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Full Length Research Paper

An inventory model with price breaks: Fuzzy approach

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In this paper Fuzzy inventory model is developed with price breaks; a joint economic lot-size model is developed under Fuzzy environment. Optimum order quantity and optimum number of price breaks are obtained through the techniques of Fuzzy mathematics. Cost function of both buyer and seller is represented by Fuzzy membership function, and model is solved for optimum results by reformulating it as Fuzzy linear programming problem. For optimum solution of proposed problem, "LINDO" software is used.

Key words: Fuzzy membership function, trapezoidal numbers, price breaks, defuzzification.

INTRODUCTION

A number of studies in the economics and marketing literature have suggested quantity discount as a mechanism to achieve co-ordination between the seller (or the manufacturer) and the buyer (or the retailer). The basic aim of co-ordination is that a quantity discount schedule can be designed such that the objectives of the seller and buyer are incentive compatible with the maximum channel gain.

Quantity discount has been proposed in two separate research streams as a tool for achieving incentive compatible co-ordination within the chain. Suppose the supply chain consists of two members: The seller who determines the wholesale price and the buyer who chooses his optimal order quantity and the retail price.

In the management literature, the channels' total transaction cost can be minimized by properly designing the quantity discount schedule so that the buyer orders the channels' economical order quantity. On the other hand, the supplier can offer the buyer a quantity discount, which induces the buyer to choose his price at the joint optimum level. This eliminates double marginalization, and increases market demand due to the lower retail price, which benefits the whole system.

However, other purpose of quantity discount is found in literature, which can be summarized as:

1. Perfect price discrimination against a single customer or a set of homogeneous customers.
2. Partial price discrimination against a set of heterogeneous customers.

Dolan (1987) investigated the effect of quantity discount on joint economic lot size models, while Monahan (1985) developed the model for the quantity discount, from the perspective of the seller with constraints imposed to ensure sufficient benefit to the buyer.

In this chapter, a joint economic lot-size model is developed under Fuzzy environment. Optimum order quantity and optimum number of price breaks are obtained through the techniques of Fuzzy mathematics. Cost function of both buyer and seller is represented by Fuzzy membership function and model is solved for optimum results by reformulating it as Fuzzy linear programming problem.

ASSUMPTIONS AND NOTATIONS

Assumptions

Following are the assumptions made for developing the model under study:

1. The simpler EOQ model with deterministic demand, no stock outs, no backlogs and deterministic lead-time, can describe the buyer's inventory policy.
2. The seller has the knowledge of the holding and ordering costs governing the buyer's inventory policy.
3. There is no competitive price reaction to the seller's discount policy.
4. The demand is fixed at uniform rate.

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5. There is a single buyer.

THE BUYER'S PROBLEM

When there is no channel of co-ordination, with the knowledge of buyer's reaction as a function of the seller's decision variable, the seller can optimize his own profit. The buyer's problem is to choose an order quantity that minimizes the total cost consisting of product cost, set-up cost and inventory cost. [Dada et. el.(1987)]:

$$C_b(Q) = pD + \frac{S_b D}{Q} + \frac{H_b D}{Q} \quad \ldots \quad (1)$$

Differentiating Equation (1) w.r.t. Q and equating it to zero, we get the buyer's optimal order quantity (EOQ) as:

$$Q^* = \sqrt{\frac{2S_b D}{H_b}} \quad \ldots \quad (2)$$

This is independent of the purchasing cost.

The total cost for the buyer is:

$$C_b(Q^*) = pD + \sqrt{2DS_b H_b} \quad \ldots \quad (3)$$

It will be noted that in this formulation, the inventory holding cost is assumed to be per unit cost. Thus, this EOQ does not depend on the wholesale price. When the inventory cost depends upon the value of the wholesale price p, then EOQ will be:

$$Q^* = \sqrt{\frac{2S_b D}{H_b p}} \quad \ldots \quad (4)$$

THE SELLER'S PROBLEM

The seller's problem is to maximize his profit by influencing the buyer's order quantity. The seller can design the wholesale price as a function of the order quantity thereby providing an incentive to the buyer to choose a different order quantity.

