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A new approach to homotopy perturbation method for solving systems of Volterra integral equations of first kind

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In this article, He's homotopy perturbation method was applied in a variant way to solve the system of Volterra integral equations of first kind. The results reveal that the proposed approach is very efficient for handling such system of integral equations. Some examples are given to show the ability of the proposed modification.

Key words: Integral equations, Volterra integral equations of first kind, homotopy perturbation method.

INTRODUCTION

The application of homotopy perturbation method (HPM) (He, 1999) was studied by many scientist and engineers because this method continuously reduces a nonlinear problem into a set of linear one which is easy to handle. To handle wide variety of linear and nonlinear problems, the method was further modified and improved by He (2000, 2003, 2004). HPM has been used to solve various types of integral equations with diverse variations. In this paper, a variant approach based on HPM was proposed to solve the system of Volterra integral equations of first kind of the form:

$$\int_0^x c_i(x,t)g_i\big(y_1(x),y_2(x),\dots,y_n(x)\big)dt = f_i(x),\quad i=1,2,3,\dots\dots.n \qquad (1)$$

where $f_i(x)$ are known functions, $c_i(x,t)$ are the kernels and g_i are linear or nonlinear functions of y_i. Exact or approximate solution of integral equations has great importance because it has wide applications in scientific research. Many researchers (Golbabai and Keramati, 2008; Biazar et al., 2009; Eslami, 2014a; Biazar et al., 2012; Biazar and Mostafa, 2011a, b; Rabbani et al., 2007; Maleknejad et al., 2007; Tahmasbi and Fard, 2008; Odibat, 2008; Maleknejad and Najafi, 2011; Biazar and Eslami, 2011; Eslami and Mirzazadeh, 2014; Eslami, 2014b; Biazar and Eslami, 2010a, b, c; 2011) have solved various types of integral equations by several methods. Biazar et al. (2008, 2009) solved system of Volterra integral equations of type (1) by HPM. Using operational matrix with block-pulse functions, Babolian and Masouri (2008) solved this type of equations. Applying Adomian method, Biazar et al. (2003) presented the solution of system of Volterra integral equations of the first kind. Making a simple modification on HPM,

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Ghorbani and Saberi-Nadjafi (2008) solved nonlinear integral equations. Biazar and Eslami (2010a, b, c) also solved the equations of type (1) by differential transform method. Volterra integral equations were also studied by several researchers with various numerical techniques. Masouri et al. (2010) presented the numerical solution of Volterra integral equation of the first kind by an expansion-iterative method. Using Runge-Kutta method, Maleknejad and Shahrezaee (2004) solved the equations of type (1). Armand and Gouyandeh (2014) presented numerical solutions of the system of Volterra integral equations. Applying decomposition method, Ngarasta et al. (2009) solved Volterra integral equations system. Saeedi et al. (2013) solved some nonlinear Volterra integral equations of the first kind numerically.

BASIC IDEAS OF HE'S HPM

To illustrate the basic ideas of He's HPM, let us consider an integral or differential operator L such that:

$$L(u) = 0 \tag{2}$$

According to homotopy technique, we can construct a homotopy $v(r;\, p)\colon \Omega \times [0,1] \to \mathbb{R}$ (where Ω is the domain) which satisfies:

$$H(v;p) = (1-p)F(v) + pL(v) = 0 \tag{3}$$

where $F(v)$ is a functional operator with known solution u_0, which can be obtained easily. Clearly, we have:

$$H(v;0) = F(v) \tag{4}$$

and

$$H(v;1) = L(v) \tag{5}$$

This shows that the changing process of embedding parameter p from 0 to unity is just that of $v(r;p)$ changing from $u_0(x)$ to the solution $u(x)$. This is known as deformation and also in topology $F(v)$ and $L(v)$ are called homotopic. So, we may assume that the solution of Equation 4 and 5 can be expressed as:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{6}$$

and putting $p = 1$, we get the approximate solution of Equation 1 as:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots \tag{7}$$

