

*Full Length Research Paper*

# **Volatility measure of Nigeria crude oil production as a tool to investigate production variability**

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The interest to carry out volatility analysis of crude oil production in Nigeria in this paper is motivated by the shortfalls in quantities of crude oil produced in recent past, given the country's high dependence on oil and its contribution to the nation's economic development. In 2016 precisely, the country experienced drastic instability in prices of crude oil at international markets and dwindling production quantities due to vandalism on oil facilities and other corrupt practices in the sector. This paper aims at using volatility measures to investigate variability of crude oil production as an assumed contributor to the economic downturn observed in the recent past in Nigeria. The data used are crude oil production data in millions of barrels collected from NNPC Statistical Bulletin. Variance of the crude oil series has been fitted with ARCH (2) model. ARCH (3) and GARCH (3,3) models are also fitted to the variance of the error obtained from ARIMA(0,1,1). ARCH and GARCH models have shown evidence of volatility in the series. The parameter estimate of the non-linear component of the bilinear model fitted to the crude oil data could not capture volatility clustering. This explains the superiority of ARCH and GARCH model over bilinear model when fitting volatile series. Synonymous with oil price volatility, evidence has it that crude oil production data are volatile. Although, Nigerian government's intervention and negotiations with Niger Delta Militants to end operational attacks on oil facilities yielded positive results, fact has been established that the two economic variables (production and price) are volatile, as contributors to the recent past economic recession in Nigeria. It is important for the stakeholders in the sector and Nigerian government to always exhibit proactive measures against illicit activities that negatively affect oil production at all times and ensure maximum control of crude oil production process and militating factors during price shocks to avoid uncontrollable economic instability.

**Key words:** Crude oil, volatility, ARIMA Model, ARCH, GARCH, BARIMA Model.

## **INTRODUCTION**

The contribution of crude oil to the growth of Nigerian economy has always triggered various interests and discourse by government, organisations, and individuals, on the operations in the upstream, midstream and downstream ventures of the petroleum sector. Through

standard agreement, the country's major petroleum industry, Nigerian National Petroleum Corporation (NNPC) has long been in a joint venture with multinational companies for exploration in the upstream. The raw production of crude has different sources which are

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sometimes described as streams. The streams may be characterised by their locations and grades: onshore (lands, swamps and shallow waters) and offshore (deep sea). The types of crude oil produced in Nigeria include; Farcodos Blend, Bonny Light, Brass Blend, Escravos Light, Qua-Iboe-Light, AGBAMI, Akpo, BONGA, ERHA, Pennington Light, Antan Blend, Amenam Blend, IMA, Okoro, Ukpokiti, Okono, EA Crude, YOHO, ABO, OBE, OKWB, Oyo Blend, Ebok, USAN, Okwibome, Asaramatoro, etc. The aggregation of products from the streams makes up the daily and monthly production capacity periodically published in the Statistical Bulletin of the Nigerian National Petroleum Corporation. Apparently, the aforementioned sources of crude oil in Nigeria do not record equal production capacity. Statistically, the sources that have recorded with a minimum of 5% each to the total production capacity in the country include; Qua-Iboe-Light, Farcodos Blend, Bonny Light, Brass Blend, Escravos Light, AGBAMI, AKPO, BONGA, ERHA and USAN. The concern here is not on the production capacity by stream, but on the variability of the total crude oil production in Nigeria for the period under study. A lot has been discussed about Nigeria crude oil price volatility with little or no research interest in the variability in the production quantity. A few literatures on price volatility include; Afees and Ismail (2012), Alhassan and Kilishi (2016) and Adegbe et al. (2019). Suffice it to say, that the country's interest in the production flow is synonymous with the market prices of crude oil. As mentioned earlier, assertions have been made in different publications by authors about Nigerian crude oil price volatility, but facts have not been established about volatility in the monthly production of crude oil. The variations in the prices of crude oil are influenced by international market mechanism, which involves demand and supply principle and market sentiment. The demand and supply principle is simple and straight from basic economic theory, while market sentiment sometimes has to do with mere belief that oil demand will increase or decrease dramatically at some point in future resulting to price variations (Kosakowski, 2018). Nigeria as a member of Organisation of Petroleum Exporting Countries (OPEC) has no control on oil price deciding factors, but could make policies and come up with control measures over factors that militate against smooth production of crude oil in the country. Moreover, the interest in the production performance is explained by the fact that petroleum proceeds are a major source of financing for Nigeria's budget vis-à-vis the dynamics between quantity and price. Therefore, despite price variation, investigation on the production instability becomes inevitable.

In 2016, there had been dwindling production of crude oil in Nigeria. This was attributed to the incidents of oil thefts, attacks on oil facilities and some deadly operations by militants (Niger Delta Avengers) and sea pirates in the Niger Delta Region of the country. The ups and downs

significantly contributed to high variations in the crude oil exploration and export. Hence, need arose for Nigerian government to have negotiation with Niger Delta Avengers, legal and institutional framework for promoting oil pipeline security and reduce reliance on oil resource for sustainable development, Yomi (2016), Amalachukw and Olaniyi (2017) and Ademola (2017). Incontrovertibly, the high and low changes in the production capacity during the period affected the monthly output of crude oil, government's proposed plans of activities for the period and stability in the economic growth.

The adoption of volatility measure is to succinctly ascertain if there exist variations in the crude oil production to the extent of affecting government economic plan in Nigeria as much as changes in prices of crude oil. Findings from many authors have established volatility in the crude oil prices. Apart from the usual published Statistics by NNPC or NBS, no research focus on ascertaining significant variability in the production flow of Nigerian crude oil and its effect on the economic planning of the country. The volatilities in the oil prices are caused by some physical and financial factors. The physical factor has to do with demand and supply mechanism, weather events, technology, geopolitics, supply interruptions (such as workers strikes, oil spills, vandalism, oil thefts), while financial has to do with the exchange rates, the interest rate, speculation, financial stress index (Algia and Abdelfatteh, 2015). Some of the physical factors are controllable by each exporting country, like in the case of Nigeria when multinational companies in the Niger Delta Region of the country were facing challenges of attacks on oil facilities. The initiative by the Federal Government of Nigeria for intervention through dialogue with the Niger Delta Militants yielded positive results, as the statistics indicated increase in crude oil production towards the last quarter of 2017. This implies that some criminal activities that drastically brought down production capacity are controllable by individual oil production country independent of global activity such as oil price. A pertinent question to be answered in this research is whether crude oil production is accounted for by volatility measures, having in mind that sharp and rapid swings in the production quantity and price of oil have effects not only on the production company, but the larger economy of a nation. In 1970s, United States played a role of a swing supplier of crude oil to ramp up production when demand for oil is rising and curtail output when the market is glutted, thus moderating oil price volatility. For some reason, Saudi Arabia took over the role for about thirty years, but in 2014, it opted out not to curtail production despite plunging oil prices, seemingly abdicating its swing price role (Maurice, 2016). At CFR workshop where a question was raised as to who plays the swing-supplier role between United States and Saudi Arabia, participants argued that US cannot play the role because its recent production has not followed oil prices. It was also