In the cost minimization problem, the seller's total cost is given by:

$$C_s(Q) = S_s\left(\frac{D}{Q}\right) - H_s\left(\frac{Q}{2}\right) \quad \ldots \quad (5)$$

$$C_s(Q^*) = (H_b S_s - H_s S_b)\left(\sqrt{D/2H_b S_b}\right) \quad \ldots \quad (6)$$

with the resulting joint channel cost:

$$J(Q^*) = C_b(Q^*) + C_s(Q^*)$$

$$= pD + (H_b(2S_b + S_s) - H_s S_b)\sqrt{D/2H_b S_b} \quad \ldots \quad (7)$$

On the other hand, when the channel members' co-ordinate to minimizes the joint cost we have:

$$J(Q) = pD + \frac{D}{Q}(S_b + S_s) + (H_b - H_s)\frac{Q}{2} \quad \ldots \quad (8)$$

The jointly optimal order quantity can be derived from Equation (7) as:

$$Q^{**} = \sqrt{\frac{2(S_b + S_s)D}{H_b - H_s}} \quad \ldots \quad (9)$$

The resulting joint channel cost at Q^{**} is given as:

$$J(Q^{**}) = pD + \sqrt{2(D(H_b - H_s)(S_b + S_s)} \quad \ldots \quad (10)$$

since $S_s > 0,\ H_s > 0$ and $Q^{**} > Q^*$.

This implies that the buyer's cost given in Equation (1) increases as its order quantity deviates from Q^*.

Also by definition, $J(Q^{**}) < J(Q^*)$. Therefore:

$$C_b(Q^{**}) + C_s(Q^{**}) \le C_b(Q^*) + C_s(Q^*),$$

Which implies that:

$$C_b(Q^{**}) - C_b(Q^*) \le C_s(Q^*) - C_s(Q^{**}).$$

That is, when the channel members coordinate to minimize the joint cost, the decrease in the seller's cost is more than the increase in the buyer's cost when the order quantity is increased from Q^* to Q^{**}. This provides the seller a supply that can be used to motivate the buyer to order a higher quantity by passing some of these savings to cover the extra buyer's cost.

Given the buyer's optimal order quantity policy, the seller's problem is to maximize his total profit by adjusting the wholesale price downward in order to motivate the buyer to change the order quantity for the benefit of the whole channel.

MATHEMATICAL MODEL WITH PRICE BREAKS

It is a common practice for the seller to offer quantity discounts to buyer to entice the buyer to increase the order quantity. The joint cost for buyer and seller can be minimized only when the buyer increases his economic order quantity.

For the single price break, the cost function of buyer is given as:

$$C_b(Q) = D\left[\frac{pQ^*}{Q} + p_1\left(\frac{Q-Q^*}{Q}\right)\right] - pD + \frac{S_b D}{Q} + \frac{H_b Q}{2}$$
$$= -D(p-p_1)\left(\frac{Q-Q^*}{Q}\right) + \frac{S_b D}{Q} + \frac{H_b D}{2} \quad (11)$$

Whereas, the discount given by seller is equals to the benefit received by the buyer. Therefore seller's cost function is given by:

$$C_s(Q) = D(p-p_1)\left(\frac{Q-Q^*}{Q}\right) + \frac{S_s D}{Q} - \frac{H_s D}{2} \quad \ldots \quad (12)$$

Differentiating Equation (11) with respect to Q, optimum order quantity is obtained as:

$$Q_b^* = \left[2D\left(S_b + (p-p_1)Q^*\right)/H_b\right]^{1/2} \quad \ldots \quad (13)$$

Equating $Q_b{}^*$ with Q^{**} we get:

$$p - p_1 = \frac{\left[\left(H_b/(H_b - H_s)\right)\left(S_b - S_s\right)\right] - H_b}{\left(2DS_b/H_b\right)} \quad \ldots (14)$$

Now the objective of seller is to minimize the quantity discount $p - p_1$. Considering buyer's EOQ, $Q_b{}^*$, the seller can minimize his cost function as:

$$\min_{p-p_1} D(p-p_1)\left((Q_b{}^* - Q^*)/Q_b{}^*\right) + \frac{DS_s}{Q_b{}^*} - \frac{H_s Q_b{}^*}{2}$$

