

Full Length Research Paper

A dynamic programming approach to optimal manpower recruitment and promotion policies for the two grade system

S. Nirmala^{1*} and M. Jeeva²

¹5, F-3, Suba vasantham Apartments, Lakshaman Nagar, West Street, Peravallur, Chennai-600082, India.

²Department of Mathematics, Ethiraj College, Chennai – 600 008, India.

Accepted 18 October 2010

Many manpower models play a dominant role in efficient design and control of manpower system. In this paper, a mathematical model has been developed with the objective of minimizing the manpower system cost during the recruitment and promotion period which are determined by the changes that take place in the system. It resulted in the form of recursive optimization, a dynamic programming, which has been found to be analogous to the Wagner – Whitin model in the production and inventory management.

Key words: Recruitment, promotion, dynamic programming and manpower planning.

INTRODUCTION

Bartholomew and Forbes (1979) have described the state of the art in various facts of manpower planning. Edwards (1983) has surveyed various models on their assumption and application and concluded that good presentations of results are more important than theoretical sophistications. Price and Piskor (1972) have developed Goal programming model of manpower planning system for financial, manning, promotion and manpower accounting.

Zanakis and Maret (1981) have formulated a Markovian goal programming model with pre-emptive priorities and provided a more flexible and realistic tool for manpower planning problems. Mehlmann (1980) has developed optimal recruitment and transition strategies for manpower systems using dynamic programming. He has formulated a dynamic programming recursion with the objective of minimizing a quadratic penalty function which reflects the importance of correct manning of each grade under preferred recruitment and transition patterns.

While the models developed in the manpower planning literature have considered financial and labor costs and

various recruitment policy objectives as indicated, surprisingly no work has been reported with the objective of minimizing the manpower system costs. Manpower system costs depend upon the various factors outlined in the following sections.

Rao (1980) has discussed a Dynamic programming approach to determine optimal manpower planning for recruitment policies. While the models developed in the manpower planning literature have considered financial and labor costs and it depends upon the various factors outlined in the following section.

Wastage occurs at any instant of time due to retirement, death, accident and dissatisfaction etc. The recruitment board of an organization starts its function as soon as a vacancy arises. The wastages of employees are immediately taken as an alert signal by the recruitment board and immediate steps are taken.

In this paper, a mathematical model has been developed with the objective of minimizing the manpower system cost during the recruitment and promotion period, which are determined by the changes that take place in the system and resulted in the form of recursive optimization. A dynamic programming model has been found to be analogous to the Wagner–Whitin (1958) model, based on the cost data, it generates the optimal

*Corresponding author. E-mail: snpreeth_15@yahoo.co.in.

recruitment and promotion schedules for future periods.

Manpower system costs

Manpower system costs depend upon the various factors. The various costs associated with manpower system consist of the following:

- (1) Recruitment and Promotion costs
- (2) Overstaffing costs
- (3) Understaffing costs
- (4) Retention costs
- (5) Wastage costs.

Recruitment and promotions costs

This is incurred in the process of recruitment and this costs arises due to (1) the expansion of company, (2) the promotion from Grade 1 to 2 at time t . (3) wastage in Grade 1. The total recruitment cost = Recruitment cost in any time ' t ' (due to expansion or wastage or both) + Recruitment due to promotion. The recruitment cost can be broadly classified into two categories, namely, fixed and variable which are proportional to the number of people recruited. The different components of recruitment costs are:

- (1) Cost of advertising,
- (2) Cost of conducting written test
- (3) Cost of information processing
- (4) Cost of manpower working on the processing of application
- (5) Cost of administrative authority which determines recruitment and promotion policies
- (6) Costs incurred in the form of payment to the interview committee members or the wages of the people on the interview committee.
- (7) T.A paid to the candidates which is optional.
- (8) Cost of medical examination done by the organization
- (9) Cost of training people
- (10) Miscellaneous expenditure, including postage, telephone calls etc.

The actual components of recruitment and promotion cost depends upon the procedure followed by the organization for recruitment while the above components are indicative only. Even though the charges are paid by applicants for processing, it is not proportional to the actual recruitment and promotion costs met by many organizations. The cost of advertising and cost of administrative authority from a fixed component is independent of people recruited or promoted based on the suitability of the candidates.

The costs like traveling expenses are paid to interviewing people and also depend on the policy of each organization in determining the number of candidates to be interviewed. According to management's

policy, if the people to be called are a predetermined ratio, which is proportional to number of candidates selected or interviewed and remains constant.

A fixed and a variable component per recruiter or promoter is applicable for all the costs like conducting written test, manpower working on the processing of applications, Information processing, medical examination and training the people. The fixed costs are higher if the selection process is in groups like military recruitment process.

Overstaffing costs

Overstaffing costs are those incurred owing to an unutilized workforce. These costs are analogous to the inventory costs in a production / inventory situation.

Understaffing costs

Understaffing costs are those resulting from decreased productivity and loss of goodwill (in a profit-motive organization) as a result of the non-availability of the workforce.

Wastage costs

The costs result from the retrenchment or retirement of the employee.

