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Full Length Research Paper

MHD Heat transfer aspects between two parallel conducting porous walls in a rotating system, with Hall currents

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An analysis is presented for heat transfer in fully developed viscous incompressible hydromagnetic flow of an ionized gas bounded by two parallel porous walls under the action of uniform transverse magnetic field, when rotated with an angular velocity about an axis perpendicular to the walls, by taking Hall effect into account. The governing equations of motion and heat transfer expressing the physical situations in terms of mathematical relations are given, assuming that the magnetic Reynolds number is small. Exact solutions are obtained for both primary and secondary velocity distributions to determine the heat transfer when temperature of the walls is prescribed to be the same. The effect of flow parameters on temperature distribution is discussed in detail, when the walls are made up of conducting porous materials. It is found that, the temperature distribution decreases as the Hall parameter increases for fixed values of rotation, suction and Hartmann number; and the rate of heat transfer is greater in case of porous walls than the non-porous walls.

Key words: Magnetohydrodynamics, Hall currents, rotating fluids, suction, heat transfer.

INTRODUCTION

The study of MHD heat transfer in fully developed viscous flow of an ionized gas with Hall currents taking into account under varied conditions and of different geometrical considerations has been the object of extensive globalization of research to understand the behaviour of the performance of fluid motion, as well as the associated temperature distribution due to its wide applications in science, engineering and technology. Because geophysicists encounter MHD phenomena in the interaction of conducting fluids and magnetic fields that are present in and around heavenly bodies, whereas Engineers employ MHD principles in the design of heat exchanger devices, pumps and flow meters, in space vehicle propulsion and in creating novel power generating systems. Several useful and attractive information in both theoretical and applications to many diversified fields like aero-space science, plasma physics and in engineering applications such as power generators, magnetohydrodynamic accelerators and electric transformers etc. Also, in several academic view points by various investigators, mention may be made of the works of Sherman and Sutton (1961), Datta and Mazumdar (1976), Mazumdar et al. (1976), Sastry and Rao (1979), Bharali and Borkakati (1982), Raju and Rao (1993), Ram (1995), Watanabe and Pop (1995), Megahed et al. (2001), Aboeldahab and Elbarbary (2001) and Sato (1961).

In spite of these studies that the effect of Hall currents in Magneto-hydrodynamic viscous flow of an ionized gas under varied conditions and of different geometrical considerations, represent an area of rapid growth in the contemporary research, still there are a few numbers of fundamental problems which are yet to be fully investigated.

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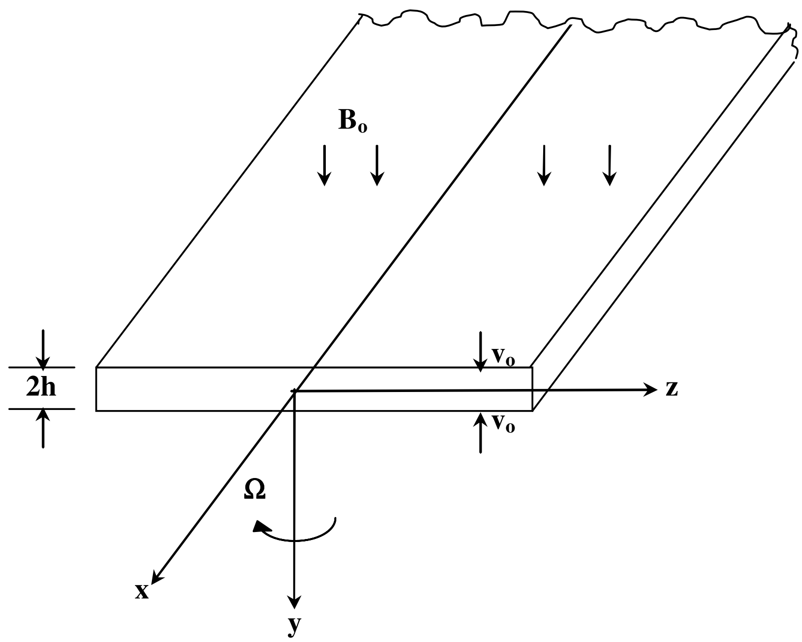

Figure 1. Physical model and coordinate system.