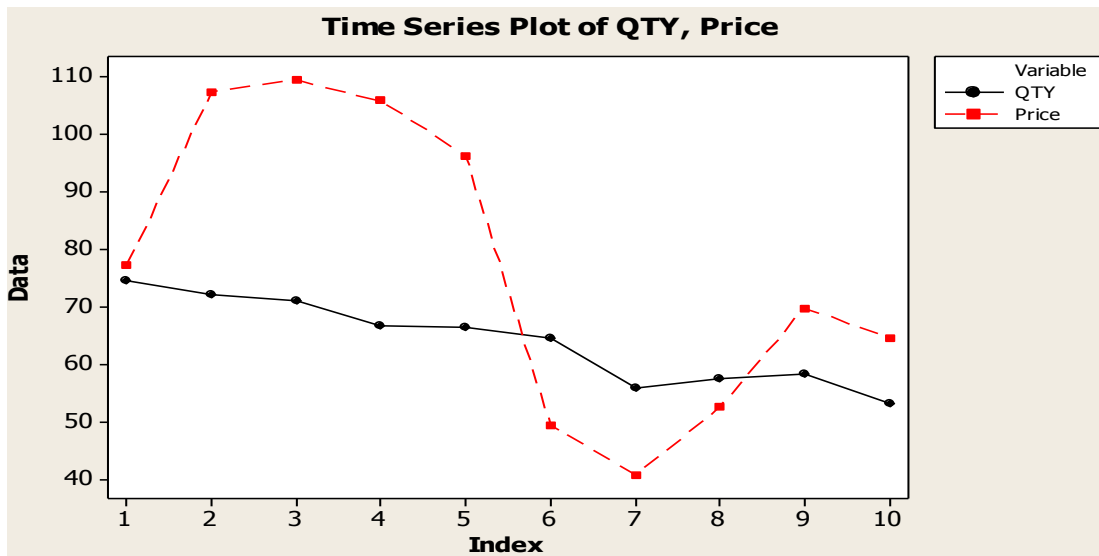


Figure 1. Multiple plots of production quantity and price.

Table 1. Descriptive statistics of crude oil quantity and price.

Var	Mean	SE	Standard Deviation	Variance	Coefficient Variance	Minimum	Maximum	Range
QTY	64.06	2.36	7.46	55.66	11.65	53.26	74.67	21.41
Price	77.36	8.21	25.95	673.39	33.54	40.76	109.45	68.69

production during the period was perceived to have observed that during the sharp downturn in oil prices after mid-2014, US oil production did not drop as expected; only in 2016 did US oil production finally started falling off. That means US oil production had no effect on the global prices as well as oil price volatility.

As shown in Figure 1, the multiple plots contain ten years data from January 2010 to August 2019. The years represent the numbers on the index axis. Although, the two series exhibit downward trend over the period, crude oil price data appear volatile and nonlinear. This does not controvert Afees and Ismail (2012), Alhassan and Kilishi (2016), and Adegbe et al. (2019) along with other literatures on the use of some classical nonlinear models in analysing volatilities. Unlike crude oil price data, the behaviour of the quantity is very uncertain about linearity, hence, the need to investigate variability through volatility measures. Though, crude oil production quantity appears smoother than price in Figure 1, findings in this research will reveal presence or absence of volatility of the crude oil quantity data.

The basic statistics in Table 1 characterises significant variations in the price. With the basic statistical measures in the table, the two economic variables seem to be at variance with each other. However, this does not negate the possibility of volatility in the crude oil production. The interest in this paper is to investigate variability in the

crude oil production through volatility measure, like it was established in the crude oil price, given the incidents of vandalism, oil thefts and other corrupt practices in the upstream venture of the sector. In other words, volatility presence in the crude oil production means there is significant variability in the crude oil data, therefore establishing strong similarity and dependence among the two economic variables as joint contributors to the economic downturn in the recent past in Nigeria. Volatility measures the dispersion or variation of some value points from its central mean value. The reason behind the idea of volatility clustering in the crude oil production is that variations were perceived to be high because of operational challenges caused by incessant attacks on multinational companies, and low during the calm period.

## REVIEW OF LITERATURE

### Related research

Many research findings in time series analysis have sought to compare the performance of linear and bilinear models when analysing some economic and financial time series. Some time series data which include; consumer price index, gross domestic product, inflation, foreign exchange, bank deposits, interest rates and other

data generated from micro and macroeconomic events have been empirically used to examine the suitability of some classical time series models. Assertions have been made on the advantage of bilinear models over linear models in fitting such economic and financial data due to some non-linearity characteristics accounted for by large positive and negative observations in most of the series. Among the assertions on bilinear performance over linear models include Maravall (1983), and Subba and Gabr (1984). Their works confirmed better performance of bilinear models. In addition to these are Usoro and Omekara (2008) whose work sought to fit linear and bilinear models to internally generate revenue of a local government area in Nigeria. The correlogram of the error terms from the two models placed bilinear models better than the ordinary linear autoregressive moving average models. One of the basic assumptions in econometric and statistical analyses is that of constant variance. In most cases, because of certain natural phenomena about the variables of interest, the assumption of constant variance (homoscedasticity) is seldom fulfilled. This is evident in the exhibition of wide swings of the series after first difference, suggesting volatility or variance of the series is not constant over time, (Gujarati and Porter 2009). Contributions have been offered on volatility measures with many models including ARCH, GARCH, EGARCH models, etc. Few to mention here include Bala and Asemota (2013), and Yayah (2013). Bala and Asemota (2013) fitted GARCH model to exchange rate, while Yayah (2013) applied the GARCH model on Nigerian Stock Index. Omotosho and Doguwa (2012) used different models to analyse inflation in Nigeria. The models included GARCH, TGARCH and EGARCH models. Isenah et al. (2013) used ARMA-GARCH model to assess the volatility of Nigerian Stock Market. From the classes of models fitted, ARMA (1,2) - GARCH (1,1) model was the best. Suliman (2012) fitted GARCH(1,1) to exchange rate volatility with empirical evidence from Arab countries. The work applied both symmetric and asymmetric models to capture facts about volatility clustering and leverage effect of the exchange rate returns. Koima et al. (2015) analysed Kenyan stock market using GARH model. The result showed evidence of volatility clustering over time. Others on volatility measures include Dana (2016), Yayah et al. (2015), Carroll and Goodman (2011) and Garrett (2009).

In addition to the above literature, there are contributions on relationship between crude oil price volatility and other economic variables. Ogundipe et al. (2014) examined effect of oil price, external reserve and interest rate on exchange rate volatility in Nigeria. Johansen co-integration technique was adopted. Vector correction mechanism used established long run relationship among the variables. The results showed that oil price volatility results to exchange rate volatility. He admitted that dwindling in oil production affects macroeconomic stability. The work did not investigate

volatility in crude oil production. On oil price volatility and economic growth, Nwanna and Eyedayi (2016) investigated impact of oil price volatility on economic growth, whereas Umar et al. (2016) carried Granger Causality test to ascertain unidirectional causality between oil price and economic growth in Nigeria. Between oil price volatility and money market rate, Emenike (2017) analysed and confirmed volatility presence in oil price and money market rate. Evidence of unidirectional volatility spillovers from crude oil price to money market rate in Nigeria was established.

### Crude oil production profile

The statistics of crude oil production quantity from the Nigerian National Petroleum Corporation Annual Statistical Bulletin (NNPC ASB) gives a clear analysis of crude oil production with dwindling quantities of production in specific periods.

Figure 2 shows a downward trend of crude oil production for the period from the data in Appendix1. The analysis indicates approximately average decreasing rate of 0.2 m (200,000) barrels of crude oil production per month from January 2010 to August 2019. Precisely, 2016 experienced significant decrease with crude oil production average of about 55.84 millions of barrels per month as against the previous years 2015, 2014, 2013, 2012, 2011 and 2010 with average productions of 64.45, 66.55, 66.71, 71.06, 72.19, and 74.67 millions respectively. This experience was in a short while, as gradual improvements were observed in 2017 and 2018 with average monthly productions of 57.48 and 58.37 millions respectively, but dropped in the first quarter of 2019 as the production recorded 51.03, 47.32 and 53.69 million barrels in January, February and March respectively. The month of April and May 2019 recorded 51.39 and 51.12 million barrels, and improved from June, July and August with 55.86, 56.64 and 59.06 million barrels respectively. So far, the average monthly production from January to August 2019 is 53.26 million barrels. The crude oil prices from May to August 2019 recorded 73.65, 69, 61 and 61 US\$/Barrel, respectively. The observation here is that even in the year 2019, the movement of prices of crude oil were experiencing decrease from May to August, while production quantities experienced gradual increase. So, one is not certain about the similarities and variations that exist among the two economic variables until facts are established using volatility measures. Looking at Figure 2, crude production data require differencing to attain stationarity. In this paper, we assess the volatility of the crude oil production return series and volatility of the error variance using ARCH and GARCH models. We intend to also investigate non-linearity characteristics using bilinear model.

