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African Journal of Environmental Science and Technology

Full Length Research Paper

Determination of the optimum tilt angle for photovoltaic modules in Senegal

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This paper deals with finding the optimum tilt angle of solar panels for solar energy applications. The objective is to maximize the output electrical energy of the photovoltaic (PV) modules. A mathematical model was used to determine the optimum tilt angle of solar collectors in Senegal on a daily and monthly basis, as well as for a specific period. Then, the configuration of the optimum tilt angle was analyzed by studying four specific cases in different typical climatic zones in Senegal. On a horizontal plane, the sun spends more time in the south than in the north. This means that in Senegal, the optimum tilt angle is often set equal to latitude. On the other hand, the optimum latitude angle is only for annual optimization. This study shows that the optimum tilt angle equal to latitude does not produce maximum output. For monthly optimum tilt angle, only the month of November gives tilt optimum angles equal to latitude. It is preferable to change the tilt angle of solar conversion systems monthly instead of fixing them, to gain more energy.

Keys words: Solar energy, solar panels, tilt angle, orientation solar panel, latitude, geometry.

INTRODUCTION

Non-renewable energy sources, such as fossil fuel, have been the major source of energy in many countries, including Senegal. However, because of the problems associated with the use of these non renewable energy sources, there is a need for alternative energy sources that are sustainable and nonpolluting. The optimum orientation of a solar conversion system obeys to a simple rule towards the Equator, which gives: orientation to the south in the northern hemisphere (azimuth angle =0°); and orientation to the north in the southern hemisphere (azimuth angle =180°) (Tiris and Tiris, 1998; Ihaddadene and Charik, 2017). Other researchers (Tripathy et al., 2017; Yadav et al., 2021) used models to

determine the optimum tilt angle and then studied the influence of the shadow of urban residential buildings on this angle. The monthly optimum tilt angle was calculated in Malaysia by Liu and Jordan method over three rural areas (Fadaeenejad et al., 2015). Using diffuse radiation methods, (Hailu and Fung, 2019) determined the optimum tilt angle in an area located in Canada. In the same context, (Kamanga et al., 2014) calculated the optimum tilt angles for a district in Malawi. In order to optimise solar isolation on solar collectors, appropriate method to determine solar tilt angles at any given time is essential to increase the efficiencies of the collectors and that of the devices connected to them (Idowu et al.,

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2013). The optimum tilt angle of solar collectors such as photovoltaic solar panels is important for conversion of solar radiation into heat or electricity (Mahdi et al., 2011). Zang et al. (2016) determined the optimum tilt angle of photovoltaic systems by considering different regional climates in China. To maximise the amount of energy captured by solar collectors, the sun tracking system is often used as an attractive technology for solar collectors or panels (Lv et al., 2018).

A number of studies have been carried out by various investigators in order to optimize the tilt angle around the world (Lewis, 1987; Saraf and Hamad, 1988; Hussein et al., 2000; Shariah et al., 2002; Skeiker, 2009). Optimization of the tilt angle has been performed for various locations in differents countries. In Turkey, Bakirci (2012) determined a general models for optimum tilt angles of solar panels. It is in this same context that (Kacira et al., 2004) used a mathematical model to determine the optimum tilt angle to make the photovoltaic production optimum. A mathematical modeling of the optimum tilt for solar collectors is presented by (Stanciu et al., 2016) for intercepting maximum solar irradiance in Romania. A study in Jordan (Mahmoud and Nabhan, 1990) compared the PV energy produced with the optimum tilt angle to the energy produced with the fixed tilt angle and showed a 5.6% gain in production with the optimum tilt angle determined. In Austria and Germany, Hartner et al. (2015) evaluated the trade-off between annual energy losses and possible electricity generation cost reductions through adapting PV installation angles for the current electricity system and for potentially higher PV penetration levels in the future. In Greece, Mehleri et al. (2010) carried out a study on the determination of the optimum tilt angle and orientation for solar photovoltaic arrays in order to maximize the incident of solar irradiance exposed on the array, for aspecific period of time. For the Asian countries, studies for the modeling of the optimum angle of inclination have also been made. Khahro et al. (2015) evaluated solar energy resources by establishing diffuse solar radiation models and obtaining optimum tilt angle for a prospective location is southern region of Sindh, Pakistan. In Iran, Moghadam et al. (2011) performed optimization of solar flat collector inclination. In Indonesia, (Handoyo et al., 2013) maximized the incidence angle to obtain the tilt optimum angle. In India, Dixit et al. (2017) proposed a particle swarm optimization estimator in order to find optimum tilt angle on annual basis. To restitute solar radiation over 4 climatic zones in Senegal, Sarr et al. (2020a) used as input parameters the meteorological and geographical parameters and also the position of the sun. Eke (2011) revealed that the average angle of inclination at which a flat surface solar collector will be mounted at fixed position in Zaria, Nigeria is 22.5°. Another approach based on in situ measurements also allows to determine the energy produced with optimum tilt angles (Li and Lam, 2007). A study also showed that the optimum tilt