The effects of Hall currents on temperature distribution due to magnetohydrodynamic viscous flow of an ionized gas between two parallel porous walls in a rotating frame of reference is definitely one of them and must be treated thoroughly to explore many geophysical flows; the problems of power production, such as MHD generators, Hall accelerators, radio wave propagation through ionized gases and plasma jets, space crafts, also the way to design nuclear reactors etc. Referring to all the earlier said works and understanding the practical importance, in this paper an attempt has been made to study the temperature distribution due to the magnetohydrodynamic viscous flow of an ionized gas between two parallel porous walls in a rotating frame of reference taking into account the effects of Hall currents, Hartmann number, rotation and porous parameters.

It is assumed that the magnetic Reynolds number is small. The governing equations of motion and heat transfer have been formulated suitably and simplified with certain assumptions to get linear differential equations and solved them analytically, by using the prescribed boundary conditions for both primary and secondary velocity distributions. Consequently, closed form solutions for temperature distribution, the corresponding mean temperature and the rate of heat transfer at the walls in the usual fashion by the dimensionless Nusselt number are obtained when the walls are made up of conducting porous materials. Also, their profiles are plotted for different sets of values of the governing parameters involved. The behaviour of the heat transport is discussed by analyzing these parameters in detail. The purpose of this theoretical study is to know how to set up appropriate models, that is, express physical situations in terms of mathematical relations and to interpret their solutions for existence, also to apply wherever necessary in the aforementioned applications.

Basic equations with boundary conditions and Mathematical analysis

The steady viscous flow of an ionized gas with constant properties between two parallel walls infinite in extent along x- and z- directions subject to uniform suction v_0 normal to these walls in a rotating frame of reference under the action of uniform transverse magnetic field is considered, taking Hall currents into account. While the fluid is driven by a constant pressure gradient $\left(-\partial p / \partial x\right)$.

Figure 1 illustrates the physical model and coordinate system choosing the origin midway between the walls. The whole system is rotated in a counter clockwise direction about y-axis, perpendicular to the walls with an angular velocity $\overline{\Omega}$ (0, Ω, 0). The x-axis is taken in the direction of hydrodynamic pressure gradient in the plane parallel to the channel walls, but not in the direction of flow. A parallel uniform magnetic field B_0 is applied in the y-direction. The height of the channel is denoted by 2 h and the width is assumed to be very large in comparison with the channel height 2 h. Further to simplify the theoretical analysis, the following assumptions are made as in Raju and Rao (1993):

(i) The density of gas is everywhere constant,
(ii) The ionization is in equilibrium which is not affected by the applied electric and magnetic fields,
(iii) The effect of space charge is neglected,
(iv) The flow is fully developed and stationary, that is $\partial/\partial t = 0$ and $\partial/\partial x = 0$ except $\partial p/\partial x \neq 0$,
(v) The magnetic Reynolds number is small (so that the externally applied magnetic field is undisturbed by the flow), namely the induced magnetic field is small compared with the applied field (Shercliff, 1965). Therefore, components in the conductivity tensor are expressed in terms of B_0,
(vi) The flow is two-dimensional, namely $\partial/\partial z = 0$. Since the walls are infinite in extent, all physical variables except pressure will depend on y only.

With these assumptions and treating that the thermal boundary conditions apply everywhere on the infinite channel walls, also neglecting the thermal conduction in the flow direction and the electron heating, the governing equations of momentum, current and energy for the steady flow of neutral fully–ionized gas in a rotating frame of reference (Spitzer, 1956) are:

$$-\left[1-s(1-\frac{\sigma_1}{\sigma_0})\right]\frac{\partial p}{\partial x}+\rho\nu\frac{d^2u}{dy^2}+B_0[-\sigma_1(E_z+uB_0)+\sigma_2(E_x-wB_0)]=2\rho\Omega \mathrm{w}-\rho\nu_0\frac{du}{dy}, \tag{1}$$

$$(s\frac{\sigma_2}{\sigma_0})\frac{\partial p}{\partial x}+\rho\nu\frac{d^2w}{dy^2}+B_0[\sigma_1(E_x-wB_0)+\sigma_2(E_z+uB_0)]=-2\rho\Omega u-\rho \mathrm{v}_0\frac{dw}{dy}, \tag{2}$$

$$\rho u c_p\frac{\partial T}{\partial x}+\rho w c_p\frac{\partial T}{\partial z}-\rho \mathrm{v}_0 c_p\frac{\partial T}{\partial y}=\frac{c_p\mu}{\Pr}\frac{\partial^2T}{\partial y^2}+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{J^2}{\sigma}, \tag{3}$$

considering constant temperature at the porous walls, the boundary conditions are

$$T(\pm h)=T_w=\text{constant}, \tag{4}$$

where c_p is the specific heat at constant pressure and $\Pr$, the Prandtl number.