BASIC IDEA OF THE NEW APPROACH

To explain the new approach, let us consider the system of Volterra integral equations of type (1). In the new approach, we split $f_i(x)$ into infinite sums as follows:

$$f_i(x) = \sum_{j=0}^{\infty} k_{ij}(x), \qquad i = 1,2,3,\ldots,n \tag{8}$$

and define

$$\varphi_i(x;p) = \sum_{j=0}^{\infty} k_{ij}(x)\, p^j, \qquad i = 1,2,3,\ldots,n \tag{9}$$

where $p\epsilon[0,1]$ is an embedding parameter. At $p = 0$, $\varphi_i(x;0) = k_{i0}$ where as $\varphi_i(x;1) = f_i(x)$ when $p = 1$. Now, we construct the homotopy $Y_i(x;\, p)\colon \mathbb{R} \times [0,1] \to \mathbb{R}$ which satisfies the following equations:

$$Y_i(x;p) = \varphi_i(x;p) + pY_i(x;p) - p\int_0^x c_i(x,t) g_i(Y(x;p))dt \qquad i = 1,2,3,\ldots,n \tag{10}$$

where $Y(x;p) = (Y_1(x;p), Y_2(x;p), \ldots, Y_n(x;p))$ and

$$Y_i(x;p) = Y_{i0} + pY_{i1} + p^2 Y_{i2} + \cdots \tag{11}$$

Thus, as p changes from 0 to 1, $Y_i(x;\, p)$ deforms continuously from $Y_{i0}(x)$to the solution $y_i(x) = \lim_{p\to 1} Y_i(x;p)$. Substituting Equation 10 into Equation 9, and equating the terms with equal powers of p, we can obtain a series of linear equations and solving them, we can get the approximate solutions.

Examples

Here, the proposed approach was applied to obtain exact solutions of some linear and nonlinear system of Volterra integral equations of first kind.

Consider the system of linear Volterra integral equations of first kind (Maleknejad et al., 2007):

$$\int_0^x \big((1 - x^2 + t^2)y_1(t) - (2x - t)y_2(t)\big)dt = -\frac{1}{3}x^3 - \frac{2}{15}x^5 \tag{12}$$

$$\int_0^x \big((x + t^2)y_1(t) - (2 + x - t)y_2(t)\big)dt = -x^2 - \frac{1}{6}x^3 + \frac{1}{3}x^4 + \frac{1}{5}x^5 \tag{13}$$

having $y_1(x) = x^2$ and $y_2(x) = x$ as the exact solutions. At first, we split $f_1(x) = -\frac{1}{3}x^3 - \frac{2}{15}x^5$ as $f_1(x) = \sum_{j=0}^{\infty} k_{1j}(x)$ with $k_{10} = x^2, k_{11} = -x^2 - \frac{1}{3}x^3 - \frac{2}{15}x^5$ and $k_{1j} = 0,\ j > 1$; and $f_2(x) = -x^2 - \frac{1}{6}x^3 + \frac{1}{3}x^4 + \frac{1}{5}x^5$ as $f_2(x) = \sum_{j=0}^{\infty} k_{2j}(x)$ with $k_{20} = x, k_{21} = -x - x^2 - \frac{1}{6}x^3 + \frac{1}{3}x^4 + \frac{1}{5}x^5$ and $k_{2j} = 0,\ j > 1$. Now, we construct the homotopies $Y_1(x;\ p): \mathbb{R} \times [0,1] \to \mathbb{R}$ and $Y_2(x;\ p): \mathbb{R} \times [0,1] \to \mathbb{R}$ which satisfy the following equations:

$$Y_1(x;p) = \varphi_1(x;p) + pY_1(x;p) - p\int_0^x \big((1 - x^2 + t^2)Y_1(t;p) - (2x - t)Y_2(t;p)\big)dt \quad (14)$$

$$Y_2(x;p) = \varphi_2(x;p) + pY_2(x;p) - p\int_0^x \big((x + t^2)Y_1(t;p) - (2 + x - t)Y_2(t;p)\big)dt \quad (15)$$

where $\varphi_1(x;p), \varphi_2(x;p)$, $Y_1(x;p)$ and $Y_2(x;p)$ as in Equations 9 and 11. Substituting Equation 11 into Equations 14 and 15, and then collecting terms of same power of *p*, we get:

$$p^0: \quad \begin{cases} Y_{10} = k_{10} \\ Y_{20} = k_{20} \end{cases} \quad (16)$$

$$p^1: \quad \begin{cases} Y_{11} = k_{11} + Y_{10} - \int_0^x \big((1 - x^2 + t^2)Y_{10}(t) - (2x - t)Y_{20}(t)\big)dt \\ Y_{21} = k_{21} + Y_{20} - \int_0^x \big((x + t^2)Y_{10}(t) - (2 + x - t)Y_{20}(t)\big)dt \end{cases} \quad (17)$$

