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Full Length Research Paper

Optimal packing of fm station progammes case study: Kaase Fm station, Kumasi, Ghana

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A FM station has a pile of programmes being broadcast in the testing phase of the station. The programmes consist of fixed and unfixed programmes. However the full complement of programmes has not been reached. To achieve full complement, the station wants to plan a new programme mix with fixed programmes retaining their original time slots. The rearrangement of the programmes is modeled as a variant of packing problem. We introduce a model, which is a modification of the generalized assignment problem with identified first-use bins of Shraideh et al. (2008). Our model seeks to minimize wasted air time that results out of the new programme arrangement. The results are obtained using the modified simple bin packing algorithm, which is also a modification of the simple bin packing algorithm of Amponsah (2003). The new arrangement of programmes produces a total of zero minutes of wasted air time.

Key words: Packing, algorithm, FM station, assignment.

INTRODUCTION

The problem of packing forms an integral part of human activities. Almost everyone is involved in packing. Domestically, when packing is done efficiently, space and time are saved.

Sphere packing problems have fascinated mathematicians since ages. In time, they have also spawned many new subclasses of equally interesting and challenging problems, with applications to diverse branches of science and engineering (Gopalan, 2010). Currently, packing problems consist of packing a set of geometric objects of fixed dimensions and shape into a region of predetermined shape while accounting for design and technological consideration of the problem (Stoyan, 2003).

Athanasio et al. (2007) studied the best way a multinational chemical company delivers orders to its customers over a multi-day planning period which included load packing into vehicles. The problem was modeled as a integer linear program with a heuristic, based on the cutting plane method, as the solution algorithm. The procedure achieved remarkable cost savings. In this paper, we model the programme arrangement of an FM radio station as a packing problem and provide a heuristic that solves it to optimality.

RELATED WORKS

The sphere-packing problem began with Sir Walter Raleigh, who asked the mathematician, Thomas Harriot, for a formula to determine the number of cannonballs in a pile on a ship's deck (Shirley, 1983). Kepler (1611) stated the conclusion on the problem of packing of spheres into a container and this has been called the Kepler's packing problem: "Equal spheres when collected in any vessel come to mutual arrangement in two modes. One mode of packing in a vessel is the cubic arrangement where each sphere in a lower layer is touched by one sphere in the upper layer. In the second mode of packing, a sphere is touched by four neighbors in the same plane. It is also touched by four spheres above and four spheres below. Hence is touched by twelve spheres. This is the tightest possible arrangement so that no other arrangement can stuff more spheres in the same container than this" (Kepler, 2006). The ratio of the filled to the unfilled space (packing fraction) in Kepler's problem is (π/18) = 0.7404.

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A packing arrangement of identical spheres that has the higher packing ratio was seen to have stable configuration. Hilbert (1901) presented to the International Mathematical Congress in Paris the Hilbert's 18th Problem on the possibility of arranging most densely in space an infinite number of equal solids of a given form and size such that the packing fraction may be as great as possible.

Gauss (1831) showed that, the face centered cubic structure is the densest lattice packing in three dimensions. However the tetrahedral packing is also a dense lattice packing. This implies that non – crystalline structure could have packing fraction higher than the cubic close packing.

Fejes (1964) was the first to use an optimization technique to produce an upper bound of the packing fraction to be 0.7754. Further upper bounds have been obtained to be 0.7797 (Rogers, 1958), 0.77836 (Lindsey, 1987) and 0.7731 (Muder, 1993).

The problem closely related to the sphere packing problem is the kissing number problem. The kissing number (contact number or coordination number), τ_n, is the highest number of equal non-overlapping spheres in R^n space that can touch a fixed sphere of the same size. τ_n is not known for $n \geq 4$ except when $n = 8$ and $n = 24$ where the arrangements are respectively found in the E_8 lattice and the Leech lattice (Gopalan, 2010).

Wyner (1967) showed that for a given n, the kissing number is bounded below as $\tau_n \geq 2^{(1-0.5\log_2 3)*n(1+o(1))}$ and Kabatiansky and Levenshtein (1978), showed that it is bounded above as $\tau_n \leq 2^{0.401*n(1+o(1))}$.