Moreover, since T_w is constant everywhere on the porous walls, so $\frac{\partial T}{\partial x}=0, \frac{\partial \mathrm{T}}{\partial \mathrm{z}}=0$, and T is finite everywhere in the fluid and hence a function of y only. Thus, Equation (3) becomes:

$$\frac{1}{\Pr}\frac{\mathrm{d}^2T}{dy^2}+\lambda\frac{dT}{dy}=-\frac{1}{c_p\mu}\frac{\mathrm{J}^2}{\sigma_0}-\frac{\Pr}{c_p}\left\{\left(\frac{\partial \mathrm{u}}{\partial \mathrm{y}}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right\}, \tag{5}$$

in the aforestated equations, Ω represents the angular velocity with which the whole system is rotated about the y-axis and $s = p_e/p$ is the ratio of the electron pressure to the total pressure. The value of s is ½ for neutral fully-ionized plasma and approximately zero for a weakly-ionized gas. u, w and E_x and E_z are x- and z- components of velocity $\overline{V}$ and electric field $\overline{E}$ respectively. Also,

$$\sigma_1=\frac{\sigma_0}{1+m^2}, \qquad \sigma_2=\frac{\sigma_0 m}{1+m^2} \quad \text{and}$$

$$m=\left[\omega_e\Big/\left(\frac{1}{\tau}+\frac{1}{\tau_e}\right)\right], \tag{6}$$

where σ_0 stands for the coefficient of proportionality between the current density J and collision term in the equation of motion of charged particles, ω_e is the gyration frequency of electron, τ and τ_e are the mean collision time between electron and ion, electron and neutral particles respectively; σ_1, σ_2 are the modified conductivities parallel and normal to the direction of electric field. The aforestated expression for m which is valid in the case of partially–ionized gas agrees with that of fully–ionized gas when τ_e approaches infinity. Then, the Equations (1), (2) and (5) have been non–dimensionalized by using the characteristic length h and velocity $u_P = -\left(\frac{\partial p}{\partial x}\right)\left(\frac{\mathrm{h}^2}{\rho\nu}\right)$.
Using the notation u, w for u/u_p and w/u_P, $\theta=\frac{T-T_w}{(u_p^2/c_p)}$ and $I_x+iI_z=\frac{J_x+iJ_z}{\sigma_0 B_0 u_p}$, and θ by $\frac{\theta}{u_m^2+w_m^2}$, I^2 by $\frac{I^2}{u_m^2+w_m^2}$, y for y/h, we obtain the following non–dimensional equations:

$$k_1+\frac{d^2u}{dy^2}-\frac{\sigma_1}{\sigma_0}\mathrm{M}^2(m_z+u)+\frac{\sigma_2}{\sigma_0}M^2(m_x-w)=2K^2w-\lambda\frac{du}{dy}, \tag{7}$$

$$k_2+\frac{d^2w}{dy^2}+\frac{\sigma_1}{\sigma_0}\mathrm{M}^2(m_x-w)+\frac{\sigma_2}{\sigma_0}M^2(m_z+u)=-2K^2u-\lambda\frac{dw}{dy}, \tag{8}$$

$$\frac{1}{\Pr}\frac{d^2\theta}{dy^2}+\lambda\frac{d\theta}{dy}=-\left\{\left(\frac{d\mathrm{u}}{d\mathrm{y}}\right)^2+\left(\frac{dw}{dy}\right)^2+M^2I^2\right\}, \tag{9}$$

where

$I^2 = I_x^2 + I_z^2$. (10)

The no-slip boundary conditions are

U = 0 and w = 0 at y = ±1, (11)

θ (± 1) = 0 and $\frac{d\theta}{dy} = 0$ at y = 0. (12)

