

Full Length Research Paper

On generalised fuzzy soft topological spaces

R. P. Chakraborty* and P. Mukherjee

Department of Mathematics, Hijli College, Hijli, Kharagpur, Paschim Medinipur-721301, West Bengal, India.

Received 7 August, 2014; Accepted 8 September, 2014

In this paper, union, and intersection of generalised fuzzy soft sets are introduced and some of their basic properties are studied. The objective of this paper is to introduce the generalised fuzzy soft topology over a soft universe with a fixed set of parameters. Generalised fuzzy soft points, generalised fuzzy soft closure, generalised fuzzy soft neighbourhood, generalised fuzzy soft interior, generalised fuzzy soft base are introduced and their basic properties are investigated. Finally, generalised fuzzy soft compact spaces are introduced and a few basic properties are taken up for consideration.

Key words: Fuzzy soft set, generalised fuzzy soft set, generalised fuzzy soft topology, generalised fuzzy soft open sets, generalised fuzzy soft closed sets, generalised fuzzy soft topological spaces, generalised fuzzy soft compact spaces.

INTRODUCTION

Most of our real life problems in engineering, social and medical science, economics, environment etc. involve imprecise data and their solutions involve the use of mathematical principles based on uncertainty and imprecision. To handle such uncertainties, Zadeh (1965) introduced the concept of fuzzy sets and fuzzy set operations. The analytical part of fuzzy set theory was practically started with the paper of Chang (1968) who introduced the concept of fuzzy topological spaces; however, this theory is associated with an inherent limitation, which is the inadequacy of the parametrization tool associated with this theory as it was mentioned by Molodtsov (1999).

In 1999, Russian researcher Molodtsov introduced the concept of soft set theory which is free from the above problems and started to develop the basics of the corresponding theory as a new approach for modelling

uncertainties. Shabir and Naz (2011) studied the topological structures of soft sets.

In recent times, many researches have contributed a lot towards fuzzification of soft set theory. In 2001, Maji et al. introduced the fuzzy soft set which is a combination of fuzzy set and soft set. Tanay and Burc Kandemir (2011) introduced topological structure of fuzzy soft set and gave an introductory theoretical base to carry further study on this concept. The study was pursued by some others (Chakraborty et al., (2014); Gain et al., 2013).

In 2010, Majumdar and Samanta introduced generalised fuzzy soft sets and successfully applied their notion in a decision making problem. Yang (2011) pointed out that some results put forward by Majumdar and Samanta (2010) are not valid in general. Borah et al. (2012) introduced application of generalized fuzzy soft sets in teaching evaluation.

*Corresponding author. Email: ramda2004@rediffmail.com. Tel: +91 9143028586.

Author(s) agree that this article remain permanently open access under the terms of the [Creative Commons Attribution License 4.0 International License](https://creativecommons.org/licenses/by/4.0/)

The objective of this paper is divided into three parts. In the first part we introduce the generalised fuzzy soft union, generalised fuzzy soft intersection, and several other properties of generalised fuzzy soft sets are studied. In the second part we introduce “generalised fuzzy soft topological spaces” over the soft universe (X, E) with a fixed set of parameter. Then we discussed some basic properties of generalised fuzzy soft topological spaces with an example and define generalised fuzzy soft open and closed sets. By this way we define the generalised fuzzy soft closure, generalised fuzzy soft points, generalised fuzzy soft neighbourhood, generalised fuzzy soft interior, generalised fuzzy soft base and also we established then some important theorems related to these spaces. Finally we define generalised fuzzy soft compactness and give some important definitions and theorems.

This paper may be the starting point for the studies on “Generalised fuzzy soft topology” and all results deduced in this paper can be used in the theory of information systems.

PRELIMINARIES

Throughout this paper X denotes initial universe, E denotes the set of all possible parameters for X , $P(X)$ denotes the power set of X , I^X denotes the set of all fuzzy sets on X , I^E denotes the collection of all fuzzy sets on E , (X, E) denotes the soft universe and I stands for $[0, 1]$.

Definition 1

A fuzzy set A in X is defined by a membership function $\mu_A : X \rightarrow [0, 1]$ whose value $\mu_A(x)$ represents the “grade of membership” of x in A for $x \in X$ (Zadeh, 1965).

If $A, B \in I^X$ then from Zadeh (1965), we have the following:

$$A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \text{ for all } x \in X.$$

$$A = B \Leftrightarrow \mu_A(x) = \mu_B(x), \text{ for all } x \in X.$$

$$C = A \vee B \Leftrightarrow \mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X.$$

$$D = A \wedge B \Leftrightarrow \mu_D(x) = \min\{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X.$$

$$E = A^c \Leftrightarrow \mu_E(x) = 1 - \mu_A(x), \text{ for all } x \in X.$$

Definition 2

Let $A \subseteq E$. A pair (f, A) is called a soft set over X

where f is a mapping from A into $P(X)$, that is, $f : A \rightarrow P(X)$ (Molodtsov, 1999).

In other words, a soft set is a parameterized family of subsets of the set X . For $e \in A$, $f(e)$ may be considered as the set of e -approximate elements of the soft set (f, A) .

Definition 3

Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over X , where $F : A \rightarrow I^X$ is a function, that is, for each $a \in A$, $F(a) = F_a : X \rightarrow [0, 1]$ is a fuzzy set on X (Maji et al., 2001).

Definition 4

Let X be the universal set of elements and E be the universal set of parameters for X (Majumdar and Samanta, 2010). Let $F : E \rightarrow I^X$ and μ be a fuzzy subset of E , that is, $\mu : E \rightarrow I = [0, 1]$. Let F_μ be the mapping $F_\mu : E \rightarrow I^X \times I$ be a function defined as follows: $\tilde{F}_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^X$ and $\mu(e) \in I^E$. Then \tilde{F}_μ is called a generalised fuzzy soft set (GFSS in short) over (X, E) .

Here, for each parameter $e \in E$, $\tilde{F}_\mu(e) = (F(e), \mu(e))$ indicates not only the degree of belongingness of the elements of X in $F(e)$ but also the degree of possibility of such belongingness which is represented by $\mu(e)$.

Example 1

Let $X = \{x_1, x_2, x_3\}$ be a set of three houses under consideration. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where $e_1 =$ expensive, $e_2 =$ beautiful, $e_3 =$ in the green surroundings. Let $\mu : E \rightarrow I = [0, 1]$ be defined as follows: $\mu(e_1) = 0.4$, $\mu(e_2) = 0.5$, $\mu(e_3) = 0.7$

We define a function $F_\mu : E \rightarrow I^X \times I$ as follows:

$$\tilde{F}_\mu = \left\{ \begin{array}{l} F_\mu(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.2, x_3 \setminus 0.5\}, 0.4), F_\mu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.7\}, 0.5), \\ F_\mu(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.5, x_3 \setminus 0.1\}, 0.7) \end{array} \right\}$$

Then \tilde{F}_μ is a GFSS over (X, E) .

