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Full Length Research Paper

Selection and resolution of function problems and their effects on student learning

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This paper examines two experienced Turkish teachers', Ahmet and Burak, selection and implementation of function problems and relates this to the quality of their students' understanding of this notion. The research findings indicate that regardless of the task quality Ahmet engages, through process-oriented teaching, his students with the notion of function and this encourages them to develop a process conception of function. In contrast, Burak makes reductions, thorough action-oriented teaching, in the task demands. He emphasises rules, procedures and the factual knowledge associated with the representational systems, and this appears to confine his students' understanding to an action conception of function. The evidence suggests that tasks should not be seen as a panacea; it is the teacher's expertise in creating task conditions (for example, establishing links between the ideas and between the representations, using process-oriented language) that may promote student learning.

Key words: Task selection, task implementation, task condition, action-oriented teaching, process-oriented teaching, action-process conceptions of function.

INTRODUCTION

The interest in examining the types of tasks in which students engage and the mathematical notions they learn from them drives from the belief that tasks shape the way students think about the subject matter and, thus, influence their learning (Doyle, 1983; Marks and Walsh, 1988; Stein and Lane, 1996). National Council of Teachers of Mathematics (2000) recognised the importance of using worthwhile tasks in teaching mathematics: "In effective teaching, worthwhile tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. … Regardless of the context, worthwhile tasks should be intriguing; with a level of challenge that invites speculation and hard work".

Doyle (1983) defined an academic task (academic task is not specific to mathematics) as a product students are expected to produce, the operations students need to know how to produce the product, and the resources available to students when they are generating the product. According to Stein et al. (1996) mathematical task refers to classroom activity that engages students with the mathematical concepts or algorithmic skills. In their views a mathematical task passes through three stages until it becomes a learning outcome. The first stage involves designing and presenting the task as it appears in the instructional materials. The second stage, 'the set up phase', entails the teacher's introduction of the task in the classroom, whilst the third stage, 'the implementation phase', embraces the process in which the task is resolved by the teaching-learning community. Stein et al. (1996) discuss two dimensions of a mathematical task: task feature and cognitive demand. Task feature may require using more than one solution strategy, multiple representations, and various forms of communication styles; or it might simply request recalling pre-presented rules and procedures and applying them to the problem at hand. Cognitive demand refers to a sort of thinking that the teachers suggest during the set up phase and the thinking process within which the teaching-learning community engage while solving the problem.

In recent years a good deal of attention has been given to the role of instructional tasks in teaching and learning mathematics. Stein and Lane (1996) examined the development of students' mathematical thinking in various contexts (for example, pre algebra, geometry, probability and statistics) and related this to the task quality and the ways tasks were implemented. They reported that there was an increase in the students' understanding when the teachers chose cognitively challenging tasks for use in the classroom and when they maintained the task demands during the set up and implementation phases. Nevertheless, it is not easy to maintain the task demand, especially, in a socially-oriented classroom environment. Stein et al. (1996) identified several factors that caused reduction in the task demands. Some of the students put pressures on the teacher to reduce ambiguity of the task, and the teacher tendencies to take over the most challenging part of the task for students. Bennet and Desforges (1988) reported similar findings noting that the decline in the task demands occurs for various reasons which include inappropriate classroom management, teachers' lack of understanding of what the students need to learn, teachers' lack of subject-matter understanding and the ways of students' thinking, and their tendencies to favour mechanical progress at the expanse of students' conceptual development. Henningsen and Stein (1997) suggest that teachers should spend appropriate amount of time over a task to facilitate students' abstraction of the mathematical ideas embedded in a task. They state that providing scaffolding and asking students to give explanations for their answers are decisive teaching inputs to retain students' high-level engagement with the mathematical tasks.

There has been a call for new studies to gain better insight of the mechanism through which student engagement with a mathematical task may lead to increased mathematical learning (Stein and Lane, 1996). It is suggested that such studies should consider the cognitive processes set into various forms of tasks and the sort of teaching discourses that support or inhibit student engagement with the tasks that are intended to be cognitively demanding. The present study contributes to a growing body of research in the field by examining two Turkish teachers' selection and implementations of function problems and relates this to their students' understanding of this notion. As it is used within this paper 'task' refers to 'function problems' that the students were given, and 'task condition' refers to 'teaching discourse's that the teachers displayed when resolving the function problems during the classes.

RESEARCH METHODOLOGY

This research employed a qualitative case study (Merriam, 1988; Yin, 2003) and used a purposeful sampling strategy to involve teachers who had different views about teaching functions, to control the students' initial achievement levels and their socio-economic backgrounds, and to consider other school-teacher related variables such as the teachers' formal qualifications in mathematics. The participants were two experienced teachers (Ahmet: 25 years teaching experience and Burak: 24 years teaching experience) and their 9th grade students (Age 15).

Data concerning the selection and implementation of function problems were collected through classroom observations and document reviews. Each teacher was observed teaching all aspects of the functions. Lessons were tape-recorded and annotated field notes were taken to document psychological and pedagogical aspects of the teachers' classroom practices and the key features of their task implementation. The researcher wrote down all the problems that the teaching-learning community resolved during the lessons. Additionally, copies of students' notebooks (one from each class) and the teachers' handouts were retained to triangulate the data collected through classroom observation.

Students' learning was investigated through pre and post tests which encouraged them to provide reasons for their answers. Clarification interviews with three students from each class were carried out after each test. The interviewees were selected in conjunction with their teachers' recommendation and on the basis of their achievements in these tests. Prior to course on functions students were given a questionnaire which explored their informal knowledge of function and manipulative skills. After the course students were given a post-test questionnaire which includes 28 items in total. Nineteen questions were the classification tasks in that the students were asked to identify whether or not the given situations represented a function. The remaining nine questions were the implementation tasks in that the students were asked to make manipulation with or on functions. The implementation tasks differed however in their focus and cognitive demands. Three items assessed students' mechanical skills such as calculating images when the pre-images are given whilst the remaining six items assessed their conceptual understanding such as reversing a function in the graphical context. The questionnaire was divided into two parts. Part I: Classification tasks and Part II: Implementation tasks and administered to the students on two consecutive sessions (first the classification tasks, and second the implementation tasks). This was to eliminate the possibility that students might get clues from completing the implementation tasks to respond to the classification tasks. Later three students from each class were interviewed on 21 questions (16 classification and 5 implementation tasks). As it had been the pattern in the delivery of the questionnaire students were interviewed firstly upon the classification task, and then they were interviewed upon the implementation tasks. The method of semi-structured interview was employed and the aspects of the clinical interview (Ginsburg, 1981) were considered to delve into the students' thinking.

