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Review

Distortion modeling of feedback single stage amplifier

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In this paper, the distortion of feedback single stage amplifier is modeled. The phasor method was exploited to obtain the distortion factor expressions of the feedback amplifier. All the nonlinearities, including intermodulation nonlinearity, are considered in the paper. The derivation is simple and easy to understand. These equations provide deep insights into the distortion modeling of the feedback circuit. The theoretical analyses are in accordance with the transistor level circuit simulation.

Key words: Analog integrated circuit, harmonic distortion, nonlinear distortion, intermodulation, feedback amplified.

INTRODUCTION

Along with the steps of the integrated circuits, the performance like linearity is playing a more and more important role in analog integrated circuits design. However, the traditional circuit design methods are not capable of modeling the nonlinearity behaviors such as distortion. Therefore, the distortion modeling of the analog integrated circuits is a real hot topic in today's circuit design world. Many literatures are related to distortion modeling of all kinds of analog circuits.

It is generally known that a few methods are suitable for modeling the distortion of memoryless nonlinearity of analog circuits. The best known one is the Volterra series method which is complex but accurate enough when the nonlinearity is weak. It is widely used to evaluate the distortion of the amplifier and other types of analog circuits (Narayanan, 1967, 1970; Khadr and Johnston, 1974; Wambacq and Sansen, 1998; Baki et al., 2006; Nichols, 2009). To reduce the complexity and give more insights to the designers, the phasor method and its variations are employed to deal with the distortion modeling of analog circuits (Wambacq et al., 1990, 1999; Hernes et al., 2003, 2005; Giustolisi et al., 2002; Palumbo and Pennisi, 2003, Palumbo et al., 2008; Cannizzaro et al., 2005, 2006). In addition, several approximate analytical methods (Ge and Chang, 2009; Chen and Hsieh, 2009; Renna and Marsili, 2008) offer the designers more choices.

As a main building block, the modeling of amplifier is one of the highlights of analog circuit's distortion. In the paper, the phasor method was exploited to investigate the distortion of feedback single stage amplifier. In general, all the nonlinear sources should be considered in the modeling. After obtaining the first-order transfer functions, the responses of the nonlinear sources can be regarded as polynomials of corresponding coefficients of these nonlinear sources and the first-order transfer functions. All the effects of nonlinear transconductance, conductance and intermodulation can be taken into account. Hence, the nonlinear coefficients of the amplifier can be calculated exactly with the help of the nonlinear small signal circuit. The rule of feedback can be found with the support of the phasor method again. Specifically, the nonlinear coefficients of the closed-loop amplifier were derived in the similar way as in (Pederson and Mayaram, 1991). By aides of these analyses, the closed form second- and third-order distortion factor expressions can be found conveniently. The accuracy of the modeling in the paper was verified by comparing the results of theoretical analyses with the transistor level circuit simulation. The equations obtained in the paper can be used to evaluate the distortion of single stage amplifier and avoid time-consuming transient simulation. In addition, these equations can be used to identify the dominant nonlinear sources.

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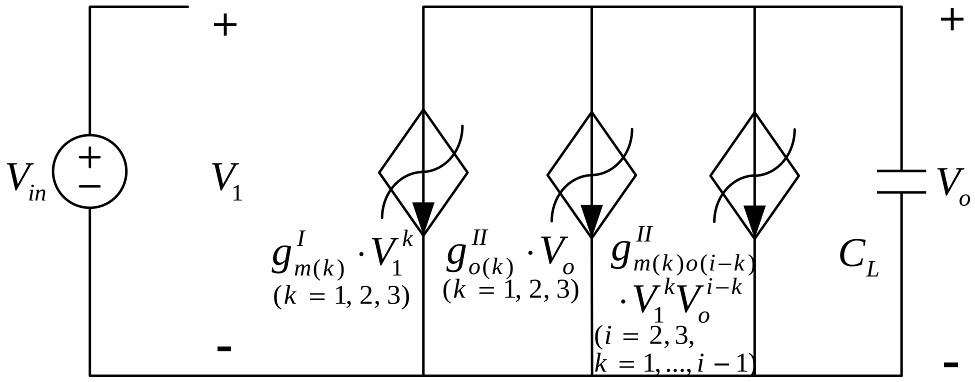

Figure 1. Nonlinear small signal circuit of single stage amplifier.