Definition 5

Let \tilde{F}_μ and \tilde{G}_δ be two GFSS over (X, E) (Majumdar and Samanta, 2010). Now \tilde{F}_μ is said to be a GFS subset of \tilde{G}_δ or \tilde{G}_δ is said to be a GFS super set of \tilde{F}_μ if μ is a fuzzy subset of δ

$F(e)$ is also a fuzzy subset of $G(e)$, $\forall e \in E$

In this case we write $\tilde{F}_\mu \subseteq \tilde{G}_\delta$.

Definition 6

Let \tilde{F}_μ be a GFSS over (X, E) (Majumdar and Samanta, 2010). Then the complement of \tilde{F}_μ , is denoted by \tilde{F}_μ^c and is defined by $\tilde{F}_\mu^c = \tilde{G}_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)$, $\forall e \in E$.

Obviously $(\tilde{F}_\mu^c)^c = \tilde{F}_\mu$.

Definition 7

Union of two GFSS \tilde{F}_μ and \tilde{G}_δ , denoted by $\tilde{F}_\mu \cup \tilde{G}_\delta$, is a GFSS \tilde{H}_ν , defined as $H_\nu : E \rightarrow I^X \times I$ such that $\tilde{H}_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) \vee G(e)$ and $\nu(e) = \mu(e) \vee \delta(e)$, $\forall e \in E$.

Let $\{(\tilde{F}_\mu)_\lambda, \lambda \in \Lambda\}$, where Λ is an index set, be a family of GFSSs. The union of these family is denoted by $\tilde{\cup}_{\lambda \in \Lambda} (\tilde{F}_\mu)_\lambda$; is a GFSS \tilde{H}_ν , defined as $H_\nu : E \rightarrow I^X \times I$ such that $\tilde{H}_\nu(e) = (H(e), \nu(e))$, where $H(e) = \vee_{\lambda \in \Lambda} (F(e))_\lambda$ and $\nu(e) = \vee_{\lambda \in \Lambda} (\mu(e))_\lambda$, $\forall e \in E$.

Definition 8

Intersection of two GFSS \tilde{F}_μ and \tilde{G}_δ , denoted by $\tilde{F}_\mu \cap \tilde{G}_\delta$, is a GFSS \tilde{M}_σ , defined as

$M_\sigma : E \rightarrow I^X \times I$ such that $\tilde{M}_\sigma(e) = (M(e), \sigma(e))$, where $M(e) = F(e) \wedge G(e)$ and $\sigma(e) = \mu(e) \wedge \delta(e)$, $\forall e \in E$.

Let $\{(\tilde{F}_\mu)_\lambda, \lambda \in \Lambda\}$, where Λ is an index set, be a family of GFSSs. The intersection of these family is denoted by $\tilde{\cap}_{\lambda \in \Lambda} (\tilde{F}_\mu)_\lambda$, is a GFSS \tilde{M}_σ , defined as

$M_\sigma : E \rightarrow I^X \times I$ such that $\tilde{M}_\sigma(e) = (M(e), \sigma(e))$, where $M(e) = \wedge_{\lambda \in \Lambda} (F(e))_\lambda$ and $\sigma(e) = \wedge_{\lambda \in \Lambda} (\mu(e))_\lambda$, $\forall e \in E$.

Example 2

Consider the GFSS \tilde{F}_μ over (X, E) given in Example 1

$$\tilde{F}_\mu = \left\{ \begin{array}{l} F_\mu(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.2, x_3 \setminus 0.5\}, 0.4), F_\mu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.7\}, 0.5), \\ F_\mu(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.5, x_3 \setminus 0.1\}, 0.7) \end{array} \right\}.$$

Let \tilde{G}_δ be another GFSS over (X, E) defined as follows:

$$\tilde{G}_\delta = \left\{ \begin{array}{l} G_\delta(e_1) = (\{x_1 \setminus 0.3, x_2 \setminus 0.5, x_3 \setminus 0.2\}, 0.2), G_\delta(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.3, x_3 \setminus 0.4\}, 0.4), \\ G_\delta(e_3) = (\{x_1 \setminus 0.8, x_2 \setminus 0.6, x_3 \setminus 0\}, 0.8) \end{array} \right\}.$$

Then $\tilde{F}_\mu \cup \tilde{G}_\delta = \tilde{H}_\nu$

$$\tilde{H}_\nu = \left\{ \begin{array}{l} H_\nu(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.5, x_3 \setminus 0.5\}, 0.4), H_\nu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.7\}, 0.5), \\ H_\nu(e_3) = (\{x_1 \setminus 0.8, x_2 \setminus 0.6, x_3 \setminus 0.1\}, 0.8) \end{array} \right\}.$$

Again $\tilde{F}_\mu \cap \tilde{G}_\delta = \tilde{M}_\sigma$

$$\tilde{M}_\sigma = \left\{ \begin{array}{l} M_\sigma(e_1) = (\{x_1 \setminus 0.3, x_2 \setminus 0.2, x_3 \setminus 0.2\}, 0.2), M_\sigma(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.3, x_3 \setminus 0.4\}, 0.4), \\ M_\sigma(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.5, x_3 \setminus 0\}, 0.7) \end{array} \right\}.$$

$$\tilde{F}_\mu^c = \left\{ \begin{array}{l} F_\mu^c(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.8, x_3 \setminus 0.5\}, 0.6), F_\mu^c(e_2) = (\{x_1 \setminus 0.4, x_2 \setminus 0.6, x_3 \setminus 0.3\}, 0.5), \\ F_\mu^c(e_3) = (\{x_1 \setminus 0.3, x_2 \setminus 0.5, x_3 \setminus 0.9\}, 0.3) \end{array} \right\}.$$

Definition 9

A GFSS is said to be a generalised null fuzzy soft set, denoted by $\tilde{\phi}_\theta$, if $\phi_\theta : E \rightarrow I^X \times I$ such that $\tilde{\phi}_\theta(e) = (F(e), \theta(e))$, where $F(e) = \bar{0} \forall e \in E$ and $\theta(e) = 0 \forall e \in E$ (Where $\bar{0}$ denotes the null fuzzy set) (Majumdar and Samanta, 2010).

Definition 10

A GFSS is said to be a generalised absolute fuzzy soft set, denoted by \tilde{I}_Δ , if $1_\Delta : E \rightarrow I^X \times I$, where $\tilde{I}_\Delta(e) = (1(e), \Delta(e))$ is defined by $1(e) = \bar{1} \forall e \in E$ and $\Delta(e) = 1, \forall e \in E$ (Where $\bar{1}(x) = 1, \forall x \in X$) (Majumdar and Samanta, 2010).

