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Management of inventory for profitability from supplier to retailers in supply chain

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Supply Chain Management (SCM) is a network of facilities and distribution options focused on cost, customer service, inventory cost, and the flow of activities within companies and organizations, with the main goal of maximizing profitability. Supply chain practices and inventory management ensure that products are delivered to customers with greater accuracy, safety, and promptness. However, the level of profitability in the supply chain when company management employs high service level drivers, conversant with road networks and quantity-based shipment consolidation for delivering finished products to retailers, needs to be explored. The objective of this study was to evaluate the optimum quantity and optimal cost required by customers for the supply chain to maximize profit. A quantity-based mathematical model with renewal theory was applied to obtain the optimal profit in the supply chain system. The results showed that with simultaneous variations in the costs of the supply chain and the retailer's and supplier's replenishment quantities, the total cost of the supply chain increased optimally with an increase in the optimal replenishment quantity of the retailer and a constant replenishment quantity of the supplier. Also, demand increases linearly as the value of the arrival rate and mean demand increase at the retailer point, resulting in increased profitability in the supply chain when the retailer orders more quantities from the supplier. A series of simulation tests showed that the model's functions are reasonably good. Therefore, enhancing levels of collaboration and communication on supply chain and inventory replenishment strategies between stakeholders in the supply chain who coordinate the flow of materials within a company and to consumers should be emphasized.

Key words: Delivery, inventory, replenishment, shipment consolidation, supply chain.

INTRODUCTION

Supply Chain Management (SCM) is a network of facilities and distribution options that perform functions such as material procurement, transformation of the materials into intermediate and finished products, and

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then distribution of the finished product to consumers (Kaur and Sharma, 2015). SCM centers on cost, customer service, and inventory cost, as well as the flow of activities in companies and organizations (Pettersson, 2013), with the main goal of minimizing supply chain costs to meet fixed and given demands (Shapiro, 2001). A supply chain strategy, therefore, describes how the business structure manages supply chain operations and evaluates the impact of its operations on the perceptions of its stakeholders (Das et al., 2012). These stakeholders include distributors, retailers, clients, and other actors involved in coordinating the progress of materials within a company and to the end consumer, between the supplier and the consumer (Chase et al., 2001; JNU, 2013). The business structures or organizations may be autonomous or semi-autonomous and perform processes associated with the flow and transformation of goods and services from the initial stage of raw materials to the end stage of users (Kaur and Sharma, 2015). SCM integrates distinct functions like purchases, inventory management, distribution, and production planning, as well as offers opportunities for cost reduction across functions, better planning for purchase and production, and improved use of capital (Oluwaseyi et al., 2017 and Ugoani and Ugoani, 2017). Inventory management in SCM is crucial for controlling stock at the data level where the business is organized, such as maintaining the correct level of stock and recording its movement (Oluwaseyi et al., 2017). Inventories are essential in developing and managing activities of raw materials, semi-finished materials, and finished goods (Kotler, 2002), and ensuring that production, market, and distribution systems are intact (Oluwaseyi et al., 2017), so that supplies are available, costs are low, and profits are maximized (Rothschild, 2006).

SCM ensures that the right product or service is distributed in the right quantities, to the proper locations, and at the appropriate time, in order to minimize system-wide costs while satisfying customer service level requirements (Kaur and Sharma, 2015). Supply chain practices and inventory management ensure that products are delivered to customers faster, with greater accuracy, safety, and promptness (O'Byrne, 2016), and with higher profitability (Adyang, 2012; Simon, 2012). Additionally, increased service quality, customer satisfaction, and service performance due to improved quality of operations in retail workshop processes have been described in supply chain practices and management (Muthoni, 2010). Zohreh and Amir (2018) reported that managing supply chain orders for profitability involves long-term order earnings, increased customer loyalty, long-term cooperation with companies, and minimizing total costs through forward flows to reduce fixed and variable costs and increase customer responsiveness. However, companies should keep their inventory value at the lowest possible cost rates to maximize profit, such as by operating supplier and customer-managed inventories and consignment inventory processing, with suppliers storing the goods at the customer location (Judit et al., 2017). In this context, the objective of the study was to determine the level of profitability in the supply chain when company management employs high service level drivers conversant with road networks and applies quantity-based shipment consolidation for delivering finished products to retailers in a specific area. This has a significant effect on the optimum order quantities and optimal cost required for the supply chain to maximize profit.

MATERIALS AND METHODS

A model for inventory replenishment and delivery planning in a supply chain consisting of a supplier and retailer, with the supplier authorized to manage the inventory level of the retailer, previously described by Teimoury et al. (2008) and Kang and Kim (2010) with slight modifications, was considered. Briefly, a quantity-based policy with renewal theory was used to obtain the long-run average cost in a coordinated supply chain system and determine the order-up-to levels of the retailers from the supplier. We considered replenishments to represent the events where the retailer received products from the supplier, and deliveries represented the events where the retailer delivered the products to the consumers. Additionally, we considered that the batch size was not the same but equaled the ordered quantity to be delivered and might vary due to stochastic demand from customers (that is, the end consumers). Demands from the supplier to the retailer and from the retailer to consumers followed a compound Poisson distribution. The replenishment (delivery) cycle denoted the time interval between two consecutive replenishments (deliveries). Also, the replenishment and delivery costs of the supplier and retailers were composed of a fixed cost, which was incurred when there was a positive replenishment quantity, and a linear variable cost, which was proportional to the quantity. This variable cost included the cost of loading products onto vehicles at the company, transporting them to the supplier, and unloading them from the vehicles at the retailer. Figure 1 shows the inventory levels at the supplier and the retailers. The reorder points of the supplier and the retailers can be easily determined to be zero.