Theoretical frameworks and data analysis

The methods of discourse and content analysis (Philips and Hardy, 2002) were used to analyse the data collected from the teacher and the student sides. The notions of task feature and cognitive demand (Stein et al., 1996) and the literature about epistemology of the function concept (Vinner, 1983; Eisenberg, 1991; Breidenbach et al., 1992; Sfard, 1992) guided our interpretation of the tasks that the teachers presented to their students. The notions of action-process conceptions of function provided a framework to interpret the students' understanding of the concept. These notions were also used to identify key features of the teaching discourses that the teachers displayed when resolving the function problems. Simply defining an action conception of a mathematical idea refers to mental or physical manipulations that the students implement to transform objects into a new ones. It entails the ability to insert an element into an algebraic function and calculate its image through step-by-step manipulations (Dubinsky and Harel, 1992; Breidenbach et al., 1992). Such understanding enables students to recognise a function, algebraic or otherwise, from memory but it does not allow them to deal with the concept in a complex situation such as a graph made up of discrete points. A process conception is considered to be at a higher level in that the possessor is not only able to internalise actions associated with the previous step but is also able to think about a function process in terms of inputs-outputs (Dubinsky and Harel, 1992; Breidenbach et al., 1992). With this quality of understanding one could interpret a function process in the light of concept definition; he/she could recognise an 'all-to-one' transformation in the algebraic and graphical contexts without a disruption that may be caused by the absence of an explicit algebraic formula. Initial codes (brief descriptions) were assigned to a data base of 308 tasks presented to the students during the observations (Ahmet: 158, Burak: 150). Since there was no difference in the teachers' selection and implementation of the problems using set-diagrams and ordered pairs were not considered. Thus, in the second phase of analysis the focus was upon tasks with either an algebraic (Ahmet: 103, Burak: 115) or graphical form (Ahmet: 40, Burak: 15). Attention was given to the critical features of the tasks which include, for instance, the cognitive demands that the tasks posed, the operational steps that the students had to carry out, and the representations students need to use to resolve them. Codes such as 'connection needs to be established…', 'addresses the univalence...', 'requests recalling pre-presented rules and applying…', and 'graphs needs to be interpreted point-by-point' were established for each problem. Repeated on different copies of the texts this eventually led to the creation of three major categories: Procedural tasks, conceptual tasks, and others. Aspects of each of these categories are illustrated in the data presentation section.

Concerning the teachers' task implementations lessons were fully transcribed and considered line by line whilst annotated field notes, copies of students' notebooks, and the teacher' handouts were used as supplementary sources. A particular attention was given to the thinking process in which the teaching-learning community engaged when resolving the problems. The first phase of analysis includes assigning codes (brief descriptions) to the data. This process created 25 categories for Ahmet including, for instance: 'Engages students with the function-related ideas in an imaginary situation', 'Uses set-diagram like scaffolding', 'Always refers to the definition of function', 'Encourages students to solve the problem in another way'…The initial analysis of Burak's task implementation produced 22 categories, such as: 'Cuts off students' mental contact with the function concept', 'Implements the vertical line test in a procedural way', 'Uses representations in a discrete manner', 'Manipulates algebraic functions like ordinary expressions'... This process was repeated on different copies of the texts and eventually led to creation of six major categories for each teacher. These categories are presented in the data presentation section.

Quantitative method was employed to provide descriptive statistics of the students' achievements in the questionnaires. Students' interviews were fully transcribed and considered line by line. The method of content analysis (Philips and Hardy, 2002) was used to interpret the students' understanding of the function concept in relation to action-process conceptions of the function. Finally, cross-case analysis (Miles and Huberman, 1994) was used to establish the relationship between the variables. Instances where the students displayed noticeable differences in their understanding of the function concept were identified and cross referenced to corresponding variables in the teachers' selection and implementation of the function problems. This was also associated with a reverse analysis – teacher selection and implementation of the function problems and student learning.

RESULTS

The results are presented in two parts. First, we consider the teachers' selection and implementation of the function problems, and secondly we examine their students' understanding of this notion. We provide firstly though a brief summary of the teachers' instructional approaches. Ahmet (the teacher of Class A) shifted between a guided-discovery and a connectionist teaching approaches (Askew et al., 1996). Using the former he acted as a facilitator and prompted his students' thinking through open-ended questions. Employing the latter Ahmet acted like a dispenser of the knowledge; nevertheless the teacher encouraged his students' understanding through the connections he established between the ideas and between the representations. Burak (the teacher of Class B) mostly employed a transmission-oriented teaching approach and communicated rules, procedures and the factual knowledge with a little connection to underlying meaning.

The actual distinction was grounded in their approaches to the essence of the function concept. Ahmet employed a process-oriented teaching through which he prioritised the concept of function, its properties and sub-notions. He used the definition of the function like a cognitive tool and provided concept-driven, clear, and explicit verbal explanations that presented a function, algebraic or otherwise, as a process transforming inputs to outputs. Ahmet utilised pedagogically strong representations, set-diagrams and ordered pairs, like a scaffold to facilitate students' accession to the function process in the algebraic and graphical representations. In contrast, Burak mostly employed action-oriented teaching through which he emphasised rules, procedures, and the factual knowledge associated with the algebraic and graphical representations of the functions. Connections between the representations and between the ideas were not established.

Teachers' selection and implementation of function problems

The two teaching orientations: Ahmet: process-oriented teaching and Burak: action-oriented teaching, were signified by the teachers' selection and implementation of the function problems. This played, apparently, a major role in producing qualitatively different learning outcomes in their students. An analysis of the data base indicated that Ahmet and Burak differed remarkably in their tendencies to use conceptual or procedural tasks (Table 1; tasks in the forms of set-diagram and ordered pairs are excluded from this categorisation). Procedural tasks were implemented through the application of rules and procedures; and they had the potential that students could develop misconceptions such as the idea that a function is an algebraic expression in a nice equation form (Vinner, 1983).

