

*Full Length Research Paper*

# Role of finite element method (FEM) in predicting transverse modulus of fiber-reinforced polymer (FRP) composites: A revelation

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The present investigation aims to validate finite element method (FEM) in micromechanical analysis of a unidirectional continuous fiber reinforced composites and experimental verification of results in case of fiber-matrix debond. Available analytical models are reviewed, compared and are seldom in agreement with each other in case of transverse modulus of unidirectional continuous fiber reinforced composites. Reasons for variation of these models are analyzed and their limitations are discussed. FEM of a square representative volume element (RVE) are developed to simulate various conditions such as matrix/fiber dominated cases (in volume and stiffness) and fiber-matrix interface debond in ANSYS v12 to facilitate comparison with the available analytical results. Numerical results are compared with the approximate as well as exact analytical models and are found to be in very close agreement with exact analytical results. To simulate fiber reinforced composite behavior close to a mathematical model of square RVE, a specimen with a combination of two metals is designed, fabricated and tested to determine the transverse modulus. FEM of a regular square RVE is modified to suit the specimen conditions such as finite dimensions relative to fiber and possible fiber-matrix interface debond. FEM results are found to be in good agreement with the experimental results and thus the validity and applicability of FEM in predicting transverse modulus of fiber reinforced composites is established.

**Key words:** Fiber-reinforced polymer (FRP) composites, micromechanics, transverse Young's modulus, fiber-matrix debond.

## INTRODUCTION

The inherent anisotropy of composites makes it compulsive to test the components case by case based on the loading pattern and application. In this process, experimental methods not only demand higher levels of skills right from fabrication to testing of the specimen but also are time taking. Alternative empirical, semi-empirical, approximate and exact analytical models that are available in micromechanical analysis to determine the mechanical properties of composites are based on certain assumptions, for mathematical simplification, offer

satisfactory results in some cases but do not cover the entire spectrum of material compositions. On the other hand, finite element method (FEM) that can cater to varying requirements in analyses is a convenient tool for providing quicker and economical solutions but needs validation by other means. The present study is a step made towards benchmarking FE, analytical and experimental analyses in case of transverse Young's modulus.

A large number of analytical models with varying

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degrees of accuracy are available for predicting the mechanical properties of unidirectional composites. They range from the simple rule of mixtures (ROM) to methods based upon the use of elastic energy principles. In general, they incorporate certain simplifications of the physical state of materials that resulted in theories which do not satisfactorily correlate with the experimental data. Unlike ROM that works perfectly for predicting longitudinal Young's modulus, the inverse ROM (IROM) fails to give satisfactory results for transverse Young's modulus (referred as transverse modulus in this paper) in all cases. This may be one of the reasons why researchers worked on developing several models for predicting the transverse modulus. Modified inverse IROM (MIROM) has taken into account the lateral contraction of matrix material under tension due to Poisson's effect and accommodated it accordingly (Isaac and Ori, 1994; Robert, 1999; Autar, 1997). It is demonstrated that a combination of ROM and IROM can be adapted to suit theoretical modeling of a composite material by considering a combination of parallel and series orientation of rectangular elements of fibers scattered over entire area of representative volume element (RVE) and proposed two models: a horizontal and a vertical models to predict transverse modulus (Jacquet et al., 2000). Halpin-Tsai and Kardos (1976) have developed a semi-empirical equation to determine the transverse modulus by taking the shape of the fiber cross section into consideration as reinforcing efficiency factor. Neilson (1970) modified the Halpin-Tsai equation by introducing a packing factor ( $\phi_{\max}$ ) for square, hexagonal and random fiber packing arrays. Hirsch (1962) model is a combination of both ROM and IROM. When the value of  $x = 0$ , the relation reduces to IROM and when  $x = 1$ , it reduces to ROM. The value of  $x$  depends on the fiber orientation with respect to the direction of loading. Kalaprasad et al. (1997) mentioned Neilson and Hirsch models in their paper and compared the available experimental data with various analytical models for short sisal-LDPE composites. Alfredo (2000) derived a closed form of expression based on simple mechanics of a repeating square cell for predicting transverse modulus. Hui-Zu and Tsu-Wei (1995) have used elasticity theory and derived expressions for exact transverse modulus of a square RVE. Also, they have extended the theory to find a solution for the case of fiber matrix debonding by using an elastic contact model. Mistou et al. (2000) made a comparative study of elastic properties of composite materials by quasi-static as well as ultrasonic methods. It is observed that the ultrasonic method of testing is efficient, accurate and easy to conduct in comparison with tests on UTM. Stagni (2001) derived a formula for evaluating the effective transverse modulus of multilayered hollow fiber composites. The author observed that under certain conditions, increase in porosity results in increased transverse modulus. Muhannad et al. (2011) compared experimental results of

longitudinal and transverse moduli with the values from four micromechanical models of a unidirectional fiber-reinforced polymer (FRP) composite (E-Glass/Epoxy) of 37% volume fraction. However, none of the analytical results matched mutually and also with the experimental results. The reason behind analytical models not concurring with each other are obvious but the experimental results not agreeing with any one of the analytical results could be due to random arrangement of fibers. Li and Wisnom (1994) reviewed typical finite element formulations and models for unidirectional composite materials and shown that finite element analysis (FEA) provides more accurate and detailed characterization of composite properties for complicated geometries and constituent property variations. Theocaris et al. (1997) proposed a simple numerical homogenization method to predict effective transverse elastic modulus of fiber reinforced composites and the results are compared with existing analytical models. The authors observed that the results of homogenization are close to the results of mesophase concept and have only limited correlation with Hashin-Rosen model. Haktan and Dilek (2007) made a study of effective thermal expansion coefficients of composite materials by micromechanical FE modeling in ANSYS. The results are compared with other analytical and experimental data. While the results of all models are in close agreement with each other in case of  $\alpha_1$ , they differed well in case of  $\alpha_2$  except that FEA results matched with some of the experimental results. The mismatch could be due to randomness of fibers in matrix. Bhaskar and Mohammed (2012) used FEA to study transverse modulus along with other mechanical properties of fiber reinforced composites. The numerical results are compared with analytical solutions of IROM and Halpin-Tsai and their FEA results are not in close agreement with Halpin-Tsai's for all volume fractions at  $E_f/E_m$  around 4. However, the authors have not validated the results by any other means.

