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Applications of ig, dg, bg - Closed type sets in topological ordered spaces

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In this paper we discuss possible applications of ig, dg and bg- closed type sets in topological ordered spaces.

Key words: dg-closed, bg-closed, ig*-closed, dg*-closed, bg*-closed sets, Closed type sets, topological ordered spaces

INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like.

Nachbin (1965) initiated the study of topological ordered spaces. Levine (1970) introduced the class of g-closed sets, a super class of sets in 1970. Veera Kumar (2000) introduced a new class of sets, called g*-closed sets in 2000, which is properly placed in between the class of closed sets and the class of g-closed sets. Veera Kumar (2002) introduced the concept of i-closed, d-closed and b-closed sets in 2001. Srinivasarao introduced ig-closed, dg-closed, bg-closed, ig*-closed, dg*-closed and bg*-closed sets in 2014. In this paper, Srinivasarao discusses the possible applications of ig, dg and bg – closed type sets in topological ordered spaces.

A topological ordered space is a triple $(X, \tau, \leq)$, where τ is a topology on X, Where X is a non-empty set and $\leq$ is a partial order on X.

Definition 1

For any $x \in X$, $\{y \in X / x \leq y\}$ will be denoted by $[x, \rightarrow]$ and $\{y \in X / y \leq x\}$ will be denoted by $[\leftarrow, x]$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be ***increasing*** if $A = i(A)$ where $i(A) = \bigcup_{a \in A} [a, \rightarrow]$ (Veera Kumar, 2002).

Definition 2

For any $x \in X$, $\{y \in X / y \leq x\}$ will be denoted by $[\leftarrow, x]$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be ***decreasing*** if $A = d(A)$, where $d(A) = \bigcup_{a \in A} [a, \leftarrow]$ (Veera Kumar, 2002).

PRELIMINARIES

Definition 1

A subset A of a topological space (X, τ) is called

1) a **generalized closed set (briefly g-closed)** (Levine, 1970) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

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2) a **g^*-closed set** (Veera Kumar, 2000) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

Definition 2

A subset A of a topological space (X, τ , ≤) (Veera Kumar, 2002; Srinivasarao, 2014) is called

1) an **i-closed** set if A is an increasing set and closed set.
2) a **d-closed** set if A is a decreasing set and closed set.
3) a **b-closed** set if A is both an increasing and decreasing set and a closed set.
4) **ig-closed** set if icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
5) **dg-closed** set if dcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
6) **bg-closed** set if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

Theorem 1: Every closed set is a g-closed set

The following example supports that a g-closed set need not be closed set in general (Veera Kumar, 2000).

Example 1

Let X = {a, b, c}, τ_2 = {ϕ , X, {a}} and ≤1 = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, ≤1) is a topological ordered space. Closed sets are ϕ , X, {b, c}. g-closed sets are ϕ , X, {b}, {c}, {a, b}, {b, c}, {c,a}. Let A={c}. Clearly A is a g-closed set but not a closed set (Veera Kumar, 2000).

Theorem 2: Every g*-closed set is a g-closed set

The following example supports that a g-closed set need not be a g*-closed set in general (Veera Kumar, 2000).

Example 2

Let X = {a, b, c}, 2 τ = {ϕ , X, {a}} and ≤1 = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, ≤1) are topological ordered spaces. g-closed sets are ϕ , X, {b}, {c}, {a,b}, {b, c}, {c, a}. g*-closed sets are ϕ , X, {b, c}.Let A={c}. Then A is a g-closed set but not a g*-closed set (Veera Kumar, 2000).

Theorem 3: Every i-closed set is an ig-closed set

The following example supports that an ig-closed set need not be an i-closed set in general (Srinivasarao, 2014).

Example 3

Let X = {a, b, c}, 2τ = {ϕ , X, {a}} and ≤2 = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2 , ≤2) is a topological ordered space (Srinivasarao, 2014).

ig-closed sets are Φ, X, {b}, {a, b}. i-closed sets are ϕ, x. Let A = {b} or {a, b}. Clearly, A is an ig-closed set but not an i-closed set.

Theorem 4: Every d-closed set is a dg-closed set

The following example supports that a dg-closed set need not be d-closed set in general (Srinivasarao, 2014).

Example 4

Let X = {a, b, c}, 2 τ = {ϕ , X, {a}} and ≤2 = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2 , ≤2) is a topological ordered space (Srinivasarao, 2014). dg-closed sets are ϕ , X, {c}, {b, c}. d-closed sets are ϕ , X, {b, c}. Let A = {c}. Clearly, A is a dg-closed set but not a d-closed set.

Theorem 5: Every b-closed set is a bg-closed set

The following example supports that a bg-closed set need not be a b-closed set in general (Srinivasarao, 2014).

Example 5

Let X = {a, b, c}, τ_2 = { ϕ , X, {a}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_2, $\leq_3$) is a topological ordered space. bg-closed sets are ϕ , X, {c}. b-closed sets are ϕ , X. Let A = {c}. Clearly A is a bg-closed set but not a b-closed set (Srinivasarao, 2014).

Theorem 6: Every bg-closed set is an ig-closed set

The converse of the above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 6

Let X = {a, b, c}, τ_1 = { ϕ , X, {a}, {b}, {a, b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_1, $\leq_1$) is a topological ordered space (Srinivasarao, 2014).

Let A = {c}. Clearly A is an ig-closed set but not a bg-closed set.