Conceptual tasks were considered to pose cognitive demands and engage the students with the concept of function, its properties and sub-notions. These problems encourage the development of a process conception of

Table 1. Task profiles that the teachers used in their instructions of the functions.

Type of tasks	Ahmet	Burak
Conceptual tasks	63	25
Procedural tasks	20	75
Others	60	30
Total (n)	143	130

function. One needs to establish connections between the ideas and between the representations to resolve conceptual tasks. Tasks that fell into the category of others did not have a clear focus nor cognitive demands as indicated above. These problems could be manipulated procedurally or they can be used to encourage students' conceptual understanding, yet this depended upon the teachers' approach. Table 1 illustrates that Ahmet prioritised conceptual tasks over the procedural ones in the ratio 3:1 whilst Burak did the reverse in the same quotient. For instance, all the graphs Burak presented to his students were smooth and continuous lines or curves. This limitation was likely to cause, and did so, students to develop a continuity misconception (Vinner, 1983). In contrast, Ahmet presented his students with many problems that were conceptually focused and cognitively challenging, for example partitioned graphs and the function problems that encouraged his students to reflect, spontaneously, upon a function process and the inverse of that process without loosing the sight of univalence. The distinction continued in the teachers' resolution of the function problems, and now we turn to illustrate key features of their task implementations.

Ahmet's task implementation

Irrespective of the task quality Ahmet created task conditions that encouraged his students to develop conceptually rich knowledge of function. Ahmet's task implementation had six crucial features (Table 2) each of which acted as a scaffold promoting his students' progress towards a process conception of function.

Prioritises concept over the procedure

Ahmet almost always prioritised function-related ideas over the rules, procedures and the factual knowledge. The indicator was that in the process of task resolution the teacher used the definition of function as a driving force, yet he illustrated the procedures as part of the routine. Students were not given ready-made rules and formulas; rather the teaching-learning community discovered them through collective reflection upon the task at hand. Consider the problem: Work out the values 'a' and 'b' for which *f: R→R f(x)=(a-2)x2+(b+1)x+5* represents a constant function.

The task statement suggests noting as to how this problem should be resolved. As we shall see in Burak's teaching (Episode B1) such problems could be implemented by the application of a rule which suggests simply eliminating the terms with *x*'s from the expression. The underlying meaning does not necessarily need to be uncovered if the instructional goal is to ensure that the student acquired the method of solution which works in similar situations. Yet, the way Ahmet implemented this problem consistently engaged his students with the idea of constant function (Episode A1):

Ahmet: … We described it (the constant function) as a relation matching all the elements in the domain to one and the same element… Yes we are keeping this definition in mind, OK. … This expression involves something that does not allow the transformation of all the real numbers to one and the same… What should we do here so that the function produces the same element irrespective of whatever we put into the *x*?
Student: The value of 'a' is 2 and the value of 'b' is -1.
Ahmet: How did you find out?
Student: … I thought like that the expression must involve just 5 so that it matches all the values of *x* to 5. … That is why I equalised the coefficients of the other terms to 0…
Ahmet: … Your friend suggests removing the terms with *x*'s from the expression. … Because if a function involves an independent variable like *x*… [It] produces different outputs for the number given for that variable. … We should fix the value of *y*, the image. We ensure it as we remove the terms involving *x*'s from the expression. ... So, we have to equalise the coefficients of the terms involving *x*'s to 0… [Through appropriate manipulations the teacher gets the function *f(x) =5*]… No matter whatever put into the *x*, say -5, 0, 4…., all goes to 5 under this function.

Notice that right at the beginning of his explanation Ahmet brings the definition to the students' attention and encourages them to discover the idea that the terms with *x's* must be removed from the expression (Bayazit, 2006; Bennet and Desforges, 1988) so that the function gives out one and the same output for every input. He refers back to the definition time and again (Breidenbach et al., 1992; Cobb et al., 1997; Doyle, 1983; Dubinsky and Harel, 1992) to ensure that his students understood the underlying meaning of 'why the variable *x* must be removed from the expression?'

Implements procedural tasks in a conceptual way

On many occasion Ahmet presented his students with the procedural tasks; yet when implementing these problems he created task conditions that engaged his students with the concept of function. Consider, for instance, the

Table 2. Key features of Ahmet's task implementation.

1. Establishes connections between the representations – Ahmet increased his students' gain from a task through the connections he established between the representations. On some occasions he took a task and examined it across the representations following the sequence which began with the set-diagrams and ordered pairs and developed through algebraic and graphical representations. In doing so Ahmet made use of the visual power of the first two representations to facilitate students' accession to the function-related ideas in the algebraic and graphical situations. On other occasions he presented the students with an algebraic function and then obtained its graphical form, or he did the reverse.
2. Prioritises concept over the procedure –The indicator was that the teacher used the definition of the function like a cognitive tool throughout the task implementation. The procedures were not the cognitive focus of Ahmet's task implementation; they were illustrated as part of the routine.
3. Implements procedural tasks in a conceptual way – Tasks were procedural in that they could be resolved by the application of rules and procedures; yet the teacher created task conditions in which students were engaged with the notion of function and the related sub-notions.
4. Enforces students' understanding through conceptual tasks – Ahmet encouraged his students' reasoning, critical thinking, and sense making through conceptually focused and cognitively challenging tasks. The tasks were conceptually focused in that they addressed the function concept, its properties and sub-notions. They were cognitively challenging in that the teaching-learning community had to use more than one strategy and they had to be flexible in their approaches to the tasks so that they could resolve the problems.
5. Encourages students' visual thinking – This feature of task implementation was manifest in Ahmet's teaching in two ways. First, on many occasions Ahmet encouraged his students to imagine a graph of a function before sketching it through point-by-point manipulations. He provided scaffolding to facilitate the students' visualisation of how the graph behaves in accord with the changes in the x and y in the corresponding algebraic expressions. Second, Ahmet gave his students a graph of function and then asked them to sketch the graph of inverse function. Resolving such problems he encouraged his students to visualise the function process behind the graph, reverse it in an imaginary situation, and then relocate it to the Cartesian space.
6. Displays multiple perspectives on a task – The teacher created task conditions that allowed students to experience the function concept and its properties from different perspectives. Common cases were identifying situations in which an algebraic or graphical relation did or did not represent a function.