In which Taylor number (rotation parameter) K defined by $K^2 = \frac{\Omega h^2}{\upsilon}$, Hartmann number M is given by $M^2 = \frac{B_0{}^2 h^2 \sigma_0}{\rho \nu}$, $k_1 = 1 - s\left(1 - \frac{\sigma_1}{\sigma_0}\right)$, $k_2 = -\frac{\sigma_2}{\sigma_0} s$ Suction number $\lambda = h v_0 / \nu$ and $m_x = E_x/(B_0 u_P)$, $m_z = E_z/(B_0 u_P)$ (13). Further, writing $q = u + iw$, $k = k_1 + ik_2$, $E = m_x + im_z$; Equations (7) and (8) can be written in complex form as:

$$\frac{d^2 q}{dy^2} + \lambda \frac{dq}{dy} + \left(\frac{-\sigma_1}{\sigma_0} M^2 + i \frac{\sigma_2}{\sigma_0} M^2 + 2iK^2 \right) q = -k - i \frac{\sigma_1}{\sigma_0} M^2 E - \frac{\sigma_2}{\sigma_0} M^2 E \qquad (14)$$

which is to be solved subject to the no-slip boundary conditions

q (± 1) = 0. (15)

Also, I_x and I_z defined by $J_X/(\sigma_0 B_0 u_P)$ and $J_Z/(\sigma_0 B_0 u_P)$ respectively, are given as

$$I = I_X + iI_Z \frac{\sigma_2 + i\sigma_1}{\sigma_0}\left(q - iE - \frac{s}{M^2} \right) + \frac{is}{M^2}. \qquad (16)$$

The non-dimensional electric field E is to be determined by boundary conditions at large x and z. Now, it is the purpose to determine the temperature distribution, mean temperature and the rate of heat transfer in the fluid flow when both the walls are made up of conducting porous materials, using the expressions for velocity fields (primary and secondary velocities) obtained from the solutions of Equations (7) and (8).

Solutions

When the side walls are made up of conducting material and short circuited by an external conductor, the induced electric current flows out of the channel. In this case no electric potential exists between the side walls. If we assume zero electric field also in the x – and z – directions, we have $m_x = 0$, $m_z = 0$. Constants in the solution are determined by these two conditions.
Solutions for u, w, I_x and I_z are all depend on ‘s’ and are obtained as

$$u = \frac{1}{2}\left[a_9 e^{a_5 y} + a_{10} e^{a_6 y} + a_{19} e^{a_{12} y} + a_{20} e^{a_{13} y} + a_{22} + a_{23} \right], \qquad (17)$$

$$w = a_{21}\left[a_9 e^{a_5 y} + a_{10} e^{a_6 y} - a_{19} e^{a_{12} y} - a_{20} e^{a_{13} y} + a_{22} - a_{23} \right], \qquad (18)$$

$$I = \left(\frac{\sigma_2 + i\sigma_1}{\sigma_0} \right)\left(q - \frac{s}{M^2} \right) + \frac{i\,s}{M^2}, \qquad (19)$$

and $q_m = a_{11} a_{22}$. (20)

Substituting the solutions obtained from Equations (14) and (16) for q and I, we have to solve:

$$-\frac{1}{\Pr}\frac{d^2\theta}{dy^2} - \lambda \frac{d\theta}{dy} = \frac{dq_*}{dy}\frac{d\bar{q}_*}{dy} + M^2 \frac{\sigma_1}{\sigma_0}\left(q_* \bar{q}_*\right) + \left(1 - \frac{\sigma_1}{\sigma_0}\right)\frac{s^2}{M^2}\frac{1}{q_m \bar{q}_m} + \frac{i\,s\,\sigma_2}{\sigma_0}\left(\frac{\bar{q}_*}{q_m} - \frac{q_*}{\bar{q}_m} \right). \qquad (21)$$

Then solving for θ using the boundary conditions (12), the expressions for temperature distribution, mean temperature and rate of heat transfer coefficient at the walls are obtained as follows:

$$\theta = a_{60} e^{a_5 y} + a_{61} e^{a_6 y} + a_{62} e^{a_{12} y} + a_{63} e^{a_{13} y} + a_{64} e^{a_{30} y} + a_{65} e^{a_{31} y} + a_{66} e^{a_{32} y} + a_{67} e^{a_{33} y} + a_{59} e^{a_{57} y} + a_{68} y + a_{58}, \qquad (22)$$