angle varies depending on the type of PV technologies used (Ayaz et al., 2017).

Senegal is located on the extreme western tip of the African continent between latitude 12 and 17° north and longitude 10 and 18° west. Solar energy is abundant in Senegal though locally measured data is not available for most locations. Internationally available satellites databases, however, provide sufficient data for most design purposes. Senegal ceives 5.5 kWh/m2/day and an average annual sunshine duration of around 3,000 h (www.aner.sn).

The research question addressed in this paper is concerned with the optimum tilt angle of solar systems to maximize electricity production. Many studies are limited to calculating the monthly or annual optimum tilt angle for a single specific zone. Our study determines the daily, monthly and yearly optimum tilt angle for Senegal and applies them on different zones chosen according to the climate contrast existing between them.

MATHEMATICAL PROCEDURE FOR CALCULATING THE OPTIMUM TILT ANGLE

In Senegal, there is no research on the determination of the optimum tilt angle. This work applies the Kassaby model to calculate the optimum tilt angle over Senegal, that is, between latitudes 12 and 17° north. This model will then be applied in four zones of Senegal (Figure 1) chosen according to their different regional climates. The percentage of daily and monthly variation of the optimum tilt angle is given for each study case. There are other methods to determine the optimum tilt angle, but we chose the Kassaby method. This method consists of determining the optimum daily $\beta_{opt,d}$ and monthly tilt angle $\beta_{opt,m}$. Our goal is to adjust the optimum tilt angle for our solar applications at least once a month to maximize photovoltaic production. The equations $\beta_{opt,d}$ and $\beta_{opt,m}$ are programmed with R software.

Main angles in solar applications

The optimum tilt angle βopt, the declination angle δ and sunset hour angle h_{ss} can be determined by the following equations (El-Kassaby, 1988):

$$\beta_{opt,d} = \phi - \tan^{-1}\left[\frac{h_{ss}}{\sin h_{ss}}\tan(\delta)\right] \tag{1}$$

$\beta_{opt,d}$ is optimum tilt angle at a particular day and ϕ is the latitude of the location.

$$\delta = -23.25\left[\cos(n + 10.5)\frac{360}{365}\right] \tag{2}$$

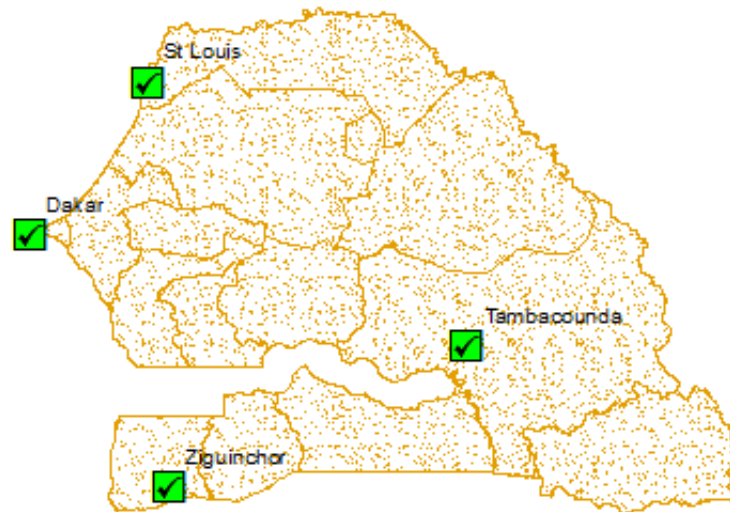$$h_{ss} = \cos^{-1}[-\tan(\phi)\tan(\delta)] \tag{3}$$

Figure 1. Typical climate zones selected for the study.