Mathematical model

A mathematical model was developed for the quantity in a coordinated supply chain system in which retailers initiate the ordered quantity from the supplier, who is replenished using small vehicles. To maintain mathematical tractability, previously described simplified frameworks (Teimoury et al., 2008; Kang and Kim, 2010; Jac-Hun, 2010) were considered. Let Q represent the size of a replenishment quantity and r denote the number of dispatches in a cycle.

The model considers the replenishment quantity of the supplier (S_Q) and the delivery quantity of the retailers (R_Q).

Definition of notations

T_i: Inter-arrival time between the arrivals of the $(n-1)^{st}$ and the n^{th} retailers, A_t: Arrival time of the n^{th} retailer ($A_t = \sum_{i=1}^{n} T_i$), λ:

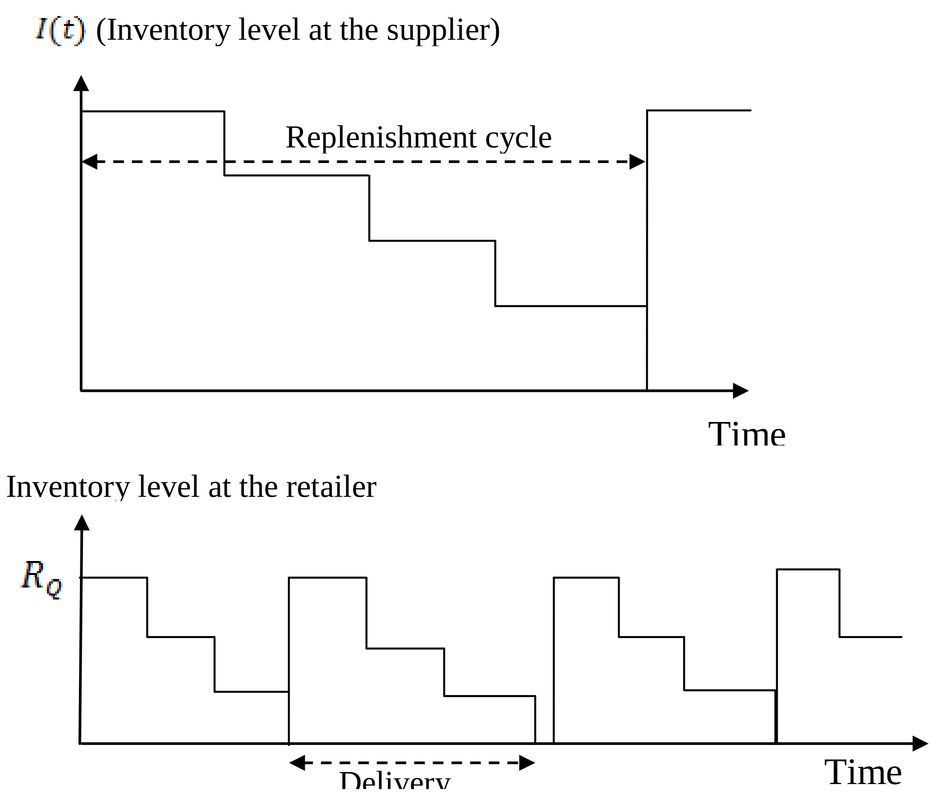

Figure 1. Inventory levels of the supplier and retailer.

Arrival rates of the customers, $\frac{1}{\lambda}$: The mean of the inter-arrival time of customers, $N(t)$: Number of orders that have arrived by time t, ($N(t) = max\{n/A \le t_t\}$); it is assumed that this follows the Poisson distribution with mean λt, d_n: Demand quantity (or weight) of the n^{th} retailer, μ: Mean of demand quantities, σ^2: Variance of the demand quantities, D_n: Cumulative demand quantity of the first n retailers ($D_n = \sum_{i=1}^{n} d_i$), $N_2(x)$: Minimum number of retailers, whose cumulative demand quantity exceeds, that is, $N_2(x) = min\{n/D_n > x\}$, $L^j(x)$: Minimum number of retailers whose cumulative demand quantity exceeds x in the j^{th} deliver cycle, S_Q: The order-up-to level of the supplier, R_Q: The order-up-to level of the retailer, h_S: The inventory holding cost per unit per unit time at the supplier, h_R: The inventory holding cost per unit per unit time at the retailer, $I_S(t)$: Inventory level of the supplier at time t, $I_R(t)$: Inventory level of the retailer at time t, C_R: The cost replenishing one unit at supplier , A_R:The fixed cost of replenishing the inventory at the retailer from the supplier, C_D: The cost of delivering one unit from the supplier to the retailer; A_D: The fixed cost of delivering of a shipment from the supplier to the retailer; K: Number of delivery cycles within replenishment cycles (a random variable), $F(x)$: Distribution of $D_{N_2(S_R)}$, the sum of demand quantities of the customers that arrive during a delivery cycle, that is, $F(x) = P\{D_{N_2(S_R)} \le x\}$, $F^{(k)}(x)$: k-fold convolution of $F(x)$, $C(S_Q, R_Q)$: The expected long-run average cost incurred when the order-up-to-levels of the supplier and the retailer are S_Q and R_Q respectively.