From this literature survey, it is apparent that available analytical models and FEA results are matching only in few cases and no evidence is available regarding validation of FEA results with exact analytical results. Also, the information about the basis for comparing analytical and FE models supported by subsequent validation is not available in the literature so far referred. The investigation regarding fiber arrangement in the test specimen as per RVE modeled in FEA seems to have been unexplored. In the present work, test specimens are specifically designed and fabricated to match the RVE modeled in FEA and transverse moduli results from experimental and FEA are compared with exact and other analytical models for mutual validation.

## REVIEW OF ANALYTICAL MODELS

Of the available analytical models, popular models are

considered for comparison and presented here for convenience.

### The IROM

The classical ROM predicts the longitudinal Young's modulus ( $E_1$ ) of a composite material accurately but the IROM (Isaac and Ori, 1994; Robert, 1999; Autar, 1997) fails to predict transverse modulus ( $E_2$ ) in general and particularly at higher fiber volume fractions. The IROM fails in the case of voids as well. Equation (1) works perfectly for those slab models (with negligible Poisson's effect) that are placed in series which is not the case with fiber reinforced composites in reality and hence the inevitable failure.

$$E_2 = \frac{E_f \cdot E_m}{E_f \cdot V_m + E_m \cdot V_f} \quad (1)$$

### Modified IROM (MIROM)

Modifications to IROM are suggested by various researchers based on specific assumptions. The MIROM as suggested by Ekvall (Isaac and Ori, 1994; Robert, 1999; Autar, 1997) and given as Equation (2) considers the Poisson's effect of matrix and the relation does not attempt to take care of the actual geometry of the composite. Even then the equation fails in fiber dominated and fiber like void cases.

$$E_2 = \frac{E_f \cdot E_m}{V_f \cdot E_m + V_m \cdot E_f (1 - \nu_m^2)} \quad \text{where} \quad E_m = \frac{E_m}{(1 - 2\nu_m^2)} \quad (2)$$

### Jacquet's horizontal and vertical models (JA-H and JA-V)

Jacquet et al. (2000) made an attempt to assess the transverse modulus of a unidirectional composite by using two novel models (horizontal and vertical) based on classical ROM. The horizontal (JA-H) and vertical (JA-V) models are given as Equations (3a) and (3b).

$$\frac{1}{E_2} = \frac{\sqrt{V_f}}{E_f \sqrt{V_f} + E_m (1 - \sqrt{V_f})} + \frac{1 - \sqrt{V_f}}{E_m} \quad (3a)$$

$$E_2 = \frac{E_f \cdot E_m}{E_m + E_f (1 - \sqrt{V_f}) / \sqrt{V_f}} + (1 - \sqrt{V_f}) E_m \quad (3b)$$

Though the treatment is simple, the assumption such as decomposing of fiber of any shape into small rectangular elements that are scattered in matrix in a regular array is unrealistic and correct results can never be expected from such models.

### Halpin-Tsai model (H-TSAI)

The semi-empirical relation for transverse modulus suggested by Halpin –Tsai (1976) is given as Equation (4)

$$E_2 = E_m \frac{(1 + \xi \cdot \eta \cdot V_f)}{(1 - \eta \cdot V_f)} \quad \text{where} \quad \eta = \frac{E_f - E_m}{E_f + \xi \cdot E_m} \quad (4)$$

and  $\xi$  is reinforcing efficiency factor for transverse loading that depends on the fiber cross section and the kind of packing geometry. The value of  $\xi$  is suggested as lying between 1 and 2 by several authors (Muhannad et al., 2011; Li and Wisnom, 1994; Theocaris et al., 1997) for prediction of  $E_T$ . The selection of  $\xi$  value on empirical basis limits the usage of this equation for a generalized case.

### Modified Halpin-Tsai model (MH-TSAI)

Neilson (1970) modified the Halpin-Tsai equation by including the maximum packaging fraction  $\phi_{\max}$  of the reinforcement and the equation transformed to

$$E_2 = E_m \frac{(1 + \xi \cdot \eta \cdot V_f)}{(1 - \eta \cdot \phi_{\max} \cdot V_f)} \quad (5)$$

where  $\phi_{\max}$  is packing factor and is given as 0.785 for square array, 0.907 for hexagonal array and 0.82 for randomly oriented. Though this relation is clear about fiber packing factor, the empirical value of  $\xi$  still limits its application for a generalized case.

### Hirsch model

Hirsch (1962) model is a combination of parallel and series models and the transverse modulus is calculated according to Equation (6).

$$E_2 = x (E_m V_m + E_f V_f) + (1-x) \frac{E_f E_m}{E_m V_f + E_f V_m} \quad (6)$$

As can be seen from the structure of the equation, this model is a combination of ROM and IROM, and the value of  $x$  depends on the fiber orientation with respect to loading direction. For transverse modulus where the angle between fiber and loading directions is  $90^\circ$ ,  $x$  becomes zero and hence this model reduces to IROM.