Proposition 1

Let \tilde{F}_μ be a GFSS over (X, E) , then the following holds:

- (i) $\tilde{F}_\mu = \tilde{F}_\mu \cup \tilde{F}_\mu$
- (ii) $\tilde{F}_\mu = \tilde{F}_\mu \cap \tilde{F}_\mu$
- (iii) $\tilde{F}_\mu \cup \tilde{\phi}_\theta = \tilde{F}_\mu$
- (iv) $\tilde{F}_\mu \cap \tilde{\phi}_\theta = \tilde{\phi}_\theta$
- (v) $\tilde{F}_\mu \cup \tilde{I}_\Delta = \tilde{I}_\Delta$
- (vi) $\tilde{F}_\mu \cap \tilde{I}_\Delta = \tilde{F}_\mu$

Proof: Follow from the respective definitions.

Proposition 2

Let \tilde{F}_μ , \tilde{G}_δ and \tilde{H}_ν be any three GFSS over (X, E) , then the following holds:

- (i) $\tilde{F}_\mu \cup \tilde{G}_\delta = \tilde{G}_\delta \cup \tilde{F}_\mu$
- (ii) $\tilde{F}_\mu \cap \tilde{G}_\delta = \tilde{G}_\delta \cap \tilde{F}_\mu$
- (iii) $\tilde{F}_\mu \cup (\tilde{G}_\delta \cap \tilde{H}_\nu) = (\tilde{F}_\mu \cup \tilde{G}_\delta) \cap \tilde{H}_\nu$
- (iv) $\tilde{F}_\mu \cap (\tilde{G}_\delta \cap \tilde{H}_\nu) = (\tilde{F}_\mu \cap \tilde{G}_\delta) \cap \tilde{H}_\nu$

Proof: Straightforward.

Proposition 3

Let \tilde{F}_μ , \tilde{G}_δ and \tilde{H}_ν be any three GFSS over (X, E) , then the following holds:

- (i) $\tilde{F}_\mu \cap (\tilde{G}_\delta \cup \tilde{H}_\nu) = (\tilde{F}_\mu \cap \tilde{G}_\delta) \cup (\tilde{F}_\mu \cap \tilde{H}_\nu)$
- (ii) $\tilde{F}_\mu \cup (\tilde{G}_\delta \cap \tilde{H}_\nu) = (\tilde{F}_\mu \cup \tilde{G}_\delta) \cap (\tilde{F}_\mu \cup \tilde{H}_\nu)$

Proof: Straightforward.

Proposition 4

Let \tilde{F}_μ and \tilde{G}_δ are two GFSS over (X, E) , then the following holds:

- (i) $(\tilde{F}_\mu \cap \tilde{G}_\delta)^c = (\tilde{F}_\mu^c \cup \tilde{G}_\delta^c)$
- (ii) $(\tilde{F}_\mu \cup \tilde{G}_\delta)^c = (\tilde{F}_\mu^c \cap \tilde{G}_\delta^c)$

Proof

- (i) Let $\tilde{F}_\mu \cap \tilde{G}_\delta = \tilde{H}_\nu$

then $\tilde{H}_\nu(e) = (H(e), \nu(e)), \forall e \in E$,

where $H(e) = F(e) \wedge G(e)$ and $\nu(e) = \mu(e) \wedge \delta(e)$.

$$\begin{aligned} \text{Thus } (\tilde{F}_\mu \cap \tilde{G}_\delta)^c &= (\tilde{H}_\nu)^c = (H^c(e), \nu^c(e)) = ((F(e) \wedge G(e))^c, (\mu(e) \wedge \delta(e))^c) \\ &= (F^c(e) \vee G^c(e), \mu^c(e) \vee \delta^c(e)) \\ &= (\tilde{F}_\mu^c \cup \tilde{G}_\delta^c) \end{aligned}$$

- (ii) Let $\tilde{F}_\mu \cup \tilde{G}_\delta = \tilde{M}_\sigma$ then

$\tilde{M}_\sigma(e) = (M(e), \sigma(e)), \forall e \in E$, where $M(e) = F(e) \vee G(e)$ and $\sigma(e) = \mu(e) \vee \delta(e)$.

$$\begin{aligned} \text{Thus } (\tilde{F}_\mu \cup \tilde{G}_\delta)^c &= (\tilde{M}_\sigma)^c = (M^c(e), \sigma^c(e)) = ((F(e) \vee G(e))^c, (\mu(e) \vee \delta(e))^c) \\ &= (F^c(e) \wedge G^c(e), \mu^c(e) \wedge \delta^c(e)) \\ &= (\tilde{F}_\mu^c \cap \tilde{G}_\delta^c) \end{aligned}$$

GENERALISED FUZZY SOFT TOPOLOGICAL SPACE

Here, we introduce the notion of generalised fuzzy soft topology over a soft universe and study some of its basic properties.

Definition 11

Let T be a collection of generalised fuzzy soft sets over (X, E) . Then T is said to be a generalised fuzzy soft topology (GFS topology, in short) over (X, E) if the following conditions are satisfied:

- (i) $\tilde{\phi}_\theta$ and \tilde{I}_Δ are in T .

- (ii) Arbitrary unions of members of T belong to T .
- (iii) Finite intersections of members of T belong to T .

The triplet (X, T, E) is called a generalised fuzzy soft topological space (GFST-space, in short) over (X, E) .

Definition 12

Let (X, T, E) be a GFST-space over (X, E) , then the members of T are said to be a GFS open sets in (X, T, E) .

Definition 13

Let (X, T, E) be a GFST-space over (X, E) . A GFSS \tilde{F}_μ over (X, E) is said to be a GFS closed in (X, T, E) , if its complement \tilde{F}_μ^c belongs to T .

Proposition 5

Let (X, T, E) be a GFST-space over (X, E) . Let T' be the collection of all GFS -closed sets. Then

- (i) $\tilde{\phi}_\theta$ and $\tilde{1}_\Delta$ are in T'
- (ii) Arbitrary intersections of members of T' belongs to T'
- (iii) Finite unions of members of T' belongs to T'

Proof

Follows from the definition of GFST-space and De-Morgan's law for GFSS which is given in Proposition 4.

Example 3

$T = \{\tilde{\phi}_\theta, \tilde{1}_\Delta\}$ forms a GFS topology over (X, E) , which is said to be the GFS indiscrete topology over (X, E) .

Example 4

Let T be the collection of all GFSS which can be defined over (X, E) . Then T forms a GFS topology over (X, E) ; it is called the GFS discrete topology over (X, E) .