For reasons of space, only three of abovementioned aspects are illustrated in the following sections.

following task: Consider the function $f(x)=x^2$, $f: x\rightarrow x^2$ from A={-1, 0, 1, 2} to B={0, 1, 2, 4, 9}; and work out the image set.

The solution of this problem is quite straightforward. One can easily resolve this problem by inserting inputs into the expression and calculating their images through step-by-step manipulations. Yet, the teacher did not prefer this kind of action-oriented teaching; rather the way he implemented the task encouraged his students to conceive $f(x)=x^2$ as a process transforming every input to an output (Episode A2):

… Look at the function; it matches x to x^2; that is, this function matches every element to its square. Yes, this is the rule of function… Since the function matches every element to its square it will match 0 to 0, because as we put 0 into the *x*… (makes manipulations)…we obtain 0. … Likewise, as we put -1 into the expression…we get 1; that is -1 goes to 1 under this function.

At this stage the teacher shifted from algebra to ordered-pairs and continued to promote his students' understanding:

… We can write down the same function in the set form [ordered pairs] like *f={(0,0), (-1,1), (1,1), (2,4)}*. Here the function matches first components to the second ones; as you see 0 goes to 0…2 goes to 4 (moves his finger between the components of the pairs). … This is another way to represent the same function. …

Notice that the teacher presents the function in two forms: $f(x)=x^2$ and $f(x)=x\rightarrow x^2$. The second one ($f(x)=x\rightarrow x^2$) suggests that the function transforms every substitute to its square; thus it would help students recognise the process of function which is implicit in $f(x)=x^2$. Ahmet uses process-oriented language in that his speech emphasises that the function transforms every substitute to its square, afterwards he describes this transformation as the rule of the function. He does not use action-oriented language which emphasises algorithmic procedures such that '*insert 1 into the expression and then take its square*'. Additionally, the teacher gives the students an opportunity to experience the same function in the ordered-pairs. It is conjectured that these features of Ahmet's tasks implementation

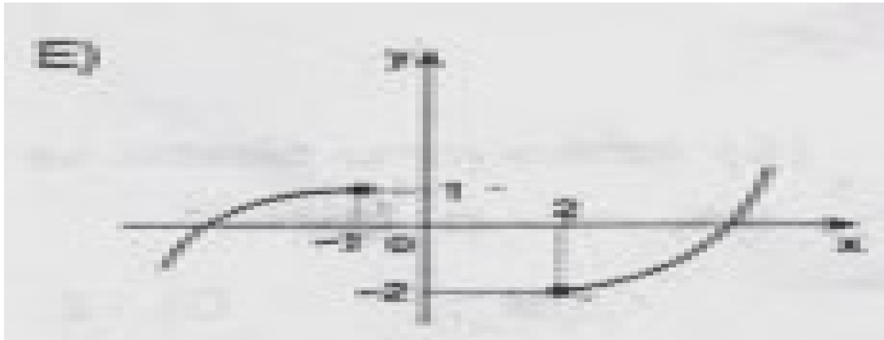

Figure 1. A graph derived from Ahmet's handout.

complement each other and eventually increases students' gain from the procedural tasks.

Displays multiple perspectives on a task

Ahmet often presented his students with the tasks which can be resolved in different ways. The teaching-learning community examined these problems from different perspectives and exhausted all the alternative strategies to solve these tasks. By this strategy Ahmet's goal was not to teach a new method of solution, but to deepen and strengthen his students' understanding of the ideas set into the tasks. The following citation illustrates this feature of Ahmet's task implementation. The teacher examined together with the students that the graph (Figure 1) did not represent a function on IR, because it omits some elements in the domain. Then, he situated the problem into the context of piecewise function and gave the explanation (Episode A3):

… Let's have a look at the issue [problem] from another perspective… If I say this is actually a graph of function, do I confuse you? … (After a short silence some students got the point and suggested that the elements between -1 and 2 could be removed from the domain)… Yes it does, but not on the set of IR…we have to redefine the domain… How could we do that? It is quite obvious, look at the graph; it tells us what we should do… It covers this part and that part of the x-axes (moves his finger on the sub-domains). … Here is a graph made of two branches with two sub-domains; so what does it mean? It means that this is, in fact, a graph of piecewise function… If we determine the domain set as $(-\infty, -1]\cup[2, \infty)$, this graph matches every element in this set to only one element on the y-axis (illustrates matching over the graph)…

The premise of this episode is the construction of a process of piecewise function which was not initially there. The teacher illuminates properties of the piecewise function (sub-domains, and branches of the graph); then he forms a single domain by unifying the sub-domains and illustrates how this function transforms elements from domain to co-domain. More importantly, the teacher gives the above explanation after he illustrated the idea that the graph does not represent a function on IR because it omits some elements in the domain. In doing so, the teacher creates opportunities for the students to experience the concept from two different perspectives. He prompts his students' flexibility in thinking the conditions where a relation does or does not represent a function.

Burak's task implementation

Burak's task implementation include six constraints (Table 3) each of which caused reductions in the task demands and played, apparently, a major role in confining his students' understanding to an action conception of function.

Prioritises procedure over the concept

This approach embraced fundamentally an explanation of how to get the correct answer in an economical way. It was manifest in Burak's teaching in two ways. First, during the set-up phase Burak emphasised what must be done – the rules and the formulas were announced and the students were explained how to use them to resolve the problems. Secondly, when resolving the problems Ahmet used concise and vague language illustrating the concept but provided explicit and elaborative descriptions to emphasise the procedures. The language was concise in that in the beginning of the task implementation Burak illustrated the function-related ideas through one or two sentences. Following such a brief introduction he provided extensive descriptions of the procedure being implemented; thus, the idea illustrated in the beginning was concealed in the whole of the teacher's verbal discourses. The language was vague in that the teacher described a function process in terms of inputs-outputs but did not point out the implicitly existing process in the situation. The following explanation was given upon the task: Work out the precise form of the constant function

Table 3. Key features of Burak's task implementation.