Assumptions of the model

To enable us achieve the quantity-based dispatching model for the coordinated supply chain, the followings are the assumptions of the model.

(a) The inventory level is under continuous review
(b) The load is dispatched whenever the size of demands is accumulated
(c) The mean and variance of the quantities is known to each supplier
(d) Inter-arrival times of the ordered quantities are mutually independent.
(e) Shortages are not allowed.
(f) Lead times for inventory replenishments are fixed and negligibly short.
(g) There are an integer number of delivery cycles in each replenishment cycle.
(h) The distances between the supplier and retailers are not very large.

Since we assumed that dispatching decisions were made on a recurrent basis, we made use of the renewal theory (Çetinkaya and Lee, 2000; Çetinkaya, 2004 Teimoury et al., 2008; Kang and Kim, 2010) to obtain an optimal solution for our problem.

Here let T_i ($i = 1, 2, \cdots, K$) be the instants that the demands had accumulated to a level of S_Q and R_Q and a dispatch took place. At a time instant T_K, inventory replenishment takes place and the replenishment arrives promptly (as we assume zero lead time).

The objective here is to obtain the optimal values of S_Q and R_Q so that the average long-run cost of the system is minimized. The average long-run cost of the system is given by using the renewal reward theorem.

$$TC(S_Q, R_Q) = \frac{\text{E[Replenishment cycle cost]}}{\text{E[Replenishment cycle length]}} \quad (1)$$

The cost of a replenishment/delivery cycle consists of the following parameters or variables; expected delivery cycle length; expected delivery quantity to the retailer in a delivery cycle; expected number of delivery cycles within a replenishment cycle; expected replenishment cycle length; expected replenishment quantity to the supplier in the replenishment cycle; expected inventory holding cost at the retailer in a delivery cycle; and expected inventory holding cost at the supplier in a replenishment cycle as components in the objective function (the expected long-run average cost).

Note that the inter-arrival times of demands $\{T_n : n \geq 1\}$ are exponentially distributed with parameter, λ. $d_n, n = 1, 2, 3, \cdots$ are random variables representing the demand quantity of the n^{th} customer, and d_n's are assumed to be identically and independently distributed and are independent of $N_1(t)$ as well.

The expectation delivery cycle length

When the inventory at the retailer dropped below a certain point, the retailer replenishes the items to bring the inventory back at a level R_Q. This implies that the inventory level of the retailer was a generative process. Since the number of customers that arrived at the retailer for a delivery cycle is $N_2(R_Q)$, from Wald's equation (Ross, 1996; Kang and Kim, 2010), the expected delivery cycle length is given by $E[T_i]E[N_2(R_Q)]$. But $E[T_i] = \frac{1}{\lambda}$ on the inter-arrival time of the customer, then the value of $E[N_2(R_Q)]$ can be estimated as $\frac{R_Q}{E[d_n]} + 1$ since $\lim_{S_R \to \infty} \frac{N_2(R_Q)-1}{R_Q} = \frac{1}{E[d_n]}$ as given previously (Ross, 1996; Kang and Kim, 2010). Thus, the expected delivery cycle length was

$$E[Delivery\ cycle\ length] = E\left[A_{N_2(R_Q)}\right] = E\left[\sum_{i=1}^{N_2(R_Q)} T_i\right] = E[T_i]E[N_2(R_Q)]$$
$$= E[T_n]\left(\frac{R_Q}{E[d_n]} + 1\right) = \frac{R_Q+\mu}{\lambda\mu} \quad (2)$$

Expected delivery quantity to the retailer in a delivery cycle

The number of customers arriving at the retailer during a delivery cycle is $N_2(R_Q)$. The delivery quantity to the retailer is $D_{N_2(R_Q)}$.

Now, $E\left[D_{N_2(R_Q)} - R_Q\right] = \frac{E[d_n^2]}{2E[d_n]}$ by the inspection paradox (Ross, 1996; Kang and Kim, 2010), the expected delivery quantity in a delivery cycle is given as:

$$E[Delivery\ quantity] = E[N_2(R_Q)] = R_Q + E\left[D_{N_2(R_Q)} - R_Q\right]$$
$$= R_Q + \frac{E[d_n^2]}{2E[d_n]} = R_Q + \frac{Var[d_n]+E[d_n]^2}{2e[d_n]} = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu} \quad (3)$$