In this paper, volatility of the crude oil production return series and volatility of the error variance were assessed

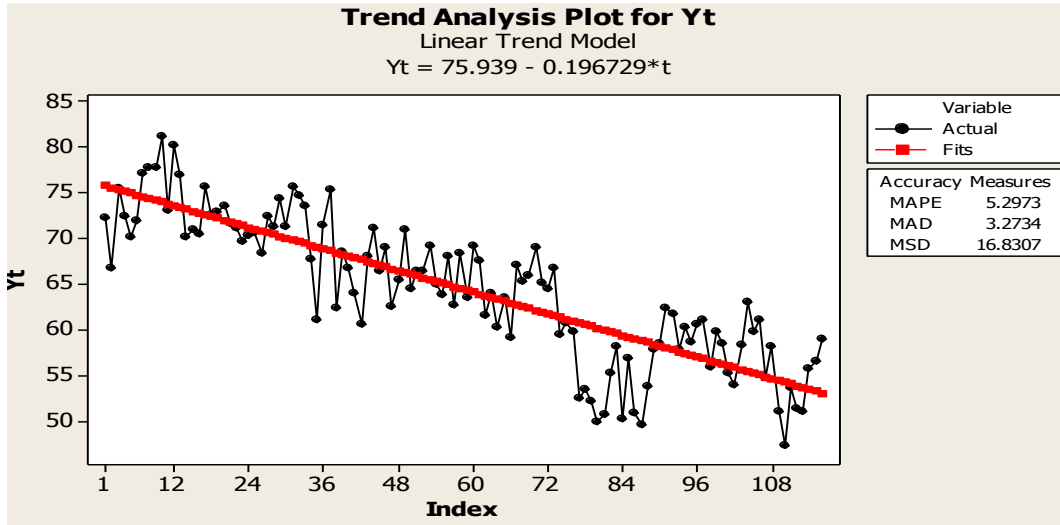


Figure 2. Trend analysis of original monthly crude oil series.

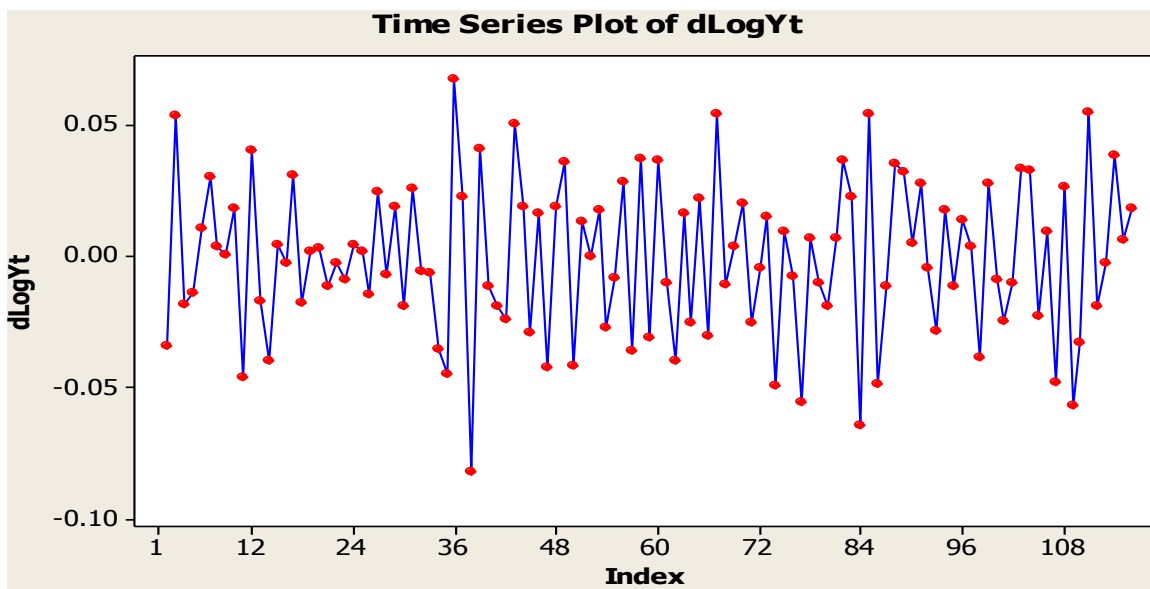


Figure 3. Plot of the return series.

using ARCH and GARCH models. We intend to also investigate non-linearity characteristics using bilinear model.

**METHOD OF ANALYSIS**

**Volatility measure from the return series**

Given  $Y_t$  = Crude oil production quantity

$$Y_t^* = \log \text{ of } Y_t, \tag{1}$$

$$dY_t^* = Y_t^* - Y_{t-1}^* \tag{2}$$

$$X_t = dY_t^* - d\bar{Y}_t^* \tag{3}$$

Equation 2 is the relative change in the crude oil production quantity also known as the return series of the crude oil production quantity, whereas Equation 3 is the mean-adjusted relative change in the crude oil production quantity. The square of  $X_t$ , that is  $X_t^2$  is now used as the measure of volatility (Gujarati and Porter, 2009).

From Figure 2, there are considerable ups and downs along the downward trend in the crude oil production

data. The ups and downs movements have vividly shown in Figure 3, with some periods of wide wings in 2013, 2016 and 2019, thus exemplifying the phenomenon of volatility clustering.

Given the following ARCH(p) model,

$$X_t^2 = \beta_0 + \beta_1 X_{t-1}^2 + \beta_2 X_{t-2}^2 + \dots + \beta_p X_{t-p}^2 + U_t \quad (4)$$

where,  $X_t^2$  = measure of volatility from the crude oil return series,  $\beta_0$  = constant,  $\beta_1, \beta_2, \dots, \beta_p$  = parameters of volatilities at  $t-1, t-2, \dots, t-p$  time periods respectively,  $U_t \sim N(0, \sigma_{U_t}^2)$ . Parameters of "4" can be estimated with ordinary least squares regression method. Volatility is detected if any of  $\beta_1, \beta_2, \dots$  is significant.

### Volatility measure from ARIMA ARCH/GARCH model

Here, we consider volatility measure from the residual term of ARIMA (p,d,q)-ARCH (p) or ARIMA (p,d,q) – GARCH (p,q), (Yayah et al., 2015). ARCH or GARCH model is fitted to the variance of the residual from the fitted ARIMA model. Normally, the distributions of autocorrelation and partial autocorrelation functions of the integrated crude oil series suggest appropriate ARIMA model for the data. A pure autoregressive model is suggested if ACF decays exponentially and PACF exhibits a significant cut-off mostly within the first two lags. For MA model, PACF decays gradually and ACF exhibits a significant cut-off within the first two lags. ARMA model is suggested if the original series (without differencing) shows cut-off both in ACF and PACF. If differencing is carried out on a series to become stationary, with significant cut-offs both in ACF and PACF, ARIMA model is suggested, and it is called Autoregressive Integrated Moving Average, ARIMA (p,d,q) model. The word "Integrated" comes in because of differencing to make the series stationary before fitting ARIMA model to data. This implies that data require stability before fitting ARIMA, ARCH and GARCH models (Kendall and Ord 1990).