$$Y_{10} = x^2 \ ; \ Y_{20} = x \quad (18)$$

$$Y_{11} = Y_{12} \dots = 0 \ ; \ Y_{21} = Y_{22} \dots = 0 \quad (19)$$

and

$$y_1(x) = x^2; \ \ y_2(x) = x \quad (20)$$

Example

Let us consider the system of nonlinear Volterra integral equations of first kind (Tahmasbi and Fard, 2008; Odibat, 2008):

$$\int_0^x \big(y_1(t) + (x - t)y_1(t)y_2(t)\big)dt = -\frac{3}{4} + \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 + e^x - \frac{1}{4}e^{2x} \quad (21)$$

$$\int_0^x \big(y_2(t) + (x - t)y_1(t)y_2(t)\big)dt = \frac{5}{4} + \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 - e^x - \frac{1}{4}e^{2x} \quad (22)$$

having $y_1(x) = x + e^x$ and $y_2(x) = x - e^x$ as the exact solutions. At first, we split $f_1(x) = -\frac{3}{4} + \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 + e^x - \frac{1}{4}e^{2x}$ as $f_1(x) = \sum_{j=0}^{\infty} k_{1j}(x)$ with $k_{10} = x + e^x, k_{11} = -\frac{3}{4} - \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 - \frac{1}{4}e^{2x}$ and $k_{1j} = 0,\ j > 1$; and $f_2(x) = \frac{5}{4} + \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 - e^x - \frac{1}{4}e^{2x}$ as $f_2(x) = \sum_{j=0}^{\infty} k_{2j}(x)$ with $k_{20} = x - e^x, k_{21} = \frac{5}{4} - \frac{x}{2} + \frac{1}{2}x^2 + \frac{1}{12}x^4 - \frac{1}{4}e^{2x}$ and $k_{2j} = 0,\ j > 1$.

Now, we construct the homotopies $Y_1(x;\ p): \mathbb{R} \times [0,1] \to \mathbb{R}$ and $Y_2(x;\ p): \mathbb{R} \times [0,1] \to \mathbb{R}$ which satisfy the following equations:

$$Y_1(x;p) = \varphi_1(x;p) + pY_1(x;p) - p\int_0^x \big(Y_1(t;p) + (x - t)Y_1(t;p)Y_2(t;p)\big)dt \quad (23)$$

$$Y_2(x;p) = \varphi_2(x;p) + pY_2(x;p) - p\int_0^x \big(Y_2(t;p) + (x - t)Y_1(t;p)Y_2(t;p)\big)dt \quad (24)$$

where $\varphi_1(x;p), \varphi_2(x;p)$, $Y_1(x;p)$ and $Y_2(x;p)$ as in Equations 9 and 11. Substituting Equation 11 into Equations 23 and 24, and then collecting terms of same power of *p*, we get:

$$p^0: \quad \begin{cases} Y_{10} = k_{10} \\ Y_{20} = k_{20} \end{cases} \quad (25)$$

$$p^1: \quad \begin{cases} Y_{11} = k_{11} + Y_{10} - \int_0^x \big(Y_{10}(t) + (x - t)Y_{10}(t)Y_{20}(t)\big)dt \\ Y_{21} = k_{21} + Y_{20} - \int_0^x (Y_{20}(t) + (x - t)Y_{10}(t)Y_{20}(t))dt \end{cases} \quad (26)$$

$$Y_{10} = x + e^x \ ; \ Y_{20} = x - e^x \quad (27)$$

$$Y_{11} = Y_{12} \dots = 0 \ ; \ Y_{21} = Y_{22} \dots = 0 \quad (28)$$

and

$$y_1(x) = x + e^x; \ \ y_2(x) = x - e^x \quad (29)$$

Example

Let us consider the system of nonlinear Volterra integral equations (Maleknejad et al., 2007) having $y_1(x) = x^2$ and $y_2(x) = x$ as the exact solutions:

$$\int_0^x (1-x^2+t^2)(y_1(t)+y_2^3(t))dt = -\frac{1}{12}x^6 - \frac{2}{15}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 \quad (30)$$

$$\int_0^x (5+x-t)(y_1^3(t)-y_2(t))dt = \frac{1}{56}x^8 + \frac{5}{7}x^7 - \frac{1}{6}x^3 - \frac{5}{2}x^2 \quad (31)$$

At first, we split $f_1(x) = -\frac{1}{12}x^6 - \frac{2}{15}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3$ as $f_1(x) = \sum_{j=0}^{\infty} k_{1j}(x)$ with $k_{10} = x^2$, $k_{11} = -\frac{1}{12}x^6 - \frac{2}{15}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$ and $k_{1j} = 0,\ j > 1$; and $f_2(x) = \frac{1}{56}x^8 + \frac{5}{7}x^7 - \frac{1}{6}x^3 - \frac{5}{2}x^2$ as $f_2(x) = \sum_{j=0}^{\infty} k_{2j}(x)$ with $k_{20} = x$, $k_{21} = \frac{1}{56}x^8 + \frac{5}{7}x^7 - \frac{1}{6}x^3 - \frac{5}{2}x^2 - x$ and $k_{2j} = 0,\ j > 1$.