Theorem 7: Every bg-closed set is a dg-closed set

The converse of the above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 7

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, $\tau_1, \leq_3$) is a topological ordered space (Srinivasarao, 2014). Let A = {a, c}. Clearly A is a dg-closed set but not a bg-closed set.

Theorem 8: Every b-closed set set is an i-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 8

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, $\tau_1, \leq_1$) is a topological ordered space (Srinivasarao, 2014). i-closed sets are ϕ, X, {c}, {b, c}. b-losed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is an i-closed set but not a b-closed set.

Theorem 9: Every b-closed set is a d-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 9

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space (Srinivasarao, 2014). d-closed sets are ϕ, X, {c}, {b, c}. b-closed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is a d-closed set but not a b-closed set.

Theorem 10: Every ig*-closed set is an ig-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 10

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, $\tau_2, \leq_1$) is a topological ordered space (Srinivasarao, 2014). ig-closed sets are ϕ, X, {c}, {b, c}. ig*-closed sets are ϕ, X, {b, c}. Let A = {c}. Clearly A is an ig-closed set but not a ig*-closed set.

Theorem 11: Every dg*-closed set is an dg-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 11

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, $\tau_2, \leq_2$) is a topological ordered space (Srinivasarao, 2014). dg-closed sets are ϕ, X, {c}, {b, c}. dg*-closed sets are ϕ, X, {b, c}. Let A = {c}. Clearly A is an dg-closed set but not a dg*-closed set. So the class of dg-closed sets properly contains the class of all dg*-closed sets.

Theorem 12: Every bg*-closed set is a bg-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 12

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly, (X, $\tau_2, \leq_3$) is a topological ordered space. bg*-closed sets are ϕ, X. bg-closed sets are ϕ, X, {c} (Srinivasarao, 2014). Let A = {c}. Clearly A is bg-closed set but not a bg*-closed set. So the class of bg-closed sets properly contains the class of all bg*-closed sets.

Theorem 13: Every bg*-closed set is an ig*-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 13

Let X = {a, b, c}, τ_3 = {ϕ, X, {a}, {b, c}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_3, $\leq_3$) is a topological ordered space (Srinivasarao, 2014). Let A = {b}. Clearly A is an ig*-closed set but not a bg*-closed set.

Theorem 14: Every bg*-closed set is an dg*-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 14

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_1, $\leq_3$) is a topological ordered space (Srinivasarao, 2014). Let A = {a, c}. Clearly A is a dg*-closed set but not a ig*-closed set. The class of all dg*-closed sets properly contains the class of all bg*-closed sets.

Theorem 15: Every i-closed set is an ig*-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 15

Let X = {a, b, c}, τ_3 = {ϕ, X, {a}, {b, c}} and $\leq_4$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_3, $\leq_4$) is a topological ordered space. ig*-closed sets are ϕ, X, {b, c}. i-closed sets are ϕ, X. Let A = {b, c}. Clearly A is a ig*-closed set but not an i-closed set (Srinivasarao, 2014). The class of all ig*-closed sets properly contains the class of all i-closed sets.

Theorem 16: Every d-closed set is a dg*-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 16

Let X = {a, b, c}, τ_4 = {ϕ, X, {a}, {b, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_4, $\leq_2$) is a topological ordered space (Srinivasarao, 2014). dg*-closed sets are ϕ, X, {b, c}. d-closed sets are ϕ, X. Let A = {b, c}. Then A is dg*-closed set but not a d-closed set. The class of all dg*-closed sets properly contains the class of all d-closed sets.

Theorem 17: Every b-closed set is a bg*-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 17

Let X = {a, b, c}, τ_6 = {ϕ, X, {a}, {b}, {a, b}{a, c}} and $\leq_7$ = {(a, a), (b, b), (c, c), (b, c), (c, a), (b, a)}. Clearly (X, τ_6, $\leq_7$) is a topological ordered space (Srinivasarao, 2014). bg*-closed sets are ϕ, X, {b}. b-closed sets are ϕ, X. Let A = {b}. Then A is bg*-closed set but not a b-closed set. The class of all bg*-closed sets properly contains the class of all b-closed sets.

Theorem 18: Every bg*-closed set is an ig-closed set

Then every bg*-closed set is an ig-closed set (Srinivasarao, 2014). The converse of above theorem need not be true. This will be justified from the following example.

Example 18

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_1, $\leq_3$) is a topological ordered space (Srinivasarao, 2014). bg*-closed sets are ϕ, X. ig-closed sets are ϕ, X, {c}, {b, c}. Let A = {c} or {b, c}. Clearly A is an ig-closed set but not a bg*-closed set. The class of all ig-closed sets properly contains the class of all bg*-closed sets.

Theorem 19: Every bg*-closed set is a dg-closed set

The converse of above theorem need not be true (Srinivasarao, 2014). This will be justified from the following example.

Example 19

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.Clearly (X, τ_1, $\leq_2$) is a topological ordered space (Srinivasarao, 2014). bg*-closed sets are ϕ, X. dg-closed sets are ϕ, X, {c}, {b, c}. Let A = {c} or {b, c}. Clearly A is a dg-closed set but not a bg*-closed set.

APPLICATIONS OF g-CLOSED SETS

We introduce the following definitions.

Definition 1

A topological ordered space (X, τ, $\leq$) is called

i) a **${}_iT_{1/2}$ space**, if every ig-closed set is closed.
ii) a **${}_dT_{1/2}$ space**, if every dg-closed set is closed.
iii) a **${}_bT_{1/2}$ space**, if every bg-closed set is closed.