The patterns of ACF and PACF suggest ARIMA (0,1,1), because ACF cut-off at lag1 of the lower bound of  $(\pm \frac{2}{\sqrt{n}})$  and PACF decays exponentially to zero. The "n" is the total number of data points in the crude oil series. The red dotted lines in the ACF and PACF indicate upper and lower limits, representing  $(\frac{2}{\sqrt{n}})$  and  $(-\frac{2}{\sqrt{n}})$  respectively. From the fitted ARIMA (0,1,1), ARCH and GARCH models are developed for the error term, which is distributed conditionally on the information available at time (t-1) as  $u_t \sim N[0, (\gamma_0 + \gamma_1 u_{t-1}^2)]$ . If  $u_t$  is distributed with zero mean and  $\text{var}(u_t) = (\gamma_0 + \gamma_1 u_{t-1}^2)$ , then the variance of  $u_t$  follows an ARCH (1) process. The general ARCH(p) model is

$$\text{Var}(u_t) = \sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \gamma_2 u_{t-2}^2 + \dots + \gamma_p u_{t-p}^2 \quad (5)$$

where  $\gamma_i$  are the parameters of the lagged term of the squared error. Hypothetically, ARCH effect is present in the series if any of  $\gamma_1, \gamma_2, \dots$  is significant from "t" and "p" values. Otherwise, there is no volatility, which would now suggest stability in the crude oil production for the period under study.

Another popular model for volatility measure is Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model. In ARCH model, the variance of the error term is dependent only on the time lag of the squared error, while in GARCH model, the variance of the error  $\sigma_t^2$  is dependent upon the time lag of the squared error term and its variance. The general GARCH (p,q) model is

$$\sigma_t^2 = \gamma_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_p u_{t-p}^2 + \phi_1 \sigma_{t-1}^2 + \phi_2 \sigma_{t-2}^2 + \dots + \phi_p \sigma_{t-p}^2 \quad (6)$$

From Equation 6, the usual test is conducted through OLS parameter estimates. ARCH/GARCH is present if the parameter(s) are significant.

### Volatility measure from BARIMA model

Bilinear time series model is a model used in fitting of time series data that have some non-linearity characteristics. The model is made up of two parts; the linear and non-linear parts. The linear part is the aggregation of the popular autoregressive and moving average processes, while the non-linear part is the product of the two processes (Usoro, 2017). The general bilinear time series model as defined by Kendall and Ord (1990), Bibi and Oyet (1991) and Iwueze (2002) is

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t \quad (7)$$

where,  $X_t$  is the bilinear process,  $X_{t-i}$  and  $\epsilon_{t-j}$  are time varying autoregressive and moving average processes respectively, with  $\phi_i$  and  $\theta_j$  as their respective parameters;  $\beta_{ij}$  are the non-linear parameters of  $X_{t-i} \epsilon_{t-j}$ ,  $1 \leq i \leq p, 1 \leq j \leq q$  and  $1 \leq i \leq P, 1 \leq j \leq Q$ ;  $\epsilon_t$  is the error term,  $\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)$ . Model 7 is a difference equation for a time varying bilinear process of order (p, q, P, Q), also known as BL(p, q, P, Q). Once ACF and PACF are obtained, the order of ARIMA model is applicable to BARIMA model. The difference is that BARIMA model has a non-linear component to complete its form; one part is linear and the other is non-linear, while the ARIMA model does not have non-linear component. The idea behind the adoption of bilinear model in this research finding is that the presence of volatility in the crude oil series could manifest in the non-linear component of the bilinear model, and it is shown if the parameter estimate of the non-linear part of the model is significant ( $\beta_{ij} \neq 0$ )

**Table 2.** Regression estimates with two independent variables.

Predictor	Coefficient	SE Coefficient	t	P
Constant	0.000593	0.0001462	4.05	0.000
$X_{t-1}^2$	0.08571	0.09198	0.93	0.353
$X_{t-2}^2$	0.19564	0.09218	2.12	0.036

**Table 3.** ANOVA of the regression model.

S.V	D.F	Sum of Squares	MS	F	P
Regression	2	0.000006691	0.000003346	2.93	0.057
Residual	110	0.000125388	0.000001140		
Total	112	0.000132079			

**Table 4.** Regression estimates with one independent variable.

Predictor	Coefficient	SE Coefficient	T	P
Constant	0.0006581	0.0001281	5.14	0.000
$X_{t-2}^2$	0.20495	0.09159	2.24	0.027

Usoro and Omekara (2008).

## ANALYSIS AND INTERPRETATION

### Estimation of parameters of ARCH model

From model 4, the parameters of ARCH(2) is estimated by ordinary least squares method with the following regression analysis and analysis of variance for overall fitness of the model. Table 2 shows the estimated coefficients of volatility measure. From Table 2, the estimated model becomes,

$$X_t^2 = 0.000593 + 0.0857X_{t-1}^2 + 0.196X_{t-2}^2 \quad (8)$$

Where  $X_t^2$  is a volatility measure. Equation 8 is the estimated ARCH (2) model. Table 3 gives overall fitness of the model. Though, the parameter estimate of the first lagged variable  $\beta_1$  is not significantly different from 0, but  $\beta_2$  is significant, given the “t” and “p” values in Table 2. The order of the model stops at 2 (two), because other coefficients after the first two lags are not insignificant. From the estimated model, at least one parameter estimate is significant, exemplifying the phenomenon of volatility clustering in the crude oil production data (Gujarati and Porter, 2009).

Further regression with only significant parameter ( $\beta_2$ ) produces the following model

$$X_t^2 = 0.000658 + 0.205X_{t-2}^2 \quad (9)$$

The analysis in Table 1 shows that the parameter of the second lagged variable is significant, given its “t” and “p” values. Table 4 provides estimate with one independent lagged variable as shown in the “t” and “p” values. With the stepwise regression, Table 2 is modified, confirming the adequacy of the model as shown in Table 5. The adequacy is achieved after dropping the first lagged variable with insignificant parameter. The probability and histogram plots in Figures 6 and 7 characterise normality of the residual values. Furthermore, the usual assumption of the error term to confirm model adequacy is met. The residual of the estimated ARCH model is a white noise process; that means presence of ARCH effect in the series.

### Parameter estimates of ARIMA (p,d,q)-ARCH(p) and GARCH(p,q)

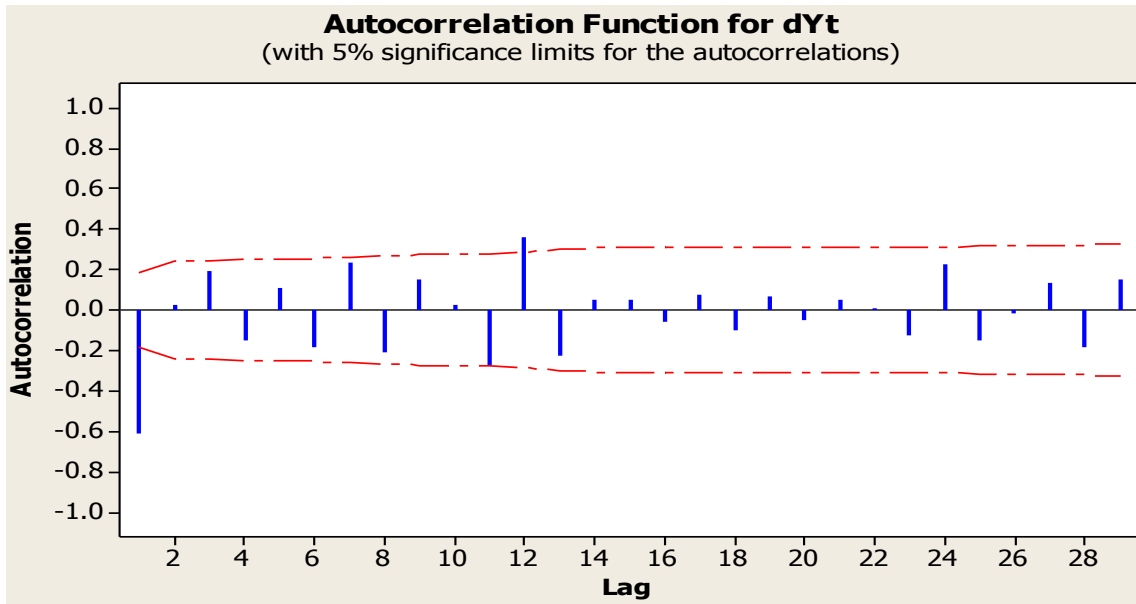
The autocorrelation and partial autocorrelation functions in Figures 4 and 5 respectively suggested ARIMA (0,1,1). Estimates in Table 6 provide the following model,