Letting $x_1 = p - p_1$ the above equation can be rewritten as:

$$\min_{x_1} Dx_1\left((Q_b{}^* - Q^*)/Q_b{}^*\right) + \frac{DS_s}{Q_b{}^*} - \frac{H_s Q_b{}^*}{2} \quad \ldots (15)$$

For $n > 1$, let $B_n(Q)$ and $S_n(Q)$ be the cost function for buyer and seller respectively. Then for second price break cost function of buyer and seller can be written as:

$$B_2(Q) = D\left[-p + \frac{pQ^*}{Q} + \frac{p_1(Q_1{}^* - Q^*)}{Q} + \frac{p_2(Q_2{}^* - Q^*)}{Q}\right]$$
$$+ \frac{S_b D}{Q} + \frac{H_b}{2}$$

Letting $x_1 = p - p_1$ and $x_2 = p - p_2$, we have:

$$B_2(Q) = -D\left[x_1(Q_1{}^* - Q^*)/Q + x_2(Q_2{}^* - Q^*)/Q\right] + \frac{S_b D}{Q} + \frac{H_b Q}{2}$$

In general:

$$B_n(Q) = -D\left[\sum_{i=1}^{n} x_i(Q_i{}^* - Q_{i-1}{}^*)/Q\right] + \frac{S_b D}{Q} + \frac{H_b Q}{2} \quad \ldots (16)$$

Where $Q_0{}^* = Q^*$.

Similarly seller's general cost function is simplified as:

$$S_n(Q) = -D\left[\sum_{i=1}^{n} x_i(Q_i{}^* - Q_{i-1}{}^*)/Q\right] + \frac{S_s D}{Q} - \frac{H_s Q}{2} \quad \ldots \quad (17)$$

To obtain the buyer's optimum order quantity for n - price breaks, differentiating Equation (16) with respect to Q and setting result equal to zero, we get:

$$Q_n{}^* = \left\{2D\left[x_1 Q^* + \sum_j (x_j - x_{j-1})Q^*_{j-1} + S_b\right]/H_b\right\}^{1/2} \quad \ldots (18)$$

FUZZY MATHEMATICAL MODEL

In fact, from the perspective of the seller and buyer, the Fuzzy model solves the problem simultaneously. It also identifies the optimal number of price breaks and optimum order quantity at each price break with the same satisfaction level to both buyer and seller.

In order to develop upper bound, U_b is for the lower bound and L_b for the buyer's cost function. Under the situation of one price break, if buyer chooses optimal order quantity, Q^*, then U_b is the minimum cost to the buyer when seller offer no quantity discount:

$$\therefore\ U_b = pD + \frac{S_b D}{Q^*} + \frac{H_b Q^*}{2} \quad \ldots (19)$$

On the contrast, if buyer chooses joint optimum order quantity, Q^{**}, then the lower bound, L_b can be obtained as:

$$L_b = -D(p-p_1)(Q^{**}-Q^*)/Q^{**} + \frac{S_b D}{Q^{**}} + \frac{H_b Q^{**}}{2} \quad \ldots (20)$$

The buyer's cost function is given by:

$$\Pi_b(x_p) = -D(p-p_1)(Q^{**}-Q^*)/Q^* + \frac{S_b D}{Q^*} + \frac{H_b Q^*}{2} \quad \ldots (21)$$

The membership function for the buyer's cost function is given by Zadeh et al. (1970), Zimmerman (1983) and Park (1987)

$$\mu_b(x_1) = \begin{cases} 0, & \Pi_b(x_1) > U_b \\ \dfrac{U_b - \Pi_b(x_1)}{U_b - L_b}, & L_b \le \Pi_b(x_1) \le U_b \\ 1, & \Pi_b(x_1) \le L_b \end{cases} \quad \ldots (22)$$

Similarly, upper and lower bound for seller's cost function can be determined as:

$$U_s = \frac{DS_s}{Q^*} - \frac{H_s Q^*}{2} \quad \ldots (23)$$

$$L_s = \frac{DS_s}{Q^{**}} - \frac{H_s Q^{**}}{2} \quad \ldots (24)$$

The seller's cost function is:

$$\Pi_s(x_1) = \frac{Dx_1(Q_b^* - Q^*)}{Q_b^*} + \frac{DS_s}{Q_b^*} - \frac{H_s Q_b^*}{2} \quad \ldots (25)$$

with the membership function:

$$\mu_s(x_1) = \begin{cases} 0, & \Pi_s(x_1) > U_s \\ \dfrac{U_s - \Pi_s(x_1)}{U_s - L_s}, & U_s \le \Pi_s(x_1) \le U_s \\ 1, & \Pi_s(x_1) \le L_s \end{cases} \quad \ldots (26)$$