Theorem 1: Every ${}_iT_{1/2}$ space is ${}_bT_{1/2}$ space

Proof

Let (X, τ, $\leq$) be ${}_iT_{1/2}$ space. Let A be bg-closed subset of X.Then A is an ig-closed set.Since (X, τ, $\leq$) is an ${}_iT_{1/2}$ space then 'A' is a closed set. Therefore every bg-closed set is a closed set. Hence (X, τ) is a ${}_bT_{1/2}$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 1

Let X = {a, b, c}, τ_2 = { ϕ , X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, $\leq_1$) is a topological ordered space. bg-closed sets are ϕ , X. Closed sets are ϕ , X, {b, c}. Here every bg-closed set is a closed set. Therefore (X, τ_2, $\leq_{3)}$ is ${}_bT_{1/2}$ space.

Theorem 2: Every ${}_dT_{1/2}$ space is ${}_bT_{1/2}$ space

Proof

Let (X, τ , $\leq$) be ${}_dT_{1/2}$ space. We show that (X, τ , $\leq$) is a ${}_bT_{1/2}$ space. Let A be bg-closed subset of X. Then A is a dg-closed subset of X. Since (X, τ , $\leq$) is ${}_dT_{1/2}$ space, we have A is a closed set. Thus every ${}_dT_{1/2}$ space is ${}_bT_{1/2}$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 2

Let X = {a, b, c}, τ_2 = { ϕ , X, {a}} and $\leq_2$ = {(a, a), (b, b),(c, c), (a, b), (c, b) }. Clearly (X, τ_2, $\leq_{2)}$ is a topological ordered space. bg-closed sets are ϕ , X . Closed sets are ϕ , X. Here every bg-closed set is a closed set. Hence (X, τ_2, $\leq_2$) is ${}_bT_{1/2}$ space. dg-closed sets are ϕ , X, {c}, {b, c }. Here {c} or {b, c} is not a closed set. Thus (X, τ_2, $\leq_2$) is not a ${}_dT_{1/2}$ space.

Theorem 3: ${}_iT_{1/2}$ space and ${}_dT_{1/2}$ space are independent notions as will be seen in the following examples

Example 3

Let X = {a, b, c}, τ_2 = { ϕ , X, {a}} and $\leq_1$ = {(a, a), (b, b),(c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, $\leq_1$) is a topological ordered space. Closed sets are ϕ , X, {b, c }. ig-closed sets are ϕ , X, {c}, {b, c} and dg-closed sets are ϕ , X. Here every dg-closed set is a closed set. Thus (X, τ_2, $\leq_{1)}$ is ${}_dT_{1/2}$ space . Let A={c}. Clearly a is an ig-closed set but not a closed set. Hence (X, τ_2, $\leq_1$) is not a ${}_iT_{1/2}$ space.

Example 4

Let X = {a, b, c}, τ_3 = { ϕ , X, {a}, {b, c}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_3, $\leq_3$) is a topological ordered space. Closed sets are ϕ , X, {a}, {b, c }. ig-closed sets are ϕ , X. dg-closed sets are ϕ , X, {c}. Here every ig-closed set is a closed. Thus (X, τ_3, $\leq_3$) is ${}_iT_{1/2}$ space. Let A={c}. Clearly A is dg-closed set but not a closed set. Hence (X, τ_3, $\leq_3$) is not a ${}_dT_{1/2}$ space.

We thus introduce the following definitions.

Definition 2

The topological ordered space (X, τ , $\leq$) is called

i) ${}_i$**$T_{i,1/2}$ space** if every ig-closed set is an i-closed set.
ii) ${}_d$**$T_{d,1/2}$ space** if every dg-closed set is an d-closed set.
iii) ${}_b$**$T_{b,1/2}$ space** if every bg-closed set is a b-closed set.
iv) ${}_C$**T_i space** if every closed set is an i-closed set.
v) ${}_c$**T_d space** if every closed set is a d-closed set.
vi) ${}_c$**T_b space** if every closed set is a b-closed set.
vii) ${}_i$**T_b space** if every i-closed set is a b-closed set.
viii) ${}_d$**T_b space** if every d-closed set is a b-closed set.

Theorem 4: Every ${}_cT_b$ space is a ${}_cT_i$ space

Proof

Let (X, τ , $\leq$) be ${}_cT_b$ space. We show that (X, τ , $\leq$) is a ${}_cT_i$ space. Let A be a closed set. Since (X, τ , $\leq$) is ${}_cT_b$ space, then A is a b-closed set. Then A is an i-closed set. Therefore every closed set is an i-closed set. Then (X, τ , $\leq$) is a ${}_cT_i$ space. Hence every ${}_cT_b$ space is a ${}_cT_i$ space. The converse of above theorem need not be true. This will be justified from the following example.

Example 5

Let X = {a, b, c}, τ_2 = { ϕ , X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, $\leq_{1)}$ is a topological ordered space. Closed sets are ϕ , X, {b, c }. i-closed sets are ϕ , X, {b, c} and closed sets are ϕ , X . Here every closed set is an i-closed set. Let A = {b, c}. Clearly A is a closed set but not a b-closed set. Thus (X, τ_2, $\leq_1$) is ${}_cT_i$ space but not ${}_cT_b$ space.