Example 5

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$

We consider the following GFSS over (X, E) defined as

$$\tilde{F}_\mu = \left\{ \begin{array}{l} F_\mu(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.5, x_3 \setminus 0.4\}, 0.2), F_\mu(e_2) = (\{x_1 \setminus 0.2, x_2 \setminus 0.3, x_3 \setminus 0.8\}, 0.5), \\ F_\mu(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.4, x_3 \setminus 0.3\}, 0.6) \end{array} \right\}$$

where $\mu \in I^E$

$$\tilde{G}_\delta = \left\{ \begin{array}{l} G_\delta(e_1) = (\{x_1 \setminus 0.3, x_2 \setminus 0.2, x_3 \setminus 0.2\}, 0.1), G_\delta(e_2) = (\{x_1 \setminus 0.1, x_2 \setminus 0.3, x_3 \setminus 0.5\}, 0.3), \\ G_\delta(e_3) = (\{x_1 \setminus 0.5, x_2 \setminus 0.3, x_3 \setminus 0.3\}, 0.5) \end{array} \right\}$$

where $\delta \in I^E$.

Let us consider $T = \{\tilde{\phi}_\theta, \tilde{1}_\Delta, \tilde{F}_\mu, \tilde{G}_\delta\}$. Then T forms a GFS topology over (X, E) .

Definition 14

Let (X, T, E) and (X, T_1, E) be two GFST-spaces. If each member of T belongs to T_1 , then T_1 is called GFS finer (larger) than T or (equivalently) T is GFS coarser than T_1 .

Example 6

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$.

Consider the GFS topology T over (X, E) given in Example 3.7

Let us consider another GFS topology $T_1 = \{\tilde{\phi}_\theta, \tilde{1}_\Delta, \tilde{F}_\mu, \tilde{G}_\delta, \tilde{H}_\nu\}$, where $\tilde{F}_\mu, \tilde{G}_\delta$ are given in Example 5.

$$\tilde{H}_\nu = \left\{ \begin{array}{l} H_\nu(e_1) = (\{x_1 \setminus 0.7, x_2 \setminus 0.6, x_3 \setminus 0.4\}, 0.5), H_\nu(e_2) = (\{x_1 \setminus 0.3, x_2 \setminus 0.5, x_3 \setminus 0.8\}, 0.6), \\ H_\nu(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.8, x_3 \setminus 0.6\}, 0.7) \end{array} \right\}$$

Then T_1 is finer than T .

Proposition 6

Let (X, T_1, E) and (X, T_2, E) be two GFST-spaces over the same universe (X, E) , then $(X, T_1 \tilde{\cap} T_2, E)$ is also a GFST-space over (X, E) .

Proof

(i) $\tilde{\phi}_\theta, \tilde{I}_\Delta \tilde{\in} T_1 \tilde{\cap} T_2.$

(ii) Let $\left\{ \left(\tilde{F}_\mu \right)_i : i \in \Lambda, \Lambda \text{ being an index set} \right\}$ be a family of GFSS in $T_1 \tilde{\cap} T_2.$ Then $\left(\tilde{F}_\mu \right)_i \tilde{\in} T_1$ and $\left(\tilde{F}_\mu \right)_i \tilde{\in} T_2, \forall i \in \Lambda,$ so $\tilde{\bigcup}_{i \in \Lambda} \left(\tilde{F}_\mu \right)_i \tilde{\in} T_1$ and $\tilde{\bigcup}_{i \in \Lambda} \left(\tilde{F}_\mu \right)_i \tilde{\in} T_2.$

Hence $\tilde{\bigcup}_{i \in \Lambda} \left(\tilde{F}_\mu \right)_i \tilde{\in} T_1 \tilde{\cap} T_2.$

(iii) Let $\tilde{F}_\mu, \tilde{G}_\delta \tilde{\in} T_1 \tilde{\cap} T_2.$ Then $\tilde{F}_\mu, \tilde{G}_\delta \tilde{\in} T_1$ and $\tilde{F}_\mu, \tilde{G}_\delta \tilde{\in} T_2.$

Since $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta \tilde{\in} T_1$ and $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta \tilde{\in} T_2,$ so $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta \tilde{\in} T_1 \tilde{\cap} T_2.$ Hence $T_1 \tilde{\cap} T_2$ forms a GFS topology over $(X, E).$

Remark: The union of two GFS topologies over (X, E) may not be a GFS topology over $(X, E),$ which follows from the following Example.

Example 7

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2, e_3\}$
 Let us consider $T_1 = \{\tilde{\phi}_\theta, \tilde{I}_\Delta, \tilde{F}_\mu\}$ and $T_2 = \{\tilde{\phi}_\theta, \tilde{I}_\Delta, \tilde{G}_\delta\}$
 be two GFS topologies over $(X, E),$ where $\tilde{F}_\mu, \tilde{G}_\delta$ are GFSS over (X, E) defined as follows

$$\tilde{F}_\mu = \left\{ \begin{array}{l} F_\mu(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.3\}, 0.1), F_\mu(e_2) = (\{x_1 \setminus 0.2, x_2 \setminus 0.3, x_3 \setminus 0.5\}, 0.3), \\ F_\mu(e_3) = (\{x_1 \setminus 0.7, x_2 \setminus 0.6, x_3 \setminus 0.1\}, 0.5) \end{array} \right\},$$

where $\mu \in I^E$

$$\tilde{G}_\delta = \left\{ \begin{array}{l} G_\delta(e_1) = (\{x_1 \setminus 0.5, x_2 \setminus 0.2, x_3 \setminus 0.4\}, 0.3), G_\delta(e_2) = (\{x_1 \setminus 0.1, x_2 \setminus 0.2, x_3 \setminus 0.3\}, 0.2), \\ G_\delta(e_3) = (\{x_1 \setminus 0.8, x_2 \setminus 0.5, x_3 \setminus 0\}, 0.4) \end{array} \right\},$$

where $\delta \in I^E$

Now, $T = T_1 \tilde{\cup} T_2 = \{\tilde{\phi}_\theta, \tilde{I}_\Delta, \tilde{F}_\mu, \tilde{G}_\delta\}$

Let $\tilde{F}_\mu \tilde{\cup} \tilde{G}_\delta = \tilde{H}_\nu$

Where

$$\tilde{H}_\nu = \left\{ \begin{array}{l} H_\nu(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.4\}, 0.3), H_\nu(e_2) = (\{x_1 \setminus 0.2, x_2 \setminus 0.3, x_3 \setminus 0.5\}, 0.3), \\ H_\nu(e_3) = (\{x_1 \setminus 0.8, x_2 \setminus 0.6, x_3 \setminus 0.1\}, 0.5) \end{array} \right\}$$

Now $\tilde{H}_\nu \tilde{\notin} T.$

Thus T is not a GFS topology over $(X, E).$

Definition 15

Let (X, T, E) be a GFST-space and \tilde{F}_μ be a GFSS over $(X, E).$ Then the generalised fuzzy soft closure of $\tilde{F}_\mu,$ denoted by $\overline{\tilde{F}_\mu}$ is the intersection of all GFS closed supper sets of $\tilde{F}_\mu.$