$$\theta_m = \frac{a_{60}}{a_5}(e^{a_5} - 1) + \frac{a_{61}}{a_6}(e^{a_6} - 1) + \frac{a_{62}}{a_{12}}(e^{a_{12}} - 1) + \frac{a_{63}}{a_{13}}(e^{a_{13}} - 1) + \frac{a_{64}}{a_{30}}(e^{a_{30}} - 1) + \frac{a_{65}}{a_{31}}(e^{a_{31}} - 1) + \frac{a_{66}}{a_{32}}(e^{a_{32}} - 1) + \frac{a_{67}}{a_{33}}(e^{a_{33}} - 1) + \frac{a_{59}}{a_{57}}(e^{a_{57}} - 1) + \frac{a_{68}}{2} + a_{58}, \qquad (23)$$

$$N_u = -\left[a_5 a_{60} e^{a_5} + a_6 a_{61} e^{a_6} + a_{12} a_{62} e^{a_{12}} + a_{13} a_{63} e^{a_{13}} + a_{30} a_{64} e^{a_{30}} + a_{31} a_{65} e^{a_{31}} \right] - \left[a_{32} a_{66} e^{a_{32}} + a_{33} a_{67} e^{a_{33}} + a_{57} a_{59} e^{a_{57}} + a_{68} \right]. \qquad (24)$$

RESULTS AND DISCUSSION

The governing equations of motion and heat transfer have been formulated based on the physical model and

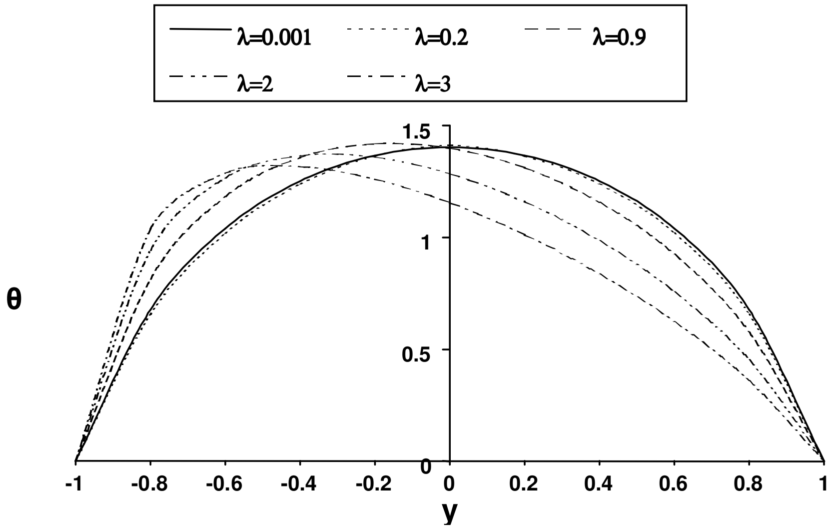

Figure 2. Temperature distribution for M=5,K=2,m=5,s=0.

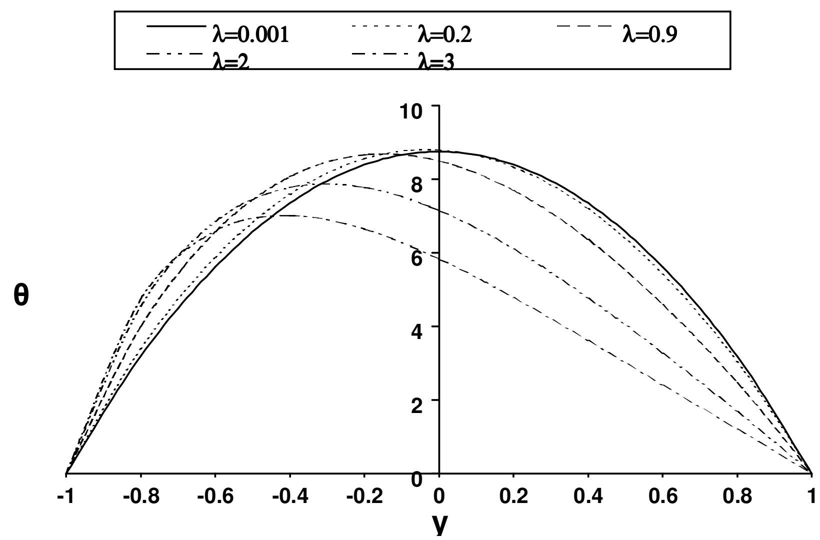

Figure 3. Temperature distribution for M=5, K=2, m=5, s=0.5.