Theorem 5: Every ${}_cT_b$ space is a ${}_cT_d$ space

Proof

Let (X, τ , $\leq$) be ${}_cT_b$ space. We show that (X, τ , $\leq$) is a ${}_cT_d$ space. Let A be a closed set. Since (X, τ , $\leq$) is ${}_cT_b$ space, then

A is a b-closed set. Then A is a d-closed set. Therefore every closed set is a d-closed set. Then (X, τ , $\leq$) is a ${}_cT_d$ space. Hence every ${}_cT_b$ space is a ${}_cT_d$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 6

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, $\tau_2, \leq_2$) is a topological ordered space. closed sets are ϕ, X, {b, c }. d-closed sets are ϕ, X, {b, c} and b-closed sets are ϕ, X . Here every closed set is a d-closed set. Let A = {b, c}. Clearly A is a closed set but not a b-closed set. Thus (X, τ_2, $\leq_1$) is ${}_cT_d$ space but not ${}_cT_b$ space.

Theorem 6: Every ${}_cT_b$ space is a ${}_iT_b$ space

Proof

Let (X, τ , $\leq$) be ${}_cT_b$ space. We show that (X, τ , $\leq$) is a ${}_iT_b$ space. Let A be an i-closed set. Then A is a closed set. Since (X, τ , $\leq$) is ${}_cT_b$ space, then A is a b-closed set.. Therefore every i-closed set is a b-closed set. Then (X, τ , $\leq$) is an ${}_iT_b$ space. Hence every ${}_cT_b$ space is a ${}_iT_b$ space. The converse of the above theorem need not be true. This will be seen in the following example.

Example 7

Let X = {a,b,c}, τ_1 = {ϕ, X, {a}, {b}, {a, b}} and $\leq_2$ = {(a, a), (b, b),(c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space. Closed sets are ϕ, X, {c}, {a, c}, {b, c }. i-closed sets are ϕ, X . b-closed sets are ϕ, X. Clearly every i-closed set is a b-closed set where as every closed set is not a b-closed set. Let A = {c} or {a, c} or {b, c}. Clearly A is a closed set but not a b-closed set. Thus (X, $\tau_2, \leq_2$) is ${}_cT_d$ space but not ${}_cT_b$ space.

Theorem 7: Every ${}_cT_b$ space is ${}_dT_b$ space

Proof

Let (X, τ , $\leq$) be ${}_cT_b$ space. We show that (X, τ , $\leq$) is a ${}_dT_b$ space. Let A be a d-closed set then A is a closed set. Since (X, τ , $\leq$) is ${}_cT_b$ space then A is a b-closed set. Thus every d-closed set is a b-closed set. Thus every ${}_cT_b$ space is ${}_dT_b$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 8

Let X = {a, b, c}, τ_1, = {ϕ, X, {a},{b},{a, b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Here closed sets are ϕ, X, {c}, {b,c}, {a,c}. d-closed sets are ϕ, X and b-closed sets are ϕ, X. Let A = {c} is not a b-closed set. Every d-closed sets is b-closed set. Thus (X, $\tau_1, \leq_1$) is a ${}_dT_b$ space but not ${}_cT_b$ space.

Theorem 8: The spaces ${}_cT_i$ and ${}_cT_d$ are independent notions as will be seen in the following examples

Example 9

Let X = {a, b, c}, τ_1, = {ϕ, X, {a}, {b}, {a, b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space. Closed sets are ϕ, x, {c}, {b,c}, {a,c}. i-closed sets are ϕ, X. b-closed sets are ϕ, x.

Clearly, every i-closed set is a b-closed set where as every closed set is not a b-closed set. Thus (X, τ_1, $\leq_2$) is an ${}_iT_b$ space but not ${}_cT_b$ space.

Example 10

Let X = {a, b, c}, τ_1, = {ϕ, X, {a},{b},{a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b,c), (a,c)}. Here closed sets are ϕ, X, {c}, {b,c}, {a,c}. d-closed sets are ϕ, X and b-closed sets are ϕ, X. Let A = {c} is not a b-closed set. Every d-closed sets is b-closed set. Thus (X, $\tau_1, \leq_1$) is a ${}_dT_b$ space but not ${}_cT_b$ space.

Theorem 9: The spaces are ${}_iT_b$ and ${}_dT_b$ are independent notions as will be seen in the following examples

Example 11

Let X = {a, b, c}, τ_1, = {ϕ, X, {a},{b},{a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a,c)}. Clearly (X, τ_1, $\leq_1$) is a topological ordered space. d-closed sets are ϕ ,x . i-closed sets are ϕ, X, {c}, {b, c}. b-closed sets are ϕ, x.

Clearly every d-closed set is a b-closed set where as every i-closed set is not a b-closed set. Thus (X, τ_1, $\leq_1$) is an ${}_dT_b$ space but not ${}_iT_b$ space.

Example 12

Let X = {a, b, c}, τ_1, = {ϕ, X, {a}, {b}, {a,b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space. Here i-closed sets are ϕ, X. d-closed sets are ϕ, X, {c}, {b, c} and b-closed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is a d-closed set but not a b-closed set. Every i-closed sets is a b-closed set where as every d-closed set is not a b-closed set. Thus (X, τ_1,$\leq_1$) is a ${}_dT_b$ space but not ${}_cT_b$ space.

Theorem 10: The spaces ${}_iT_b$ and ${}_cT_i$ are independent notions as will be seen in the following example

Example 13

Let X = {a, b, c}, τ_1, = {ϕ, X, {a}, {b}, {a, b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space. Here i-closed sets are ϕ, X. Closed sets are ϕ, X, {c}, {b, c}, {a, c} and b-closed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is a closed set but not a b-closed set. Every i-closed sets is a b-closed set where as every closed set is not a b-closed set. Thus (X, τ_1,$\leq_1$) is a ${}_iT_b$ space but not ${}_cT_i$ space.