### Morais model

Morais (Alfredo, 2000) derived a closed-form micromechanical expression for predicting the transverse modulus of a square RVE. He claims that his results

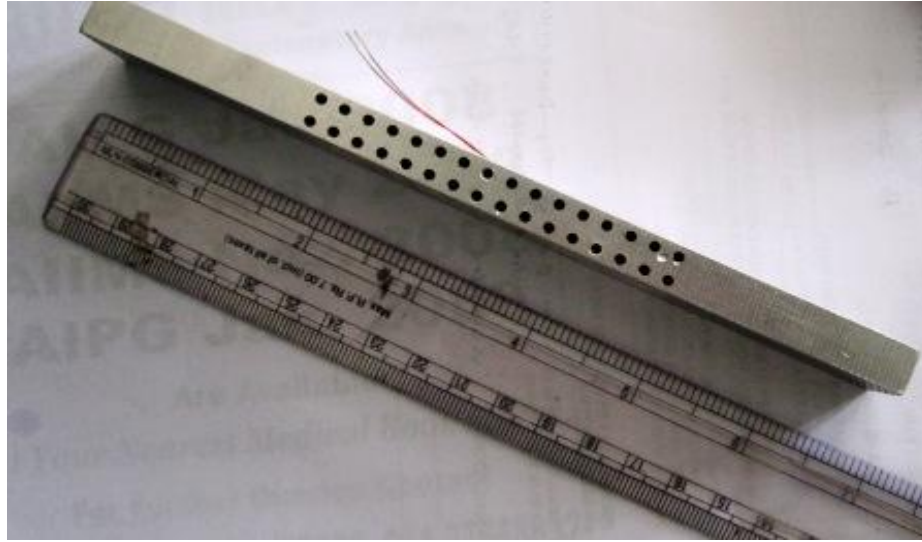


Figure 1. Specimen with holes drilled across thickness.

match with 3-D FE result of a hexagonal unit cell. It is observed that Morais' expression is a modification of Jacquet's vertical model by introducing Poisson's effect of matrix into the relation.

$$E_2 = \frac{\sqrt{V_f}}{\frac{\sqrt{V_f}}{E_f} + \frac{(1-\sqrt{V_f})(1-\nu_m^2)}{E_m}} + (1-\sqrt{V_f}) \frac{E_m}{(1-\nu_m^2)} \quad (7)$$

Hui-Zu and Tsu-Wei (1995) developed an elastic constant model to predict transverse modulus, Poisson's ratio and shear modulus of unidirectional fiber composites with interfacial debond. The elastic deformation formula of the fiber under contact pressure is derived by using elasticity theory. Results are presented for two limiting cases of perfect bonding and fiber like void.

In exact analytical approach, the whole body is considered as one entity and the equilibrium equations of one infinitely small element within the body are integrated to the boundaries to get an exact solution. However, in cases where integration is not possible due to mathematical limitations such as physical discreteness, the solution is not possible always. Apart from the geometric limitations of the material body, there are other issues such as material discontinuity, complex material combinations and loading pattern etc., which are difficult to idealize and the accuracy of the solution ultimately depends on the order of the equation making the analytical methods more and more case sensitive.

## EXPERIMENTAL INVESTIGATION

### Preparation of test specimen

Analytical methods used for determining mechanical properties of composite materials require a reference for their validity and

obviously experimental results provide the answer. Test specimens prepared invariably differ from theoretical models in many ways and there is bound to be disagreement between experimental results and analytical outcome. Fabrication of composites with conventional fibers and matrices close to the mathematical model is relatively difficult due to minute fiber diameter and since it is only for validation of methodology, metals are chosen as constituent materials for the present study. The isotropic nature of metals and the ease with which a given geometrical accuracy can be achieved on metals are reasons for choosing different metals to prepare a metal composite for the present study. The aim is to make a test specimen close to the mathematical model with a purpose to establish a verifiable relation between theory and practice. Aluminum, copper and mild steel are chosen for preparing the composite specimens with aluminum as matrix and the rest as fibers.

Three categories of composite specimens are prepared viz., copper-aluminum, mild steel-aluminum, and voids-aluminum (fiber like voids). Aluminum flats of 175\*25\*10 mm dimensions are taken as specimen blanks. While maintaining the length and width of the specimen as per ASTM D 3039/D3039M-08 (2008), the thickness of the specimen is taken as per the machining requirements. 2 mm diameter holes are drilled across 10 mm thick faces (along 25 mm width) as shown in Figure 1. Drilling is done on an NC machine taking sufficient care to maintain spacing between the holes. Each sample accommodated 32 holes (16 in each row) and the spacing of holes is according to the machining limitations. The fiber volume fraction achieved by this arrangement is 12.566%. An initial attempt to drill 1 mm diameter holes to achieve higher fiber volume fraction resulted in breaking of too many drill bits even on numerically controlled machine and hence the decision to go for higher diameter. Before going for 2 mm diameter holes, it is ensured that 2 mm diameter copper and mild steel wires are available commercially.

Twenty-five millimeter long pieces are cut from copper and mild steel wire rolls in sufficient numbers. Wires are driven into the holes by gentle tapping with a nylon mallet. Moderate force was needed to drive each fiber piece into a hole that is an indication of generous contact between the male and female surfaces. This ensured sufficient gripping due to the interference fit of the assembly between fibers and matrix without any mechanical bonding. For each category of the composite three specimens are prepared bringing the total number of specimens to nine. These specimens



Figure 2. Test set-up on UTM.

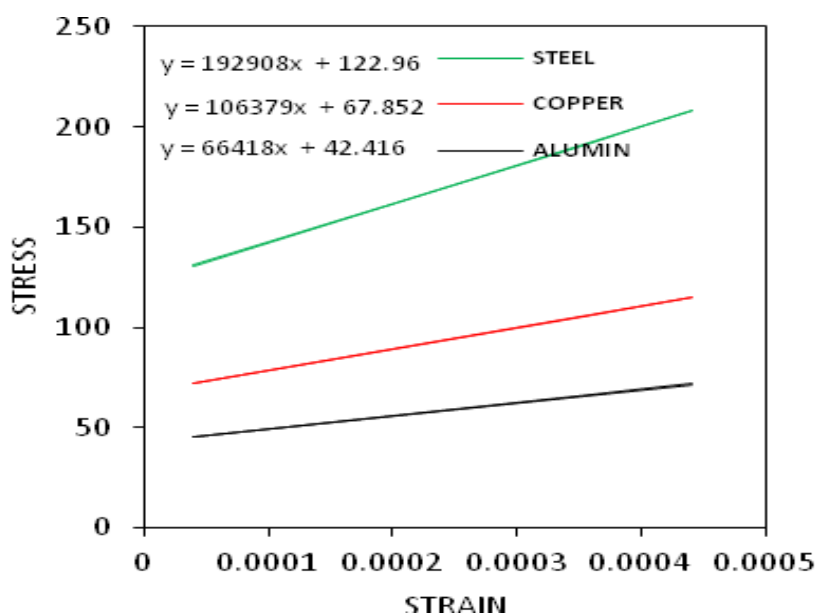


Figure 3. Stress-strain plot for one set of constituent materials.

are tension tested on a micro-computer controlled electronic UTM of 400 kN capacity as shown in Figure 2 at a cross head speed of 1 mm/min. An electronic extensometer of 1  $\mu$ m least count is used to measure the extension and the test data is recorded automatically.