The monthly optimum tilt angle is given by :

$$\beta_{opt,m} = \phi - \tan^{-1}\left[\frac{\sum_{n_1}^{n_2}\frac{24}{\pi}I_0\left[1+0.034\cos\left(\frac{2\pi n}{365}\right)\right]\sin(\delta)h_{ss}}{\sum_{n_1}^{n_2}\frac{24}{\pi}I_0\left[1+0.034\cos\left(\frac{2\pi n}{365}\right)\right]\cos(\delta)\sin(h_{ss})}\right] \quad (4)$$

The sunset hour angle for the tilted surface is expressed as:

$$h_{ss} = min\left[\cos^{-1}\left(-\tan(\phi)\tan(\delta)\right), \cos^{-1}\left(\phi - \beta_{opt,m}\right)\tan(\delta)\right] \quad (5)$$

Where ''min'' means the smaller of the two items in the bracket.

RESULTS AND DISCUSSIONS

Daily optimum tilt angle

For photovoltaic solar applications, it is important to know the tilt angle. The latter is calculated according to the temporal variation of the movement of the sun in the sky. The results obtained for different latitude angles are shown in Table 1.

The results show that for a given latitude, the optimum tilt angle ($\beta_{opt,d}$) varies from one day to another. The optimum tilt angle is sometimes greater than latitude and sometimes less than latitude. This phenomenon depends on the day and the month considered. Only the month of February gives optimum tilt angles, which are greater than the latitude for all days.

Application for four typical climatic zones for daily optimum tilt angle

To take into account the climatic zoning, we consider four zones (Figure 1) which are Dakar (14.733° N, 17.467° W) located in the Cape Verde Peninsula, St Louis (16.050° N, 16.450° W) located in the Senegal River Delta, Tambacounda (13.767° N, 13.683° W) located in-land in the south-eastern Senegal and Ziguinchor (12.550° N, 16.267° W) located in the southwest part of Senegal (Sarr et al., 2020b).

Figure 2 shows the results of the variation of the daily optimum tilt angle at Dakar. Except for a few rare days on Dakar where we find optimum tilt angles equal to the latitude, these areas show optimum tilt angles different from the latitude. In Dakar there is a difference of 20.05% between the maximum value of the optimum tilt angle and its minimum value. Latitude has a variance of 11.14% from the minimum value of the optimum tilt angle while this difference is 10.03% from the maximum value of the optimum tilt angle. This difference is significant from the point of view of efficiency. Solar power plants must take into account the optimum tilt angle to optimize their production.

Figure 3 shows the results for St Louis. The difference between the maximum value of the optimum tilt angle and the minimum value of it in St Louis is 18.74%. Between latitude and the minimum value of the optimum tilt angle, the variance is 9.75% while between the maximum value of the optimum tilt angle and latitude the difference is 9.96%.

Figure 4 shows the results for the Tambacounda zone. In Tambacounda, there is a variation of 23.43% between the maximum and minimum values of the optimum tilt angle. Compared to the latitude, the difference is 13.84% from the minimum value of the optimum tilt angle and 11.13% from its maximum value.

The findings for Ziguinchor are shown in Figure 5. The results showed that in Ziguinchor there are a difference of 23.01% between the maximum and minimum values of

Table 1. Daily optimum tilt angle $\beta_{opt,d}$ for different latitude angles of Senegal.