1. Prioritises procedure over the concepts – Resolving the function problems Burak prioritised procedures over the concept. On some occasions concepts were partially used, yet they were not the driving force in the teacher's task implementations.
2. Makes interference: The teacher diverted his students' attention from the concept of function and engaged them with procedures or other mathematical ideas.
3. Implements conceptual tasks in a procedural way: The teacher manipulated the conceptual tasks through the application of rules and procedures. No attempt was made to discern the meaning embedded in such problems.
4. Does not care continuity and consistency in the task demands performed one after another – Burak engaged the students with the procedural tasks and then gave them conceptual tasks; yet what the students learned when resolving procedural tasks was far from supporting them to tackle the cognitive demands posed by the conceptual tasks. When the consecutive tasks were conceptually interrelated Burak did not illustrate this so that he would help the students develop a better understanding.
5. Does not establish connections between the representations – The representation systems were not used in connection to each other although it was crucial to establish such connections between the representations to facilitate the students' understanding of the problems at hand.
6. Oversimplifies the task demands: The teacher totally ignored the task demands and manipulated the functions like an ordinary algebraic expression.

$f(x)=(4-2n)x+(2n+3)$, and sketch the graph of it. Burak explains (Episode B1):

Burak: … We described it (the constant function) like a fixed minded person; did we not? … Whatever we say; he never changes his mind. Yes, constant function is like a fixed minded person; no matter whatever we put into the x we come up with the same image. … Let's remember the algebraic form of the constant function; it will help us so much for the solution of the problem. What was it? In general we represented it as $f(x)=a$, $a\in R$... So, could we say that the algebraic form of a constant function involves just a number; this number would be an integer, a natural number, or a rational number… …have a look at the formula, $f(x)=(4n-2)x+(2n+3)$. In this formula there are two terms; one is the constant term, 2n+3, and the other is a term involving x, $(4n-2)x$. … So, first of all we should get rid of the term containing x; because if this is the constant function…it must not involve x. How can we do that…?

Students: We would equalise the coefficient of x to 0…

Burak: Yes exactly, this is what we must do here. We should equalise the coefficient of x, 4n-2, to 0. …

It is clear that the teacher talks about the process of constant function in terms of inputs and output but he does not emphasise it in an explicit manner such that this function transforms every input to one and the same output. As the instruction develops he focuses students' attention upon the visual properties of $f(x)=a$ ($a\in R$) and emphasises rules and factual knowledge without any connection to underlying meaning. Additionally, the teacher offers an analogy of '*a fixed minded person*' but does not link it to the notion of constant function presenting, for instance, human mind as a constant function (a process) receiving all the ideas (inputs), reasoning (processing) them, and then reaching at a single conclusion (an output).

Implements conceptual tasks in a procedural way

Within this study, conceptual tasks were considered as the ones which addressed the definition of the function. Burak presented his students with such problems; nevertheless when resolving them he totally ignored the concept and engaged his students with the procedures. To illustrate this aspect of Burak's task implementation two cases are provided below. We note as a background that by the time Burak resolved these problems he had not examined algebraic relations in the light of concept definition. After teaching how to calculate images when the pre-images are given, or vice versa, Burak performed three problems which were epistemologically identical to:

Given the function $f: R\text{-}\{2\}\rightarrow B$ $B\subset R$, $f(x)=\dfrac{3}{2x-m}$; and work out the value of m.

Burak explains (Episode B2):

Burak: In this type of questions they (domain and co-domain) are very important. …what is the element removed from the domain…

Students (shouting out): It is 2…

Burak: So, what is the number 2? It is the root of the expression in the denominator; it makes the denominator zero... That is why it has been excluded… … to find the value of *m* we should equalise the expression *2x-m* to zero. … [Constructs the equation *2x-m=0*]… …what was the number satisfying this equation? We determined it; it was 2; actually this number is given in the question. It has been excluded from the domain; because it makes the denominator zero. So, in this equation, we should put 2 into *x* and solve the equation… (Obtains the value of m through appropriate manipulations)…

Resolving the problem Burak does not refer to concept definition nor does he indicate that this function produces an image for every real number apart from 2. He stresses time and again a factual knowledge that the element 2 is excluded from the domain because it makes the denominator zero. The teacher bypasses the underlying meaning of this factual knowledge. He does not illustrate, for instance, that they should fix the value of *m* using the element 2 so that the function produces an output for every element of the domain. The following exchange shows how the procedure emphasised in the above episode became the focus of reflection in the following part of the lesson. It occurred when the students were asked to:

Consider the function $f: R\text{-}\{-3\}\rightarrow A,\ A\subset R$,

$f(x)=\dfrac{-5}{2x-m+7}$; and work out the value of *m*.

Burak explains (Episode B3):

Burak: Who would like to solve it? … [Invites a student to the board] …
Student: (Establishes the equation *2x-m+7=0*).
Burak: Could you remember the previous question?
Student: I should put -3 into the *x* in this equation, because that number makes the denominator zero…
Burak: Yes, this is what we did before. We should substitute -3 into *x* in this equation…2 times -3 makes -6; there is another number here, 7; so -6 plus 7 makes 1. What happened now, *-m* plus 1 equals to 0; this is a very simple linear equation. When we take *-m* to other side…we get the value of *m* as 1. Yes, this is the result; OK. We are thinking of such problems in two parts. … The first is the domain set; the number left out of the domain is very important. … In the second step, we are substituting that number into the *x* in the denominator and solving the equation. … C [asking a student] have you understood…?
Student: Yes, I have, I have; simply we are equalising the expression [in the denominator] to zero and then putting the number removed from the domain into the *x*…

This exchange substantiates that the teaching-learning community displays a collective reflection (Cobb et al., 1997) over the problem; nevertheless, the focus of reflection is the algorithmic procedure, not the concept of function; and we see that the students are learning what is emphasised by the teacher.

Makes interference

Interference refers to teaching acts that divert students' attention from the notion of function and engage them with the algorithmic procedures or irrelevant ideas. It is internal to the teacher's approach to a task rather than to the epistemology of the task being implemented. In Burak's teaching interference occurred at two levels: deliberate interference and un-deliberate interference. The former was intentional in that the teacher was keen to solve certain type of problems that his students would encounter their kinds in the exams. Burak presented ten (10) problems to his students which were epistemologically similar to:

Consider the function *f:R→R 2·(fx)=(x+1)·f(x+1) and f(1)=2*; and work out the sum of *f(2)+f(-20)*.