Example 14

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2,$\leq_1$) is a topological ordered space. Closed sets are ϕ, X, {b, c }. i-closed sets are ϕ, X, {b, c} and b-closed sets are ϕ, X . Here every closed set is an i-closed set. Let A = {b, c}. Clearly A is an i-closed set but not a b-closed set. Thus (X, τ_2,$\leq_1$) is ${}_cT_i$ space but not ${}_iT_b$ space.

Theorem 11: The spaces ${}_dT_b$ and ${}_cT_d$ are independent notions as will be seen in the following examples

Example 15

Let X = {a, b, c}, τ_1, = {ϕ, X, {a}, {b}, {a,b}} and $\leq_6$ = {(a, a), (b, b), (c, c), (b, a), (a, c), (b, c)}. Clearly (X, τ_1, $\leq_6$) is a topological ordered space. Here closed sets are ϕ, X, {b, c} . d- closed sets are ϕ, X and b-closed sets are ϕ, X. Let A = {b, c}. Clearly A is a closed set but not a d-closed set. Every d-closed sets is a b-closed set. Thus (X, τ_1,$\leq_6$) is a ${}_dT_b$ space but not ${}_cT_d$ space.

Example 16

Let X = {a,b,c}, τ_2 = {ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2, $\leq_2$) is a topological ordered space. Closed sets are ϕ, X, {b, c }. d-closed sets are ϕ, X, {b, c}. b-closed sets are ϕ, X. Clearly every closed set is a d-closed set where as every d-closed set is not a b-closed set.Let A = {b,c}. Clearly A is a closed set but not a b-closed set. Thus (X, τ_2, $\leq_2$) is ${}_cT_d$ space but not ${}_dT_b$ space.

Theorem 12: The spaces ${}_iT_{i,1/2}$ and ${}_bT_{b,1/2}$ are independent notions as will be seen in the following examples

Example 17

Let X = {a, b, c}, τ_6, = {ϕ, X, {a}, {b}, {a,b}, {a, c}} and $\leq_7$ = {(a, a), (b, b), (c, c), (b, c), (c, a), (b, a)}. Clearly (X, τ_6, $\leq_7$) is a topological ordered space. Here i-closed sets are ϕ, X, {a, c} . ig- closed sets are ϕ, X, {a, c} and b-closed sets are ϕ, X bg- closed sets are ϕ, X, {b}. Clearly every ig-closed set is an i-closed set. So(X, τ_6, $\leq_7$) is ${}_iT_{i,1/2}$ space. Let A = {b} . Clearly A is a bg-closed set but not a b-closed set. Thus (X, τ_6, $\leq_7$) is not a ${}_bT_{b,1/2}$ space.

Example 18

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2,$\leq_1$) is a topological ordered space. i-closed sets are ϕ, X, {b, c }. ig-closed sets are ϕ, X, {c}, {b, c} and b-closed sets are ϕ, X. Clearly every bg-closed set is a b-closed set. Let A = {c}. Clearly A is an ig-closed set but not an i-closed set.

Hence (X, τ_2, $\leq_1$) is ${}_bT_{b,1/2}$ space but not a ${}_iT_{i,1/2}$space.

Theorem 13: The spaces ${}_dT_{d,1/2}$ and ${}_bT_{b,1/2}$ are independent notions as will be seen in the following examples

Example 19

Let X = {a,b,c}, τ_2 = {ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2, $\leq_2$) is a topological ordered space. d-closed sets are ϕ, X, {b, c }. dg-closed sets are ϕ, X, {c}, {b, c}. bg-closed sets are ϕ, X and b-closed sets are ϕ, X. Clearly every bg-closed set is a b-closed set. Let A = {c}. Clearly A is a dg-closed set but not a d-closed set. Hence (X, τ_2, $\leq_2$) is a ${}_bT_{b,1/2}$ space but not a ${}_dT_{d,1/2}$ space.

Example 20

Let X = {a, b, c}, τ_6, = {ϕ, X, {a}, {b}, {a,b}, {a, c}} and

$\leq_7$ = {(a, a), (b, b), (c, c),(b, c), (c, a), (b, a)}. Clearly (X, τ_6, $\leq_7$) is a topological ordered space. Here d-closed sets are ϕ, X, {b}, {b, c} . dg- closed sets are ϕ, X, {b}, {b, c} and b-closed sets are ϕ, X bg- closed sets are ϕ, X, {b} . Clearly every dg-closed set is a d-closed set. So(X, τ_6, $\leq_7$) is $_dT_{d,1/2}$ space. Let A = {b} . Clearly A is a bg-closed set but not a b-closed set. Thus (X, τ_6, $\leq_7$) is not a $_bT_{b,1/2}$ space.

Theorem 14: The spaces $_iT_{i,1/2}$ and $_bT_{b,1/2}$ are independent notions as will be seen in the following examples

Example 21

Let X = {a, b, c}, τ_6, = {ϕ, X, {a}, {b}, {a,b}, {a, c}} and $\leq_7$ = {(a, a), (b, b), (c, c),(b, c), (c, a), (b, a)}. Clearly (X, τ_6, $\leq_7$) is a topological ordered space. Here i-closed sets are ϕ, X, {a, c} . ig- closed sets are ϕ, X, {a, c} and b-closed sets are ϕ, X bg- closed sets are ϕ, X, {b} . Clearly every ig-closed set is an i-closed set. So (X, τ_6, $\leq_7$) is $_iT_{i,1/2}$ space. Let A = {b}. Clearly A is a bg-closed set but not a b-closed set. Thus (X, τ_1, $\leq_6$) is not a $_bT_{b,1/2}$ space.