N	Date	Latitude					
		12	13	14	15	16	17
January							
1	1	11.83	12.81	14	15.56	15.81	16.79
15	15	12.19	13.17	14	15.19	16.17	17.1
31	31	11.69	12.67	13.64	14.69	15.67	16.64
February							
32	1	12.19	13.17	14	15.19	16.18	17.1
46	15	13.41	14.41	15.41	16.41	17.41	18.41
59	28	12.87	13.86	15.41	15.87	16.86	17.84
March							
60	1	10.82	11.82	12.8	13.82	14.82	15.81
74	15	12.76	13.75	14.68	15.76	16.75	17.72
90	31	13.11	14.1	15.08	16.11	17.11	18.09
April							
91	1	10.84	11.83	12.81	13.84	14.83	15.82
105	15	13.39	14.39	15.38	16.39	17.34	18.39
120	30	11.68	12.66	13.63	14.69	15.67	16.63
May							
121	1	12.79	13.78	14.7	15.78	16.78	17.74
135	15	10.44	11.44	12.44	13.44	14.44	15.44
151	31	11.72	12.71	13.69	14.73	15.71	16.67
June							
152	1	11.79	12.77	13.81	14.79	17.77	16.74
166	15	10.5	11.5	12.5	13.5	14.5	15.5
181	30	10.57	11.57	12.57	13.57	14.57	15.57
July							
182	1	11.98	12.96	14	14.98	15.96	17
196	15	11.68	12.66	13.63	14.68	15.67	16.63
212	31	11.19	12.18	13.13	14.19	15.18	16.15
August							
213	1	12.24	13.23	14.05	15.25	16.23	17.16
227	15	11.85	12.83	14	14.86	15.84	16.82
243	31	10.44	11.44	12.44	13.44	14.44	15.44
September							
244	1	13.37	14.37	15.36	16.37	17.37	18.36
258	15	12.14	13.12	14	15.14	16.12	17.04
273	30	11.83	12.81	14	14.84	15.82	16.79
October							
274	1	10.48	11.48	12.48	13.48	14.48	15.48
288	15	11.34	12.33	13.28	14.34	15.33	16.3
304	31	10.55	11.55	12.55	13.55	14.55	15.55
November							
305	1	11.41	12.39	13.34	14.41	15.39	16.36
319	15	11	11.2	12.97	14.01	15	15.99
334	30	11.58	12.56	13.51	14.58	15.57	16.53

Table 1. Contd.

| December						
335	1	11.11	12.1	13.07	14.11	15.1	16.08
349	15	13.14	14.14	15.12	16.14	17.14	18.13
365	31	13.56	14.56	15.56	16.56	17.56	18.56
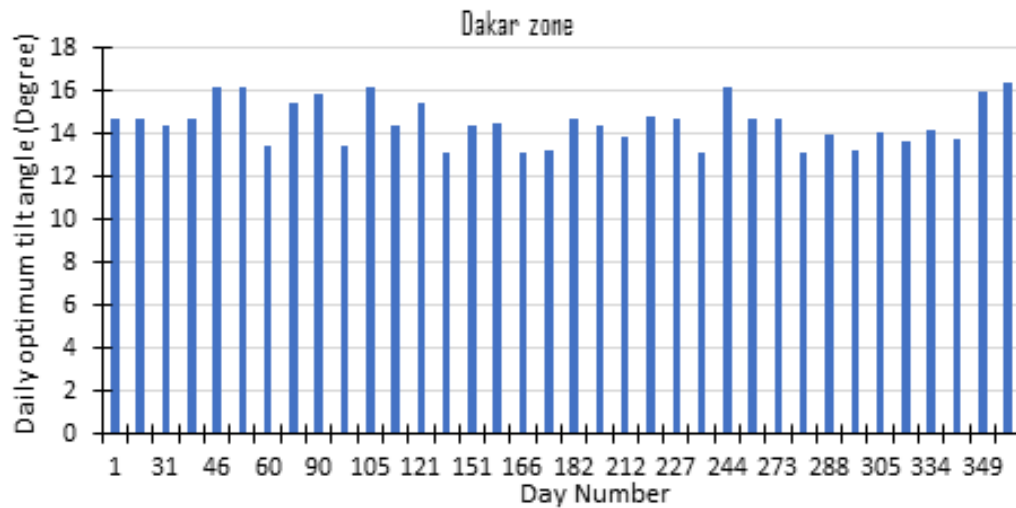
Figure 2. The variation of daily optimum tilt angle, $\beta_{opt,d}$ with the day number n for Dakar.
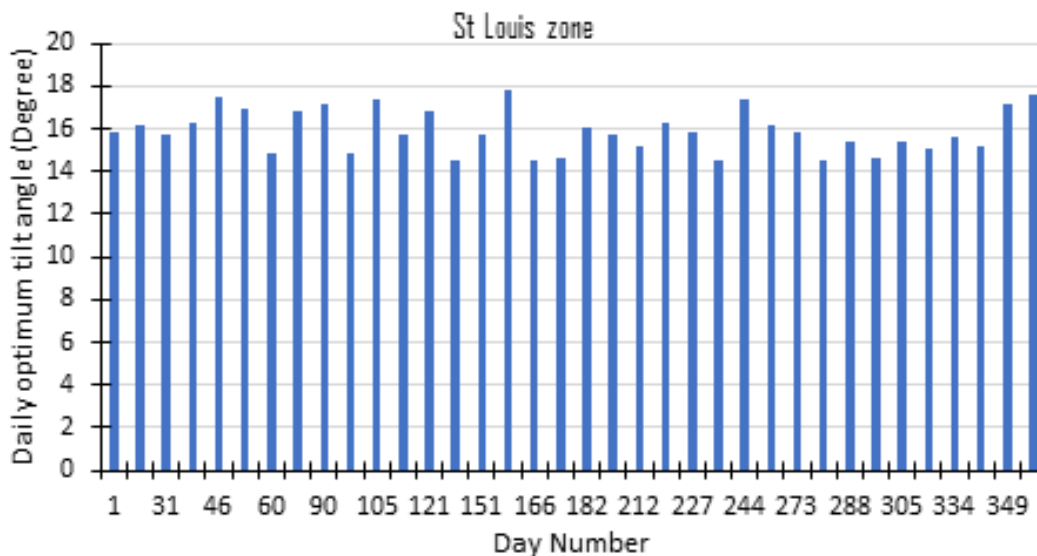
Figure 3. The variation of daily optimum tilt angle, β_(opt,d) with the day number n for St Louis.