Expected number of delivery cycles within a replenishment cycle

To calculate the expected long-run average cost we first obtained the expected value and the variance of the number of delivery cycles within a replenishment cycle. From Equation 2 we considered that $D_{N_2(R_Q)}$ follows the Poisson distribution with parameter",

$$\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$$

that is,

$$E\left[D_{N_{2(R_Q)}}\right] = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$$

by allowing the value of $D_{N_{2(R_Q)}}$ to be less than or equal to R_Q. We know from the assumption that $F^{(k)}(S_Q)$ is the distribution function of the Poisson distribution with parameter $k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)$, since $F^{(k)}(S_Q)$ is the $k-fold$ convolution of the Poisson with

$$\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$$

Since k can be expressed as $K = min\left\{k/\sum_{j=1}^{k} D_{Ij(R_Q)} > S_Q\right\}$, and the event $\{K \geq k\}$ is equivalent to $\left\{\sum_{j=1}^{k-1} D_{Ij(R_Q)} \leq S_Q\right\}$ and $P\{K \geq k\} = P\left\{\sum_{j=1}^{k-1} D_{Ij(R_Q)} \leq S_Q\right\} = F^{(k-1)}(S_Q)$. Therefore, the distribution function of K is expressed as:

$$P\{K \leq k\} = 1 - F^{(k)}(S_Q) = 1 - \sum_{i=0}^{S_Q} \frac{k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)^i esp\left(-k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)\right)}{i!} \quad (4)$$

The equation represents the distribution function of the $(S_Q + 1)$, stage Erlang (Gamma) distribution with mean $\frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2}$ and variance $\frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2}$.

Therefore, we approximated the expected value and the variance of the number of delivery cycles within a replenishment cycle as:

$$E[K] = \frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2} \quad (5)$$

and

$$Var[K] = \frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2} \quad (6)$$

Expected replenishment cycle length

A replenishment cycle has k delivery cycles. The from Wald's equation (1996), the expected replenishment cycle length was calculated by multiplying the expected delivery cycle length by the expected number of delivery cycles within a replenishment cycle.

Thus,

$$E\left[\sum_{j=1}^{K} X_i\right] = E[K]E[X]$$

where $X_1, X_2, \ldots$ were independent and identically distributed random variables with finite expectations and K was a stopping time for $X_1, X_2, \ldots$ such that $E[K] < \infty$. The stopping time for $X_1, X_2, \ldots$ if the event $\{K = k\}$ was independent of $X_{k+1}, X_{k+2}, \ldots, k \geq 1$. From Equations 2 and 4, the approximate expected replenishment cycles was as follows:

$$E[Replenishment\ cycle\ length] = E[Delivery\ cycle\ length].E[K]$$

$$= \frac{2(S_Q+1)(R_Q+\mu)}{\lambda(2\mu R_Q+\mu^2+\sigma^2)} \quad (7)$$

Expected replenishment quantity to the supplier in a replenishment cycle

There are K delivery cycles in the replenishment cycle; we calculated the expected replenishment quantity in a replenishment cycle by multiplying the expected delivery quantity in a delivery cycle by the expected number of delivery cycles within a replenishment cycle. From Equations 3 and 5, it can be given as:

$$E[Replenishment\ quantity] = E[K]E\left[D_{2(R_Q)}\right] = S_Q + 1 \quad (8)$$

Expected inventory holding cost at the retailer in a delivery cycle

The inventory level of the retailer in a delivery cycle was expressed as:

$$I_R(t) = \begin{cases} R_Q & if\ 0 \leq t < F_1 \\ R_Q - D_1 & if\ F_1 \leq t < F_2 \\ R_Q - D_2 & if F_2 \leq t < F_3 \\ \vdots & \\ R_Q - D_{\square_2(R_Q)-1} & if\ F_{N_2(R_Q)-1} \leq t \leq F_{N_2(R_Q)} \end{cases}$$

The expected inventory holding cost at the retailer in a delivery cycle was calculated as follows:

$$h_R E\left[R_Q T_1 + (R_Q - D_1)T_2 + (R_Q - D_2)T_3 + \cdots + \left(R_Q - D_{N_2(R_Q)-1}\right)T_{N_2(R_Q)}\right]$$

$$= h_R E\left[R_Q \sum_{i=1}^{N_2(R_Q)} T_i - \sum_{i=1}^{N_2(R_Q)-1} D_i T_{i+1}\right]$$

$$= h_R R_Q E[T_i]E[N_2(R_Q)] - h_R E\left[E\left[\sum_{i=1}^{N_2(SR_Q)-1} D_i T_{i+1}\right]\right]$$

$$= h_R R_Q E[T_i]E[N_2(R_Q)] - h_R E\left[E\left[\sum_{i=1}^{N_2(R_Q)-1} D_i T_{i+1}/N_2(R_Q) = m+1\right]\right]$$

$$= h_R R_Q E[T_i]E[N_2(R_Q)] - h_R E\left[E\left[\sum_{i=1}^{m} D_i T_{i+1}/N_2(R_Q) = m+1\right]\right]$$

$$= h_R R_Q E[T_i]E[N_2(R_Q)] - h_R E[T_i]E\left[E\left[\sum_{i=1}^{m} D_i/N_2(R_Q) = m+1\right]\right]$$

We assumed that $d_i, i = 1, \cdots, N_2(R_Q) - 1$, which follows an exponential distribution and the cumulative demand quantities, $D_1, \cdots, D_m$, for $m = N_2(R_Q) - 1$, are mutually independent random variables following the uniform distribution with range $(0, R_Q)$ from the relationship between the arrival times of the Poisson arrival process and the uniform distribution (Ross, 1996; Kang and Kim, 2010).