$$X_t = -0.1304 + 0.5536\epsilon_{t-1} \quad (10)$$

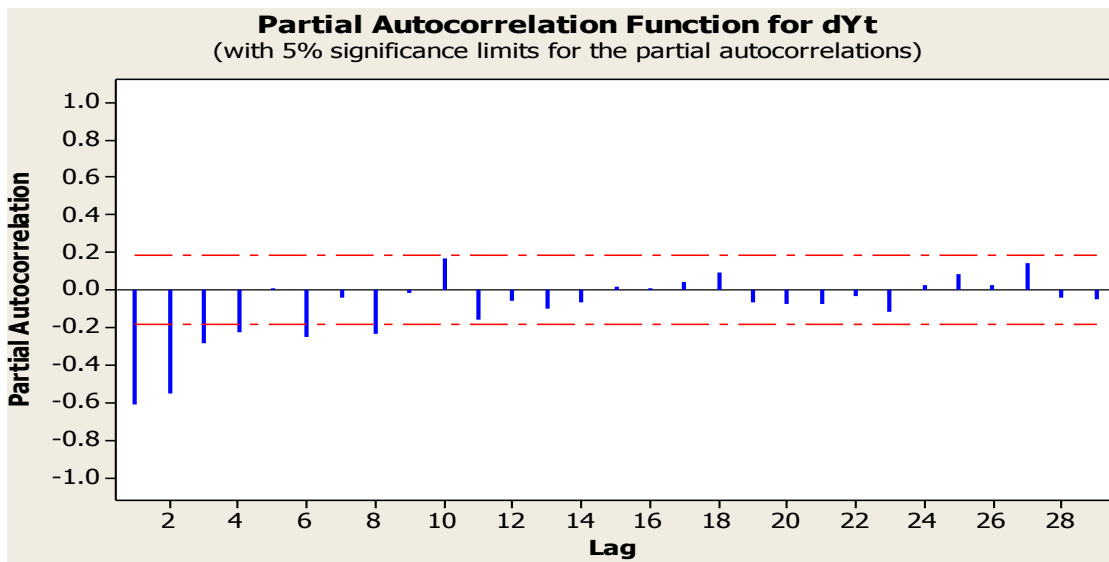
The estimates for the residual in Table 6, with p-values less than 0.05 indicate significant of the residual values (also see Table 7). This further implies that the residual is auto-correlated, which ought not to be. The auto-correlation and partial autocorrelation functions in Figures 8 and 9 exhibit some cut-offs at certain lags, indicating a

**Table 5.** ANOVA of the regression model.

S.V	D.F	Sum of Squares	MS	F	P
Regression	1	0.000005701	0.000005701	5.01	0.027
Residual	111	0.000126377	0.000001139		
Total	112	0.000132079			



**Figure 4.** Autocorrelation functions of the stationary crude oil data.



**Figure 5.** Partial autocorrelation functions of the stationary crude oil data.

non-white noise process of the residual. This account for the volatile nature of the crude oil production data, thereby



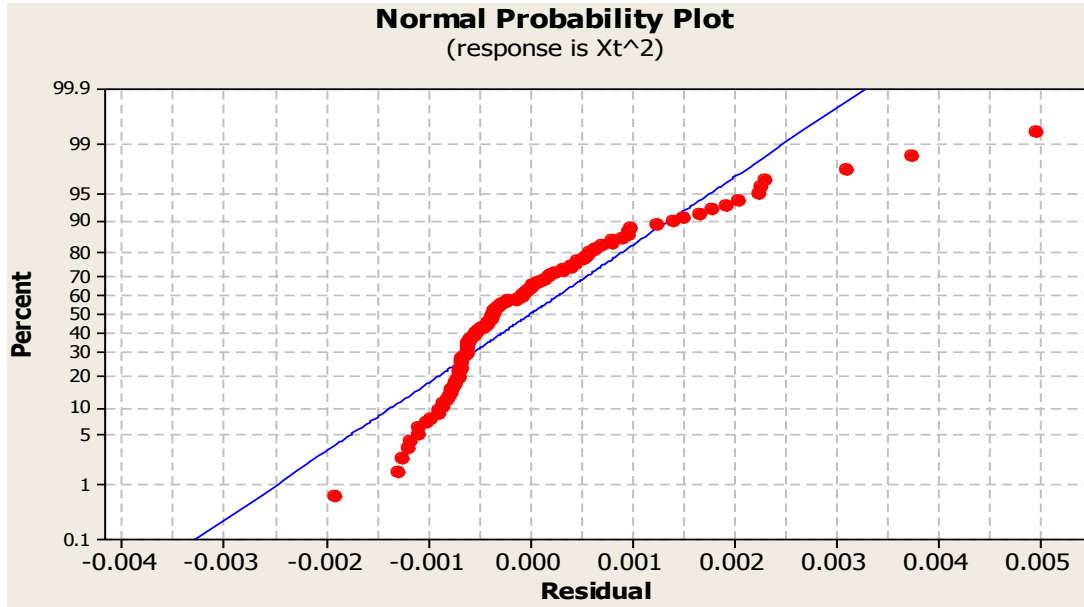


Figure 6. Normal probability plot of residual.

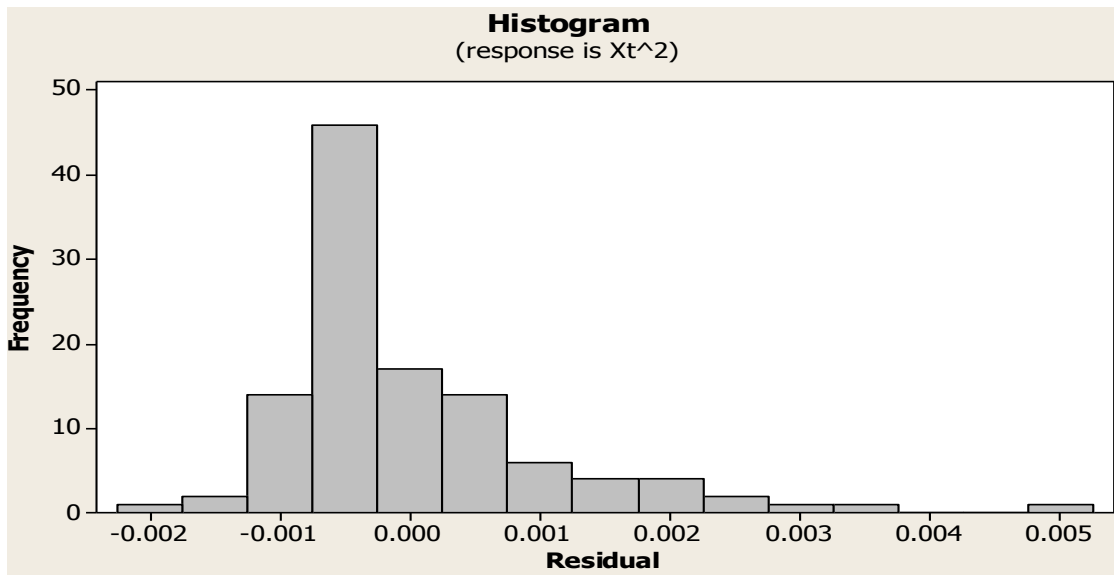


Figure 7. Histogram of the residual.