the optimum tilt angle. This variance is 13.00% between the latitude and the minimum of the optimum tilt angle while it is 11.50% between the maximum of the optimum tilt angle and the latitude.

Monthly optimum tilt angle

Table 2 represents the monthly optimum tilt angle $\beta_{opt,m}$ for different latitude angles of Senegal. The optimum tilt
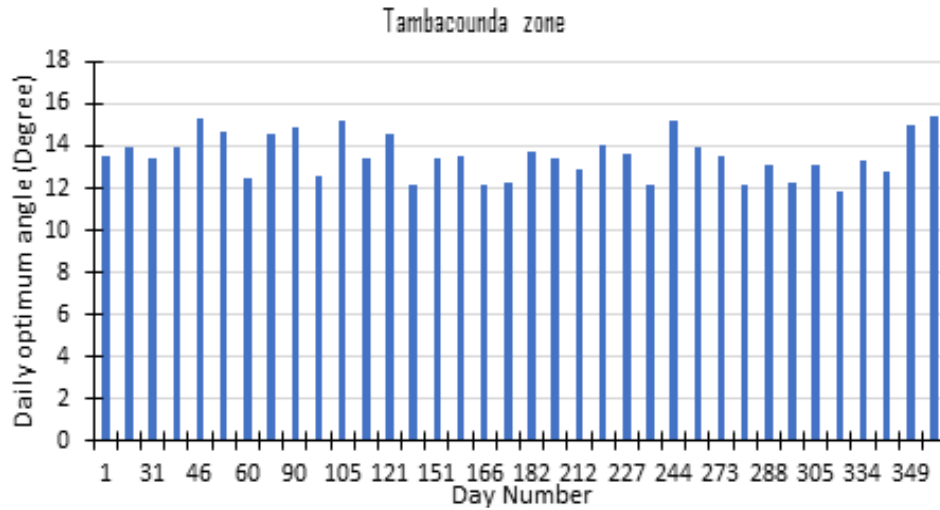
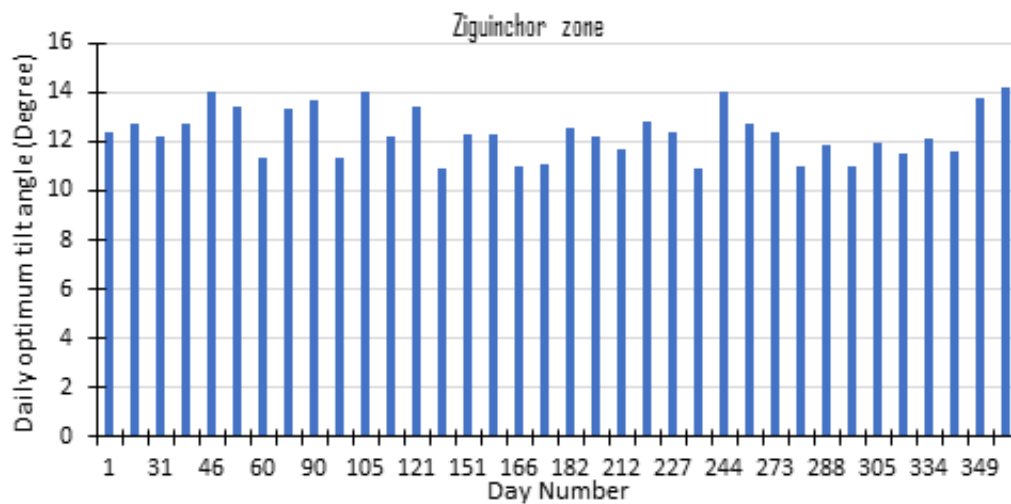**Figure 4.** The variation of daily optimum tilt angle, β_(opt,d) with the day number n for Tambacounda.