Retention costs

There are certain costs which are involved in retaining an employee in an organization. These costs consist of (i) probation costs, (ii) training and development costs, and (iii) internal mobility costs.

Probation costs are those incurred owing to the learning effect of an employee during a probationary period. The training and development costs are different from the recruitment costs and are incurred owing to the development programmes which an employee undergoes during the course of his service to the organization. Internal mobility costs are the costs involved in demotion or transfer of an employee within the organization.

MATHEMATICAL MODEL

The following assumptions are made while formulating the manpower planning problem to determine optimal recruitment and promotion policies:

- (a) The recruitment and promotion size are known and fixed.
- (b) Recruitment and promotion at a particular grade is considered.
- (c) Recruitment, promotion and overstaffing costs are known and fixed. But overstaffing cost not allowed for Grade 2.

(d) Understaffing is not allowed in both the grades.

Notations

R (t) Recruitment in any period t. S (t) : Fixed recruitment cost in period t.
 P(t) : Promotion at any period t. Q (t) : Cost of promotion / period.
 i(t) : Cost of overstaffing per recruiter or promoter per period.
 I(t) : Number of people recruited / promoted in an earlier period for the requirements of period t. $x_1(t)$: Number of people recruited in period t at Grade 1.
 $x_2(t)$: Number of people recruited in period t at Grade 2. $\mathcal{Y}(t)$: Number of people promoted in period t from Grade 1 to Grade 2. v_1 : Variable cost of recruitment at Grade 1 / employee recruited. v_2 : Variable cost of recruitment at Grade 2 / employee recruited. u : Variable cost of promotion at Grade 1 to Grade 2.
 Overstaffing cost not allowed for Grade 2 since it was for higher level, not necessary. Since, we need to satisfy all requirements on time, so that understaffing is prohibited. The cost structure consists of the following components in period t.

(a) The requirement cost in period t is given by the concave function:

$$S(t) \delta[x_i(t)] + v_i x_i(t), \quad i=1,2.$$

$$\text{where } \delta[x_i(t)] = \begin{cases} 0, & \text{if } x_i(t) = 0 \\ 1, & \text{if } x_i(t) > 0 \end{cases}$$

(b) The promotional cost in period t is given by:

$$Q(t) \mathcal{Y}(t) + u y(t)$$

$$\text{where } \mathcal{Y}(t) = \begin{cases} 0, & \text{if } y(t) = 0 \\ 1, & \text{if } y(t) > 0 \end{cases}$$

(c) The overstaffing cost is i(t) I(t). The total cost of recruitment for the T – period planning interval is:

$$\sum_{t=1}^T [S(t) \delta[x_i(t)] + v_i x_i(t) + i(t) I(t)] \tag{1}$$

(d) The total cost of promotion for the T – period planning interval is:

$$\sum_{t=1}^T [Q(t) \mathcal{Y}(t) + u y(t) + i(t) I(t)] \tag{2}$$

Thus the total cost of recruitment and promotion for the T – period planning interval is:

$$\sum_{t=1}^T [S(t) \delta[x_i(t)] + Q(t) \mathcal{Y}(t) + v_i x_i(t) + u y(t)], \quad i=1,2 \tag{3}$$

Here we take $i(0) = I(0) = 0$ without loss of generality. The problem is to minimize this sum, subject to the constraint that all recruitments and promotions must be met on time, and since the variable cost of recruitment and promotions are constant. Thus we have that $v_i \sum_{t=1}^T R(t)$ and $u \sum_{t=1}^T P(t)$ are constant in Equation (3)

Thus the problem may therefore be stated as:

Minimize

$$\sum_{t=1}^T [S(t) \delta[x_i(t)] + Q(t) \mathcal{Y}(t) + i(t) + I(t)], \quad i = 1,2$$

Subject to $\sum_{k=1}^t x_{ik} = \sum_{k=1}^t R_k, \quad t=1,2,\dots,T; \quad i=1,2 \text{ and}$
 $\sum_{k=1}^t y_k = \sum_{k=1}^t P_k, \quad t=1,2,\dots,T.$

DYNAMIC PROGRAMMING FORMULATION

Theorem

The well-known Wagner–Whitin model is characterized to determine economic lot size with this model. The fixed recruitment and promotion cost is analogous to the set-up cost and the overstaffing cost is analogous to the cost of carrying inventory in an inventory system. The propositions of Wagner – Whitin model, which facilitate formulation of Dynamic programming recursion are thus given.

Theorem 1

There exist an optimal program such that:

$$I(t) x_i(t) = 0 \text{ for all } t \text{ and } i=1,2.$$

Theorem 2

The minimum cost policy has the property that the recruitment cost x takes the values 0, R(t), R(t) + R(t+1), . . . , R(t) + R(t+1) + . . . + R(T) and the promotion cost y

Table 1. It provides hypothetical data for a 5 year planning period of a manpower system.

Year	R	P	S in 000's	Q in 000's	I in 000's
1	79	41	728	540	15
2	34	10	705	220	12
3	52	14	698	385	16
4	61	38	714	412	14
5	25	8	708	398	16

takes the values 0, P(t), P(t) + P(t+1), . . . , P(t) + P(t+1)+ . . . + P(T).

