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Full Length Research Paper

Conflict resolution using statistical approach

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Conflict can be described as a condition in which actions of one person prevent or compel some outcome at the resistance of the other. Quite often this can be seen as "two or more competing, often incompatible, responses to same event". In this paper, a statistical approach to conflict resolution using the concept of bargaining game theory is presented. The approach gives chances of failure that are minimal since any offer made in a conflict situation is tied to the likelihood of it being accepted as it takes into considerations the demands from the other party. The approach presents a fair way of solving a conflict without affecting a system. An employer-employee relationship was used to illustrate the application of the approach.

Key words: Bargaining game theory, conflict, ultimatum game.

INTRODUCTION

In the recent past, formal models and quantitative analysis have come a long way towards explaining how strategic actors bargain in a variety of conflict settings. In a conflict, for instance, in the political setting or international relations, bargaining plays a central role in understanding and solving any conflict and thus the masterly of the concept of bargaining is very important, (Banks, 1990; Bennett, 1996; Huth and Allee, 2002; London, 2002; Powell, 1987; Powell, 1996). To understand the basics of logic of bargaining in the face of conflicting interests, Game theory has played a key role to this end. Political scientist have employed for instance, bargaining models to analyze effects of open and closed rules on the distributive politics of legislative appropriation to the study of war initiation and termination (Baron and Ferejohn, 1989; Mansfield et al., 2000).

Most conflicts are mostly triggered by the differences in opinions and interpretation of an idea. It is therefore important that these differences are understood in terms of there magnitude in a conflict and taken care of before any bargaining can commence. This gives the opinion of each party an unbiased attention since they fairly assign time/attention based on their contribution in a conflict.

Further, theoretical studies of the bargaining problem have pointed to the importance of asymmetric information and the "reservation values" of players in distributional politics. In general, it is important to understand the effects of substantive variables on the bargaining process. The theoretical models tell us something about the path by which these variables may influence outcomes.

To fully understand the bargaining problem, research has been conducted on the empirical relationship between substantive variables of interest, such as regime type, economic interdependence, institutional rules, legislative composition and bargaining outcomes (McCarty and Poole, 1995; Werner 1999). However, lacking an explicit model of the process that generates the empirical data, and leaving out the choice-based path by which these variables influence decisions, it is often the case that selection and omitted variable bias plague the analysis (King et al., 1994). In particular, Signorino (1999, 2002) demonstrates that traditional linear and categorical estimation techniques can lead to faulty inferences when the strategic data generating process is ignored during estimation.

For effective bargaining there is every need to integrate theoretical models and statistical methods (Ramsay and Signorino, 2006). A statistical tool that supports theoretical consistent inferences about the relationship between substantive variables, the bargain struck, and the probability of bargaining failure is needed. An Ultimatum bargaining games model which is a statistical model has been developed which address the substantive variables,

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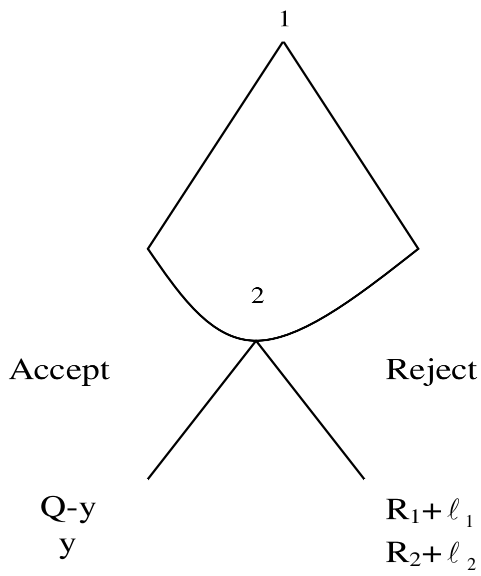

Figure 1. The Ultimatum game.

the bargain struck and the probability of bargaining failure, Ramsay and Signorino (2006).The model presents the relationship between the variables that affect the players' utilities and the outcomes of the bargaining in a strategic setting.

Using the Ultimatum bargaining model, we present a case of conflict in a social context where the contested opinions are seen as the regressor variable(s) and the outcome are seen as the dependent variable(s).