Thus,

$E[D_i]$ as $\frac{R_Q}{2}$,

and

$$E\left[\sum_{i=1}^{m} D_i T_{i+1}/N_2(R_Q) = m+1\right] = mE[D_i T_{i+1}] = mE[D_i]E[T_{i+1}] = m\frac{1}{\lambda}\frac{R_Q}{2}.$$

Therefore,

$$E\left[E\left[\sum_{i=1}^{m} D_i T_{i+1}/N_2(R_Q) = m+1\right] = E\left[m\frac{1}{\lambda}\frac{R_Q}{2}\right] = \frac{1}{\lambda}\frac{R_Q}{2}E[m]\right] = \frac{1}{\lambda}\frac{R_Q}{2}E[N_2(R_Q) - 1]$$

since $E[N_2(R_Q)] = \frac{R_Q}{\mu+1}$, the expected inventory holding cost at the retailer in a delivery cycle was estimated as:

$$\begin{aligned} h_R R_Q E[T_i]E[N_2(R_Q) - 1] &- h_R\frac{1}{\lambda}\frac{R_Q}{2}E[N_2(R_Q) - 1] \\ &= h_R\left(R_Q\frac{1}{\lambda}\left(\frac{R_Q}{\mu} + 1\right) - \frac{1}{\lambda}\frac{R_Q}{2}\frac{R_Q}{\mu}\right) \\ &= \frac{h_R R_Q(R_Q + 2\mu)}{2\lambda\mu} \end{aligned} \quad (9)$$

Expected inventory holding cost at the supplier in a replenishment cycle

The inventory level of the supplier in a replenishment cycle was expressed as:

$$I_S(t) = \begin{cases} S_Q & if\ 0 \leq t < F_{Lj(R_Q)} \\ S_Q - D_{Lj(R_Q)} & if\ F_{Lj(R_Q)} \leq t < \sum_{j=1}^{2} F_{Lj(R_Q)} \\ S_Q - \sum_{j=1}^{2} D_{Lj(R_Q)} & if\ \sum_{j=1}^{2} F_{Lj(R_Q)} \leq t < \sum_{j=1}^{3} F_{Lj(R_Q)} \\ \vdots & \\ S_Q - \sum_{j=1}^{K-1} D_{Lj(R_Q)} & if\ \sum_{j=1}^{K-1} F_{Lj(R_Q)} \leq t \leq \sum_{j=1}^{K} F_{Lj(R_Q)} \end{cases}$$

The expected inventory holding cost at the supplier in a replenishment cycle was given as:

$$h_S E\left[S_S F_{Lj(R_Q)}\right] + \left(S_Q - D_{Lj(R_Q)}\right)F_{L^2(R_Q)} + \cdots + \left(S_Q - \sum_{j=1}^{K-1} D_{Lj(R_Q)}\right)F_{L^K(R_Q)}$$

$$= h_S E\left[S_Q \sum_{j=1}^{K} F_{Lj(R_Q)} - \sum_{i=2}^{K}\left\{F_{Lj(R_Q)} \sum_{j=1}^{2} D_{Lj(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K]E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{Lj(R_Q)} \sum_{j=1}^{i-1} D_{Lj(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K]E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{Lj(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{ij(R_Q)}\right\} / K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[F_{N_2(R_Q)}\right] E\left[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{ij(R_Q)} / K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E[F_{N_2(R_Q)}] E\left[E\left[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{ij(R_Q)} / K = k\right]\right]$$

But

$$E\left[E\left[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{ij(R_Q)} / K = k\right]\right] = E\left[E\left[\sum_{j=1}^{k-1} (k-j) D_{ij(R_Q)} / K = k\right]\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] E\left[E\left[\sum_{j=1}^{k-1} (k-j) / K = k\right]\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] E\left[\frac{K^2 - K}{2}\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] \frac{Var[K] + E[K]^2 - E[K]}{2}$$

Hence, the expected inventory holding cost at the supplier in a cycle was given as:

$$h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[F_{N_2(R_Q)}\right] E\left[D_{ij(R_Q)}\right] \frac{Var[K] + E[K]^2 - E[K]}{2}$$
$$= \frac{h_S(S_Q+1)(S_Q+\mu)(4\mu S_Q + 4\mu R_Q + 2\mu^2 + 2\sigma^2 - 8\mu)}{4\lambda\mu(2\mu R_Q + \mu^2 + \sigma^2)} \quad (10)$$

Hence, the total average long-run cost was obtained by adding Equations 3, 5, 8, 9, and 10 divided by the expected Replenishment Cycle Length (Equation 7). That is, substituting the total sum of Equations 3, 5, 7, 8, 9, and 10 in replenishment cycle cost in Equation 1 to give:

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{h_S(\sigma^2-\mu^2-6\mu)}{4\mu} + \lambda\mu(C_R + C_D) \quad (11)$$

Since all demands at the planned period will be eventually satisfied through the replenishment and delivery processes, the cost terms related to the unit replenishment cost (C_R) and the unit delivery cost (C_D) were not affected by the decision variables (that is, the order-up- to-levels). This implies that the same quantity should be replenished and delivered regardless of the order-up-to levels. Our objective here was to minimize the average long-run cost and the minimization problem was given by:

$$\min_{Q,T} TC(S_Q, R_Q)$$
$$\text{Subject to } S_Q, R_Q \geq 0$$

We then gave a cost analysis of the quantity-based model. The optimal values of $S_Q{}^*$ and $R_Q{}^*$ was obtained in analytical form. We obtained the optimal solution for the best lower bound of the average long-run cost. Thus, when we consider the optimal solution for the average long-run cost $TC(S_Q{}^*, R_Q{}^*)$ in Equations 3 to 5 and from Equation 11, we have:

$$\frac{\partial C(S_Q, R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} \quad (12)$$

and

$$\frac{\partial C(S_Q, R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} \quad (13)$$