Example 22

Let X = {a, b, c}, τ_2 = {ϕ, X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_2, $\leq_1$) is a topological ordered space. i-closed sets are ϕ, X, {b, c }. ig-closed sets are ϕ, X, {c}, {b, c} and b-closed sets are ϕ, X . Clearly every bg-closed set is a b-closed set. Let A = {c}. Clearly A is an ig-closed set but not an i-closed set.

Hence (X, τ_2, $\leq_1$) is $_bT_{b,1/2}$ space but not a $_iT_{i,1/2}$space.

Theorem 15: Every $_iT_b$ space is an $_bT_{b,½}$ space

Proof

Let be (X, τ, $\leq$) iT_b space. Now we (X, τ, $\leq$) is a $_bT_{b,1/2}$ space. Let A be a bg-closed set. Then A is an ig-closed set. Since (X, τ, $\leq$) is $_iT_b$ space then A is a b-closed set. Therefore every bg-closed set is a b- closed set Hence every $_iT_b$ space is an $_bT_{b,1/2}$ space.

The converse of the above theorem need not be true. This will be justified from the following example.

Example 23

Let X = {a, b, c}, τ_1, = {ϕ, X, {a},{b}, {a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b,c),(a,c)}. Here i-closed sets are ϕ, X, {c},{b,c}. b-closed sets are ϕ, X and bg-closed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is an i-closed set but not a b-closed set. Every bg-closed sets is a b-closed set. Thus (X, τ_1, $\leq_1$) is a $_bT_{b,1/2}$ space but not $_iT_b$ space.

Theorem 16: Every $_dT_b$ space is an $_bT_{b,½}$ space

Proof

Let be (X, τ, $\leq$) $_dT_b$ space. Now we (X, τ, $\leq$) is a $_bT_{b,1/2}$ space. Let A be a bg-closed set. Then A is a dg-closed set. Since (X, τ, $\leq$) is $_dT_b$ space then A is a b-closed set. Therefore every bg-closed set is a b- closed set Hence every $_dT_b$ space is an $_bT_{b,1/2}$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 24

Let X = {a, b, c}, τ_1, = {ϕ, X, {a}, {b}, {a,b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_2$) is a topological ordered space. Here b-closed sets are ϕ, X. d-closed sets are ϕ, X, {c}, {b, c} and bg-closed sets are ϕ, X. Let A = {c} or {b, c}. Clearly A is a d-closed set but not a b-closed set. Every bg-closed sets is a b-closed set where as every d-closed set is not a b-closed set. Thus (X, τ_1, $\leq_1$) is a $_bT_{b,1/2}$ space but not $_dT_b$ space.

Theorem 17: Every $_iT_b$ space is an $_iT_{1/2}$ space

Proof

Let (X, τ, $\leq$) be $_iT_b$ space. we show that (X, τ, $\leq$) is a $_iT_{1/2}$ space. Let A be a ig-closed set. Since (X, τ, $\leq$) is $_iT_b$ space then A is a b-closed set. Then A is a closed set. Thus every i-closed set is a closed set. Thus every $_iT_b$ space is $_iT_{1/2}$ space.

The converse of the above theorem need not be true. This will be justified from the following example.

Example 25

Let X = {a, b, c}, τ_1, = {ϕ, X, {a},{b},{a, b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b,c), (a,c)}. Here ig-closed sets are ϕ, X, {c},{b,c}. b-closed sets are ϕ, X and closed sets are ϕ, X, {c}, {b, c}, {c, a}. Let A = {c} or {b, c}. Clearly A is an ig-closed set but not a b-closed set. Every ig-closed sets is a closed set. Thus (X, τ_1, $\leq_1$) is a $_iT_b$ space but not $_iT_{1/2}$ space.

Theorem 18: Every $_{d}T_{b}$ space is an $_{d}T_{1/2}$ space

Proof

Let (X, τ , ≤) be $_{d}T_{b}$ space. we show that (X, τ , ≤) is a $_{d}T_{1/2}$ space. Let A be a dg-closed set. Since (X, τ , ≤) is $_{d}T_{b}$ space then A is a b-closed set. Then A is a closed set. Thus every dg-closed set is a closed set. Thus every $_{d}T_{b}$ space is $_{d}T_{1/2}$ space.

The converse of the above theorem need not be true. This will be justified from the following example.

Example 26

Let X = {a, b, c}, τ_1, = { ϕ , X, {a},{b},{a,b}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_{2}$) is a topological ordered space. Here b-closed sets are ϕ , X . dg-closed sets are ϕ , X, {c}, {b, c} and closed sets are ϕ , X, {c}, {b, c}, {c, a}. Let A = {c} or {b, c}. Clearly A is a dg-closed set but not a b-closed set. Every dg-closed sets is a closed set where as every d-closed set is not a b-closed set. Thus (X, τ_1,$\leq_1$) is a $_{d}T_{1/2}$ space and not a $_{d}T_{b}$ space.