DISTORTION MODELING OF SINGLE-STAGE AMPLIFIER

Nonlinear coefficient of the open loop amplifier

To obtain the nonlinear coefficients of the open-loop amplifier, the small signal circuit of the amplifier as shown in Figure 1 was exploited. At first, the nonlinear sources in this figure were neglected to calculate the linear response of the amplifier. Then, the high-order coefficients or responses of the amplifier can be derived by means of the phasor method.

With the help of the nonlinear small signal circuit in Figure 1, the KCL equation of the output node was written as

$$\sum_{k=1}^{3} g_{m(k)} V_{in}^{k} + \sum_{k=1}^{3} g_{o(k)} V_{o}^{k} + \sum_{\substack{k=1 \\ i=2,3}}^{i-1} g_{m(k)o(i-k)} V_{in}^{k} V_{o}^{i-k} + j\omega C_L V_o = 0 \quad (1)$$

Providing that the input signal $V_{in} = x_s e^{j\omega t}$, the output signal as can be represented as $V_o = \sum_{k=1}^{3} h_k(j\omega) x_s^k e^{jk\omega t}$. The $h_k(j\omega)$,*k=1,2,3* are frequency dependent nonlinear coefficients to be evaluated. Substituting these signals into Equation (1) and equating the terms with same exponential factor, the equations for calculating these nonlinear coefficients are obtained. They are written as:

$$g_{m1} x_s e^{j\omega t} + g_{o1} h_1(j\omega) x_s e^{j\omega t} + j\omega C_L h_1(j\omega) x_s e^{j\omega t} = 0 \quad (2)$$

$$g_{m2} x_s^2 e^{j2\omega t} + g_{o1} h_2(j\omega) x_s^2 e^{j2\omega t} + g_{o2}[h_1(j\omega) x_s e^{j\omega t}]^2 + g_{m1o1} x_s e^{j\omega t} h_1(j\omega) x_s e^{j\omega t} + j2\omega C_L h_2(j\omega) x_s^2 e^{j2\omega t} = 0 \quad (3)$$

$$\begin{aligned} & g_{m3} x_s^3 e^{j3\omega t} + g_{o1} h_3(j\omega) x_s^3 e^{j3\omega t} + g_{o3}[h_1(j\omega) x_s e^{j\omega t}]^3 + g_{m1o1} x_s e^{j\omega t} h_2(j\omega) x_s e^{j2\omega t} + g_{m1o2} x_s e^{j\omega t}[h_1(j\omega) x_s e^{j\omega t}]^2 \\ & + g_{m2o1}[x_s e^{j\omega t}]^2 h_1(j\omega) x_s e^{j\omega t} + j3\omega C_L h_3(j\omega) x_s^3 e^{j3\omega t} = 0 \end{aligned} \quad (4)$$

With the aid of the Equations (2) to (4), the first-, second- and third-order nonlinear coefficients were derived and they are

$$h_1(j\omega) = -\frac{g_{m1}}{g_{o1} + j\omega C_L} \quad (5)$$

$$h_2(j\omega) = -\frac{g_{m2} + g_{o2}[h_1(j\omega)]^2 + g_{m1o1} h_1(j\omega)}{g_{o1} + j2\omega C_L} \quad (6)$$

$$h_3(j\omega) = -\frac{g_{m3} + g_{o2} 2 h_1(j\omega) h_2(j\omega) + g_{o3} h_1^3(j\omega) + g_{m1o1} h_2(j\omega) + g_{m1o2} h_1^2(j\omega) + g_{m2o1} h_1(j\omega)}{g_{o1} + j3\omega C_L} \quad (7)$$

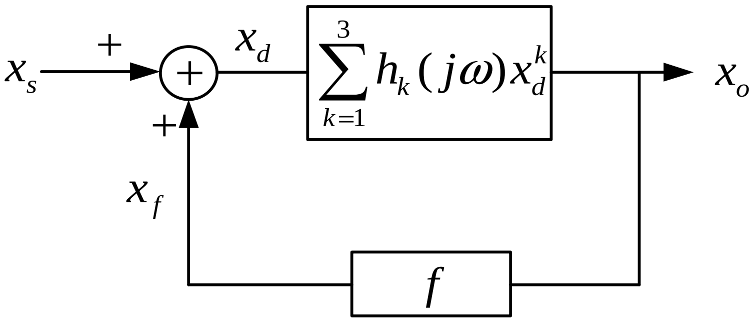

Figure 2. Block diagram of the closed-loop amplifier.