suggesting volatility measures as provided in Table 8.

$$\sigma_t^2 = 9.563 + 0.13567U_{t-1}^2 - 0.2853U_{t-2}^2 + 0.21083U_{t-3}^2 \tag{11}$$

$$\sigma_t^2 = 10.129 - 17.23U_{t-1}^2 + 19.78U_{t-2}^2 + 76.25U_{t-3}^2 + 17.37\sigma_{t-1}^2 - 19.81\sigma_{t-2}^2 - 76.09\sigma_{t-3}^2 \tag{12}$$

Equations 11 and 12 are estimates of ARCH (3) and GARCH (3,3) respectively.

The above estimates indicate  $U_t^2$  and  $\sigma_t^2$  are significant each at (t-3). At least one parameter estimate is significant in each of the ARCH and GARCH models, indicating presence of volatility clustering in the crude oil production data. If the insignificant parameter estimates in Equations 11 and 12 are dropped, the models become

$$\sigma_t^2 = 11.074 + 0.20841u_{t-3}^2 \tag{13}$$

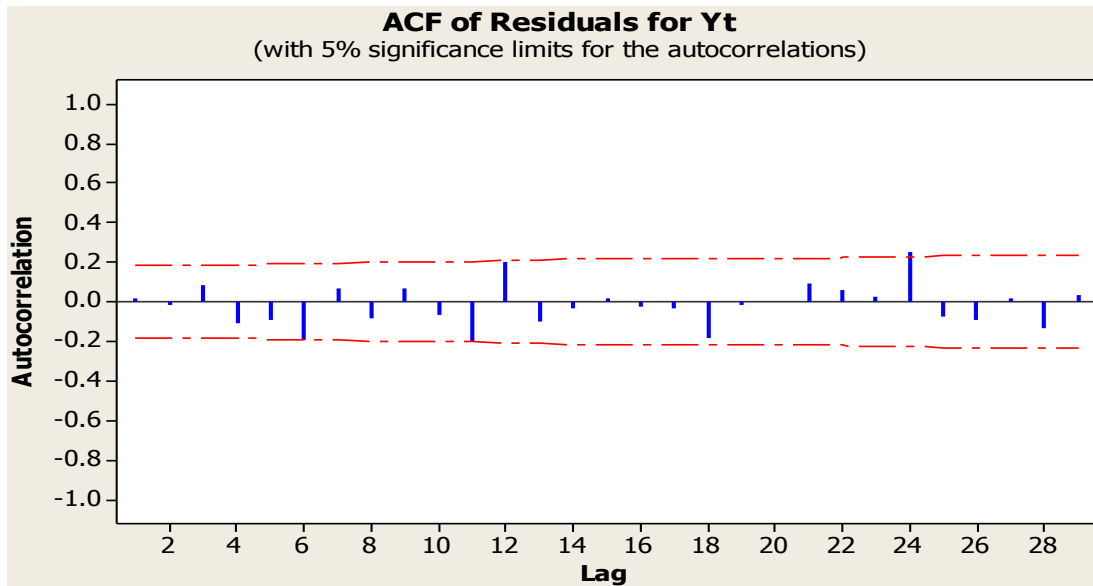
$$\sigma_t^2 = 11.621 + 78.44U_{t-3}^2 - 78.28\sigma_{t-3}^2 \tag{14}$$

**Table 6.** Parameter estimates of ARIMA(0,1,1)-ARCH(3) and GARCH(3,3).

ARIMA(0,1,1)	Coefficient	SE Coefficient	t	p
Constant	-0.1304	0.1588	-0.82	0.413
MA(1)	0.5536	0.0797	6.95	0.000

**Table 7.** Box-Pierce (Ljung-Box) chi-square statistic for the residual.

Lag	12	24	36	48
Chi-Square	20.7	38.1	61.2	77.7
DF	10	22	34	46
P-Value	0.023	0.018	0.003	0.002



**Figure 8.** Autocorrelation function of the residual.

The above autocorrelation and partial autocorrelation functions in Figures 10, 11, 12, and 13 confirm a pure white noise process for residual of models 13 and 14. Hence, volatility clustering in the series is captured by the ARCH and GARCH models.

**Parameter estimates of BARIMA (p, q, d, P, Q) model**

Here, we consider bilinear model BL(p,q,d,P,Q), where p and q represent the order of the linear autoregressive and moving average processes, P and Q represent the order of the nonlinear autoregressive and moving average processes, d represents the order of difference. The choice of order of bilinear models is derived from the usual ARIMA process, where autocorrelation and partial

autocorrelations become the tools for model suggestion. From Equation 7,

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t$$

The first two expressions on the RHS of Equation 7 give the usual ARMA (p, q) model for a complete linear process without difference (d). It is the sum of the autoregressive and moving average processes. The last expression is the product of the two processes, which accounts for nonlinear component of the series. The aggregation of the linear part and nonlinear part produces bilinear model. From Equation 7, if  $i = 0$ , which expresses the pure moving average of the series, the

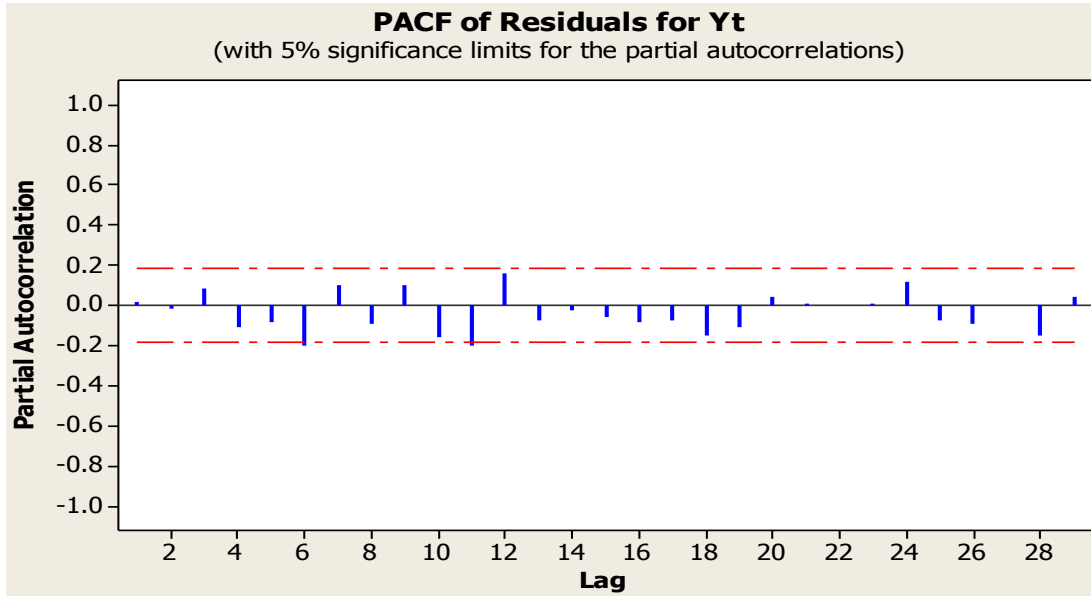


Figure 9. Partial autocorrelation function of the residual.

Table 8. Parameter estimates of ARCH(3) and GARCH(3,3).