Theorem 19: The spaces $_{c}T_{i}$ and $_{d}T_{b}$ are independent notions as will be seen in the following examples

Example 27

Let X = {a, b, c}, τ_1, = { ϕ , X, {a}, {b},{a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_1, $\leq_1$) is a topological ordered space. Here b-closed sets are ϕ , X, {c}. d-closed sets are ϕ , X . Closed sets are ϕ , X, {c}, {b, c}, {a, c}. i-closed sets are ϕ , X, {c}, { b, c}. Let A = {a, c}. Clearly A is a closed set but not an i-closed set. .Every d- closed set is a b-closed set where as every closed set is not an i-closed set. Thus (X, τ_1,$\leq_1$) is a $_{d}T_{b}$ space and not a $_{c}T_{i}$ space.

Example 28

Let X = {a, b, c}, τ_8, = { ϕ , X, {a,b}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_8, $\leq_3$) is a topological ordered space. Here b-closed sets are ϕ , X, {c} . d-closed sets are ϕ , X, {c}, {b, c} and closed sets are ϕ , X, {c}. i-closed sets are ϕ , X, {c}. Let A = {b, c}. Clearly A is a d-closed set but not a b-closed set. Every closed sets is an i-closed set where as every d-closed set is not a b-closed set. Thus (X, τ_1,$\leq_1$) is a $_{c}T_{i}$ space and not a $_{d}T_{b}$ space.

Theorem 20: The spaces $_{d}T_{d,1/2}$ and $_{i}T_{i,1/2}$ are independent notions as will be seen in the following examples

Example 29

Let X = {a,b,c}, τ_4 = { ϕ , X, {a}, {a, c}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_4, $\leq_3$) is a topological ordered space. dg-closed sets are ϕ , X, {a, b }. d-closed sets are ϕ , X . ig-closed sets are ϕ , X, {b} and i-closed sets are ϕ , X {b}. Clearly every ig-closed set is an i-closed set. Let A = {a, b }. Clearly A is a dg-closed set but not a d-closed set. Hence (X, τ_4, $\leq_3$) is a $_{i}T_{i,1/2}$ space but not a $_{d}T_{d,1/2}$ space.

Example 30

Let X = {a,b,c}, τ_2 = { ϕ , X, {a}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2, $\leq_1$) is a topological ordered space. dg-closed sets are ϕ , X. d-closed sets are ϕ , X . ig-closed sets are ϕ , X, {c}, {b, c} and an i-closed sets are ϕ , X, {b, c}. Clearly every dg-closed set is a d-closed set. Let A = {c}. Clearly A is an ig-closed set but not a i-closed set. Hence (X, τ_2, $\leq_2$) is a $_{d}T_{d,1/2}$ space but not a $_{i}T_{i,1/2}$ space.

Theorem 21: The spaces $_{d}T_{d,1/2}$ and $_{d}T_{b}$ are independent notions as will be seen in the following examples

Example 31

Let X = {a,b,c}, τ_4 = { ϕ , X, {a}, {a, c}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_4, $\leq_3$) is a topological ordered space. dg-closed sets are ϕ, X, {a, b}. d-closed sets are ϕ , X. b-closed sets are ϕ , X . Clearly every d-closed set is a b-closed set. Let A = {a, b}. Clearly A is a dg-closed set but not a d-closed set. Hence (X, τ_4, $\leq_3$) is a $_{d}T_{b}$ space but not a $_{d}T_{d,1/2}$ space.

Example 32

Let X = {a, b, c}, τ_1, = { ϕ , X, {a},{b},{a,b}} and $\leq_4$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_1, $\leq_4$) is a topological ordered space. Here b-closed sets are ϕ , X . d-closed sets are ϕ , X, {c, a}. dg- closed sets are ϕ , X, {c, a}. Let A = {a, c}. Clearly A is a d-closed set but not a b-closed set. Every dg-closed set is a d-closed set where as every d-closed set is not a b-closed set. Thus (X, τ_1,$\leq_4$) is a $_{d}T_{d,1/2}$ space and not a $_{d}T_{b}$ space.

Theorem 22: The spaces $_{i}T_{i,1/2}$ and $_{c}T_{d}$ are independent notions as will be seen in the following examples

Example 33

Let X = {a, b, c}, τ_1, = { ϕ, X, {a}, {b},{a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_1, $\leq_1$) is a topological ordered space. Here d-closed sets are ϕ, X. i-closed sets are ϕ, X, {c}, {b, c} .ig- closed sets are ϕ, X, {c}, {b, c}. Let A = {c} or {b, c} or {a, c}. Clearly A is a closed set but not a d-closed set. Every ig-closed set is an i-closed set where as every closed set is not a d-closed set. Thus (X, τ_1,$\leq_1$) is a $_{i}T_{i,1/2}$ space and not a $_{c}T_{d}$ space.

Example 34

Let X = {a,b,c}, τ_2 = { ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2, $\leq_2$) is a topological ordered space. ig-closed sets are ϕ, X, {b}, {a, b }. d-closed sets are ϕ, X, {b, c}. i-closed sets are ϕ, X and closed sets are ϕ, X, {b, c}. Clearly every closed set is a d-closed set. Let A = {b} or {a, b}. Clearly A is an ig-closed set but not an i-closed set. Hence (X, τ_2, $\leq_2$) is a $_{c}T_{d}$ space but not a $_{i}T_{i,1/2}$ space.