Figure 5. The variation of daily optimum tilt angle, β_(opt,d) with the day number n for Ziguinchor.

Table 2. Monthly optimum tilt angle β_(opt,m) for different latitude angles of Senegal.

Lat.	January	February	March	April	May	June	July	August	September	October	November	December	Year
12	11.63	11.75	11.71	11.62	11.63	11.97	11.89	12.24	12.38	12.36	12	12.18	11.95
13	12.63	12.75	12.71	12.62	12.63	12.97	12.89	13.24	13.38	13.36	13	13.18	12.95
14	13.63	13.75	13.71	13.62	13.63	13.97	13.89	14.24	14.38	14.36	14	14.18	13.95
15	14.63	14.75	14.71	14.62	14.63	14.97	14.89	14.24	15.38	15.36	15	15.18	14.86
16	15.63	15.75	15.71	15.62	15.63	15.97	15.89	16.24	16.38	16.36	16	16.18	15.95
17	16.63	16.75	16.71	16.62	16.63	16.97	16.89	17.24	17.38	17.36	17	17.18	16.95

angle for the month of November is equal to the latitude. For this month, a solar collector tilted at an angle equal to the latitude will receive solar radiation normally. Indeed, for the other months, photovoltaic modules need a
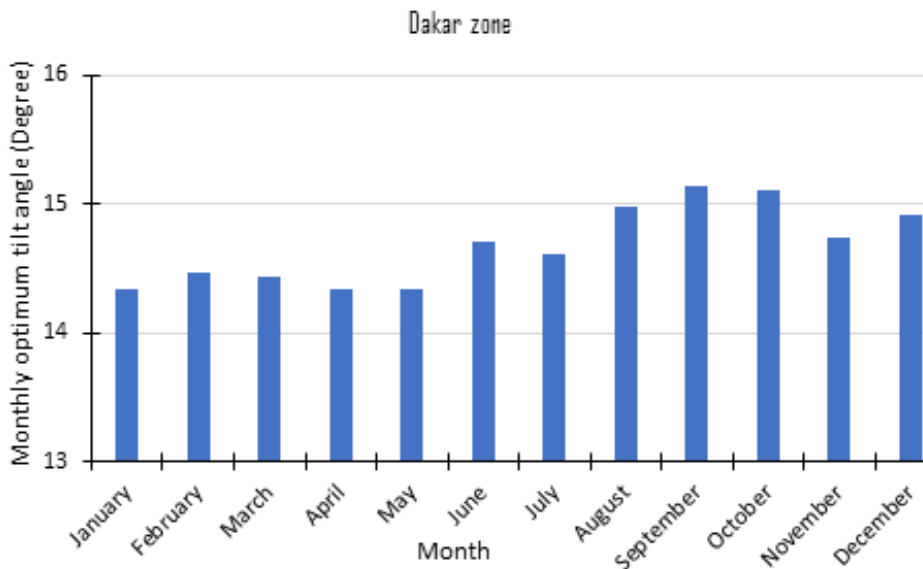
Figure 6. The variation of monthly optimum tilt angle β_(opt,m) for Dakar zone.