Burak believed that the students had to experience this kind of problems to succeed in the local and national exams (Episode B4):
… You would see such problems in the university exam leaflets as well. … If you want to succeed in those exams you have to learn how to cope with such problems…
Key aspects of Burak's implementation of this problem can be seen in Episode B5:
…We have to give attention to three things: the rule of function, here it is *2·(fx)=(x+1)·f(x+1)*; known value, here it is *f(1)=2*; and the unknown value… In this question we know *f* of 1… Yet, we need to find *f(2)* and *f(-20)*. To do this we have to make use of what have been given…the rule of function and the known value. … Let's start by inserting 1 into the expression [substitutes 1 into the expression, *2·(fx)=(x+1)·f(x+1)*, and gets the value of *f(2)* as 2]. … We are going to work out *f* of -20 in a similar way. … We are going to substitute integers from 0 till -20… (Constructs an equation system as follows)…

$$x=0\Rightarrow 2f(0)=1f(1)$$
$$x=-1\Rightarrow 2f(-1)=0f(0)$$
$$x=-2\Rightarrow 2f(-2)=-1f(-1)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x=-20\Rightarrow 2f(-20)=-19f(-19)$$

… …could you see the pattern…in this equation system? Remember…we need to find the value of *f(-20)*… Do you have any idea? … I will give you a hint; in such situations we usually conduct additions or multiplication…on the both side of equations. … [Conducts the multiplication and gets the following equation]…

$$2f(0)\cdot 2f(-1)\cdot 2f(-2)\cdot\ldots\cdot 2f(-20)=1f(1)\cdot 0f(0)\cdot(-1)f(-1)\cdot\ldots\cdot(-19)\cdot f(-19)$$

. …
Now, look at the right side of the equation... There is zero there; it makes the right side of the equation zero, OK.

So, it has finished; …the value of *f(-20)* is zero, because as we leave it alone on the left side… Tell me now what is the sum of *f(2)+f(-20)*? …

Burak makes a considerable effort to ensure that his students acquired the method of solution that works in similar situations. In this respect, he provides sound heuristics by decomposing the problem into three parts and explaining how to use the algorithms and the known values to get the unknown values. The way that the teacher implements the problem could enforce students' inductive reasoning – seeing a pattern in the equation system. Nevertheless, in every step of the task implementation Burak keeps his students detached from the idea of function. When implementing such problems the spoken language is the basic mean to maintain students' engagement with the concept (Bayazit, 2006). This can be achieved by using process-oriented language which refers to the definition of the function and emphasises the transformation such that 'this function transforms -20 to 0'. Burak uses however action-oriented language which emphasises algorithmic procedures or describes the transformation in terms of inputs-outputs, such that *"…the value of f(-20) is zero…"*.

The teacher made un-deliberate interference when he was using students' prior knowledge as a metaphor to promote their understanding of the problem at hand. The following excerpt illustrates this aspect of Burak's teaching. It was given when the students were asked to sketch the graph of *f(x)=4,* (Episode B6):

… In the previous years you learned how to sketch the graphs of lines parallel to the coordinate axes. … When asked to sketch the graph of *y=2*, you were marking the point 2 on the y-axis and then drawing a parallel line…through this point. Here, we are going to do the same thing; look at the rule of function…; it is *f(x)=4*. … So, we are going to mark the point 4 on the y-axis and then sketch a parallel line passing through this point. … Yes, we could say, in general, that every line parallel to the x-axis represents a constant function. I suggest you to note it on your notebook…

The way that the teacher implements the task adds noting to the students' previous knowledge. Burak refreshes students' knowledge of how to sketch parallel lines to the coordinate axis and, then, sets up the goal: "*…here we are going to do the same thing…*" This sequence of teaching acts does not allow students to understand the qualitative distinction between a straight line (a static geometrical figure) and a graph of constant function (a dynamic process doing an 'all-to-one' transformation). This approach might even lead students to develop a misconception such that 'there is no difference between a graph of an equation and that of a constant function'.

Learning outcomes

The two classes of students (Ahmet's students: class A, Burak's students: class B; classes are identified by the initials of the teachers' names) were largely comparable in their initial knowledge: their informal knowledge of function and their manipulative skills. The former include students' understanding of a relationship between two varying quantities, a transformation represented by a (function) box, and an implicit process in a set of ordered pairs. For instance, 68% of the class A and 78% of the class B gave a correct answer to the question: You are asked to double the circumference of a circle. How would you do that? Give your answer with the underlying reasons.

Correct answers emphasised that the perimeter of a circle depends upon its radius and suggested that the radius should be multiplied by 2 a feature confirmed through the interviews with three students from each class. Belgin's answer is typical:

Belgin: …I cannot change 2 and π, because they are constants. I can change only r; and when I double it the circumference…automatically increases twice.

The two groups were also equally competent in manipulating algebraic expressions and reading a graph represented a real world situation. For instance, all the students excluding four (2 from each class) calculated the value $2x^2-3x+5$ when $x=2$.

The post-test and the follow-up interviews indicated that there was no difference in the group performances in making mechanical manipulations with the functions. Both groups produced a hundred percent correct answer when they were asked to work out the images of -1 and 3 under a function $f(x)=2x-5$. In responding to a question: Consider the functions $f: R\to R$, $f(x)=2x+1$ and $g: R\to R$, $g(x)=x^2-1$, and work out the value of $(gof^{-1})(9)$, 89% of the Class A and 93% of the Class B obtained the rule of composite function (gof^{1}) by replacing each occurrence in $g(x)$ by the rule of $f^{-1}(x)$. Nevertheless, students' performances declined when they were given conceptual tasks; and this was more dramatic in the achievement of Class B students compared to that of Class A students. The examination of the students' understanding draws upon their responses to three questions. The first item assessed their understanding of the concept within the algebraic situations.

Item 1: Does the relation $y=\frac{x+7}{x-1}$ represent a function on IR; give your answer with the underlying reasons. Students' answers were analysed in terms of correctness and the clarity of their explanations (Table 4). Table 4 shows that class A students outperforms class B students with a margin of 20% in detecting the element (1) for which the function is not defined – an indicator of a process conception of function. More than half of class B students (56%) placed greater reliance upon the rule of function – these students made calculations or expressed that the situation was a function because it was given with a formula whilst only 18% of class A students did so. Three interviewees (Okan and Serap: class A, and Aylin: class B) indicated a strong process conception of function.

Table 4. Students' understanding of the function concept in the algebraic context.