THE ULTIMATUM GAME

Assuming a scenario of two players in a bargaining arena as shown in Figure 1, where the two players must divide a contested prize, which is represented as Q. Let the prize $Q \subset \Re_{+}$ be compact and convex, with lower and upper bounds $\underline{Q} < \bar{Q}$. Without loss of generality, shift the bounds of the prize $\left[\underline{Q}, \bar{Q}\right] = \left[0, Q^{*}\right]$.

The game then proceeds as follows: Player 1 first offers some division of the prize (Q*- y; y), where player 1's allocation is Q*-y and player 2's is y. Player 2 then decides whether to accept or reject player 1's offer. If player 2 accepts, they divide the prize according to player 1's offer. If player 2 rejects the offer, they receive some reservation amount, which may differ between the players.

Assuming each player's utility for bargaining failure has two components: one that is public knowledge and one that is private, then, we denote player 1's reservation value as $R_1 + \ell_1$ and player 2's as $R_2 + \ell_2$, where R_i is player i's publicly observable reservation value and ℓ_i is private information. Let nature draw the type ℓ_i of each player i from a well defined probability distribution.

Assume that each player has a well defined prior beliefs about the distribution of these types and that each type is drawn independently and identically distributed (i.i.d) from the cumulative distribution function F_i (.), with a corresponding everywhere positive density f_i (.), mean $\mu_i = 0$ variance $\sigma_i^2 < \infty$. We also assume the f_i's are continuously differentiable. Each player's strategy can be characterized by a mapping from types into actions: $\sigma_i : \ell_i \rightarrow A^i$, $i = \{1, 2\}$, where A^i defines the action set for player i. Since player 1 is making the ultimatum offer, A^1 ={y: y ∈ [0;Q^*]}, player 2 is then left to accept or reject the offer, so A^2 = {accept; reject}.

If its is further assumed that both players' utilities are strictly increasing and continuous in their amount of the disputed good, and by the random utility structure, the public and private components of the players' utilities are additively separable. That is; assuming:

u_1 (y; accept) = Q*- y
u_2 (y; accept) = y
u_1(y; reject) = $R_1 + \ell_1$
u_2(y; reject) = $R_2 + \ell_2$

Equilibrium in the statistical Ultimatum game has player 1 making an offer that balances and maximizes the marginal utility of increasing the probability that an offer is accepted and the marginal utility of a larger amount of y. Player 2, knowing her own type, chooses the alternative that maximizes her utility.

UNIQUENESS OF EQUILIBRIUM AND ITS EXISTENCE IN AN ULTIMATUM GAME

In a game, players will set strategies that map a random variable to their action space so as to win the game, such as a traditional Bayesian game or random utility model. The player's actions are however probabilistic rather than deterministic.

Noting that a Nash equilibrium of a statistical Ultimatum bargaining game, where each player knows the other has random utilities, is equivalent to a perfect Bayesian Nash equilibrium of an underlying Bayesian game, where the types of the players are private information, we use the well-known game theoretic tools to begin to specify both our theoretical predictions and our empirical estimator. If the perfect Bayesian Nash equilibrium (PBE) of this underlying game can be shown to be unique, then we can solve for the equilibrium strategies and characterize an equilibrium probability distribution over observable outcomes. It is this characteristic of the Ultimatum model that will allow for its structural estimation.

Proposition 1: If F_{ℓ_2} is log-concave, then there exists a unique perfect Bayesian-Nash equilibrium to the statistical Ultimatum game.

Assuming player 1 has made an offer y, player 2 chooses between that offer and her reservation value $R_2 + \ell_2$. Generally in any equilibrium, player 2 plays the cut-point strategy:

$$s_2(y, \ell_2) = \begin{cases} accept & if \ \ y \geq R_2 + \ell_2 \\ reject & if \ \ y < R_2 + \ell_2 \end{cases}$$

Player 1 does not observe ℓ_2, but must assess the probability that player 2 will accept or reject his offer:

$$\begin{aligned} \Pr(accept / y) &= \Pr(y \geq R_2 + \ell_2) \\ &= \Pr(\ell_2 \leq y - R_2) \\ &= F_{\ell_2}(y - R_2) \end{aligned}$$