Theorem 23: The spaces $_{i}T_{i,1/2}$ and $_{i}T_{b}$ are independent notions as will be seen in the following examples

Example 35

Let X = {a,b,c}, τ_2 = { ϕ, X, {a}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_2, $\leq_2$) is a topological ordered space. ig-closed sets are ϕ, X, {b}, {a, b }. b-closed sets are ϕ, X. i-closed sets are ϕ, X. Clearly every i-closed set is b-closed set. Let A = {b} or {a, b}. Clearly A is an ig-closed set but not an i-closed set. Hence (X, τ_2, $\leq_2$) is a $_{i}T_{b}$ space but not a $_{i}T_{i,1/2}$ space.

Example 36

Let X = {a, b, c}, τ_1, = { ϕ, X, {a},{b},{a,b}} and $\leq_1$ = {(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)}. Clearly (X, τ_1, $\leq_1$) is a topological ordered space. Here b-closed sets are ϕ, X . i-closed sets are ϕ, X, {b}.ig- closed sets are ϕ, X, {b}, {a, b}. Let A = {b} or {a, b} . Clearly A is an ig-closed set but not an i-closed set. Every i-closed set is a b-closed set where as every ig-closed set is not an i-closed set. Thus (X, τ_1,$\leq_1$) is a $_{i}T_{b}$ space and not a $_{i}T_{i,1/2}$ space.

Theorem 24: Every $_{d}T_{d,1/2}$ is a $_{d}T_{1/2}$ space

Proof

Let (X, τ , $\leq$) be $_{d}T_{d,1/2}$ space. we show that (X, τ , $\leq$) is a $_{d}T_{1/2}$ space. Let A be a dg-closed set. Since (X, τ , $\leq$) is $_{d}T_{d,1/2}$ space then A is a d-closed set. Then A is a closed set. Thus every dg-closed set is a closed set. Thus every $_{d}T_{d,1/2}$ space is $_{d}T_{1/2}$ space.

The converse of the above theorem need not be true. This will be justified from the following example.

Example 37

Let X = {a,b,c}, τ_4 = { ϕ, X, {a}, {a, c}} and $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Clearly (X, τ_4, $\leq_2$) is a topological ordered space. dg-closed sets are ϕ, X, {b, c }. Closed sets are ϕ, X, {b}, {b, c}. d-closed sets are ϕ, X . Clearly every dg-closed set is a closed set. Let A = {b, c}. Clearly A is a dg-closed set but not a d-closed set. Hence (X, τ_4, $\leq_2$) is a $_{d}T_{1/2}$ space but not a $_{d}T_{d,1/2}$ space.

Theorem 25: Every $_{i}T_{i,1/2}$ is a $_{i}T_{1/2}$ space

Proof

Let (X, τ , $\leq$) be $_{i}T_{i,1/2}$ space. we show that (X, τ , $\leq$) is a $_{i}T_{1/2}$ space. Let A be an ig-closed set. Since (X, τ , $\leq$) is $_{i}T_{i,1/2}$ space then A is an i-closed set. Then A is a closed set. Thus every ig-closed set is a closed set. Thus every $_{i}T_{i,1/2}$ space is $_{i}T_{1/2}$ space. The converse of the above theorem need not be true. This will be justified from the following example.

Example 38

Let X = {a,b,c}, τ_4 = { ϕ, X, {a}, {a, c}} and $\leq_6$ = {(a, a), (b, b), (c, c), (b, a), (a, c), {b, c) }. Clearly (X, τ_4, $\leq_6$) is a topological ordered space. ig-closed sets are ϕ, X, {b, c}. Closed sets are ϕ, X, {b}, {b, c}. i-closed sets are ϕ, X. Clearly every ig-closed set is a closed set. Let A = {b, c}. Clearly A is an ig-closed set but not an i-closed set. Hence (X, τ_4, $\leq_2$) is a $_{i}T_{1/2}$ space but not a $_{i}T_{i,1/2}$ space.

Theorem 26: Every $_{b}T_{b,1/2}$ is a $_{b}T_{1/2}$ space

Proof

Let (X, τ , $\leq$) be $_{b}T_{b,1/2}$ space. We show that (X, τ , $\leq$) is a $_{b}T_{1/2}$ space. Let A be a bg-closed set. Since (X, τ , $\leq$) is $_{b}T_{b,1/2}$ space then A is a b-closed set. Then A is a closed set. Thus every bg-closed set is a closed set. Thus every ${}_bT_{b,1/2}$ space is ${}_bT_{1/2}$ space.

The converse of the above theorem need not be true. This will be seen in the following example.

Example 39

Let X = {a,b,c}, τ_2 = {ϕ, X, {a}, {a, c}} and $\leq_3$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}. Clearly (X, τ_2, $\leq_3$) is a topological ordered space. bg-closed sets are ϕ, X, {c}. Closed sets are ϕ, X, {b, c} . b-closed sets are ϕ, X . Clearly every dg-closed set is a closed set. Let A = {b, c}. Clearly A is a dg-closed set but not a d-closed set. Hence (X, τ_4, $\leq_2$) is a ${}_dT_{1/2}$ space but not a ${}_dT_{d,1/2}$ space.

CONCLUSION

In this paper, we introduced ${}_iT_{i,1/2}$, ${}_dT_{d,1/2}$, ${}_bT_{b,1/2}$, ${}_iT_{1/2}$, ${}_dT_{1/2}$, ${}_bT_{1/2}$, new class of spaces using g-closed type sets in topological ordered spaces and studied various relationships between them.

Conflict of Interest

The authors have not declared any conflict of interest.

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