#### Determination of Young's modulus of constituent materials

Though, Young's modulus of constituent materials could be taken from the standard data, it is opined that the values once again be

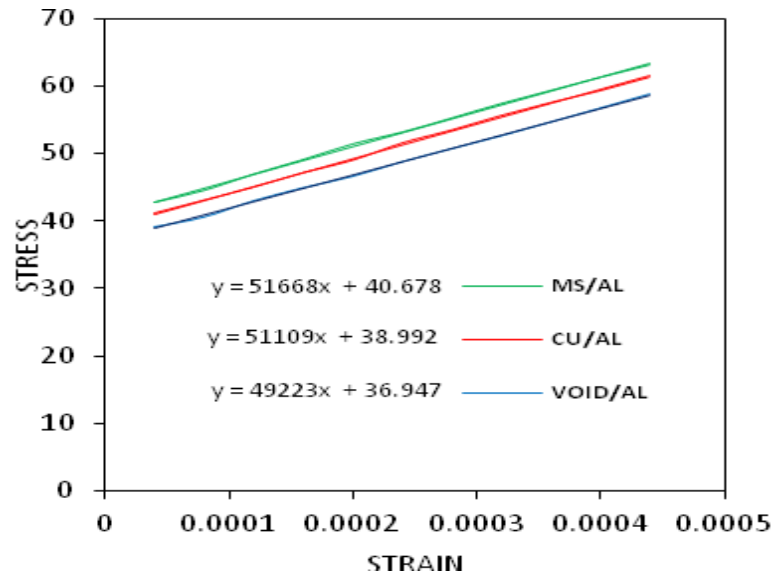
experimentally determined for closeness and subsequent substitution in analytical and FE models since the aim of the present study is validation. 175\*25\*10 mm aluminum blanks are tested on UTM and for copper and steel wires, three point bending tests are conducted on a 2-ton electronic tensometer (model: METM 2000 ER-I). For each material, three samples are tested and the value of E is calculated from the slope of linear portion of stress-strain plot obtained. The test data is tabulated in Table 1. Figure 3 shows one set of stress-strain plots for three types of specimens with corresponding trend line equations indicated along.

**Table 1.** Results of constituent materials tested for longitudinal Young's modulus.

Specimen description	TEST 1 $E_1$ (GPa)	TEST 2 $E_1$ (GPa)	TEST 3 $E_1$ (GPa)	Average value
Aluminum blank	67.501	65.521	66.418	66.48
Copper wire	106.379	105.199	107.624	106.40
Mild steel wire	195.966	191.416	192.908	193.43

**Table 2.** Results of composite specimens tested for transverse modulus.

Composite material description	Test 1 GPa	Test 2 GPa	Test 3 GPa	Average $E_2$
Mild steel-aluminum	51.668	51.364	51.429	51.487
Copper-aluminum	51.036	51.109	51.261	51.135
Void like fiber-aluminum	49.469	49.223	49.545	49.412

**Figure 4.** Stress-strain plots for one set of composite material.

#### Determination of transverse modulus of composite specimens

Three samples for each category of composites are tested and the values of transverse moduli ( $E_2$ ) are calculated from the slope of the linear portion of stress-strain plots obtained from the recorded data. The test results are tabulated in Table 2. Figure 4 shows one such set of plots with corresponding trend line equations.

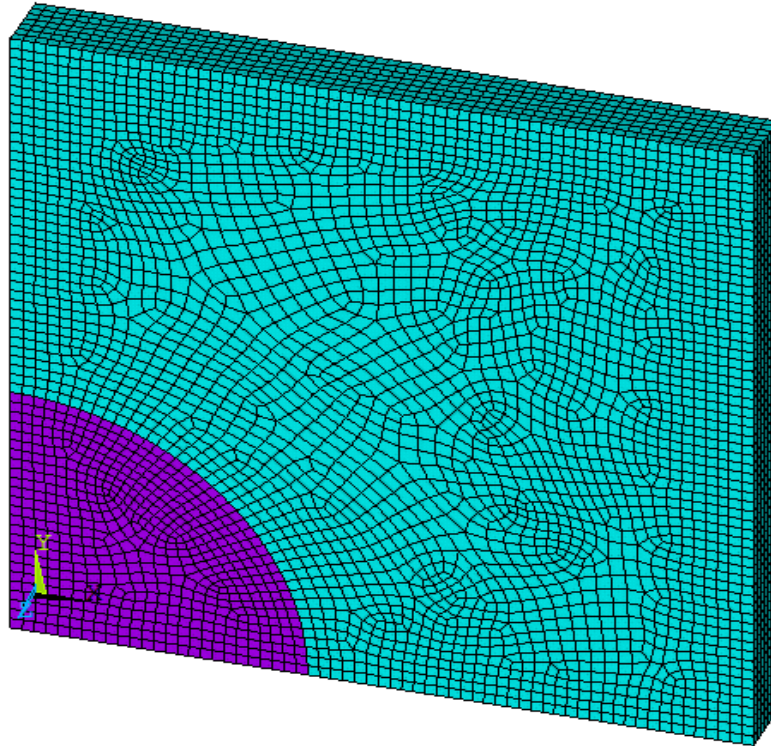
#### Numerical approach

In numerical approach, though approximate, the theme of treatment is same at local as well as global levels. Decomposing any shape and effectively encompassing the complete material geometry through finite number of fundamental elements is the basic principle behind the numerical method. Formation of local governing equations in terms of geometry, material property and loading pattern while simultaneously maintaining local-global connectivity, numerical approach will always come up with a solution. However, validation of this solution requires bench marking that forms the

basis of the present work. Numerical errors are reduced by proper choice of the element and its size and the accuracy of the outcome can be checked by testing for convergence.