Now, we construct the homotopies $Y_1(x;\,p): \mathbb{R} \times [0,1] \to \mathbb{R}$ and $Y_2(x;\,p): \mathbb{R} \times [0,1] \to \mathbb{R}$ which satisfy the following equations:

$$Y_1(x;p) = \varphi_1(x;p) + pY_1(x;p) - p\int_0^x (1-x^2+t^2)(Y_1(t;p)+Y_2^3(t;p))dt \quad (32)$$

$$Y_2(x;p) = \varphi_2(x;p) + pY_2(x;p) - p\int_0^x (5+x-t)(Y_1^3(t;p)-Y_2(t;p))dt \quad (33)$$

where $\varphi_1(x;p), \varphi_2(x;p)$, $Y_1(x;p)$ and $Y_2(x;p)$ as in Equations 9 and 11. Substituting Equation 11 into Equation 32 and 33, and then collecting terms of same power of *p*, we get:

$$p^0: \quad \begin{cases} Y_{10} = k_{10} \\ Y_{20} = k_{20} \end{cases} \quad (34)$$

$$p^1: \quad \begin{cases} Y_{11} = k_{11} + Y_{10} - \int_0^x (1-x^2+t^2)(Y_{10}(t)+Y_{20}^3(t))dt \\ Y_{21} = k_{21} + Y_{20} - \int_0^x (5+x-t)(Y_{10}^3(t)-Y_{20}(t))dt \end{cases} \quad (35)$$

$$Y_{10} = x^2\,;\ Y_{20} = x \quad (36)$$

$$Y_{11} = Y_{12} \ldots = 0\,;\ Y_{21} = Y_{22} \ldots = 0 \quad (37)$$

and

$$y_1(x) = x^2;\ \ y_2(x) = x \quad (38)$$

Example

Finally, consider the system of nonlinear Volterra integral equations (Tahmasbi and Fard, 2008) with $y_1(x) = \frac{x^2}{4} + 1$, $y_2(x) = \frac{x^2}{3} + \frac{3}{2}$ and $y_3(x) = \frac{x}{2} + \frac{2}{3}$ as the exact solutions:

$$\int_0^x \left((5+x-t)y_1(t) + (\tfrac{1}{2}x^2+t)y_2(t)y_3(t)\right)dt = \frac{1}{48}x^6 + \frac{19}{270}x^5 + \frac{19}{72}x^4 + \frac{7}{6}x^3 + x^2 + 5x \quad (39)$$

$$\int_0^x \left(\left(\tfrac{1}{2}x^2+t\right)y_1(t) + (3+x-t)y_2(t) + \tfrac{1}{4}(x^2-t^2)y_3(t)\right)dt = \frac{1}{24}x^5 + \frac{35}{288}x^4 + \frac{17}{18}x^3 + \frac{5}{4}x^2 \quad (40)$$

$$\int_0^x \left(ty_1(t)y_2(t) - xty_2^2(t) - 5y_3(t)\right)dt = -\frac{1}{54}x^7 + \frac{1}{72}x^6 - \frac{1}{4}x^5 + \frac{17}{96}x^4 - \frac{9}{8}x^3 - \frac{1}{2}x^2 - \frac{10}{3}x \quad (41)$$