Now consider the optimization problem for player 1, given player 2's strategy. His expected utility is:

$Eu_1(y/Q^*) = F_{\ell_2}$(y- R2). (Q* - y) + (1- F_{ℓ_2}(y -R2)). (R1 + ℓ_1); (1)

By the first order condition (F.O.C) and the log-concavity of f_{ℓ_2}, 1's optimal offer is the unique y* that implicitly solves:

$$y^* = Q^* - R_1 - \ell_1 - \frac{F_{\ell_2}(y^* - R_2)}{f_{\ell_2}(y^* - R_2)} \quad (2)$$

However, $0 \leq y^* \leq Q^*$ and some times y^* will be outside the feasible set. We can then show that an end-point (0 or Q^*) is optimal and in any PBE, player 1 plays:

$$s_1(\ell_1 / R_1, R_2, Q^*, F_{\ell_2}(.)) = \begin{cases} Q^*, & \ell_1 \leq -R_1 - \dfrac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} \\ y^*, & -R_1 - \dfrac{F_{\ell_2}(Q^* - R_2)}{f_{\ell_2}(Q^* - R_2)} < \ell_1 < Q^* - R_1 - \dfrac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \\ 0, & \ell_1 \geq Q^* - R_1 - \dfrac{F_{\ell_2}(-R_2)}{f_{\ell_2}(-R_2)} \end{cases} \quad (3)$$

Empirical model of the Ultimatum game

The application of the Ultimatum game in empirical analysis requires that a distribution for the ℓ_i, and the appropriate likelihood are specified given the dependent variable(s).

Assume we have data on both player 1's and player 2's actions that is, assume we can measure and code y and Q^* for each observation, as well as whether player 2 accepted or rejected the offer. Let the public portion of the players' reservation values be $R_1 = \beta X$ and $R_2 = \gamma Z$, where X and Z are sets of substantive regressors. Our interest is in estimating the effects of X and Z on y and player 2's decision.

Since the outcome of the bargaining model consists of two dependent variables that is, 1's offer and 2's decision, then the probability model is a joint density over these random variables. The estimates can be obtained by assuming that the types of players 1 and 2 are drawn i.i.d. from a logistic distribution.

Proposition 2: If F_{ℓ_2} is logistic, then it is log-concave.

Considering player 2's decision with a logistic error term in the random utility equation, then the probability that player 2 accepts the offer y is just the logit probability.

$$\Pr(accept / y) = \wedge(y - Z\gamma) \quad (4)$$

For player 1, logistic distribution of y^* implies that;

$$y^* = Q^* - \beta X - \ell_1 - \frac{\wedge(y^* - \gamma Z)}{\lambda(y^* - \gamma Z)} \quad (5)$$

where $\wedge(.)$ is the logit cumulative distribution function (c.d.f) and $\lambda(.)$ is the logit p.d.f.. Solving for y^* gives:

$$y^* = Q^* - \beta X - \ell_1 - 1 - \omega\left(e^{(Q^* - \beta X - \gamma Z - \ell_1 - 1)}\right) \quad (6)$$

Where; ω is Lambert's ω, which solves transcendental functions of the form $z = \omega e^w$ for ω. Lambert's ω is useful here because it is known to have nice properties. First, Lambert's ω is single valued on R_+. Second, ω's first and second derivatives exist and are well behaved, making it easy to show that y^* is a monotonic function of ℓ_1 and allowing for the derivation of the probability density function for equilibrium offers.

The density $f_{y^*}(y^* / \beta X, \gamma Z, Q^*)$ is then:

$$f_{y^*}\left(y^*\right)=\frac{e^{\left(Q^*-1-\beta X-e^{\left(y^*-\gamma Z\right)}-y^*\right)}.\left(1+e^{y^*-\gamma Z}\right)}{\left(1+e^{\left(Q^*-1-\beta X-e^{\left(y^*-\gamma Z\right)}-y^*\right)}\right)^2} \quad (7)$$

and

$$F_{y^*}\left(y^*\right)=\frac{1}{\left(1+e^{\left(Q^*-1-\left(e^{\left(y^*-\gamma Z\right)}+\beta X+y^*\right)\right)}\right)} \quad (8)$$

The constraint on the action space of player 1, however, implies that the observed y^* is censored both from above and below.