Function concept	Class A		Class B	
	Frequency	Percent	Frequency	Percent
Not a function (ref to definition)	14	50	8	29
Function (ref to definition)	6	21	3	11
Function (a statement or ref to rule)	5	18	15	56
No response	3	11	1	4
Total (n %)	28	100	27	100

Table 5. Students' understanding of the constant function in the algebraic context.

Constant function	Class A		Class B	
	Frequency	Percent	Frequency	Percent
Correct (ref to definition)	24	85.7	13	48.1
Incomplete	0	0	7	25.9
Incorrect	3	10.7	1	3.7
No response	1	3.6	6	22.2
Total (n %)	28	100	27	100

These students not only detected the element, 1, for which the function was not defined, but also they redefined the domain set as IR-{1} on which the relation did represent a function. Aylin's answer is typical:
Aylin: … It means that an element of the domain has not been matched… If the domain had been given as R-{1}…it would be a function. … It is conjectured that the remaining three students possessed an action conception of function. Erol and Serap made step-by-step calculations whilst Belgin acted with her concept image and disclosed a misconception that an algebraic function must be in the form of a nice linear equation form:
Belgin: …it is not a function, because it is different from the others [linear expressions]. For instance, here (refers to *y=2x+5*) there is an *x,* but in this problem [$y = \frac{x+7}{x-1}$] there is an x is in the denominator…

The second item explored the students' understanding of the concept in a situation where the function(s) were not stated with the algebraic expressions.

Item 2: Given the functions *f: R→R f(x) =5* and *g: R→R g(x) =3*. What is the value of (fog) (7)? Give your answer with the underlying reasons.

Students' answers were analysed in terms of correctness and completeness (Table 5). Once again the concept-driven explanations dominated the reasons given by the students of class A (86%) and exceeded those given by the students of class B in ratio of 2 to 1. These students articulated an 'all-to-one' transformation each function does. Half of class B students produced no answer or shifted away from the problem after satisfying the composition protocol such that *(fog)(7)=f(g(7))*. It is worth noting that the class difference in this task can not be explained by the differences in their understanding of the composite function; since as indicated earlier both classes were equality competent (class A: 89%, class B: 93%) in calculating the image of 9 under (gof^{-1}) when the functions $f(x)=2x+1$ and $g(x)=x^2-1$ were given.

The interviews drew details that complimented the class difference identified through the questionnaire. Three students from calss A (Okan, Demet, and Erol) and one from class B (Aylin) recognised the implicitly existing processes in the situations and articulated that a constant function does an 'all-to-one' transformation. Okan's response is typical:
Okan: …no need to make calculations; these are constant functions; whatever we give for *x*, *g(x)* matches it to 3… *f(x)* transforms every input to 5…so the answer 5.

It is conjectured that Serap from class B was in transition towards a process conception of function; since although she recognised an implicitly existing process in the situations her thinking missed the very essence of the constant function, an 'all-to-one' transformation. Belgin expressed that she needed algebraic formulas to carry out the composition, thus she was considered at the action level.
Belgin: If there is a formula I would compose them, but… (Silence)… Normally we should get first the value of g (7)… This is the procedure; but there is no function here; I mean there is no formula…
The third item investigated the students' understanding in

the graphical context.

Item 3: Does the following graph made of five discrete points represent a function? Give your answers with the underlying reasons (Figure 2).

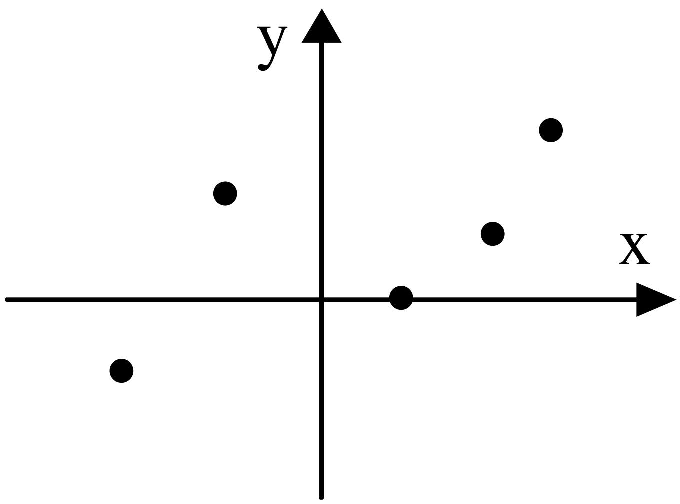

Figure 2. A graph of a relation made of disjointed points.

Students' answers were analysed in terms of correctness, clarity of the reasons they gave and the methods they used (Table 6). Table 6 shows again that class A students outperform those of class B in ratio of 2 to 1 in specifying the domain set and illustrating a 'one-to-one' matching over the graph. This quality of understanding is considered as an indicator of a process conception of function. Those who considered that the graph was not a function were considered at three levels. The largest group within each class displayed a continuity misconception; these students joined the graph with curves and broken-lines and then claimed that the graph they had sketched represented a function. Notice that 12 students in class B revealed this misconception compared to 4 students in class A.

The interview results indicated that three students (Okan and Demet: class A and Aylin: class B) had a process conception. These students specified the domain and illustrated the transformation over the graph. For instance Aylin said:

Aylin: It would be a graph of function; if the domain involves a limited number of elements; let me determine it [marks the inflections of the graph on the x-axis and illustrates the matching over the graph… Yet, if we involve one more element in the domain…(it) does not represent a function. … I mean, the domain is very crucial…

Two students (Erol: class A and Serap: class B) were in transition towards a process conception of function. These students considered that the situation did not satisfy the univalence condition; however they could not recognise the function process defined on the split domain. Belgin from class B is considered at the action level because she acted with her concept image and disclosed a continuity misconception.

Belgin: In my view, the graph of function should be a smooth line or curve… I should join them… (silence)…I have not seen any graph like that…I must join them in some way…

Up to this point students' understanding of the function concept in the algebraic and graphical contexts were illustrated by presenting quantitative and qualitative data in a harmonic way. To make the interpretation of the interview results easier the interviewees' development are summarised in Table 7. It is worth noting that if a student indicated an action conception in one algebraic/graphical situation and displayed a process conception in the other, he/she was regarded at the stage of move toward a process conception of function. Table 7 shows two students from class A possessed a process conception of function in the algebraic and graphical situations compared to only one from class B. One from each class was at move towards a process conception of function with a student, Belgin, indicated an action conception in both contexts.