Using the buyer's and seller's membership functions as a constraint, problem can be reformulated as follows (Lai and Hwang, 1994; Cox, 1995):

$$\text{Max } \alpha$$

subject to the constraints:

$$\begin{aligned} &\alpha \le \mu_b(x_1) \\ &\alpha \le \mu_s(x_1) \quad \ldots (20) \\ &x_1, \alpha \ge 0 \end{aligned} \quad \ldots (27)$$

Where α is aspiration level. … (21)

Similarly, model can be extended for n price breaks by replacing

Q_n^* for Q. Therefore, buyer's cost function for Fuzzy model with n-price breaks is simplified as:

$$\Pi_b(x_n) = -D\left[\sum_i x_i(Q_i^* - Q_{i-1}^*)/Q_n^*\right] + \frac{DS_b}{Q_n^*} + \frac{H_b Q_n^*}{2} \quad \ldots (28)$$

$$\Pi_s(x_n) = D\left[\sum_i x_i(Q_i^* - Q_{i-1}^*)/Q_n^*\right] + \frac{DS_s}{Q_n^*} + \frac{H_s Q_n^*}{2} \cdots \quad (29)$$

… (22)

Membership function for buyers $\Pi_b(x_n)$ is obtained as:

$$\mu_b(x_n) = \begin{cases} 0, & \Pi_b(x_n) > U_b \\ \dfrac{U_b - \Pi_b(x_n)}{U_b - L_b}, & L_b \le \Pi_b(x_n) \le U_b \\ 1, & \Pi_b(x_n) \le L_b \end{cases} \quad \ldots (30)$$

… (23)

Membership function for seller's $\Pi_s(x_n)$ is obtained as: … (24)

$$\mu_s(x_n) = \begin{cases} 0, & \Pi_s(x_n) > U_s \\ \dfrac{U_s - \Pi_s(x_n)}{U_s - L_s}, & U_s \le \Pi_s(x_n) \le U_s \\ 1, & \Pi_s(x_n) \le L_s \end{cases} \quad \ldots (31)$$

… (25)

Using Equations (30) and (31), the Fuzzy model can be constructed for n-price break as:

$$\text{Max } \alpha$$

subject to the constraints:

$$\begin{aligned} &\alpha \le \mu_b(x_n) \quad \ldots (26) \\ &\alpha \le \mu s(x_n) \\ &x_i, \alpha \ge 0 \quad \text{for } i = 1,2,3,\ldots,n \end{aligned} \quad \ldots (32)$$

Since Equation (32) cannot be solved analytically, therefore, “LINDO” software is used for solving Equation (32).

CONCLUSION

In this paper, an inventory model is developed for price breaks; its optimum solution is obtained through Fuzzy techniques. As it is a common practice for the seller to offer quantity discounts to buyer in order to entice the buyer to increase the order quantity. To minimize joint cost for buyer and seller, it is suggested that the buyer would have to increase his economic order quantity.

Notations: D, Total yearly number of units demanded at the retail level; Q, order size by a buyer; Q^*, optimal order quantity (EOQ) in the absence of co-ordination; Q^{**}, optimal order quantity with channel co-ordination; H_s, seller's holding cost per unit; H_b, buyer's holding cost per unit; S_s, seller's set up cost; S_b, buyer's set up cost; n, The number of price breaks; **p,** price of items up to Q*; p_i, unit price for i^{th} break, i=1,2,3… n; Q_i *, total order quantity for price i^{th} break; $x_i = p - p_i$, quantity discount for price i^{th} break; $\Pi_b(x_n)$, buyer's cost function for the Fuzzy model in the n-price break case; $\Pi_s(x_n)$, seller's cost function for the Fuzzy model in the n-price break case; $\mu_b(x_n)$, buyer's membership function for the Fuzzy model in the n-price break case; $\mu_s(x_n)$, seller's membership function for the fuzzy model in the n-price break case.

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