An RVE in the form of a square unit cell in cross-section is adapted for analysis and a one eighth unit cell (one-fourth in cross-section and half in longitudinal direction) is modeled by taking the advantage of symmetry. The geometry of FE model and the composite's constituent properties are so selected to cover sufficiently a large range of fiber volume fractions ( $V_f = 0.1$  to  $0.72$ ) and material combinations ( $E_f/E_m = 100:1$  to  $1:100$ ) in order to compare FE results with available analytical results. The dimensions of the cell are  $250 \times 250 \times 10$  mm and fiber radius is calculated as per the required volume fraction using the relation  $r = \sqrt{4 \cdot V_f \cdot 250 \cdot 250 / \pi}$ . For convenience of analysis, scaling up of cell size is done without loss of proportionality. Processing of the required steps in FEM such as generation of element matrices, assembling of system equations and solving them under prescribed constraints for nodal deformations of any structural problem is a built in feature of ANSYS software, provided the problem is clearly





**Figure 5.** FE model showing mesh with element edge length 4.

defined and discretization is made to the satisfaction of results, which is a proven tool universally used by many researchers for the analysis of composite materials. Hence, the present problem is modeled in ANSYS straight away without resorting to any other programming routines. SOLID 20 NODE 95 element of ANSYS is used to create FE mesh which is a quadratic brick element that is best suited for curved boundaries. Mesh refinements are made with different element edge lengths and convergence is verified at maximum mismatch conditions ( $E_f/E_m = 100:1$  and  $V_f = 0.72$ ). It is observed that at element edge lengths 4 and below the results have converged for this model. FE model with an element edge length 4 is shown in Figure 5.

Symmetric boundary conditions are applied on negative faces of the Cartesian coordinate system which can be observed in Figure 5. Multipoint constraints are imposed on the boundary planes  $x$ ,  $y$  and  $z$  to ensure uniform strain in respective directions. A uniform tensile load of 1 MPa is applied on the  $x$ -face to observe a uni-axial state of stress that facilitates usage of simple Hooke's law for calculating Young's modulus, while the fibers are parallel to  $z$ -axis. A similar model as above, with appropriate changes for fiber-matrix debond case, is developed. Boundary conditions and loading are kept without any change. This model is necessitated as the fibers in the specimen are not bonded to the matrix by any means.

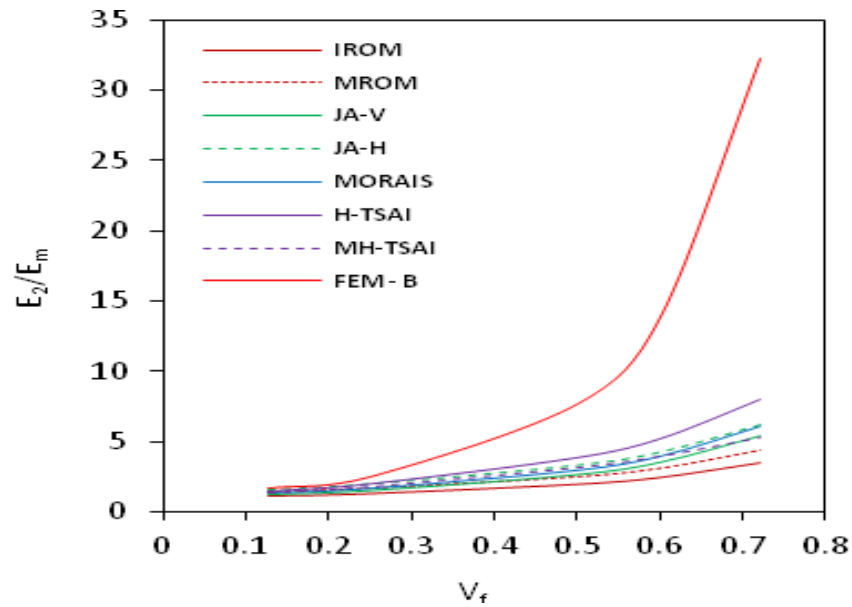
## RESULTS AND DISCUSSION

### Comparison of analytical and FE models in fiber dominated cases

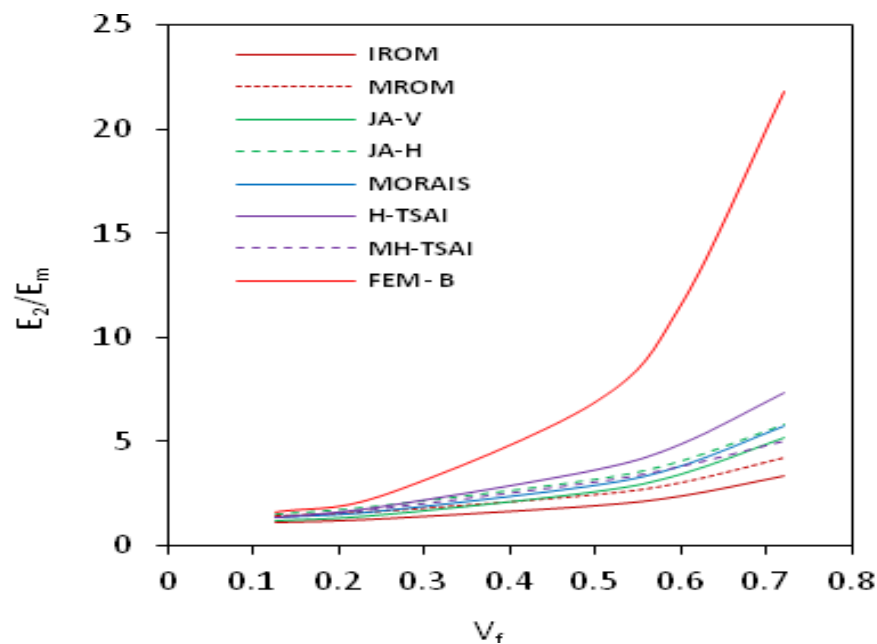
Variation of normalized transverse modulus ( $E_2/E_m$ ) with respect to  $V_f$  for  $E_f$  to  $E_m$  ratios of 100:1, 50:1, 21.19:1