DISCUSSION

Teaching refers to instructional acts taken to help students construct knowledge. It is a complex cognitive skill delivered in a dynamic classroom environment (Leinhardt, 1988). Learning is a cumulative process that the individuals develop through interacting with the external or internal stimuli. Establishing links between these two distinct fields of research is more difficult than it is assumed. There are many internal and external factors, which may include students' cognitive levels, their attitudes towards mathematics, and their socio-economic backgrounds, that would interfere the mediating process between the teacher classroom practices and the student learning (Weinert et al., 1989; Meijnen et al., 2003). Impossibility of eliminating all these factors does not permit linking teaching and learning to each other in the sense of cause-effect relationship. It is suggested however that the complexity associated with teaching and learning can be handled to some extent and the relationship between the two could be established through examining the teachers' selection and implementation of the mathematical tasks (Hiebert and Wearne, 1993).

This paper illustrates the impacts of teachers' selection and implementation of the function problems on their students' understanding of this concept. Overall, findings suggest that procedural tasks, when implemented with little connection to underlying meaning, are likely to confine students' understanding to an action conception of function and create misconceptions. Almost 50% of Burak's students, drawing upon his emphasis upon

Table 6. Students' understanding of the function concept in a graph made of five discrete points.

Function concept	Class A		Class B	
	Frequency	Percentage	Frequency	Percentage
Function (ref to definition, specifies the domain)	17	60.7	8	29.6
Not a function (concerning the univalence)	2	7.1	2	7.4
Not a function (continuity restriction)	4	14.3	12	44.4
Not a function (other reasons)	2	7.1	1	3.7
No response	3	10.7	4	14.8
Total (n %)	28	100	27	100

Table 7. Summary of the interviewees' development of the function concept.

Representation	Class A			Class B		
	Okan	Demet	Erol	Aylin	Serap	Belgin
Algebraic situations	P	P	A→P	P	A→P	A
Graphical situations	P	P	A→P	P	A→P	A

A: An action conception of function, A→P: Transition towards a process conception of function, and P: A process conception of function.

smooth and continuous graphs, revealed a continuity misconception with their desire to link graphs made of discrete points. In contrast, more than half of Ahmet's students (61%) recognised the process of function defined on five split domain an illustrated a 'one-to-one' matching over the graph. This was substantiated by the interviews in which two students from Ahmet's class displayed the same quality of understanding. It appears that Ahmet's provision of conceptual tasks (for example, graphs in pieces, graphs in strange shapes) and resolution of these problems through process-oriented teaching practices prompted his students' understanding towards a process conception of function.

Tasks can shape the way students think about the subject matter and influence their learning (Doyle, 1983; Stein and Lane, 1996). Evidence suggests that although tasks may promote thinking, it is the task conditions created by the teachers through their models of implementation that are more influential in supporting students' abstraction and processing of the ideas set into the task. In this paper, it can be seen in Episodes A1 and B1 that the two teachers implement epistemologically the same problems but emphasise different things. Ahmet engages his students with the notion of constant function. Unlike Burak, he does not set up an easily accessible goal (get rid of the terms with *x* from the expression) but prompts his students' thinking by providing concept-driven explanations: "…there is something that does not allow the transformation of all the real numbers to one and the same element". In contrast, bringing $f(x)=a$ ($a \in R$) to the students' attention, Burak emphasises factual knowledge, a constant function does not involve x, but he does not encourage his students to establish the underlying reason for such knowledge. The impacts of these can be seen in the student data. In the questionnaire (Table 5) 86% of Ahmet's students composed two constant functions and articulated that a constant function does an 'all-to-one' transformation whereas less than half of Burak's students revealed the same quality of understanding. In the interviews three students from Ahmet's class indicated a full understanding of the process of the constant functions whilst only one from Burak's class did so.

Also two more cases which support the argument is brought to attention that it is not the task but the task condition which plays decisive role on student learning. The following problem is considered:

Given the function $f: R\text{-}\{2\} \rightarrow B$ $B \subset R$, $f(x)=\dfrac{3}{2x-m}$; and work out the value of *m*.

In this study, this task is considered conceptually focused because it addresses the concept definition. Episode B2 shows however, that when implementing the problem Burak emphasises a factual knowledge – the element 2 makes the denominator zero, so 2 should be substituted into *x* in *2x-m=0* and get the value of *m*. The task condition Burak created does not communicate ideas about the concept of function and its properties nor does it encourages students to conceive the expression as a process producing an output for every input apart from 2. In contrast, Ahmet uses a procedural task (see Episode A2) – the task is procedural because it can be resolved substituting simply inputs into the *x* and calculating their images through step-by-step manipulations – but he creates a task condition that enforces his students' conception of the expression $f(x)=x^2$ as a process transforming every input to its square. Also, the impacts of qualitative distinction in the teachers' task implementations can be seen on their students' learning. In the questionnaire 50% of Ahmet's students considered that the relation $y=\frac{x+7}{x-1}$ did not represent a function because it omitted an element (1) in the domain whilst 29% Burak's students did so. Two interviewees from class A and only one from class B detected the element 1 for which the function was not defined. The gap between the class performances tells a lot as is considered in the study that class B students experienced epistemologically the same problems during the lessons (Episodes B2 and B3) but class A students did not.

To sum up, the evidence suggests that conceptual tasks should not be seen as a panacea. It is the conditions associated with task resolution that engage students with the subject matter and, thus, may help them make progress in learning. Within the function contexts, the prolific task conditions appear to include using process-oriented language consistent with the epistemology of the function concept, establishing connections between the ideas and between the representations, applying continuity and consistency in successive task demands, encouraging students' visual thinking, displaying multiple perspective on a task, and using the definition of function as a cognitive tool when solving the problems.

Conclusion

Since this study employed a qualitative inquiry the findings cannot be generalised beyond the research sample. An experience of the Turkish education system knows however that most teachers act like Burak (Episodes B4 and B5) and favour certain type of problems that the students could encounter their kinds in the local and national exams. This pressure might be greater than that noted in this study and enforcing teachers to make reductions in the task demands during their classroom practices. Therefore, the effects of examination system on teacher selection and implementation of mathematical problems needs to be investigated, and the consequences should be considered with regard to curriculum design and the classroom teachings. This is the issue worthy of further research.

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