Clearly, $\overline{\tilde{F}_\mu}$ is the smallest GFS closed set over (X, E) which contains $\tilde{F}_\mu.$

Example 8

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$

Let us consider the following GFSS over $(X, E).$

$$\begin{aligned} \tilde{F}_\mu &= \{F_\mu(e_1) = (\{x_1 \setminus 0.7, x_2 \setminus 0.3, x_3 \setminus 0.2\}, 0.4), F_\mu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.1, x_3 \setminus 0.4\}, 0.5)\} \\ \tilde{G}_\delta &= \{G_\delta(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.5\}, 0.5), G_\delta(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.3, x_3 \setminus 0.1\}, 0.3)\} \\ \tilde{H}_\nu &= \{H_\nu(e_1) = (\{x_1 \setminus 0.7, x_2 \setminus 0.4, x_3 \setminus 0.5\}, 0.5), H_\nu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.3, x_3 \setminus 0.4\}, 0.5)\} \\ \tilde{J}_\sigma &= \{J_\sigma(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.3, x_3 \setminus 0.2\}, 0.4), J_\sigma(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.1, x_3 \setminus 0.1\}, 0.3)\} \end{aligned}$$

Let us consider the GFS topology $T = \{\tilde{\phi}_\theta, \tilde{I}_\Delta, \tilde{F}_\mu, \tilde{G}_\delta, \tilde{H}_\nu, \tilde{J}_\sigma\}$ over $(X, E).$

Now,

$$\begin{aligned} \tilde{F}_\mu^c &= \{F_\mu^c(e_1) = (\{x_1 \setminus 0.3, x_2 \setminus 0.7, x_3 \setminus 0.8\}, 0.6), F_\mu^c(e_2) = (\{x_1 \setminus 0.4, x_2 \setminus 0.9, x_3 \setminus 0.6\}, 0.5)\} \\ \tilde{G}_\delta^c &= \{G_\delta^c(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.6, x_3 \setminus 0.5\}, 0.5), G_\delta^c(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.7, x_3 \setminus 0.9\}, 0.7)\} \\ \tilde{H}_\nu^c &= \{H_\nu^c(e_1) = (\{x_1 \setminus 0.3, x_2 \setminus 0.6, x_3 \setminus 0.5\}, 0.5), H_\nu^c(e_2) = (\{x_1 \setminus 0.4, x_2 \setminus 0.7, x_3 \setminus 0.6\}, 0.5)\} \\ \tilde{J}_\sigma^c &= \{J_\sigma^c(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.7, x_3 \setminus 0.8\}, 0.6), J_\sigma^c(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.9, x_3 \setminus 0.9\}, 0.7)\} \end{aligned}$$

Clearly, $\tilde{F}_\mu^c, \tilde{G}_\delta^c, \tilde{H}_\nu^c, \tilde{J}_\sigma^c$ are GFS closed sets. Let us consider the following GFSS over $(X, E).$

$$\tilde{M}_\eta = \{M_\eta(e_1) = (\{x_1 \setminus 0.4, x_2 \setminus 0.5, x_3 \setminus 0.4\}, 0.6), M_\eta(e_2) = (\{x_1 \setminus 0.3, x_2 \setminus 0.6, x_3 \setminus 0.7\}, 0.3)\}$$

Then the GFS closure of $\tilde{M}_\eta,$ denoted by $\overline{\tilde{M}_\eta}$ is the intersection of all GFS closed sets containing $\tilde{M}_\eta.$

That is, $\overline{\tilde{M}_\eta} = \tilde{J}_\sigma^c \tilde{\cap} \tilde{I}_\Delta = \tilde{J}_\sigma^c.$

Theorem 1

Let (X, T, E) be a GFST-space. Let \tilde{F}_μ and \tilde{G}_δ are GFSS over (X, E) . Then

- (1) $\overline{\tilde{\phi}_\theta} = \tilde{\phi}_\theta, \overline{\tilde{1}_\Lambda} = \tilde{1}_\Lambda$
- (2) $\tilde{F}_\mu \subseteq \overline{\tilde{F}_\mu}$
- (3) \tilde{F}_μ is GFS closed if and only if $\tilde{F}_\mu = \overline{\tilde{F}_\mu}$
- (4) $\overline{\overline{\tilde{F}_\mu}} = \overline{\tilde{F}_\mu}$
- (5) $\tilde{F}_\mu \subseteq \tilde{G}_\delta \Rightarrow \overline{\tilde{F}_\mu} \subseteq \overline{\tilde{G}_\delta}$
- (6) $\overline{\tilde{F}_\mu \cup \tilde{G}_\delta} = \overline{\tilde{F}_\mu} \cup \overline{\tilde{G}_\delta}$
- (7) $\overline{\tilde{F}_\mu \cap \tilde{G}_\delta} \subseteq \overline{\tilde{F}_\mu} \cap \overline{\tilde{G}_\delta}$

Proof

(1) and (2) are obvious.
 (3) Let \tilde{F}_μ be a GFS closed set. By (2) we have $\tilde{F}_\mu \subseteq \overline{\tilde{F}_\mu}$. Since $\overline{\tilde{F}_\mu}$ is the smallest GFS closed set over (X, E) which contains \tilde{F}_μ , then $\overline{\tilde{F}_\mu} \subseteq \tilde{F}_\mu$. Hence $\tilde{F}_\mu = \overline{\tilde{F}_\mu}$.

Conversely, let $\tilde{F}_\mu = \overline{\tilde{F}_\mu}$. Since $\overline{\tilde{F}_\mu}$ is a GFS closed set, then \tilde{F}_μ is a GFS closed set over (X, E) .

(4) Since $\overline{\tilde{F}_\mu}$ is a GFS closed set therefore, by (3) we have $\overline{\overline{\tilde{F}_\mu}} = \overline{\tilde{F}_\mu}$.

(5) Let $\tilde{F}_\mu \subseteq \tilde{G}_\delta$. Then every GFS closed super set of \tilde{G}_δ will also contain \tilde{F}_μ . That is, every GFS closed super set of \tilde{G}_δ is also a GFS closed super set of \tilde{F}_μ . Hence the intersection of GFS closed super sets of \tilde{F}_μ is contained in the GFS intersection of GFS closed super set of \tilde{G}_δ . Thus $\overline{\tilde{F}_\mu} \subseteq \overline{\tilde{G}_\delta}$.