Model	Coefficient	SE Coefficient	t	P
<b>ARCH(3)</b>				
Constant	9.563	2.604	3.67	0.000
$U_{t-1}^2$	0.13567	0.09358	1.45	0.150
$U_{t-2}^2$	-0.2853	0.09358	-0.30	0.761
$U_{t-3}^2$	0.21083	0.09276	2.27	0.025
<b>GARCH(3,3)</b>				
Constant	10.129	2.597	3.90	0.000
$U_{t-1}^2$	-17.23	36.48	-0.47	0.633
$U_{t-2}^2$	19.78	35.76	0.55	0.581
$U_{t-3}^2$	76.25	35.73	2.13	0.035
$\sigma_{t-1}^2$	17.37	36.50	0.48	0.635
$\sigma_{t-2}^2$	-19.81	35.78	-0.55	0.581
$\sigma_{t-3}^2$	-76.09	35.75	-2.13	0.036

above model becomes,

$$X_t = \mu + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=0}^0 \sum_{j=1}^Q \beta_{0j} X_t \epsilon_{t-j} + \epsilon_t \quad (15)$$

Further justification to fit BARIMA Model to the series includes; David (2019), Anthony (2018), and Anthony and Awakessien (2018). Equation 15 is Bilinear Moving Average, BARMA (0, q, d, 0, Q) or BMA (q, d, Q).

For a special case, where  $j = 1, "15"$ , parameters are estimated as shown in Table 9 with the following model

$$X_t = -0.002151 - 0.0102193\epsilon_{t-1} - 0.01462X_t\epsilon_{t-1} + \epsilon_t \quad (16)$$

Equation 16 is the bilinear analogous to ARIMA (0,1,1) earlier estimated.

Tables 9, 10 and Figure 12 appear to give good estimate and model adequacy. Figure 13 exhibits autocorrelation of the residual term with some spikes. The residual is not a white noise process. This is explained by the non-significant parameter of the nonlinear part of the estimated bilinear model. Unlike ARCH and GARCH

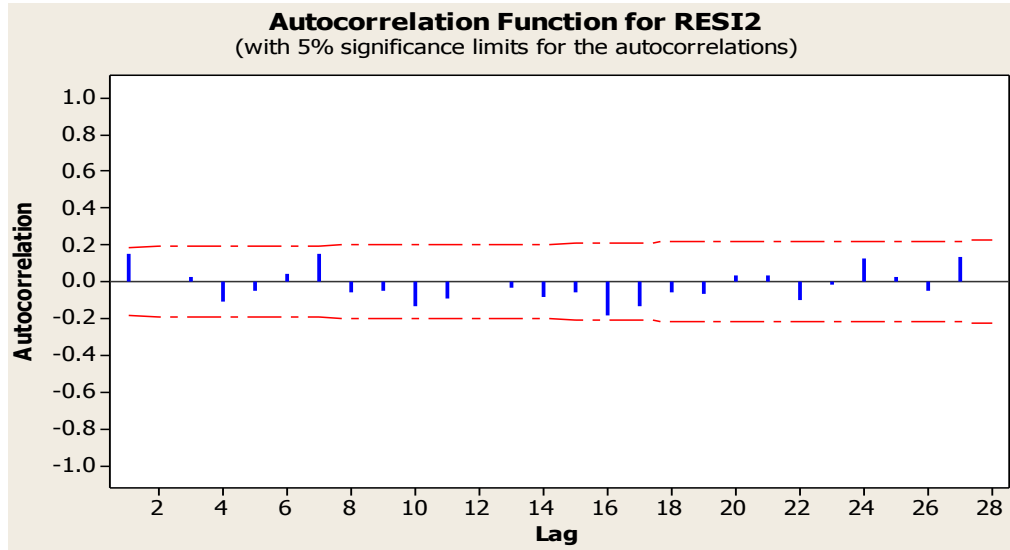


Figure 10. Autocorrelation function of the residual of Model 13.

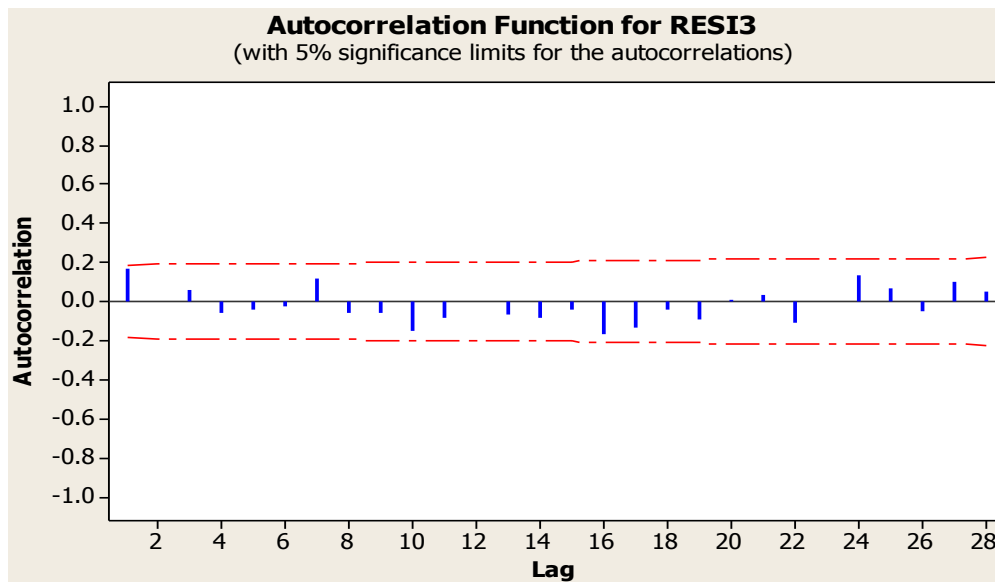


Figure 11. Autocorrelation Function of the residual of Model "14"

models, the estimated bilinear model could not take care of the volatile nature of the crude oil series. For the crude oil production quantity, ARCH and GARCH models adopted are superior to bilinear model.

**Conclusion**

There is no gainsaying the fact that Nigeria dependence on oil as the mainstay of the economy has always been generating much concern by the government on the

crude oil production and sales as the major determining factors to amount of revenue generated from the country's petroleum sector. The country's budget proposal in every fiscal year is usually prepared on the basis of the estimated oil production capacity and projected international market price. The dwindling prices of crude oil at international market triggered much researches and publications on crude oil price volatility. The concern in this paper was to consider the measure of variability of crude oil production in view of the reduction in the exploration quantity between the years 2010 and