simplified by the assumptions; where the resulting differential equations are solved analytically with the help of the prescribed boundary conditions. Exact solutions are obtained for temperature distribution when the walls are made up of conducting porous materials by making use of the developed solutions for both the primary and secondary velocity distributions. Further, the expressions for mean temperature and the rate of heat transfer (Nusselt number) at the walls are also determined. The corresponding profiles are presented after obtaining the computational values for different sets of the governing parameters involved in the study and discussed the behaviour of heat transport in detail numerically. Figures (2) and (3) show the temperature distribution θ for the cases of weakly ionized gas(s = 0) and neutral fully-ionized plasma(s = ½), respectively. From Figure (2), it is observed that, for fixed values of rotation parameter K, Hall parameter m and Hartmann number M, as suction number λ increases, θ decreases near the upper wall up to channel center line and from center of the channel to lower wall it increases and then decreases, but near the lower wall it increases. The discussion for temperature distribution for s = ½ is displayed in Figure (3), and its behaviour remains the same as that corresponding to the case when s = 0.

Figures (4) and (5) exhibit the temperature θ for the

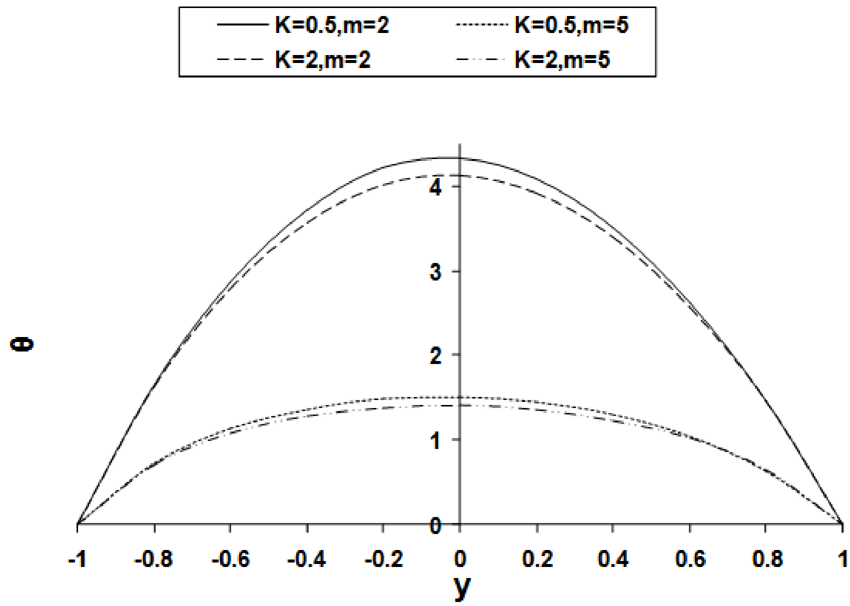

Figure 4. Temperature distribution for M=5, λ=0.2, s=0.

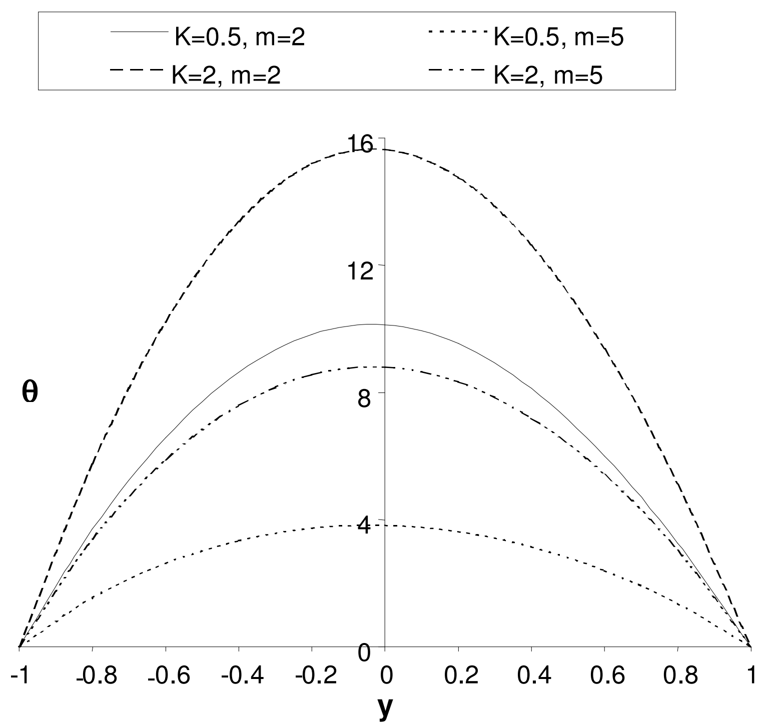

Figure 5. Temperature distribution for M=5,λ=0.2,s=0.5.