It was noted that the cost function $TC(S_Q, R_Q)$ was strictly convex for any positive S_Q and R_Q. Thus the unique global minimum for any positive S_Q and R_Q can be obtained by solving:

$$\frac{\partial C(S_Q, R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} = 0$$

and

$$\frac{\partial C(S_Q, R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} = 0$$

That is, for

$$\frac{\partial C(S_Q, R_Q)}{\partial S_S} = 0,$$

we got

$$0 = -\frac{\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2}$$

$$\frac{h_S}{2} = \frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)}$$

$$(S_Q+1)^2 = \frac{2\lambda\mu A_R(R_Q+\mu) + \lambda(\sigma^2-\mu^2)A_R}{2h_S(R_Q+\mu)}$$

$$= \frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}$$

$$S_Q = \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1 \quad (14)$$

For value of R_Q,

$$\frac{\partial C(S_Q, R_Q)}{\partial R_Q} = 0$$

$$0 = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2}$$

$$0 = \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) - \lambda(\sigma^2-\mu^2)A_R + (h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q}{2(S_S+1)(S_R+\mu)^2}$$

$$(h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q = 2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R$$

$$(R_Q+\mu)^2 = \frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}$$

$$R_Q = \sqrt{\frac{\{2\lambda\mu A_D - h_R\,\mu^2 + \lambda\,(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda\,(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu \tag{15}$$

The optimal pair was then given by

$$\left(R_Q{}^*, S_Q{}^*\right) = \left(\sqrt{\frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu, \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1\right) \tag{16}$$

Note, the demand compound Poisson and the demand quantities followed an exponential distribution. Thus, the approximated cost function was derived from Equation 11 by letting $\mu^2 = \sigma^2$. Therefore,

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\mu^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)} + \frac{\lambda(\mu^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} +$$
$$\frac{(h_S+h_R)(R_Q+\mu)}{2} + \frac{h_S(\mu^2-\mu^2-6\mu)}{4\mu} + \lambda\,\mu\,(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} - \frac{3h_S}{2}$$
$$+\lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} - \frac{h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{h_S(R_Q+\mu)}{2} + \frac{h_R(R_Q+\mu)}{2} - \frac{3h_S}{2} +$$
$$\lambda\,\mu\,(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} + h_S\left\{\frac{S_Q+R_Q+\mu}{2} - 1\right\} + \frac{h_R}{2}\left\{R_Q + \mu - \frac{\mu^2}{R_Q+\mu}\right\} + \lambda\mu(C_R+C_D) \tag{17}$$

The optimal pair was then given by

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = 0, \text{ we get } -\frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{h_S}{2} = 0$$

$$\frac{\lambda\mu A_R}{(S_Q+1)^2} = \frac{h_S}{2} \Rightarrow (S_Q+1)^2 = \frac{2\lambda\mu A_R}{h_S}$$

$$S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 \tag{18}$$

For value of R_Q,

$$0 = \frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R \mu^2}{2(R_Q+\mu)^2} + \frac{(h_S+h_R)}{2} = 0$$

$$\frac{2\lambda\mu A_D - h_R \mu^2}{2(R_Q+\mu)^2} = \frac{(h_S+h_R)}{2} \Rrightarrow (R_Q + \mu)^2 = \frac{2\lambda\mu A_D - h_R \mu^2}{(h_S+h_R)}$$

$$R_Q^* = \sqrt{\frac{2\lambda\mu A_D - h_R \mu^2}{(h_S+h_R)}} - \mu \tag{19}$$

$$(R_Q^*, S_Q^*) = \left(\sqrt{\frac{2\lambda\mu A_D - h_R \mu^2}{(h_S+h_R)}} - \mu\, , \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1\right) \tag{20}$$

The corresponding optimal costs was,

$$TC(R_Q,S_Q) = \frac{\lambda\mu A_R}{(\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+1)} + \frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}}-\mu+\mu\right)} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu}{2} - 1\right\} +$$

$$\frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}} - \mu + \mu - \frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-\mu+\mu}\right\} + \lambda\mu(C_R + C_D)$$

$$TC(R_Q,S_Q) = \frac{\lambda\mu A_R}{\left(\sqrt{\frac{2\lambda\mu A_R}{h_S}}\right)} + \frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}\right)} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}-1+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}}{2} - 1\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}} - \frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}}\right\} + \lambda\mu(C_R + C_D)$$

$$TC(R_Q^*,S_Q^*) = \frac{\sqrt{2\lambda\mu A_R h_S}}{2} + \lambda\mu A_R\frac{\sqrt{(2\lambda\mu A_D-h_R\mu^2)(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}}+\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}}-3}{2}\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D-h_R\mu^2}{(h_S+h_R)}} - \frac{\mu^2\sqrt{2\lambda\mu A_D-h_R\mu^2(h_S+h_R)}}{2\lambda\mu A_D-h_R\mu^2}\right\} + \lambda\mu(C_R + C_D) \tag{21}$$