(6) Since $\tilde{F}_\mu \subseteq \overline{\tilde{F}_\mu \cup \tilde{G}_\delta}$ and $\tilde{G}_\delta \subseteq \overline{\tilde{F}_\mu \cup \tilde{G}_\delta}$, so by (5) $\overline{\tilde{F}_\mu} \subseteq \overline{\tilde{F}_\mu \cup \tilde{G}_\delta}$ and $\overline{\tilde{G}_\delta} \subseteq \overline{\tilde{F}_\mu \cup \tilde{G}_\delta}$. Thus, $\overline{\tilde{F}_\mu \cup \tilde{G}_\delta} \subseteq \overline{\overline{\tilde{F}_\mu} \cup \overline{\tilde{G}_\delta}}$.

Conversely, as $\tilde{F}_\mu \subseteq \overline{\tilde{F}_\mu}$ and $\tilde{G}_\delta \subseteq \overline{\tilde{G}_\delta}$. So $\tilde{F}_\mu \cup \tilde{G}_\delta \subseteq \overline{\tilde{F}_\mu} \cup \overline{\tilde{G}_\delta}$, by (5), $\overline{\tilde{F}_\mu \cup \tilde{G}_\delta}$ is a GFS closed set over (X, E) being the union of two GFS closed sets.

Then $\overline{\tilde{F}_\mu \cup \tilde{G}_\delta} \subseteq \overline{\overline{\tilde{F}_\mu} \cup \overline{\tilde{G}_\delta}}$. Thus, $\overline{\tilde{F}_\mu \cup \tilde{G}_\delta} = \overline{\overline{\tilde{F}_\mu} \cup \overline{\tilde{G}_\delta}}$.

(7) Since $\tilde{F}_\mu \cap \tilde{G}_\delta \subseteq \tilde{F}_\mu$ and $\tilde{F}_\mu \cap \tilde{G}_\delta \subseteq \tilde{G}_\delta$, so by (5) $\overline{\tilde{F}_\mu \cap \tilde{G}_\delta} \subseteq \overline{\tilde{F}_\mu}$ and $\overline{\tilde{F}_\mu \cap \tilde{G}_\delta} \subseteq \overline{\tilde{G}_\delta}$. Thus $\overline{\tilde{F}_\mu \cap \tilde{G}_\delta} \subseteq \overline{\tilde{F}_\mu} \cap \overline{\tilde{G}_\delta}$.

Definition 16

Let \tilde{F}_μ be a GFSS over (X, E) . We say that $(e_x^\alpha, e_\lambda) \in \tilde{F}_\mu$ read as (e_x^α, e_λ) belongs to the GFSS \tilde{F}_μ if $F(e)(x) = \alpha$ ($0 < \alpha \leq 1$) and $F(e)(y) = 0, \forall y \in X \setminus \{x\}, \mu(e) > \lambda$.

Proposition 7

Every non-null GFSS \tilde{F}_μ can be expressed as the union of all the generalised fuzzy soft points which belong to \tilde{F}_μ .

Proof: Obvious.

Definition 17

A GFSS \tilde{F}_μ in a GFST-space (X, T, E) is called a generalised fuzzy soft neighbourhood [GFS-nbd, in short] of the GFSS \tilde{G}_δ if there exists a GFS open set \tilde{H}_ν such that $\tilde{G}_\delta \subseteq \tilde{H}_\nu \subseteq \tilde{F}_\mu$.

Definition 18

A GFSS \tilde{F}_μ in a GFST-space (X, T, E) is called a generalised fuzzy soft neighbourhood of the generalised fuzzy soft point $(e_x^\alpha, e_\lambda) \in \tilde{1}_\Lambda$ if there exists a GFS open set \tilde{G}_δ such that $(e_x^\alpha, e_\lambda) \in \tilde{G}_\delta \subseteq \tilde{F}_\mu$.

Proposition 8

Let (X, T, E) be a GFST-space over (X, E) . Then

- (1) Each $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$ has a GFS nbd;
- (2) If \tilde{F}_μ and \tilde{G}_δ are GFS nbd of some $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$, then $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta$ is also a GFS nbd of (e_x^α, e_λ) .
- (3) If \tilde{F}_μ is a GFS nbd of $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$ and $\tilde{F}_\mu \subseteq \tilde{G}_\delta$, then \tilde{G}_δ is also a GFS nbd of (e_x^α, e_λ) .

Proof

- (1) is obvious.
- (2) Let \tilde{F}_μ and \tilde{G}_δ be the GFS neighbourhoods of $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$, then there exist $\tilde{H}_\nu, \tilde{N}_\sigma \in T$ such that $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu \subseteq \tilde{F}_\mu$ and $(e_x^\alpha, e_\lambda) \in \tilde{N}_\sigma \subseteq \tilde{G}_\delta$. Now $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu$ and $(e_x^\alpha, e_\lambda) \in \tilde{N}_\sigma$ implies that $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu \tilde{\cap} \tilde{N}_\sigma$ and $\tilde{H}_\nu \tilde{\cap} \tilde{N}_\sigma \in T$; so we have $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu \tilde{\cap} \tilde{N}_\sigma \subseteq \tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta$. Thus $\tilde{F}_\mu \tilde{\cap} \tilde{G}_\delta$ is a GFS neighbourhood of (e_x^α, e_λ) .
- (3) Let \tilde{F}_μ be a GFS neighbourhood of $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$ and $\tilde{F}_\mu \subseteq \tilde{G}_\delta$. By definition there exists a GFS open set \tilde{H}_ν such that $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu \subseteq \tilde{F}_\mu \subseteq \tilde{G}_\delta$. Thus $(e_x^\alpha, e_\lambda) \in \tilde{H}_\nu \subseteq \tilde{G}_\delta$. Hence \tilde{G}_δ is a GFS neighbourhood of (e_x^α, e_λ) .

Definition 19

Let (X, T, E) be a GFST-space. Let \tilde{F}_μ be a GFSS over (X, E) and $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$. Then (e_x^α, e_λ) is said to be a generalised fuzzy soft interior point of \tilde{F}_μ if there exists a GFS open set \tilde{G}_δ such that $(e_x^\alpha, e_\lambda) \in \tilde{G}_\delta \subseteq \tilde{F}_\mu$.

Definition 20

Let (X, T, E) be a GFST-space. Let \tilde{F}_μ be a GFSS

over (X, E) . The generalised fuzzy soft interior of \tilde{F}_μ , denoted by \tilde{F}_μ^0 is the union of all generalised fuzzy soft open subsets of \tilde{F}_μ .

Clearly, \tilde{F}_μ^0 is the largest generalised fuzzy soft open set over (X, E) which contained in \tilde{F}_μ .

Example 9

Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$.