Theorem 3

There exist an optimal program such that if R is satisfied by some x(t**) and P is satisfied by some y(t**), t** < t*, then R(t) and P(t), t = t** + 1, . . . , t* - 1 are also satisfied by x(t**) and y(t**).

Theorem 4

Given that I(t) = 0 for period t, it is optimal to consider periods 1 to t – 1 by themselves. Let F(t) denote the minimal cost program for periods 1 to t, then:

$$F(t) = \min_{1 \leq j < t} [\min_{1 \leq j < t} [S(j) + Q(j) + \sum_{n=j}^{t-1} \sum_{k=h+1}^t i(h)[R(k) + P(k)] + F(j-1)], S(t) + Q(t) + F(t-1)] \quad (1)$$

The above recursion, stated in words, means that the minimum cost for the first 't' periods comprised a fixed recruitment and promotion cost in period j, plus the charges for satisfying requirements R(k) and promotion P(k), k = j+1, . . . , t by recruiting and promoting manpower in period j, which results in overstaffing cost, plus the cost of adopting an optimal policy in periods 1 to j-1 taken by themselves. We state below the manpower planning horizon theorem analogous to the Wagner – Whitin planning horizon theorem, which further simplifies determination of optimal policies.

The manpower planning horizon theorem

If the minimum in (1) occurs for j = t** < t* at any period t, then in periods t > t* it is sufficient to consider only t** ≤ j ≤ t. If t* = t** then it is sufficient to consider programmes such that x(t*) > 0 and y(t*) > 0.

Wagner–Whitin algorithm can be made use of to determine the optimal recruitment and promotion policies. The algorithm at period t*, t* = 1, 2, . . . N may be stated as:

1) Consider the policies of recruiting and promoting at

(period t**, t** = 1,2, . . . t*.

(2) Determine the total cost of these t* different policies by adding the fixed recruitment cost, promotion cost and overstaffing costs associated with the recruitment and promotion at period t** and the cost of acting optimally for periods 1 to t** - 1 considered by themselves. The latter cost has been determined previously in the computations for periods t = 1, 2, . . . t* - 1. (3) From the t* alternatives, select the minimum cost policy for periods 1 to t* considered independently. (4) Proceed the process to period t*+1 or stop if t* = N.

Numerical illustration

Table 1 shows the hypothetical data for a 5 year planning period of a manpower system.

Table 2 summarizes the calculations of the manpower planning presented in Table 1.

Thus the optimal policy may be stated as follows: (1) Recruit and promote in period 4, x₄ + y₄ = 86+46=132 and use the optimal policy for periods 1 to 4, implying (2) Recruit and promote in period 2, x₂ + y₂ = 86+24=100 and use the optimal policy for periods 1 to 2, implying (3) Recruit and promote in period 1, x₁ + y₁ = 79+41=120. The total cost of this policy is 4308.

Conclusion

In this paper an attempt has been made to obtain the optimal number of recruits and promotions made so that the total cost incurred is minimum in the manpower planning system along with the various costs like recruitment costs, promotion costs, overstaffing costs, wastage costs and retention costs. There are two types of cost have been taken into account namely fixed and variable costs. The model has been found to be analogous to the Wagner-Whitin model in a production or inventory situation. The major limitation of the model is the fact that it is considered in isolation from the various constraints and operating policies under which a manpower system operates. As another constraint of the model is that, it is assumed that there is no overstaffing in the higher grade. This model can also be discussed without this constraint as further work.

Table 2. It summarizes the calculations of the manpower planning problem presented in Table1.

Year	1	2	3	4	5
S	728	705	698	714	708
Q	540	220	385	412	398
i	15	12	16	14	16
R	79	34	52	61	25
P	41	10	14	38	8
	1268	1928*	3011	3846*	4952
		2193	2720*	5690	4308*
				4595	
Minimum cost	1268	1928	2720	3846	4308
Optimum policy	1	2	2,3	4	4,5

REFERENCES

- Bartholomew DJ, Andrew FF (1979). *Statistical Techniques for Manpower Planning*, John Wiley and Sons, New York.
- Edwards JS (1983). A survey of manpower planning models and their application, *J. Oper. Res. Soc.*, 34: 1031-1040.
- Rao P (1980). A Dynamic Programming Approach to Determine Optimal Manpower Recruitment Policies. *J. Oper. Res. Soc.*, 41: 983-988
- Price WL, Piskor WG (1972). The application of goal programming to manpower planning. *INFOR*, 10: 221-231.
- Zanakis SH, Maret MW (1981). A Markovian goal programming approach to aggregate manpower planning. *J. Oper. Res. Soc.*, 32: 55-63.
- Mehlmann A (1980). An approach to optimal recruitment and transition strategies for manpower systems using dynamic programming. *J. Oper. Res. Soc.*, 31: 1009-1015.
- Wagner HM, Whitin TM (1958). Dynamic version of the economic lot size model. *Mgmt Sci.*, 5: 89-96.