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Short Communication

Some properties of upper fuzzy order

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In this paper we shall study some properties for upper fuzzy subgroups, some lemma and theorem for this subject.

Key words: Fuzzy set, upper fuzzy subgroups, upper normal fuzzy subgroups, upper fuzzy order.

INTRODUCTION

The concept of a fuzzy subset was introduced by Zadeh (1965). Fuzzy subgroup and its important properties were defined and established by Rosenfeld (1971). There are many authors who have studied about it. After this time it was necessary to define upper fuzzy subgroups and upper normal fuzzy subgroups. The notion of the definition of an upper fuzzy subgroup was introduced by Rosenfeld. Many researchers Abd – Allah et al. (1996), Chengyi (1998), Dib and Hassan (1998), Tang and Zhang (2001), Syransu and Ruy (1998), Massa'deh (2008) studied the properties of groups and subgroups by the definition of fuzzy subgroups.

The aim of this paper is to study and prove some properties and theorem for upper fuzzy order. In this paper, G is a group with identity e. Z, N are the integer, and natural number respectively.

Preliminaries

Definition [8]: let X be a set. A fuzzy set μ of X is just a function
$\mu : X \rightarrow [0, 1]$.

Definition [6]: let G be a group and μ be a fuzzy set on G. μ is said to be an upper fuzzy subgroup of G, if for all $x, y \in G$
(i) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(x^{-1}) = \mu(x)$.

Definition [5]: An upper fuzzy subgroup μ of a group G is called upper normal fuzzy subgroups if

$\mu(x^{-1}yx) \leq \mu(y)$, for all $x, y \in G$.
(Equivalently, $\mu(xyx^{-1}) = \mu(y)$ for all $x,y \in G$)
(Equivalently, $\mu(xy) = \mu(yx)$ for all $x,y \in G$).

Definition [5]: Let λ be an upper fuzzy subgroup of G. For any $x \in G$, the smallest positive integer n such that $\lambda(x^n) = \lambda(e)$ is called an upper fuzzy order of x. If there does not exist such n then x is said to have an infinite upper fuzzy order. We shall denote the upper fuzzy order of x by $O(\lambda(x))$.

Example Let G = { e , a , b , ab } be the Klein four group and let

$\mu = \{(e, 1/4), (a, 3/4), (b, 3/4), (ab, 1/4)\}$ be an upper fuzzy subgroup, then

$O(\mu(ab)) = 1$ and $O(\mu(a)) = 2$.

Proposition: Let μ be an upper fuzzy subgroup of the group G. Then for any integer n and $x \in G$, we have $\mu(x^n) \leq \mu(x)$.

RESULTS

Proposition: Let G be a group and let μ be an upper fuzzy subgroup of the group G; let $x \in G$ be of finite order

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k ; if $r \in N$ and k are relatively prime, then $\mu(x^r) = \mu(x)$.

Proof :-

Since r, k is relatively prime, then by Bizout Theorem There exists $a, b \in Z$ such that $1 = ar + bk$; therefore $\mu(x) = \mu(x^{ar+bk}) = \mu(x^{ar} \cdot x^{bk}) = \mu(x^{ar}) \leq \mu(x^{r}) \leq \mu(x)$. Then we get $\mu(x) \leq \mu(x^r) \leq \mu(x)$, therefore $\mu(x^r) = \mu(x)$.

Lemma: Let μ be an upper fuzzy subgroup of the group G, for $x \in G$. If

$\mu(e) = \mu(x^n)$ for some $n \in Z$, then $O(\mu(x))$ divides n .

Proof :-

Suppose that $O(\mu(x)) = k$, then by Euclidean Algorithm, there exists

$a, b \in Z$. Such that $n = ka + b$; $0 \leq b < k$; thus
$\mu(x^b) = \mu(x^{n-ka}) = \mu(x^n (x^k)^{-a}) \leq \max\{\mu(x^n), \mu(x^k)^{-a}\}$
$\leq \max\{\mu(e), \mu(x^k)\} = \max\{\mu(e), \mu(e)\} = \mu(e)$.
Thus $\mu(x^b) \leq \mu(e)$; also $\mu(x^b) \geq \mu(e)$, then we get $\mu(x^b) = \mu(e)$.
Then $b = 0$; also $n = ka$ which is given $O(\mu(x))$ divides n.

Theorem: Let μ be an upper fuzzy subgroup of the group G, and let $O(\mu(x)) = k$, such that $x \in G$. If $t \in Z$ with $d = (t, k)$, then $O(\mu(x^t)) = k | d$.