Let us consider the following GFSS over (X, E)

$$\begin{aligned} \tilde{F}_\mu &= \{F_\mu(e_1) = (\{x_1 \setminus 0.7, x_2 \setminus 0.3, x_3 \setminus 0.2\}, 0.4), F_\mu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.1, x_3 \setminus 0.4\}, 0.5)\} \\ \tilde{G}_\delta &= \{G_\delta(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.4, x_3 \setminus 0.5\}, 0.5), G_\delta(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.3, x_3 \setminus 0.1\}, 0.3)\} \\ \tilde{H}_\nu &= \{H_\nu(e_1) = (\{x_1 \setminus 0.7, x_2 \setminus 0.4, x_3 \setminus 0.5\}, 0.5), H_\nu(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.3, x_3 \setminus 0.4\}, 0.5)\} \\ \tilde{J}_\sigma &= \{J_\sigma(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.3, x_3 \setminus 0.2\}, 0.4), J_\sigma(e_2) = (\{x_1 \setminus 0.5, x_2 \setminus 0.1, x_3 \setminus 0.1\}, 0.3)\} \end{aligned}$$

Let us consider the GFS topology $T = \{\tilde{\phi}_\theta, \tilde{I}_\Delta, \tilde{F}_\mu, \tilde{G}_\delta, \tilde{H}_\nu, \tilde{J}_\sigma\}$ over (X, E) .

Let us consider the following GFSS over (X, E) .

$$\tilde{N}_\psi = \{N_\psi(e_1) = (\{x_1 \setminus 0.6, x_2 \setminus 0.5, x_3 \setminus 0.4\}, 0.5), N_\psi(e_2) = (\{x_1 \setminus 0.6, x_2 \setminus 0.3, x_3 \setminus 0.8\}, 0.7)\}$$

Then the generalised fuzzy soft interior of \tilde{N}_ψ , denoted by \tilde{N}_ψ^0 , is the union of all generalised fuzzy soft open subsets of \tilde{N}_ψ .

$$\text{That is, } \tilde{N}_\psi^0 = \tilde{\phi}_\theta \cup \tilde{J}_\sigma = \tilde{J}_\sigma.$$

Theorem 2

Let (X, T, E) be a GFST-space. Let \tilde{F}_μ and \tilde{G}_δ are GFSS over (X, E) . Then

- (1) $(\tilde{\phi}_\theta)^0 = \tilde{\phi}_\theta, (\tilde{I}_\Delta)^0 = \tilde{I}_\Delta$
- (2) $(\tilde{F}_\mu)^0 \subseteq \tilde{F}_\mu$
- (3) \tilde{F}_μ is GFS open if and only if $(\tilde{F}_\mu)^0 = \tilde{F}_\mu$
- (4) $(\tilde{F}_\mu^0)^0 = \tilde{F}_\mu^0$

$$(5) \tilde{F}_\mu \subseteq \tilde{G}_\delta \Rightarrow \tilde{F}_\mu^0 \subseteq \tilde{G}_\delta^0$$

$$(6) \tilde{F}_\mu^0 \cap \tilde{G}_\delta^0 = (\tilde{F}_\mu \cap \tilde{G}_\delta)^0$$

$$(7) \tilde{F}_\mu^0 \cup \tilde{G}_\delta^0 \subseteq (\tilde{F}_\mu \cup \tilde{G}_\delta)^0$$

Proof: Straightforward.

Theorem 3

A GFSS \tilde{F}_μ over (X, E) is a GFS open set if and only if \tilde{F}_μ is a GFS nbd of each GFSS \tilde{G}_δ contained in \tilde{F}_μ .

Proof

First suppose \tilde{F}_μ is a GFS open and \tilde{G}_δ is any GFSS contained in \tilde{F}_μ . We have $\tilde{G}_\delta \subseteq \tilde{F}_\mu \subseteq \tilde{F}_\mu$, it follows that \tilde{F}_μ is a neighbourhood of \tilde{G}_δ .

Conversely, suppose \tilde{F}_μ is a neighbourhood for every GFSS contained it. Since $\tilde{F}_\mu \subseteq \tilde{F}_\mu$ there exist a GFS open set \tilde{H}_ν such that $\tilde{F}_\mu \subseteq \tilde{H}_\nu \subseteq \tilde{F}_\mu$.

Hence $\tilde{F}_\mu = \tilde{H}_\nu$ and \tilde{F}_μ is GFS open.

Theorem 4

Let (X, T, E) be a GFST-space and \tilde{F}_μ be a GFSS over (X, E) . Then

$$(1) (\overline{\tilde{F}_\mu})^c = (\tilde{F}_\mu^c)^0$$

$$(2) (\tilde{F}_\mu^0)^c = (\overline{\tilde{F}_\mu^c})$$

Proof

$$(1) (\overline{\tilde{F}_\mu})^c = (\tilde{\cap} \{ \tilde{G}_\delta \mid \tilde{G}_\delta \text{ is GFS closed set and } \tilde{F}_\mu \subseteq \tilde{G}_\delta \})^c \\ = (\tilde{\cup} \{ \tilde{G}_\delta^c \mid \tilde{G}_\delta^c \text{ is GFS open set and } \tilde{G}_\delta^c \subseteq \tilde{F}_\mu^c \}) \\ = (\tilde{F}_\mu^c)^0$$

$$(2) (\tilde{F}_\mu^0)^c = (\tilde{\cup} \{ \tilde{G}_\delta \mid \tilde{G}_\delta \text{ is GFS open set and } \tilde{G}_\delta \subseteq \tilde{F}_\mu \})^c$$

$$= (\tilde{\cap} \{ \tilde{G}_\delta^c \mid \tilde{G}_\delta^c \text{ is GFS closed set and } \tilde{G}_\delta^c \subseteq \tilde{F}_\mu^c \}) \\ = (\overline{\tilde{F}_\mu^c})$$

Definition 21

Let (X, T, E) be a GFST-space. A subfamily \mathfrak{R} of T is said to be a GFS base for T if every member of T can be expressed as the union of some members of \mathfrak{R} .

Lemma: Let (X, T, E) be a GFST-space and \mathfrak{R} be a GFS base for T over (X, T, E) . Then T equals the collection of all unions of elements of \mathfrak{R} .

Proof: Obvious.

Proposition 9

Let (X, T, E) be a GFST-space and $\mathfrak{R} \subseteq T$. Then \mathfrak{R} is a GFS base for T if and only if for any $(e_x^\alpha, e_\lambda) \in \tilde{I}_\Delta$ and any GFS open set \tilde{G}_δ containing (e_x^α, e_λ) , there exists $\tilde{R}_\mu \in \mathfrak{R}$ such that $(e_x^\alpha, e_\lambda) \in \tilde{R}_\mu$ and $\tilde{R}_\mu \subseteq \tilde{G}_\delta$.

Proof: Straightforward.

GENERALISED FUZZY SOFT COMPACT SPACES

The closed and bounded sets of real line were considered an excellent model on which to fashion the generalised version of compactness in topological space. The concept of compactness for a fuzzy topological space has been introduced and studied by many mathematicians in different ways. The first among them was Chang (1968).

In this article, we introduce and study compactness in generalised fuzzy soft topological perspective.