and 5.5:1 (Hui and Tsu-Wei, 1995) are shown in Figures 6 to 9. It is observed that, in Figures 6 and 7, the analytical models predict lower values of  $E_2$  with increasing  $V_f$  in comparison to FE model. IROM model predicts the least value for  $E_2$ , while FE model predicts the highest and the rest of the models are positioned in between. It is also observed that the differences in values of  $E_2$  between each of these models and FE model is progressively increasing with increasing  $V_f$ . As mentioned earlier, IROM works with assumption of slab models where fiber and matrix are assumed to be placed in series, which is why this model predicts least values and obviously fails to give accurate results. Though Poisson's effect of matrix is considered in MIROM, the basic assumption of IROM remains within and hence the results are marginally improved in this case. JA-H and JA-V models (Jacquet et al., 2000) decompose fibers of any shape into minute square or rectangular blocks to overcome the deficiencies of IROM that resulted in marginal improvement. However, these results are still below FEM values.

Halpin-Tsai model (Halpin-Tsai and Kardos, 1976) has an empirical term  $\xi$  introduced into the relation whose value depends on the fibers' cross sectional shape, fiber volume fraction and fiber packing. It is suggested in the literature that the value of  $\xi$  be found from experiments for a given set of known constituents of a composite which varies from 1 to 2. The values of  $E_2$  for



**Figure 6.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 100:1$ ).

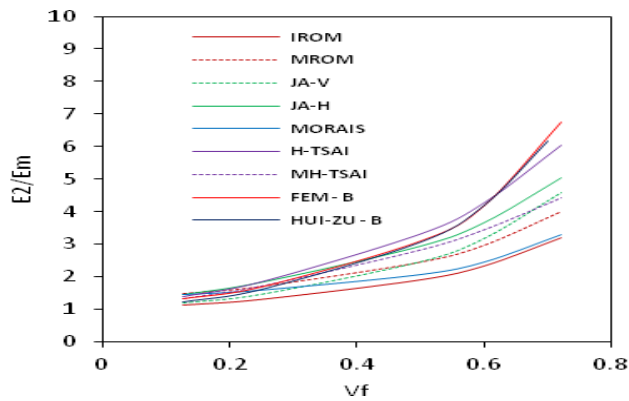


**Figure 7.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 50:1$ ).

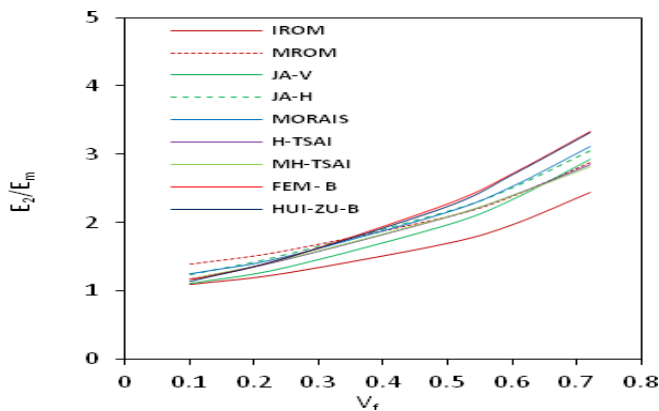
Halpin-Tsai model in the present study are calculated by taking  $\xi=2$  as suggested in the literature for circular fibers. It is observed that Halpin-Tsai predicts higher values than all other analytical models considered so far here. Since the value of  $\xi$  provided here is not for a particular case, these values cannot be treated as exact for any general case. MH-Tsai (Neilson, 1970) relation takes care of the fiber packing due to the term  $\phi$  but the

$V_f$  still has no role in it and hence the values of  $E_2$  obtained cannot be accurate at all volume fractions. Morais' model (Alfredo, 2000) which appears to be in line with JA-V equation by considering the Poisson's effect of matrix has shown further improvement, but the assumptions regarding decomposition of fibers that form the basis of JA-V makes this model still unrealistic. It is also observed that the magnitude of  $E_2$  has not been





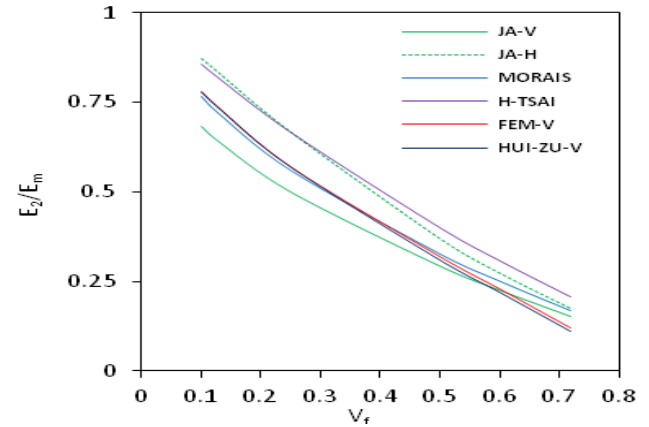
**Figure 8.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 21.19:1$ , bonded).