This was lower bound of (R_Q^*, S_Q^*) for any positive values of R_Q and S_Q, that is,

$$TC\left(R_Q{}^*, S_Q{}^*\right) \geq \frac{\sqrt{2\lambda\mu A_R h_S}}{2} + \lambda\mu A_R \frac{\sqrt{(2\lambda\mu A_D - h_R \mu^2)(h_S + h_R)}}{2\lambda\mu A_D - h_R \mu^2} + h_S \left\{ \frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}} + \sqrt{\frac{2\lambda\mu A_D - h_R \mu^2}{(h_S + h_R)}} - 3}{2} \right\}$$

$$+ \frac{h_R}{2} \left\{ \sqrt{\frac{2\lambda\mu A_D - h_R \mu^2}{(h_S + h_R)}} - \frac{\mu^2 \sqrt{2\lambda\mu A_D - h_R \mu^2 (h_S + h_R)}}{2\lambda\mu A_D - h_R \mu^2} \right\} + \lambda\mu(C_R + C_D)$$

(22)

for any $R_Q, S_Q \geq 0$.

RESULTS AND DISCUSSION

From the models developed, the optimal values of S_Q and R_Q that minimized the expected long-run average cost obtained as follows:

(a) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$, $S_Q{}^* = 0$ and $R_Q{}^* = 0$.

It was realized that the cost for replenishing and delivering products was less than the cost of holding inventories. This implies both the supplier and the retailer used a policy which satisfied the requirement from downstream members of the supply chain without carrying inventory but with immediate replenishments from upstream members.

b) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D \geq \mu(h_{\square} + 2h_R)$, $S_Q{}^* = 0$ and $R_Q{}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$

The supplier did not hold inventory since the cost of holding inventories at the supplier was greater than the cost of replenishing products from the outside supplier. In this case, there was a single delivery cycle within a replenishment cycle.

c) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D \geq \mu(h_S + 2h_R)$, $S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and $R_Q{}^* = 0$.

The retailer does not hold inventory since the cost of holding inventories at the retailer was greater than the cost of delivering products from the supplier to the retailer, when needed. In this case, there may be multiple delivery cycles within a replenishment cycle, that is, replenishment occurs when the cumulative demands exceeds the order-up-to level of the supplier while delivery occurs when there is demand at retailer.

d) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$,

$S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and $R_Q{}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$.

Both members hold inventories, since the cost of holding inventories was less that the cost of replenishment or delivery. If the order-up-to level of the supplier was smaller than that of the retailer, then there was a single delivery cycle within a replenishment cycle. Otherwise, they could be multiple delivery cycles within a replenishment cycle.

For serial simulation testing of the models to illustrate the behavior of the parameters, we varied one parameter at a time while keeping the others at based values. The results showed that from the computed values of the retailer and supplier optimal replenishment quantity and minimum total cost of the supply chain, the optimality replenishment quantity of retailer, supplier and minimum total cost of the supply chain increased with increase in the parameters. The values of $R_Q{}^*$, $\square_Q{}^*$ and $TC\left(R_Q{}^*, S_Q{}^*\right)$ were rounded to the nearest two decimal places. Analyses of the variation of relevant costs of the supply chain with respect to simultaneous variations of retailer's and supplier's replenishment quantities are presented in Table 1.

The models showed that with varying individual inventory parameters such as fixed replenishment cost (A_R) increasing from 200 to 400 Frs, fixed delivery cost (A_D) from 10 to 40, inventory cost per unit of the retailer (h_R) from 1 to 6, inventory cost per unit at the supplier (h_S) from 1 to 5, arrival rate (λ) from 1 to 6 and mean of the demand size (μ) from 1to 6, the retailer's and supplier's optimal replenishment quantity and minimum relevant total cost of the supply chain were observed to increased due to the carrying cost. At fixed replenishment cost (A_R) of 200Frs, fixed delivery cost (A_D) of 10, replenishment cost per unit (C_R), and delivery cost per unit (C_D) to be 1 and $\lambda = \mu = h_R = h_S = 1$, the value of the minimum relevant cost of the supply chain was 1257.53 Frs which agrees with minimum total cost of the supply chain previously described by Shao-Fu et al. (2006). The least total cost of supply chain (1257.53 Frs) was obtained at fixed replenishment cost

Table 1. Variation of the optimality of replenishment quantity of the retailer and supplier and total relevant cost.