cases of s = 0 and s = ½, respectively. From Figure (4) it is observed that when all other parameters are held fixed, the temperature θ decreases as m increases, also it holds good for the same result as rotation increases. From Figure (5), for fixed values of λ, M and K, as m increases θ decreases but it increases as K increases for fixed λ, M and m. This is because of the formation of thin boundary layers, and heavy reductions in the velocity

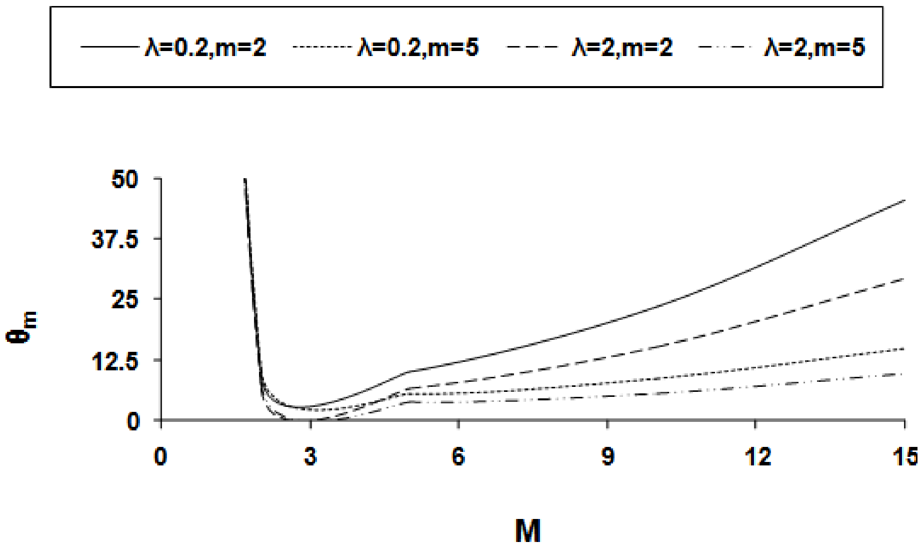

Figure 6. Mean temperature for K=2, s=0.5.

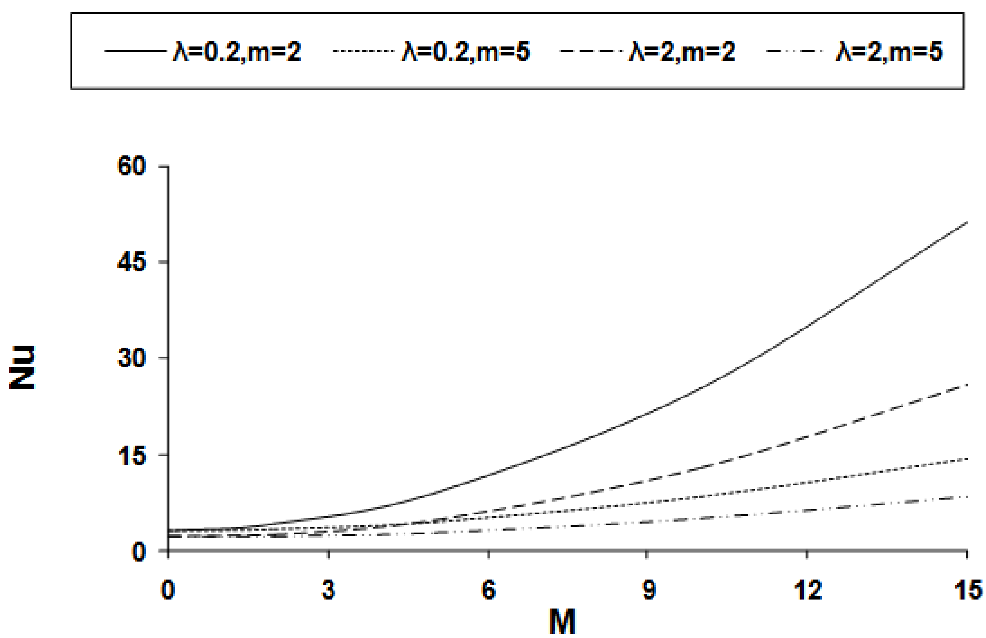

Figure 7. Nusselt number for K=2, s=0.