Remark: Suppose player 1 plays the strategy s_1 in equation 3, then the distribution of y^* is the truncated distribution of the unconstrained y^*, where the truncation points are from below at 0 and above at Q^*

Take variables $\delta_k, k \in \{0, y, 1\}$ such that $\delta_0 = 1$ if y = 0, $\delta_y = 1$ if 0 < y < Q^* and $\delta_1 = 1$ if y = Q^*. That is, a censored model with a "latent" best offer in the constraint set. Otherwise there is the best feasible offer, at, a boundary point.

Taking player 2's acceptance as $\delta_{accept} = 1$ if she accepted the offer and $\delta_{accept} = 0$ if she rejected the offer and assuming we have data on both player 1 and player 2 actions (that is, y and δ_{accept}), then the likelihood would be:

$$L=\prod_{i=1}^{n}\Pr\left(y^* < 0\right)^{\delta_0}.\Pr\left(y^* = y\right)^{\delta_y}$$

$$.\left(1-\Pr\left(y^* < Q^*\right)\right)^{\delta_1} X \ \Pr\left(accept\right)^{\delta_{accept}}$$

$$.\Pr\left(reject\right)^{1-\delta_{accept}}$$

This gives the log-likelihood function for our data in terms of distributions already derived, which are functions of our regressors, and which explicitly models the Ultimatum game. Estimates of β and γ are obtained using maximum likelihood estimation.

APPLICATION OF BARGAINING BEHAVIOUR TO INDUSTRIAL CONFLICT RESOLUTIONS

The theory of bargaining is important due to its nature of cutting across the various disciplines. The concept has been employed in areas like multinational cooperation and states over terms of foreign investment, to the resolution of territorial disputes, to social issues in relationships.

At this point, however, we examine the application of the bargaining model to conflict resolution in a society with keen emphasis to its statistical interpretation. As discussed in the Ultimatum game, there are basically two players to a game. This could be seen as parties to any dispute, that is, the proponent of a given view and the opposer of the given view. Thus considering player 1 as the proponent of a given view and player 2 as the opposser to the ideas or views as presented or adduced by player 1, then we can apply the bargaining model to a conflict resolution set up.

For instance, in an industrial strike, player 1 may be the employer and player 2 may be the employees. If there exist a conflict between the employer and the employees, one could expect that there exists grounds for some misunderstandings. If the issues are well defined, it is possible to quantify them or even model them. Suppose the conflict between the employer and the employee is on salaries. The employer may make an offer after taking into consideration a number of factors, e.g., economic factors, motivational factors etc; let us take all these factors to be the variables. It is possible that among these factors there are those which are public and those which are private. In uniqueness of equilibrium and its existence in an Ultimatum game, these factors were identified as player's utility for bargaining failure and have been denoted by $R_i + \ell_i$. Where R_i are the publicly known variables and ℓ_i are the private variables. Similarly, the employees will be making demands with both public and private variables. To avoid any conflict, if a demand is adduced by the employees to the employer, we propose a model that will give an employer an opportunity to make an offer that will be acceptable to the employees.

For illustration, let us take the variables that the employer will be taking into consideration in order to make any offer to be denoted by $\sum_i X$ and the variables possibly considered by employees in making a given demand to be denoted by $\sum_i Z$. Of critical concern is for the estimation of the effects of these variables on the outcome that is, on y (employer's allocation or demand) and the decision on the employees (accept or reject an offer). We assume that the concerns of both parties are drawn i.i.d from a logistic distribution.

Let $R_1 = \beta \sum_i X$ be public reservation values for employer

employer and $R_2 = \gamma \sum_i Z$ be the public reservation values for employees. Then the probability that the employees accept the offer y will be given by equation 4.

But the optimal offer by the employer y^* will be given equation 5. Solving for y^* will give by the best offer so that the employees will accept the offer and a conflict will be settled which will present an equilibrium strategy for the employer.

Conclusion

The logit bargaining model can appropriately be used to address and mitigate failures in a conflict by enabling the parties to make reasonably acceptable offers and demands. The bargaining games can be applied to a number of situations to assist in solving a conflict.

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