Definition 22

A family Ω of generalised fuzzy soft sets is a cover of a generalised fuzzy soft set \tilde{F}_μ iff \tilde{F}_μ is contained in the union of members of Ω . That is, $\tilde{F}_\mu \subseteq \tilde{\cup} \{ (\tilde{G}_\delta)_\lambda : (\tilde{G}_\delta)_\lambda \in \Omega, \lambda \in \Lambda \}$

It is a generalised fuzzy soft open cover iff each member of Ω is generalised fuzzy soft open set. A sub cover of Ω is a subfamily of Ω which is also a cover.

Definition 23

Let (X, T, E) be a generalised fuzzy soft topological space and \tilde{F}_μ be a generalised fuzzy soft set, over (X, E) is called generalised fuzzy soft compact (henceforth GFS compact) if each generalised fuzzy soft open cover of \tilde{F}_μ has a finite sub cover. Also, generalised fuzzy soft topological space (X, T, E) is called generalised fuzzy soft compact if each generalised fuzzy soft open cover of \tilde{I}_Δ has a finite sub cover.

Definition 24

A family Ω of generalised fuzzy soft sets has the finite intersection property if the intersection of the members of each finite sub family of Ω is not the generalised null fuzzy soft set.

Theorem 5

A generalised fuzzy soft topological space is GFS compact if and only if each family of GFS closed sets with the finite intersection property, has a generalised non-empty fuzzy soft intersection.

Proof

Let (X, T, E) be a GFS compact space and let $\Omega = \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}$ be a collection of GFS closed subsets of (X, T, E) with the FIP and suppose, if possible, $\tilde{\cap} \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\} = \tilde{\phi}_\theta$. Then $[\tilde{\cap} \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}]^c = \tilde{I}_\Delta$ or $\tilde{\cup} \{(\tilde{G}_\delta^c)_\lambda : \lambda \in \Lambda\} = \tilde{I}_\Delta$ by De-Morgan law. This means that $\{(\tilde{G}_\delta^c)_\lambda : \lambda \in \Lambda\}$ is open cover of \tilde{I}_Δ , since $\{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}$ are GFS closed. Since (X, T, E) is GFS compact, we have that $\tilde{\cup}_{\lambda=1}^n \{(\tilde{G}_\delta^c)_\lambda : \lambda \in \Lambda\} = \tilde{I}_\Delta$ and so by De-Morgan law $[\tilde{\cap}_{\lambda=1}^n \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}]^c = \tilde{I}_\Delta$ which implies that $\tilde{\cap}_{\lambda=1}^n \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\} = \tilde{\phi}_\theta$. However, this contradicts the FIP. Hence we must have $\tilde{\cap} \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\} \neq \tilde{\phi}_\theta$.

Conversely, let $\Omega = \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}$ be a GFS open cover of \tilde{I}_Δ so that

$$\tilde{I}_\Delta = \tilde{\cup} \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}$$

$$\tilde{\phi}_\theta = [\tilde{\cup} \{(\tilde{G}_\delta)_\lambda : \lambda \in \Lambda\}]^c = \tilde{\cap} \{(\tilde{G}_\delta^c)_\lambda : \lambda \in \Lambda\}$$

Thus, $\{(\tilde{G}_\delta^c)_\lambda : \lambda \in \Lambda\}$ is a collection of GFS closed sets with generalised non-null fuzzy soft intersection and so by hypothesis this collection does not have the FIP. Hence there exists a finite number of GFSS $\{(\tilde{G}_\delta^c)_\lambda : \lambda = 1, 2, \dots, n\}$ such that $\tilde{\phi}_\theta = \tilde{\cap} \{(\tilde{G}_\delta^c)_\lambda : \lambda = 1, 2, \dots, n\} = [\tilde{\cup} \{(\tilde{G}_\delta)_\lambda : \lambda = 1, 2, \dots, n\}]^c$. This implies that $\tilde{I}_\Delta = \tilde{\cup} \{(\tilde{G}_\delta)_\lambda : \lambda = 1, 2, \dots, n\}$.

Hence (X, T, E) is GFS compact.

Conclusion

The proposed work is basically a theoretical one. If these theoretical back-ups are properly nurtured and fruitfully developed, it will definitely usher in various nice applications in the fields of Engineering Science, Medical Science and Social Sciences, by skilfully analyzing and interpreting imprecise data mathematically.

Conflict of Interest

The author(s) have not declared any conflict of interest.

ACKNOWLEDGEMENT

The first author is thankful to University Grants Commission, New Delhi, for sponsoring this work under grants of Minor Research Project.

REFERENCES

- Borah MJ, Neog TJ, Sut DK (2012). Application of generalized fuzzy soft sets in teaching evaluation. *Int. J. Latest Trends Comput.* 3:25-29. <http://ojs.excelingtech.co.uk/index.php/IJLTC>
- Chakraborty RP, Mukherjee P, Gain PK (2014). A note on fuzzy soft semi open sets and fuzzy soft semi continuous functions. *J. Fuzzy Math.* 22(4): 973-089
- Chang CL (1968). Fuzzy topological spaces. *J. Mathematical Anal. Applications*, 24:182-190. <http://www.journals.elsevier.com/journal-of-mathematical-analysis-and-applications>
- Gain PK, Mukherjee P, Chakraborty RP, Pal M (2013). On some structural properties of fuzzy soft topological spaces. *Int. J. Fuzzy Mathematical Archive*. 1:1-15. <http://www.researchmathsci.org/IJFMAeditorial.html>
- Maji PK, Biswas R, Roy AR (2001). Fuzzy soft sets. *J. Fuzzy Math.* 9: 589-602. <http://maths.swjtu.edu.cn/fuzzy>
- Majumdar P, Samanta SK (2010). Generalised fuzzy soft sets, *Comput. Math. Appl.* 59:1425-1432. <http://www.journals.elsevier.com/computers-and-mathematics-with-applications/>

- Molodtsov D (1999). Soft set theory-First results. *Comput. Math. Appl* 37:19-31. <http://www.journals.elsevier.com/computers-and-mathematics-with-applications/>
- Shabir M, Naz M (2011). On soft topological spaces, *Comput. Math. Appl.* 61:1786-1799. <http://www.journals.elsevier.com/computers-and-mathematics-with-applications/>
- Tanay B, Burc MK (2011). Topological structure of fuzzy soft sets. *Comput. Math. Appl.* 61:2952-2957. <http://www.journals.elsevier.com/computers-and-mathematics-with-applications/>
- Yang HL (2011). Notes on "Generalised fuzzy soft sets". *J. Math. Res. Expos.* 31:567-570. <http://www.oriprobe.com/journals/sxyjypl.html>
- Zadeh LA (1965). Fuzzy sets. *Inform. Control* 8:338-353. <http://www.journals.elsevier.com/information-and-computation/>