distributions due to very high coriolis forces causing less temperature distribution. Which shows that a strong coriolis force can change the mode of action of the magnetic field in presence of Hall and porous parameters. In both the cases (that is, when s = 0 and s = ½) it is found that, for fixed λ, M and K, θ decreases as m increases. From Figure (6), it is observed that, when s = ½ and for fixed values of λ and K, as m increases, there exists a H_a* such that as H_a (≠0) increases up to H_a*, the mean temperature θ_m increases and then decreases beyond H_a*. In particular this H_a* is found to be 2.5 for K = 2. Also, θ_m decreases as λ increases for fixed K, M and m. When s=0, it is concluded that by keeping all other parameters fixed, θ_m decreases in both the cases of increasing values of m and λ.

For different sets of parameters, when s = 0 and s = ½, the rate of heat transfer coefficient (Nusselt number, Nu) is plotted in Figures (7) and (8) respectively. For fixed

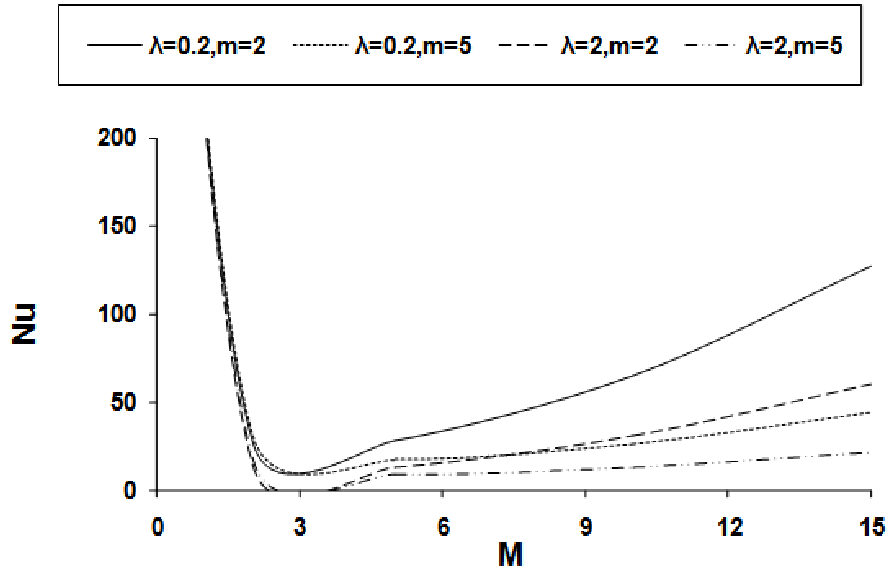

Figure 8. Nusselt number for K=2, s=0.5.

values of K, M and m as λ increases, Nu decreases. Also, it decreases as m increases for fixed K, M and λ. When s = ½ it is noticed that, the Nusselt number behaves almost similarly as mean temperature corresponding to the case for s = ½. Further, it is to be noted that the Nusselt number at the lower wall, is equal and opposite to that at the upper wall. While in both cases that is, for s = 0 and s = ½, we observe that the effect of increasing Hartmann number M is to decrease the rate of heat transfer coefficient Nu for fixed values of λ, m and K.

Conclusions

Here, the value of Pr is fixed at 1 for all graphs. We first note that in the absence of suction number, $\theta(y) = \theta(-y)$ implying that the temperature distribution is symmetrical about the channel-center line y = 0. But in the presence of suction number as expected, we find that there is a discrepancy in the symmetry of the temperature distribution.

This discrepancy increases as λ increases. Also, when λ = 0, K = 0 and m = 0 (that is, for non-porous walls, without rotation and Hall currents) these results are in good agreements with the solution corresponding to R_E = - 1(Flow meter) as given in Equation (44) by Cramer and Pai (1973). It is found that, the temperature distribution decreases as the Hall parameter increases. Further, we note that the rate of heat transfer coefficient is greater in case of porous walls than the non-porous walls in the presence of Hartmann number, Hall and rotation parameters. On the other hand, the rate of heat transfer coefficient at the channel porous walls is affected by the combination of Hall, rotation and suction parameters with the Hartmann number.

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