No	A_R	A_D	h_R	h_S	λ	μ	$R_Q{}^*$	$S_Q{}^*$	$TC(R_Q{}^*, S_Q{}^*)$
1	200	10	1	1	1	1	2.08	19.00	1257.53
2	200	20	1	1	1	1	3.42	19.00	1790.99
3	200	30	1	1	1	1	4.43	19.00	2197.20
4	200	40	1	1	1	1	5.28	19.00	2538.60
5	200	10	1	2	1	3	1.12	23.50	7470.92
6	200	20	1	2	1	4	2.93	27.28	16684.87
7	200	30	1	2	1	5	4.57	30.62	28787.06
8	200	40	1	2	1	6	6.17	33.64	43866.62
9	200	10	2	1	3	2	4.11	47.99	22056.83
10	200	20	2	2	3	4	6.58	47.99	101686.70
11	200	30	2	2	3	5	9.58	53.77	175025.50
12	200	40	2	1	3	6	15.35	83.85	230729.80
13	200	10	3	1	1	3	4.30	68.28	70130.64
14	200	20	3	2	4	5	7.04	62.25	240925.20
15	400	30	3	3	6	4	11.23	79.00	877486.60
16	400	40	3	4	4	2	7.47	39.00	212234.60
17	400	10	4	3	5	4	2.93	72.03	388129.60
18	400	20	3	2	3	6	5.06	83.85	398420.40
19	400	30	6	3	4	2	5.12	45.189	204968.20
20	400	40	4	5	6	5	10.99	68.28	172659.00

$(A_R) = 200\,Frs$, fixed delivery cost $(A_D) = 10$, unit inventory holding cost for the retailer $(h_R) = 1$, arrival rate $(\lambda) = 1$, mean demand size $(\mu) = 5$, optimal replenishment quantity of retailer $(R_Q^*) = 2.08$ and optimal replenishment quantity of supplier $(S_Q^*) = 19.00$. The lowest retailer replenishment quantity $(R_Q{}^* = 1.12)$ was at $A_R = 200Frs$, $A_D = 10$, $h_R = 1$, $h_S = 2$, λ = 1 and μ = 1, and lowest supplier replenishment quantity $(S_Q{}^* = 19.00)$ at $A_R = 200Frs$, $A_D = 10$ to 40; $h_R = 1$, $h_S = 1$, λ = 1 and μ = 1 while the minimum cost of the supply chain $(TC(R_Q{}^*, S_Q{}^*) = 1257.53)$ was at $A_R = 200Frs$, $A_D = 10$; $h_R = 1$, $h_S = 1$, λ = 1 and μ = 1. The highest retailer replenishment quantity $(R_Q{}^* = 15.35)$ was at $A_R = 200Frs$, $A_D = 40$, $h_R = 2$, $h_S = 1$, λ = 3 and μ = 6, and highest supplier replenishment quantity $(S_Q{}^* = 83.85)$ at $A_R = 200Frs$ and 400 Frs, $A_D = 20 \text{ and } 40$; $h_R = 2 \text{ and } 3$, $h_S = 1 \; and \; 2$, λ = 3 and μ = 6 while the highest cost of the supply chain $(TC(R_Q{}^*, S_Q{}^*) = 1257.53)$ was at $A_R = 400Frs$, $A_D = 40$; $h_R = 4$, $h_S = 5$, λ = 6 and μ = 5.

In agreement with the report of Shao-Fu *et al.*, (2006), the present study showed that a general increase in the optimal order quantity of the retailer and supplier and maximum profitable cost of supply chain can be obtained when there is an increase in arrival rate (λ), mean demand size (μ), fixed replenishment cost (A_R) and fixed delivery cost (A_D).

Conclusion

A supply chain consisting of a single supplier and a single retailer was considered, in which the supplier ordered quantity from the manufacturer and released part to the retailer on an order. An integrated inventory replenishment and shipment delivery planning model was proposed for a case of compound Poisson demands with distribution-free demand quantity using the renewal theory. After developing several properties for obtaining a closed-form expression for approximated long-run average cost, the order-up-to level of each member of the supply chain that minimizes the long-run average cost was determined. The result showed that the total relevant cost of the supply chain increased optimally with an increase in the optimal replenishment quantity of the retailer and a constant replenishment quantity of the supplier. Also, keeping the replenishment cost, unit inventory cost of the retailer, arrival rate, and constant mean demand size while increasing the unit inventory cost of the supplier and varying the fixed delivery cost, the total relevant cost of the supply chain and the optimal order quantity value of the retailer increased while the optimal order quantity of the supplier was constant. Also, there was variation of total relevant cost of the supply chain with respect to the simultaneous variation of retailer's and supplier's replenishment quantity. A general increase in the optimal order quantity of the retailer and supplier, and the total relevant cost of the supply chain was obtained with increase in the arrival rate (λ) and mean demand size (μ), and also when there was increase in the fixed replenishment cost (A_R), fixed delivery cost (A_D), retailer inventory holding cost and supplier inventory holding cost.

The variation in optimal replenishment quantities and total relevant costs of the supplier, retailer, and supply chain was linear with respect to the arrival rate and mean demand size, respectively. As the demand increased linearly at the retailer's point with an increase in arrival rate and mean demand size, there were higher retailer quantities ordered of items and consequently an increase in the total relevant cost of the supply chain. The study highlights managerial benefits for companies from supply chain strategies related to reduced stock-outs and overstock situations for improved operational efficiency and optimization of inventory replenishment in the supply chain between suppliers and retailers.

Savings at various cost levels leading to optimal profitability following improved supply chain management would be achieved through minimizing irrelevant inventory activities and reducing transportation costs. A series of simulation tests show that the models developed in the study functions are reasonably good. Enhancing the levels of collaborations and communication on supply chain and inventory replenishment strategies between stakeholders in the supplier chain who coordinate the flow of materials within a company and to the consumer including suppliers, distributors, and retailers should be emphasized.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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