Proof :-

Suppose that $O(\mu(x^t)) = n$, we get
$\mu((x^t)^{k|d}) = \mu((x^t)^{ka}) = \mu(x^{ka})$; for some integer a
$\leq \mu(x^k) = \mu(e)$

By Lemma 3.2 ; $n | k | d$ and $d = (t, k)$
Then there exists $b, c \in Z$ such that $bt + ck = d$; therefore

$$\begin{aligned}\mu(x^{nd}) &= \mu(x^{n(bt+ck)}) = \mu(x^{nbt} \cdot x^{nck}) \\ &\leq \max\{\mu((x^b)^{nt}), \mu((x^c)^{nk}\} \\ &= \max\{\mu((x^b)^{nt}), \mu(((x^c)^n)^k\} \\ &\leq \max\{\mu(x^b), \mu(x^c)^k\} \\ &\leq \max\{\mu(e), \mu(e)\} = \mu(e)\end{aligned}$$

Therefore $k | nd$ by lemma 3.2; this means that $k | d | n$, consequently $n = k | d$.

Proposition: Let μ be an upper fuzzy subgroup of the group G, let $O(\mu(x)) = m$; $x \in G$. If $k \in Z$ such m, n relatively prime, then $\mu(x^k) = \mu(x)$.

Proof :-

Since m, k relatively prime, $(m, k) = 1$
Then there exists $a, b \in Z$ such that $ma + kb = 1$

$$\begin{aligned}\mu(x) = \mu(x^{ma+kb}) &= \mu((x^m)^a \cdot (x^k)^b) \\ &\leq \max\{\mu(x^m)^a, \mu(x^k)^b\} \\ &\leq \max\{\mu(x^m), \mu(x^k)\} \\ &\leq \max\{\mu(e), \mu(x^k)\} \\ &= \mu(x^k) \\ &\leq \mu(x)\end{aligned}$$

Therefore $\mu(x^k) = \mu(x)$.

Lemma: Let μ be an upper normal fuzzy subgroup of the group G. Then

$O(\mu(x)) = O(\mu(y^{-1} x y))$ for all $x, y \in G$.

Proof :-

Let $x, y \in G$, then we have $\mu(x^m) = \mu(y^{-1} x^m y) = \mu((y^{-1} x y)^m)$, $\forall m \in Z$. Thus $O(\mu(x)) = O(\mu(y^{-1} x y))$.

Remark: If μ is not upper normal fuzzy subgroup of the group G, then above lemma is not true.

Example: Let G be the Dihedral group

$G = \{e, a, b, a^2, ab, ba\}$ and let
$\mu = \{(e, 1/5), (a, 4/5), (b, 1/2), (a2, 4/5), (ab, 4/5), (ba, 4/5)\}$
Since $O(\mu(b)) = 1 \neq 2 = O(\mu(a{-1} b a))$.

Proposition: Let G be a finite group and let μ, λ be an upper fuzzy subgroups, if $\lambda \subseteq \mu$ and $\mu(e) = \lambda(e)$. Then $O(\mu(x)) | O(\lambda(x)) \ \forall x \in G$ such that $O(\lambda(x))$ is finite.

Proof :-

Suppose that $O(\lambda(x)) = k$, then
$\mu(e) = \lambda(e) = \lambda(x^n) \leq \mu(x^n)$, since $\mu(e) \leq \mu(x^n)$ and $\mu(e) = \mu(x^n)$. Thus, $O(\mu(x)) | n$ (by lemma 3.2).

Lemma: Let μ be an upper fuzzy subgroup of the group G and let $x, y \in G$, such that $(O(\mu(x)), O(\mu(y))) = 1$ [that is, $O(\mu(x))$, $O(\mu(y))$ relatively prime] and $xy = yx$. If $\mu(e) = \mu(xy)$ then $\mu(x)$ and $\mu(y) = \mu(e)$.

Proof :-

Suppose that O (μ (x)) = k and O (μ (y)) = m, then μ (e) = μ (xy) $\geq$ μ ($(xy)^n$) = μ ($x^n y^n$), since μ ($x^n y^n$) = μ (e) but

μ (x^n) = μ ($x^n y^n y^{-n}$) $\leq$ max { μ ($x^n y^n$), μ (y^{-n}) } = max { μ ($x^n y^n$), μ (y^n) } = max { μ (e), μ (e) } = μ (e)

Therefore μ (x^n) = μ (y^n) = μ (e), then $k \mid n$ by lemma 3.2. Since k , m relatively prime [(k , m) = 1] thus k = 1 this means that μ (x) = μ (e); the same proof for

μ (y) = μ (e).

Proposition: Let μ be an upper fuzzy subgroup of a cyclic group G and let x , y be two generators of , then O (μ (x)) = O (μ (y)).

Proof:-

Suppose that G is a finite cyclic group with O(G) = m , since G is generated by x and y. Then we get O(x) = O (y) = m, on the other hand y = x n ; n $\in$ Z and we must have (m , n) = 1; thus O (μ (x)) = O (μ (x n)) = O (μ (y)) = n (by theorem 3.3). Note if G is an infinite group, then y = x -1.

Conclusions

We have studied in this paper the definition of the upper fuzzy order over an arbitrary group. Some proposition, lemma and examples given for it and this proposition and lemma are generalization for some properties and theorem in group theory.

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