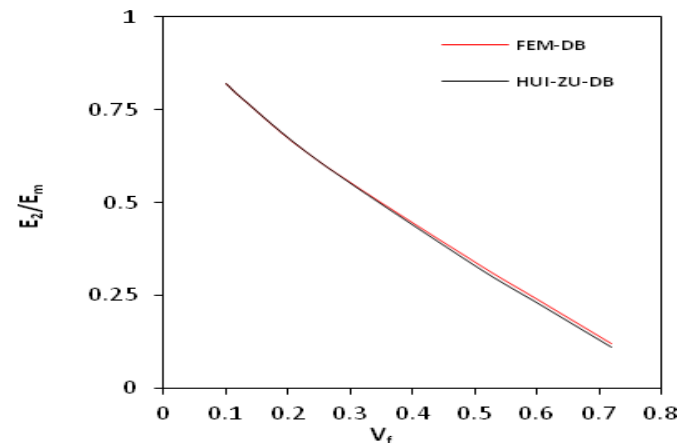
**Figure 9.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 5.5:1$ , bonded).

affected by the variation of  $E_f/E_m$  from 100 to 50 in analytical models as much as it has been in FE model. Rather, it could be stated that FE model responds more sensitively to variation of  $E_f/E_m$  compared to all analytical models referred here in case of transverse modulus.

The mutual disagreement of analytical models with FE one necessitates verification of the genuinity with an exact analytical model or experimental result. Hui-Zu and Tsu-Wei (1995) have evolved an exact elasticity model for two composites and determined  $E_2$  for a range of volume fractions from 10 to 70%. The composite combinations are glass/epoxy with  $E_f/E_m = 73.1/3.45$  (21.19:1) and alumina/aluminum with  $E_f/E_m = 379/68.9$  (5.5:1). Accordingly,  $E_2$  has been determined, for the same combinations of  $E_f/E_m$  used by Hui-Zu, with FE and other analytical models. Figures 8 and 9 show the comparison of all the available models in these two cases. Interestingly, for all values of  $V_f$ , Hui-Zu and FE models are in very close agreement for both the composites, whereas other analytical models are not so in general and at higher volume fractions in particular. It can be inferred that FE model is a reliable model for a perfectly bonded case.



**Figure 10.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 5.5:1$ , fiber like void).

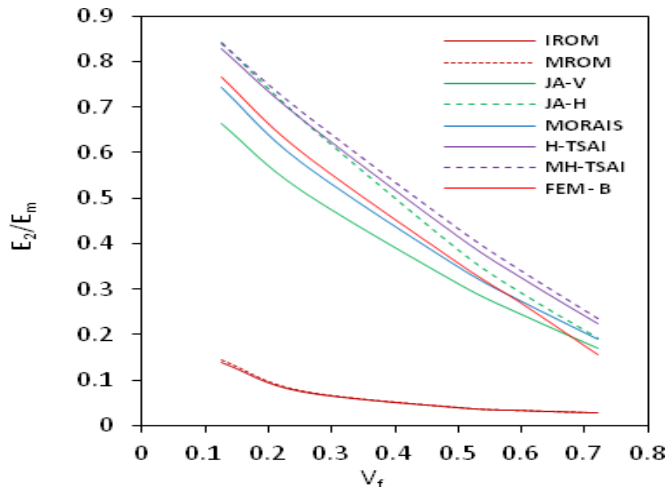


**Figure 11.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 5.5:1$ , total debond).

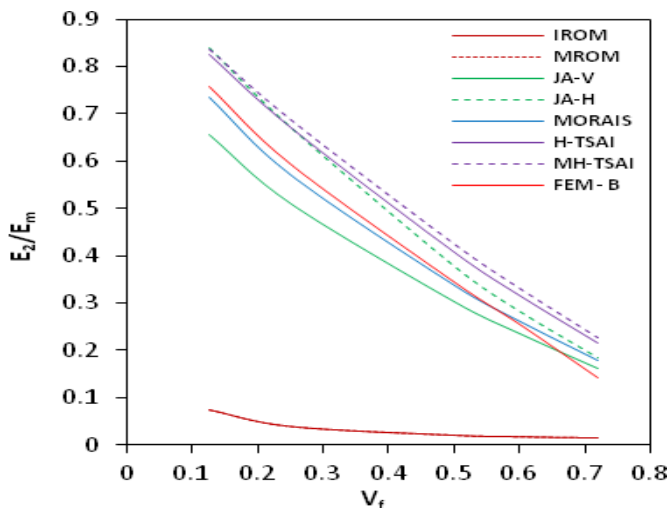
Further investigation into the cases of fiber like void and fiber-matrix debond is done and the results are compared. Figures 10 and 11 show similar comparisons made for fiber like void and fiber-matrix debond cases, respectively. Those analytical models which ever can yield results in case of fiber like void are compared in Figure 10. Since the other analytical models excepting Hui-Zu's have no provision for total debond, they do not appear in Figure 11. IROM and MIROM have no provision to deal with these two cases and hence do not appear for comparison in both these cases. It is observed that, even in cases of fiber like void and fiber-matrix debond, there is very close agreement between FE and Hui-Zu models that further confirms FEM's reliability.

### Comparison of analytical and FE models in matrix dominated cases

Very close agreement between FE and Hui-Zu results in

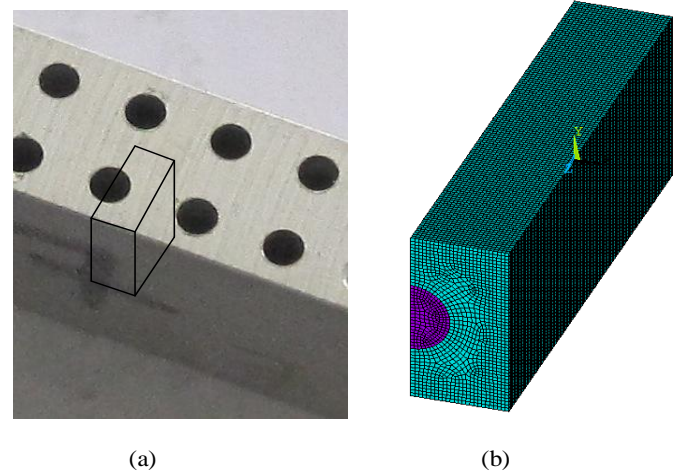


**Figure 12.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 1: 50$ ).



**Figure 13.** Variation of normalized  $E_2$  with  $V_f$  (for  $E_f/E_m = 1: 100$ ).

all cases viz., perfectly bonded, fiber like void and total debond, confirms the consistency and dependability of these two models. This outcome leads the discussion towards the necessity of comparing FE model with experimental results for further confirmation of its validity. Analytical models' disagreement with FE and Hui-Zu models in cases of  $E_f/E_m > 1$  prompted the authors to study their behavior in matrix dominated cases ( $E_f/E_m < 1$ ) as a matter of academic interest. Figures 12 and 13 show the variation of normalized  $E_2$  with  $V_f$  for cases where  $E_f/E_m < 1$ . It can be observed that in matrix dominated cases also IROM and MIROM fail to predict values anywhere near other analytical models' which is their built-in deficiency. Other analytical models are predicting values either above or below but not close to FE model that further raises the ambiguity regarding their consistency.