It is apparent that only simple manipulation is needed in the derivation of these open-loop nonlinear coefficients of the single stage amplifier. In fact, the high-order coefficients are only the polynomials of the first-order response. By this means, all the nonlinear sources, including intermodulation nonlinearity, are considered at the same time. The derivation guarantees the accuracy of these coefficients as much as possible. Therefore, these coefficients can be used directly with the rule of feedback obtained subsequently in the study to get the distortion factors of the feedback amplifier.

Nonlinear coefficient of the closed-loop amplifier

In order to evaluate the distortion of feedback amplifier, the rule of feedback nonlinear system is needed to be found. In other words, the nonlinear coefficients of the closed-loop amplifier should be calculated. Hence, the relationship between the nonlinear coefficients of the closed-loop system and the ones of the open loop amplifier has to be identified. The block diagram used to find these coefficients of feedback nonlinear system is shown in Figure 2.

The same as previously mentioned, the input signal and the output signal of the closed-loop amplifier were set as $V_{in} = x_s e^{j\omega t}$ and $V_o = \sum_{k=1}^{3} h_k^f(j\omega)x_s^k e^{jk\omega t}$, respectively. Then the error signal can be written as

$$V_d = [1 - fh_1^f(j\omega)]x_s e^{j\omega t} - f\sum_{k=2}^{3} h_k^f(j\omega)x_s^k e^{jk\omega t} \quad (8)$$

Observing Figure 2, the relationship between the input and output of the nonlinear amplifier can be represented as $x_o = h_1 x_d + h_2 x_d^2 + h_3 x_d^3$. Inserting Equation (8) into this expression, the equation describing the relationship of nonlinear coefficients of open-loop and closed-loop amplifier can be obtained. It is written as

$$\sum_{k=1}^{3} h_k^f(j\omega)x_s^k e^{j\omega t} = \sum_{i=1}^{3} h_i(j\omega)\{[1 - fh_1^f(j\omega)]x_s e^{j\omega t} - f\sum_{k=2}^{3} h_k^f(j\omega)x_s^k e^{jk\omega t}\}^i \quad (9)$$

Neglecting the second- and third-order terms, Equation (9) changes to

$$h_1^f(j\omega)x_s e^{j\omega t} = h_1(j\omega)[1 - fh_1^f(j\omega)]x_s e^{j\omega t} \quad (10)$$

From Equation (10), the linear response of the feedback amplifier can be written as

$$h_1^f(j\omega) = \frac{h_1(j\omega)}{1 + h_1(j\omega)f} \quad (11)$$

Equating the second-order terms in Equation (9), the equation changes

$$h_2^f(j\omega)x_s^2 e^{j2\omega t} = -h_1(j2\omega)fh_2^f(j\omega)x_s^2 e^{j2\omega t} + h_2(j\omega)\{[1 - fh_1^f(j\omega)]x_s e^{j\omega t}\}^2 \quad (12)$$

The second-order nonlinear coefficient of the closed-loop amplifier was derived and written as:

$$h_2^f(j\omega) = \frac{h_2(j\omega)}{[1 + h_1(j2\omega)f][1 + h_1(j\omega)f]^2} \quad (13)$$

Using the similar steps as aforementioned analyses, the third-order nonlinear coefficient was obtained. It is:

$$h_3^f(j\omega) = \frac{h_3(j\omega)}{[1 + h_1(j3\omega)f][1 + h_1(j\omega)f]^3}\{1 - \frac{2fh_2^2(j\omega)}{h_3(j\omega)[1 + h_1(j2\omega)f]}\} \quad (14)$$

After obtaining the nonlinear coefficients of open-loop amplifier previously, the coefficients of closed-loop amplifier were gotten immediately by using Equations (11) to (14).

Closed form distortion factor expressions of the feedback amplifier

The distortion factor is defined as the ratio of high-order frequency component to fundamental component. Therefore, with the aides of Equations (11) to (14), the second- and third-order distortion factors expressions of the feedback amplifier are:

$$
\begin{aligned}
HD_2^f &= \frac{x_s}{2}\left|\frac{h_2^f(j\omega)}{h_1^f(j\omega)}\right| \\
&= \frac{x_s}{2}\left|\frac{h_2(j\omega)}{h_1(j\omega)}\frac{1}{[1+h_1(j2\omega)f][1+h_1(j\omega)f]}\right|
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
HD_3^f &= \frac{x_s^2}{4}\left|\frac{h_3^f(j\omega)}{h_1^f(j\omega)}\right| \\
&= \frac{x_s^2}{4}\left|\frac{h_3(j\omega)}{h_1(j\omega)}\frac{1}{[1+h_1(j3\omega)f][1+h_1(j\omega)f]^2}\left\{1-\frac{2fh_2^2(j\omega)}{h_3(j\omega)[1+h_1(j2\omega)f]}\right\}\right|
\end{aligned} \tag{16}
$$