Borndorfer (2003) argued that packing constraints are one of the most common problem characteristics in combinatorial optimization. They come up in problems of bin packing, vehicle and crew scheduling, VLSI and network design, and frequency assignment. The study of such combinatorial optimization problems has yielded deep structural and algorithmic results.

Johnson (1973) showed that the algorithmic strategy that orders items in descending order and places them sequentially in the first bin in which they fit is never sub-optimal by more than 22% and further that no efficient bin packing algorithm can be guaranteed to do better than 22%. Amponsah (2003), proposed the simple bin packing algorithm (SIBINPA) for the solution of the bin packing problem. The SIBINPA arranges the items in ascending order of magnitude. Starting with the item of largest magnitude, it packs the items from the leftmost bin to the rightmost bin. It then returns from the rightmost bin to the leftmost bin. The values of items in each bin should not exceed the bin capacity. The algorithm steps for the SIBINPA are shown as:

Step 1: Arrange in ascending order the time length required to pack each of the items to be assigned to the bins (programme days).
Step 2: Insert the items in descending time magnitude from first bin to the last bin making sure bin capacity is not exceeded for each item input.
Step 3: Reverse the order of the bins and go to step 2.
Step 4: Repeat processes in step 2 and 3 until all the items are packed

In this paper, we provide a modification of the simple bin packing algorithm to solve the problem of programme arrangement in an FM station.

PROBLEM STATEMENT

Kaase FM Station in Kumasi, Ghana, has recently been set up with the necessary resources and has been licensed to broadcast transmission. The station is on a test transmission and has piled up a set of programmes they are broadcasting. Management do not have the full complement of programme broadcasts to last from 6 a.m. to 12 midnight each day. They have sought to obtain sponsorship for additional programmes. The programmes consist of fixed and unfixed programmes; so management want to rearrange the pile of programme outline so as to have a new programme mix whereby the fixed programmes will maintain their air-times while the unfixed programmes can be mixed with the new programmes. Spaces created in the new programme arrangement will be used to look for sponsorship. Table 1 shows the current pile of programmes. Programme activities for Sundays are excluded since the entire programme spaces for the day have fixed programmes.

A BIN PACKING PROBLEM FORMULATION

Shraideh et al. (2008) introduced a discrete multi criteria optimization problem of assigning contract jobs to primary and secondary workers with the condition that secondary workers could only be assigned jobs if the primary workers did not have the capacity for the job at hand. The problem combined the characteristics of bin packing problem (BPP) and generalized assignment problem (GAP). The multi criteria problem was subsequently reduced to two optimization problems consisting of a new variant of bin packing problem, called generalized assignment problem with identified first used bins (GAPIFB) and the generalized assignment problem (GAP). The use of secondary agents is allowed only when the primary agents are not capable to treat all the tasks. The objective function of GAPIFB is to minimize the number of secondary agents. Let the binary U_i represent primary or secondary agents, where, for $i \in \{1, 2, ..., N\}$, U_i represents a primary agent and for $i \in \{N+1, N+2, ..., N+M\}$, U_i represents a secondary agent. Let, Z = Number of task types, M = Available secondary agents; N = Available primary

Table 1. Current programme arrangement from Monday to Saturday.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Home news(30)*	Home news(30)*	Home news(300*	Home news(30)*	Home news(30)*	Home news(30)*
Morning show(180)*	Morning show(180)*	Morning show(120)*	Morning show(120)*	Morning show(150)*	World sports(120)
Busy time(180)*	Busy time(180)*	EkwansoKosekose(60)	Time with NCCE(60)	AhemfoAsoe(30)	MmofraKyepen(60)
Jazz music(60)*	Francophone(60)	Busy time(180)*	Busy time(60)*	Periscope(120)	Youth forum(60)
Kumasi monsom(60)	Total sports(60)	Reggae(60)	Governance hour(60)*	Gospel(60)	Women's avenue(60)
Drive time(180)*	Drive time(170)*	TeteAmamere(60)	Busy time II(60)*	ObaaPa (60)	Highlife(90)
This is life(60)	Abrabo Mu Nsem(60)	Drive time(180)*	Health matters(60)	Drive time(180)*	Metro news(60)*
Reggae(145)	Reggae(120)	AkuafoMfaAdwene(60)	KyereW'adwen(60)*	AkuafuoBadwa(60)	Nwomkro(120)
	Gospel music(120)	Slow gospel(30)	Metro drive(180)*	Weekend groove(210)*	ELS(90)
		Heart to heart(120)*	Kenkan me(30)		LNM(120)*
			Smooth joints(30)		Reggae sunsplash(180)
			Reggae(85)		

* fixed programmes. The time duration (minutes) for each programme is shown in brackets.

agents, CAP_i = Capacity of agent *i*, QT_j = Quantity of tasks of type ***j*** T_{ij} = Needed time for primary or secondary agent *i* to treat a task of type *j*, and X_{ij} = The number of tasks of type *j* assigned to primary or secondary agent *i*.