**Figure 14.** Modified RVE (a) Highlighted on the specimen; (b) Modified FE model.

### Verification of FE with experimental results in case of fiber-matrix de-bond

Though square RVE is acceptable for the present FE analyses, further discussions have led to a situation that prompted the authors to go for suitable modifications of RVE. Thus, FE model of a regular square RVE is modified to suit the specimen conditions such as finite dimensions across the thickness and width and for possible fiber-matrix interface debond. Figures 14(a) and (b) show the modified FE model used for final analysis. Table 3 shows the transverse moduli of the subject composites ( $V_f = 0.12566$ ) determined from experimental and FE studies. Since the fibers are only inserted into the matrix and not glued, the composites in the present experiment fall into the category of debonded fiber-matrix. Hence, the experimental results are compared with debond case of FE results. The error in case of fiber like void is within the acceptable limits. However, the errors in the cases of other two composites (MS/AL and CU/AL) are slightly higher but not too objectionable. This could be attributed to the type of fit effected between the fiber and matrix surfaces during assembly of fibers in matrix.

Relative closeness of experimental and FE results confirms the dependability of FEM in predicting transverse modulus of fiber reinforced composites. Also this study enables the authors to state that FEM can be extended in micromechanical analysis of composites to study those cases where conventional analytical models are unable to address. Summing up, it can be said that the study not only verifies FEM with experimental results for validation and vice versa as well.

### Conclusions

A test specimen that can be exactly modeled in analytical

**Table 3.** Experimental and FEA results of transverse modulus (GPa) of the subject composites.

Composite material	Experimental results (average)	FEA modified debond case	% error
Mild steel-Aluminum	51.487	49.270	4.30
Copper-Aluminum	51.135	49.217	3.75
Fiber like void-Aluminum	49.412	48.437	2.01

form is designed, fabricated and tested for transverse Young's modulus for three different material combinations viz., mild steel-aluminum, copper-aluminum and fiber like void-aluminum. Identical conditions are simulated using application 3-D FEM in ANSYS software. FE results are found to be in close agreement with exact analytical results available in the literature and also in good agreement with experimental results. Hence, the aim of exploring the capability of FEM for micromechanical analysis of fiber reinforced composites is accomplished.

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## REFERENCES

- Alfredo BM (2000). Transverse moduli of continuous fiber reinforced polymers, *Compos. Sci. Technol.* 60:997-1002.
- Autar KK (1997). *Mechanics of Composite Materials*, CRC Press.
- Bhaskar P, Mohammed RH (2012). Analytical Estimation of elastic properties of polypropylene fiber matrix composite by finite element analysis, *Scientific Research. Adv. Mater. Phys. Chem.* 2:23-30.
- Haktan KZ, Dilek K (2007). A numerical study on the coefficients of thermal expansion of fiber reinforced composite materials. *Sci. Direct Compos. Struct.* 78:1-10.
- Halpin-Tsai JC, Kardos JL (1976). The Halpin-Tsai equations: A review. *Polymer Eng. Sci.* 16(5).
- Hirsch TJ (1962). Modulus of elasticity of concrete affected by elastic moduli of cement paste matrix and aggregate. *J. Am. Con. Inst.* 59:427.
- Hui-Zu S, Tsu-Wei C (1995). Transverse elastic moduli of unidirectional fiber composite with fiber/matrix interfacial debonding. *Compos. Sci. Technol.* 53:383-391.
- Isaac MD, Ori I (1994). *Engineering Mechanics of Composite Materials*. Oxford University Press.
- Jacquet E, Trivaudey F, Varchon D (2000). Calculation of the transverse modulus of a unidirectional composite material and of the modulus of an aggregate: Application of rule of mixtures. *Compos. Sci. Technol.* 60:345-350.
- Kalaprasad G, Joseph K, Thomas S, Pavithran C (1997). Theoretical modeling of tensile properties of short sisal fiber-reinforced low-density polyethylene composites. *J. Mater. Sci.* 32:4261-4267.
- Li DS, Wisnom MR (1994). Finite element micromechanical modeling of unidirectional fiber reinforced metal-matrix composites. *Compos. Sci. Technol.* 51(4):545-563.
- Mistou S, Karama M, Morgado E, Diez J, Schwarz R, Macarico A, Koynov S (2000). Comparative study on the determination of elastic properties of composite materials by tensile tests and ultra sound measurement. *J. Compos. Mater.* 34(20):1696-1709.
- Muhannad ZK, Mustafa SA, Hyder MA (2011). Mechanical properties comparison of four models, failure theories study and estimation of thermal expansion coefficients for artificial E-glass polyester composite. *Eng. Tech. J.* 29(2):278-292.
- Neilson LE (1970). Generalized Equation for the Elastic Moduli of Composite Materials. *J. Appl. Phys.* 4:4626.
- Robert MJ (1999). *Mechanics of Composite Materials*, Taylor and Francis.
- Stagni L (2001). Effective transverse elastic moduli of a composite reinforced with multilayered hollow cored fibers. *Compos. Sci. Technol.* 61:1729-1734.
- ASTM International (2008). Standard test method for tensile properties of polymer matrix composites materials, D3039/D3039M-08.
- Theocaris PS, Stavroulakis GE, Panagiotopoulos PD (1997). Calculation of effective transverse elastic moduli of fiber-reinforced composites by numerical homogenization. *Compos. Sci. Technol.* 57(5):573-586.