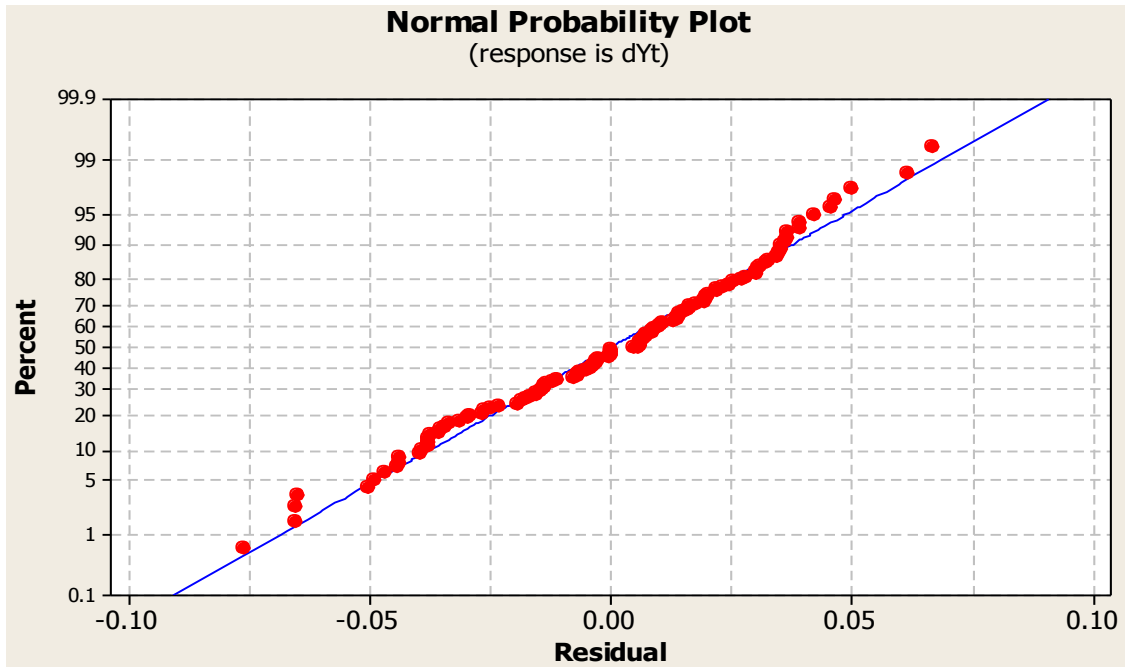


Figure 12. Normal probability plot of residual of the BARIMA (0,1,1,0,1).

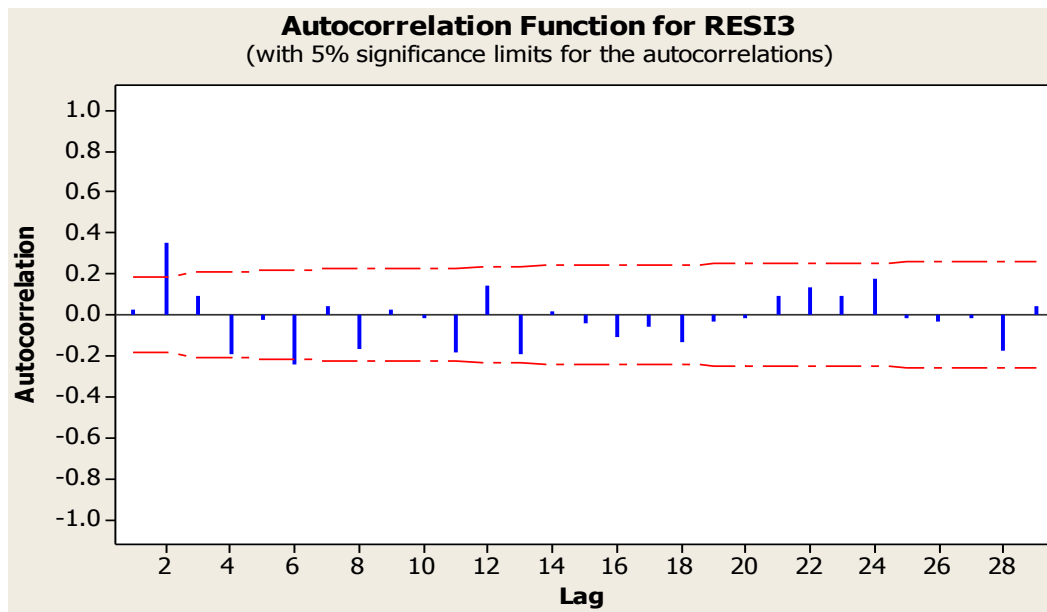


Figure 13. Autocorrelation function of the residual of Model 16.

Table 9. Parameter estimates of BARIMA (0,1,1,0,1) Model.

BARIMA(0,1,1,0,1)	Coefficient	SE Coefficient	t	P
Constant	-0.002151	0.003420	-0.63	0.531
$\epsilon_{t-1}$	-0.0102193	0.0007571	-13.50	0.000
$X_t \epsilon_{t-1}$	-0.01462	0.01364	-1.07	0.286

**Table 10.** ANOVA of the BARIMA (0,1,1,0,1) regression model.

S.V	D.F	Sum of Squares	MS	F	P
Regression	2	0.175498	0.087749	100.00	0.000
Residual	111	0.097402	0.000877		
Total	113	0.272900			

2019. Variability was perceived to have been accounted for by the drastic reduction in crude oil production in the years 2013, 2016 and first quarter of 2019 as shown in Figure 1 above. These periods were very challenging to Nigerian Government and multinational companies that were engaged in the exploration activities due to increasing cases of attacks on oil facilities in the Niger Delta Region of Nigeria.

From the analysis, the ARCH and GARCH models fitted to the squared crude oil data and error variance have shown evidence of volatility in the crude oil production data. The bilinear time series model fitted to the crude oil data was purposely to display non-linearity characteristics in the data, which in most cases are explained by high variability in series. The non-linear parameter of the bilinear model is not significant. Long term decrease in the quantity was observed during the period. This does not indicate absence of shocks in the production quantity of crude oil. Rather, it shows the superiority of ARCH and GARCH models over bilinear model. The parameter of the nonlinear part of the bilinear model could have been significant if the variability in the series was probably explosive in nature. ARCH and GARCH models have overcome the limitation of bilinear, as the research findings confirm evidence of volatility in crude oil production quantity, though not in the same volatility level with crude oil prices as evident in Figure 1. The observed decrease in the production was very alarming in 2016. Although, there was alarming cases of attacks on oil facilities during the period Nigeria experienced recession, one cannot negate the fact that the two economic variables contributed to the economic downturn in 2016 and part of 2017. The improvement in the crude oil production quantity in the second and third quarters of 2017 was the product of government dialogue and negotiations with the group leaders of Niger Delta Militants and government commitment in the amnesty programme. The presence of volatility in the crude oil production quantity as shown in the ARCH and GARCH models reveals variability in the crude oil production and its effect on the economic development of Nigeria. The two variables constituted a major threat to the nation's economic stability during the period of shocks in production quantity and price. This further informs government, policy makers and the producers of oil in Nigeria that whenever shocks are anticipated in crude oil price, more efforts should be precipitated to ensure maximum stability in production quantity to cushion effect

of prices shocks and guide against economic instability at such times.

## CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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**Appendix 1.** Nigeria monthly crude oil production in millions of barrels.

<b>Year</b>	<b>2010</b>	<b>2011</b>	<b>2012</b>	<b>2013</b>	<b>2014</b>	<b>2015</b>	<b>2016</b>	<b>2017</b>	<b>2018</b>	<b>2019</b>
<b>Month</b>										
Jan	72.29	77	70.71	75.3	71.05	67.63	66.71	56.95	61.19	51.03
Feb	66.78	70.23	68.38	62.36	64.5	61.68	59.58	50.9	56.03	47.32
March	75.57	70.93	72.4	68.56	66.48	64.04	60.87	49.57	59.75	53.69
April	72.42	70.49	71.28	66.82	66.48	60.39	59.8	53.79	58.56	51.39
May	70.15	75.66	74.43	64.01	69.25	63.5	52.63	57.96	55.35	51.12
June	71.92	72.63	71.3	60.56	65.06	59.19	53.49	58.6	54.08	55.86
July	77.07	72.94	75.66	68.07	63.82	67.04	52.26	62.46	58.42	56.64
Aug	77.7	73.49	74.65	71.12	68.1	65.39	50.04	61.82	63.05	59.06
Sep	77.81	71.56	73.56	66.52	62.69	65.93	50.86	57.92	59.86	
Oct	81.2	71.18	67.78	69.08	68.32	69.08	55.3	60.34	61.2	
Nov	73	69.73	61.17	62.65	63.6	65.14	58.28	58.75	54.76	
Dec	80.13	70.41	71.45	65.43	69.21	64.44	50.25	60.66	58.2	