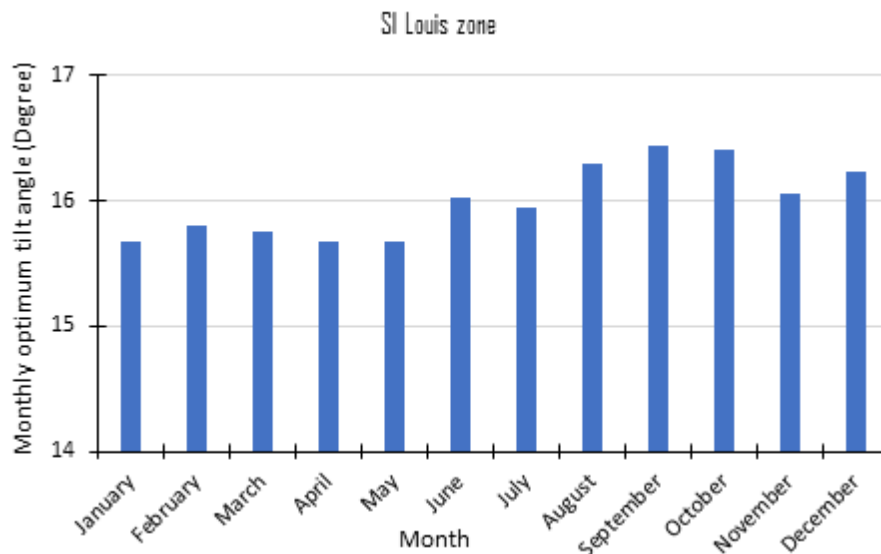**Figure 7.** The variation of monthly optimum tilt angle β_(opt,m) for St Louis zone.

monthly adjustment to produce a maximum of electricity. As a result, energy losses will occur if we take the latitude throughout the year.

Application for four typical climatic zones for monthly optimum tilt angle

The $\beta_{opt,m}$ variation for the four typical climatic zones of Senegal is plotted in Figures 6, 7, 8 and 9. For these different zones too, only the month of November gives

angle values equal to the latitude. We noticed that the optimum tilt angle varies from one month to another and also from zone to zone.

Figure 6 shows the results of the monthly variation of the optimum tilt angle at Dakar. There is a difference of 5.29% between the maximum and minimum values of the optimum tilt angle. Between the latitude and the minimum of the optimum tilt angle this difference is 2.71%. The variance is 2.64% between the maximum of the optimum tilt angle and the latitude.
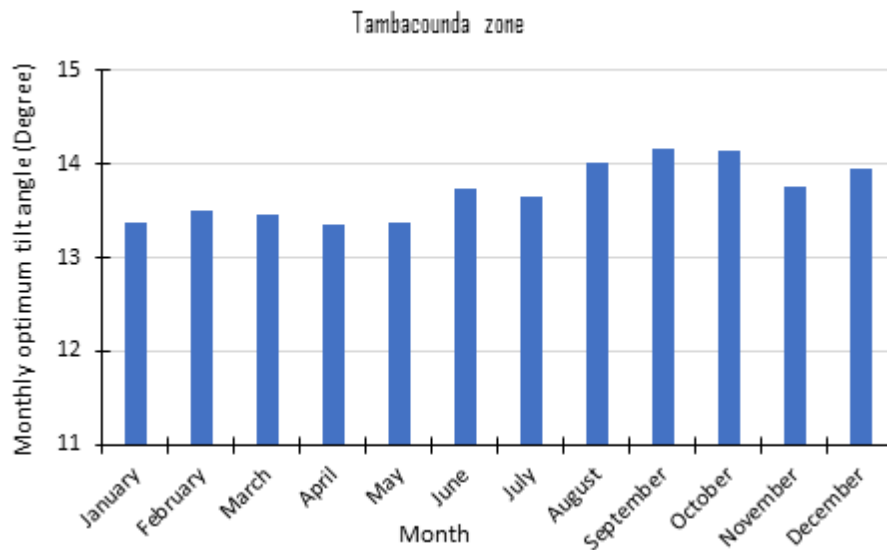
At St Louis the results of the monthly variation of the