The GAPIFB is thus formulated as:

$$Min \sum_{i=1}^{N+M} U_i \tag{1}$$

Subject to:

$$\sum_{j=1}^{z} X_{ij} * T_{ij} \leq CAP_i * U_i \qquad \forall i \in \{1, 2, ..., N+M\} \tag{2}$$

$$\sum_{i=1}^{N+M} X_{ij} = QT_j \qquad \forall j \in \{1, 2, ..., Z\} \tag{3}$$

$$\sum_{i=1}^{N} U_i = N \tag{4}$$

The objective function (1) seeks to minimize the number of secondary agents used to treat all tasks. Constraint (2) ensures that the capacity of agents is not violated. Equation (3) ensures that all tasks are allocated and each task is assigned to only one agent. Equation (4) ensures that all primary agents are included in solution.

For the GAP problem, consider the following:
Let, L = Number of secondary agents to be hired.
Put:

$i \in \{1, 2, ..., L\}$ and $j \in \{1, 2, ..., Z\}$.

The formulation of the GAP is then:

$$Min \sum_{i=1}^{N+L} \sum_{j=1}^{Z} X_{ij} * T_{ij} \tag{5}$$

subject to

$$\sum_{j=1}^{Z} X_{ij} * T_{ij} \leq CAP_i \qquad \forall i \in \{1, 2, ..., L\} \tag{6}$$

$$\sum_{i=1}^{L} X_{ij} = QT_j, \qquad \forall j \in \{1, 2, ..., Z\} \tag{7}$$

The objective function (5) minimizes the total treatment time for all contracts. Constraint (6) ensures agent capacity is not violated. Equation (7) ensures that the distributed quantity of tasks of type k is less than the received quantity of that type.

Modification of the GAPIFB

Here, we introduce a new optimization model, which uses some of the characteristics of the GAPIFB-GAP formulation and will satisfy the problem of arrangement and redistribution of programme spaces for the FM station with the condition that fixed programmes should remain as currently scheduled. Let:

$NC = \text{Number of all tasks(programmes)}$

$A_{isj} = \text{Quantity of programme } j \text{ of type } s \text{ assigned to bin } i$

$\text{k} = \text{Number of bins}$

$\text{QT}_{is} = \text{Quantity of programmes of type } s \text{ in bin } i$

$\text{n}_\text{i} = \text{Number of fixed programmes in bin } i$

$\text{m}_\text{i} = \text{Number of unfixed programmes in bin } i$

$r_\text{i} = \text{Number of unused spaces in bin } i$

$\text{N} = \text{Number of fixed programmes}$

$M = \text{Number of unfixed programmes}$

$c = \text{Capacity of the bins}$

$T_{ij} = \text{Needed time to pack fixed programme } j \text{ into bin } i$

$\text{V}_{ij} = \text{Needed time to pack unfixed programme } j \text{ into bin } i$

$U_{ij} = \text{Boolean 1, if fixed programme } j \text{ is used in bin } i\text{, and 0, otherwise.}$

$X_{ij} = \text{Boolean 1, if unfixed programme } j \text{ is used in bin } i\text{, and 0, otherwise}$

$\text{W}_{ij} = \text{Time left in block } j \text{ for bin } i \text{ after block has been sealed}$

The objective function is:

$$Min \sum_{i=1}^{k}\sum_{j=1}^{r_i} W_{ij} \tag{8}$$

Subject to:

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i} U_{ij} = N \tag{9}$$

$$\sum_{i=1}^{k}\sum_{j=1}^{m_i} X_{ij} = M \tag{10}$$

$$\sum_{j=1}^{n_i}(U_{ij} * T_{ij}) + \sum_{j=1}^{m_i}(X_{ij} * V_{ij}) + \sum_{j=1}^{r_i} W_{ij} = c \quad \forall i = 1,2,\ldots,k \tag{11}$$

$$\sum_{j=1} A_{isj} \le 1 \qquad \forall i = 1,2,\ldots,k \;\; \forall s = 1,\ldots,QT_{is} \tag{12}$$

$$A_{isj} = \{0,1\} \tag{13}$$

The objective function (8) seeks to minimize the wasted time in the bins. Equations (9) and (10) ensure that all programme items are packed. Equation (11) ensures that the capacity of the bins is not violated. Constraint (12) ensures each item is packed once. Equation (13) is binary condition

Proposed bin packing algorithm

The Simple Bin Packing algorithm (SIBINPA) was modified to suit the structure of the proposed programme outline of the Kaase FM Station. When the fixed programmmes for a day's bin are inserted, blocks of programme spaces are left between the fixed programmes. Thus a day's bin may contain multiple blocks as compared to the packing problem solved by the Simple Bin Packing Algorithm for which a day's bin corresponds a single block. The Modified Simple Bin Packing Algorithm (MOSIBINPA) is a hybrid of the SIBINPA of Amponsah (2003), and first fit decreasing algorithm. The MOSIBINPA procedure first arranges fixed programmes in their respective positions and the blocks of spaces are filled by unfixed and new programmes.

In this method, the Simple Bin Packing is applied until a fraction, β, of the total sum of value of the items are arranged. First fit decreasing algorithm is then applied to pack the rest of the items into the Bins. The steps of MOSIBINPA are given as:

Step 1: Categorize programmes into fixed and unfixed programmes.
Step 2: The fixed programmes are arranged in their respective time slots thereby creating blocks in the various days' programmes.
Step 3: Arrange in ascending order the time length required to pack each of the unfixed programme items, which have not yet been assigned to any of the programme days (bins).
Step 4: Pack items in descending order of magnitude from first bin (Monday) to the last bin (Saturday).
Step 5: Reverse the order of the bins and go to step 4. Items are place in unfilled blocks at the same level in all the bins. If a block of a bin cannot accommodate a particular item we move to the next Bin and place item in a block that is of the same level as the previous block. If an item cannot be accommodated by any block on the

Table 2. New programme mix for Kaase FM Radio Station.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Home news (6:00) *	Home news (6:00)*	Home news (6:00)*	Home news (6:00)*	Home news (6:00)*	Home news (6:00)*
Morningshow (6:30) *	morningshow (6:30) *	Morningshow (6:30) *	morningshow(6:30)	Morningshow (6:30)*	Reggae sound splash (6:30)
Busy time (9:30) *	Busy time (9:30)*	Busy time (8:30) *	Free space (8:30) [a]	Reggae (9:00)	Reggae (9:30)
Periscope (12:30)	World sport (12:30)	Nwomkro (11:30)	Busy time (9:30) *	Gospel music (11:00)	Free space (11:55) [a]
This is life(13:30)	Kumasi mosom (13:30)	Jazz music (12:30)	Free space (10:30)	Smooth joint(13:00)	Metro news (13:00)*
Francophone (14:30)	Total sports(14:30)	Abrabo mu Nsem (13:30)	Busy time2 (11:30)	Governance hour(15:00)	Highlife (14:00)
Free time(15:30)[a]	Free time(15:30) [a]	AkuafoAdwene (14:30)	Time with NCCE (12:30)	Free space (16:00)	Reggae (15:30)
Drive time(16:30)*	Drive time(16:30)*	Free space (15:30) [a]	Free space (13:30)	Drive time (16:30) *	Ekwansokose (16:55)
Health matters(19:30)	Kyerewadwene (19:30)	Drive time(16:30)	Drive time (16:30)*	Free space (19:20) [a]	TeTeAmamere (17:55)
Mmofrakyepen(20:30)	AkuafoBadwa (20:30)	Gospel(19:30)	Kenkan Me(19:30)	Weekend groove (20:30)*	Free space (18:55) [a]
Youth forum(21:30)	Women's avenue(21:30)	Obaa Pa(20:30)	AhenfoAsoe (20:00)	Close down(0:00)	ELS(20:30)*
Slow gospel(22:30)	Reggae(22:30)	Free space(21:30) [a]	Free space(20:30) [a]		LNM(22:00)*
Free space(23:00) [a]	Free space(23:30) [a]	Heart to heart(22:00)	Close down(0:00)		Close down(0:00)
Close down(0:00)	Close down(0:00)	Close down(0:00)			

* fixed programmes; [a] free airtimes that will be used for new programmes that require sponsorship. The start time of each programme is provided in bracket beside the programme.

current level in the entire bins move to the next level of unfilled blocks.