At first, we split $f_1(x) = \frac{1}{48}x^6 + \frac{19}{270}x^5 + \frac{19}{72}x^4 + \frac{7}{6}x^3 + x^2 + 5x$ as $f_1(x) = \sum_{i=0}^{\infty} k_{1j}(x)$ with $k_{10} = \frac{x^2}{4} + 1$, $k_{11} = \frac{1}{48}x^6 + \frac{19}{270}x^5 + \frac{19}{72}x^4 + \frac{7}{6}x^3 + \frac{3}{4}x^2 + 5x - 1$ and $k_{1j} = 0,\ j > 1$; and $f_2(x) = \frac{1}{24}x^5 + \frac{35}{288}x^4 + \frac{17}{18}x^3 + \frac{5}{4}x^2 + \frac{9}{2}x$ as $f_2(x) = \sum_{i=0}^{\infty} k_{2j}(x)$ with $k_{20} = \frac{x^2}{3} + \frac{3}{2}$, $k_{21} = \frac{1}{24}x^5 + \frac{35}{288}x^4 + \frac{17}{18}x^3 + \frac{11}{12}x^2 + \frac{9}{2}x - \frac{3}{2}$ and $k_{2j} = 0,\ j > 1$; and $f_3(x) = -\frac{1}{54}x^7 + \frac{1}{72}x^6 - \frac{1}{4}x^5 + \frac{17}{96}x^4 - \frac{9}{8}x^3 - \frac{1}{2}x^2 - \frac{10}{3}x$ as $f_3(x) = \sum_{i=0}^{\infty} k_{3j}(x)$ with $k_{30} = \frac{x}{2} + \frac{2}{3}$, $k_{31} = -\frac{1}{54}x^7 + \frac{1}{72}x^6 - \frac{1}{4}x^5 + \frac{17}{96}x^4 - \frac{9}{8}x^3 - \frac{1}{2}x^2 - \frac{23}{6}x - \frac{2}{3}$ and $k_{3j} = 0,\ j > 1$.

Now, we construct the homotopies $Y_1(x;\,p): \mathbb{R} \times [0,1] \to \mathbb{R}$, $Y_2(x;\,p): \mathbb{R} \times [0,1] \to \mathbb{R}$ and $Y_3(x;\,p): \mathbb{R} \times [0,1] \to \mathbb{R}$ which satisfy the following equations:

$$Y_1(x;p) = \varphi_1(x;p) + pY_1(x;p) - p\int_0^x \left((5+x-t)Y_1(t;p) + (\tfrac{1}{2}x^2+t)Y_2(t;p)Y_3(t;p)\right)dt \quad (42)$$

$$Y_2(x;p) = \varphi_2(x;p) + pY_2(x;p) - p\int_0^x \left(\left(\frac{1}{2}x^2 + t\right)Y_1(t;p) + (3 + x - t)Y_2(t;p) + \frac{1}{4}(x^2 -)Y_3(t;p)\right)dt \quad (43)$$

$$Y_3(x;p) = \varphi_3(x;p) + pY_3(x;p) - p\int_0^x \left(tY_1(t;p)Y_2(t;p) - xtY_2^2(t;p) - 5Y_3(t;p)\right)dt \quad (44)$$

where $\varphi_1(x;p), \varphi_2(x;p)$, $Y_1(x;p)$ and $Y_2(x;p)$ as in Equations 9 and 11. Substituting Equation 11 into Equations 42, 43 and 44 and then collecting terms of same power of *p*, we get:

$$p^0: \quad \begin{cases} Y_{10} = k_{10} \\ Y_{20} = k_{20} \\ Y_{30} = k_{30} \end{cases} \quad (45)$$

$$p^1: \quad \begin{cases} Y_{12} = k_{11} + Y_{10} - \int_0^x \left((5 + x - t)Y_{10}(t) + (\frac{1}{2}x^2 + t)Y_{20}(t)Y_{30}(t)\right)dt \\ Y_{21} = k_{21} + Y_{20} - \int_0^x \left(\left(\frac{1}{2}x^2 + t\right)Y_{10}(t) + (3 + x - t)Y_{20}(t) + \frac{1}{4}(x^2 - t^2)Y_{30}(t)\right)dt \\ Y_{31} = k_{31} + Y_{30} - \int_0^x \left(tY_{10}(t)Y_{20}(t) - xtY_{20}^2(t) - 5Y_{30}(t)\right)dt \end{cases} \quad (46)$$

$$Y_{10} = \frac{x^2}{4} + 1\,;\; Y_{20} = \frac{x^2}{3} + \frac{3}{2};\; Y_{30} = \frac{x}{2} + \frac{2}{3} \quad (47)$$

$$Y_{11} = Y_{12} \ldots = 0\,;\; Y_{21} = Y_{22} \ldots = 0;\; Y_{31} = Y_{32} \ldots = 0 \quad (48)$$

and

$$y_1(x) = \frac{x^2}{4} + 1,\; y_2(x) = \frac{x^2}{3} + \frac{3}{2},\; y_3(x) = \frac{x}{2} + \frac{2}{3} \quad (49)$$

Conclusion

Based on HPM, an analytic approach for solving the system of Volterra type integral equations of first kind was developed. Evaluating the examples, it was observed that the proposed approach is straightforward in calculations and very effective in both linear and nonlinear cases. Moreover, in most of the cases, it gives exact solutions at the first approximation.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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