Tambacounda zone

Figure 8. The variation of monthly optimum tilt angle β_(opt,m) for Tambacounda zone.
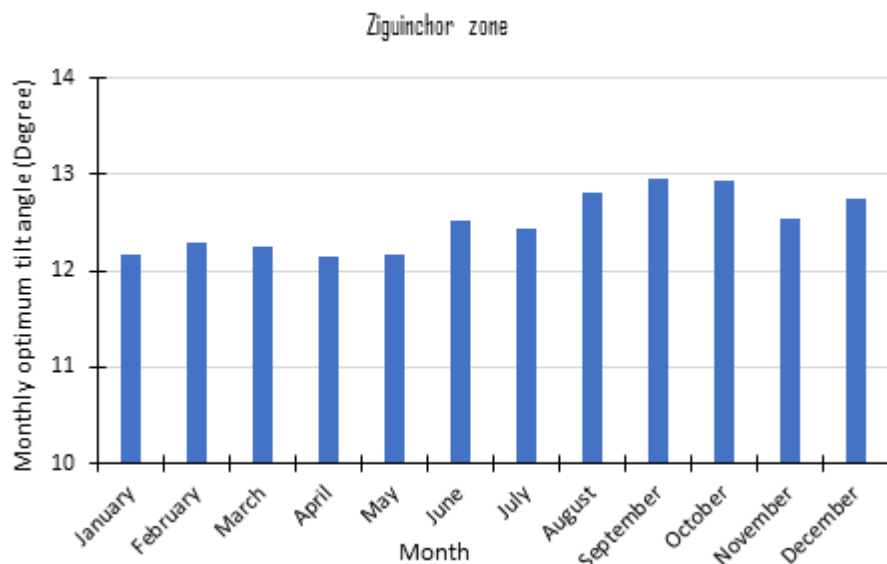
Ziguinchor zone

Figure 9. The variation of monthly optimum tilt angle β_(opt,m) for Ziguinchor zone.

optimum tilt angle are presented in Figure 7. The variance between the maximum and minimum values of the optimum tilt angle is 4.64%. The variance between the latitude and minimum of the optimum tilt angle is 2.37%. The variance between the maximum of the optimum tilt angle and the latitude is 2.32%.

In Tambacounda (Figure 8), the monthly variation between the maximum and minimum values of the optimum tilt angle is 5.68%. In fact, this difference represents 2.93% between latitude and the minimum of the optimum tilt angle, while it is 2.84% between the maximum of the optimum tilt angle and latitude.

In the Ziguinchor area (Figure 9), there is a monthly variation of 6.14% between the maximum and minimum values of the optimum tilt angle. However, between the latitude and the minimum of the optimum tilt angle the difference is 3.17% while it is 3.07% between the maximum of the optimum tilt angle and the latitude.

Conclusion

The optimum tilt angle plays an important role in improving the energy collection of solar collectors. In this

study, the optimum values of tilt angles for solar collectors in Senegal were determined using a mathematical model. The results showed a daily variation of the tilt angle of 20.05% in Dakar, 18.74% in St Louis, 23.43% in Tambacounda and 23.01% in Ziguinchor. On the other hand, the monthly variation of the optimum tilt angle is 5.29% in Dakar, 4.64% in St Louis, 5.68% in Tambacounda and 6.14% in Ziguinchor. The annual optimum tilt angle is approximately equal to the latitude of the location. Comparing these four climatic zones, we note that the variation is less important in St Louis and Dakar than in Ziguinchor and Tambacounda. The best orientation for solar collectors in Senegal is due south. To increase the efficiency of solar collector use, it is recommended, if possible, to mount the solar collector at the average monthly tilt angle and adjust the tilt once a month, as indicated by this study.

CONFLICTS OF INTEREST

The authors have not declared any conflict of interest.

ACKNOWLEDGEMENTS

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