)]

Step 7: Now we know that, if $\tilde{m} = (m_1, m_2, m_3)$ and $\tilde{n} = (n_1, n_2, n_3)$ are two triangular fuzzy numbers, then the distance between two triangular fuzzy numbers $\tilde{m}$ *and* $\tilde{n}$ can be calculated using vertex method (Chen, 2000) as follows (Tables 11 and 12);

$$d_v(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{3}\left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2\right]}$$

Step 8: In this step we calculate the closeness coefficient to rank the suppliers. The closeness coefficient is the distance to the FPIS (S^*) and the FNIS (S^-) simultaneously by taking the relative closeness to the FPIS.

The closeness coefficient (CC_i) for each supplier is calculated as,

$$CC_i = \frac{d_i^-}{d_i^- + d_i^*}, \; i = 1,2,\ldots,m$$

It can be noted that $CC_i = \begin{cases} 1 & if\ S_i = S^* \\ 0 & if\ S_i = S^- \end{cases}$

So, CC_i approaches 1 when the supplier S_i is closer to FPIS (S^*) and farther from FNIS (S^-) (Table 13).

Step 9: The supplier can be ranked based on the values of closeness coefficient, to describe the status of each supplier the interval $[0,1]$ is divided into five sub-intervals. Five linguistic variables are defined to divide the assessment status of suppliers into five classes. The decision rule is shown below;

$CC_i \in [0, 0.2)$ – Class I – Not recommended
$CC_i \in [0.2, 0.4)$ – Class II – Recommended with high risk
$CC_i \in [0.4, 0.6)$ – Class III – Recommended with low risk
$CC_i \in [0.6, 0.8)$ – Class IV – Approved
$CC_i \in [0.8, 1.0]$ – Class V – Approved and highly recommended.

Table 13. Computation of d_i^-, d_i^* and CC_i.

	d_i^*	d_i^-	$d_i^* + d_i^-$	$CC_i = \frac{d_i^-}{d_i^- + d_i^*}$
S_1	1.4878	1.0897	2.5775	0.4227
S_2	1.4231	1.2329	2.656	0.464
S_3	1.471	1.0916	2.5626	0.425
S_4	1.4437	1.2201	2.6638	0.458

Based on the value of CC_i, $S_2 > S_4 > S_3 > S_1$.

Since all the suppliers S_1, S_2, S_3 and S_4 belong to Class III, so they can be recommended with low risk. Management of the company can select supplier S_2 followed by supplier S_4. The ranking order of supplier is $S_2 > S_4 > S_3 > S_1$ and management can take decision accordingly.

Conclusion

This paper proposes a multiple criteria decision model in fuzzy environment for vendor selection problem. This is considered as one of the critical decision making process for effective operation of supply chain. The crisp approach of vendor rating schemes and allied techniques result in unreasonable selection that affects the performance of enterprise in the long run. Therefore, a fuzzy approach capable of capturing vagueness associated with subjective perception of decision makers has been proposed. Further, biasness in decision making process has been avoided using weightages for decision makers based on their proficiency in the problem under consideration. In real practice, the decision makers are hardly equally important as they usually come from varied background. A novel technique of determining weightages for decision makers has been proposed. The procedure uses ranking of a decision maker by other decision makers; hence conflict resolution becomes quite easy. The weightages of decision makers have been integrated with the general frame work of fuzzy hierarchical multi criteria decision making process. The complete process has been elicited with the help of real life case study. The case study problem deals with selection or ranking of four suppliers by three decision makers based on four criteria. The method uses the concept of fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS) for solving the problem. The suppliers are ranked on the basis of value of closeness coefficient which is calculated for each supplier. According to the closeness coefficient, the ranking order of four alternatives has been determined as $S_2 > S_4 > S_3 > S_1$. So, supplier 'S_2' can be chosen for the case study problem. In fact, supplier 'S_2' is one of the leading manufacturers of general equipment and accessories treated as fast moving items in steel companies. Although the study concentrates on steel industry, the methodology is quite general and can be adopted in any industry if the criteria and alternatives are clearly defined.

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