Step 6: Repeat process in steps 4 and 5 until β = $\frac{3}{5}th$ of the total sum of the time value of items have been packed.

Step 7: Pack the rest of the items by using the first fit decreasing algorithm as follows: Start from the first block of the first bin and start packing items from the unused block space. If an item cannot be accommodated by the block move to the next block that still has space in the same bin. If an item cannot be accommodated by any of the unused blocks that still has space in a particular bin, move to the next bin and pack the items starting from the first unused block space. Continue in this manner to the last bin.

Step 8: Repeat the process in step 7 until all the items has been arranged.

RESULTS

A matlab program code was written and executed on Vista Pentium D dual core, 2.3G CPU, 1G ram and 120G hard disk spaces. Matlab programming software for Windows version 7.5.342 (R2007b) was used. The program calls for the input of capacities of the fixed and unfixed programmes in matrix format. This is followed by another matrix of capacities of fixed and unfixed programmes with the capacities of unfixed programmmes set to zero. The number of programme days (bins) and the blocks in each bin are entered. The MOSIBINPA program provides an optimal arrangement of programs as shown in Table 2. In accordance with step 5 of the MOSIBINPA procedure, we defined a random variable β = (0, 1) to be the fraction of total sum of values of items (program times). The algorithm changes procedure from SIBINPA procedure to first fit decreasing algorithm at a given value of β. The algorithm was used to find the value of β for which we have zero wasted air time space and the programme arrangement corresponding to this state was the optimal arrangement. The value of β for the optimal arrangement was obtained to be 3/5.

DISCUSSIONS

Dyckchoff (1990) noted that cutting and packing problems have the same logical structure such that the space of a pattern to be cut is analogous to the space in an object (container) to be filled by items. The solution types of cutting and packing algorithms are either object/item oriented solution type or pattern oriented. The MOSIBINPA solution algorithm for the Kaase FM packing problem is of the first solution type where items (programmes) are packed into objects (days of the week) as containers. Simple packing problems in this first category are solved by exact methods such as branch and bound and dynamic programming algorithms (Hopper, 2000). While Eilon and Christofides (1971) developed a branch and bound method which is item oriented; Fukunaka and Korf (2007) developed an object oriented branch and bound method for packing problems. Majority of packing problems are constrained and hence are more complex and such complex packing problems are solved by approximate methods using bin packing algorithms and other heuristic methods. Genetic algorithm (GA) is the most common metaheuristics used to solve packing problems. However, these metaheuristics bring little improvements in the solution quality compared with standard packing techniques. GA is better suited for small to medium sized problems at the expense of significant computational time (Hopper, 2000). Our FM. packing problem is a highly constrained problem because of the block partitions introduced by the insertion of the fixed programmes. Our algorithm therefore does not rely on exact solution methods but is a modification of the SIBINPA algorithm of Amponsah (2003) which itself is a variant of the standard FFD algorithm of the bin packing problem

The efficiency of container packing is measured in the percent of space utilization by minimizing the empty space left between items in the container (Thapatsuwan et al., 2007). In the final solution to the FM programme packing problem, the algorithm achieves 100% efficiency when β is set to 3/5. To the best of our knowledge, this is the first time FM programme arrangement is being modeled as a packing problem.

Conclusion

In this paper, we have adapted the model of Shraideh et al. (2008) to obtain a model that can be used to provide optimal arrangement of fixed and unfixed programmes to generate spaces for new programmes. We also extended the simple bin packing algorithm (SIBINPA) of Amponsah (2003). The resulting modified simple bin packing algorithm (MOSIBINPA) is our proposed packing algorithm.

Due to poor arrangement of programmes, some FM stations are not able to maximize the income they are supposed to get through advertisement and programme sponsorship. In this paper, we have presented a highly effective algorithm to help FM stations arrange their programmes.

For β = 3/5, the MOSIBINPA procedure provides programme arrangement that makes maximum use of all the airtime available.

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