Using the aforementioned expressions with Equations (5) to (7) directly, the distortion of the feedback amplifier was evaluated. To give the designers more understanding of the distortion behaviors, Equations (15) to (16) were changed into expressions with poles and zeros. Using the definition $\omega_o = g_{o1}/C_L$ and $A_0 = -g_{m1}/g_{o1}$, the nonlinear coefficients of the open-loop amplifier can be represented as

$$
h_1(j\omega) = -\frac{g_{m1}}{g_{o1}+j\omega C_L} = \frac{A_0}{1+j\dfrac{\omega}{\omega_o}} \tag{17}
$$

$$
h_2(j\omega) = -\frac{1}{g_{o1}(1+j2\dfrac{\omega}{\omega_o})}\sum_{k=0}^{2}\frac{g_{m(2-k)o(k)}A_0^k}{(1+j\dfrac{\omega}{\omega_o})^k} \tag{18}
$$

$$
h_3(j\omega) = -\frac{1}{g_{o1}(1+j3\dfrac{\omega}{\omega_o})}\sum_{k=0}^{3}\frac{g_{m(3-k)o(k)}A_0^k}{(1+j\dfrac{\omega}{\omega_o})^k} + \delta(j\omega) \tag{19}
$$

where

$$
\delta(j\omega) = \frac{2g_{o2}A_0 + g_{m1o1}(1+j\dfrac{\omega}{\omega_o})}{g_{o1}^2\prod_{k=1}^{3}(1+jk\dfrac{\omega}{\omega_o})}\sum_{k=0}^{2}\frac{g_{m(2-k)o(k)}A_0^k}{(1+j\dfrac{\omega}{\omega_o})^k}
$$

Substituting the aforementioned nonlinear coefficients into Equations (15) and (16) yields the closed-form expressions of the second- and third-order distortion factors as

$$
HD_2^f = \frac{x_s}{2}\left|\frac{1}{T_0^2 g_{o1}}\sum_{k=0}^{2}\frac{g_{m(2-k)o(k)}A_0^{k-1}}{(1+j\dfrac{\omega}{\omega_o})^{k-2}}\frac{1}{[1+j2\dfrac{\omega}{T_0\omega_o}][1+j\dfrac{\omega}{T_0\omega_o}]}\right| \tag{20}
$$

$$
HD_3^f \approx \frac{x_s^2}{4}\left|\left[\frac{1}{T_0^3 g_{o1}}\sum_{k=0}^{3}\frac{g_{m(3-k)o(k)}A_0^{k-1}}{(1+j\dfrac{\omega}{\omega_o})^{k-3}}\frac{1}{[1+j3\dfrac{\omega}{T_0\omega_o}][1+j\dfrac{\omega}{T_0\omega_o}]^2}-\Delta(j\omega)\right]\left[1+\frac{2}{A_0 g_{o1}}\frac{\Gamma(j\omega)}{(1+j2\dfrac{\omega}{T_0\omega_o})}\frac{(1+j3\dfrac{\omega}{\omega_o})}{(1+j2\dfrac{\omega}{\omega_o})}\right]\right| \tag{21}
$$

where the functions Δ and Γ are:

$$
\Delta(j\omega) = \frac{\delta(j\omega)}{T_0^3 A_0}\frac{(1+j3\dfrac{\omega}{\omega_o})(1+j\dfrac{\omega}{\omega_o})^3}{[1+j3\dfrac{\omega}{T_0\omega_o}][1+j\dfrac{\omega}{T_0\omega_o}]^2}
$$

$$
\Gamma(j\omega) = \frac{\left[\sum_{k=0}^{2}\dfrac{g_{m(2-k)o(k)}A_0^k}{(1+j\dfrac{\omega}{\omega_o})^k}\right]^2}{\sum_{k=0}^{3}\dfrac{g_{m(3-k)o(k)}A_0^k}{(1+j\dfrac{\omega}{\omega_o})^k} - g_{o1}(1+j3\dfrac{\omega}{\omega_o})\delta(j\omega)}
$$

SIMULATION AND DISCUSSION

The accuracy of the proposed method was verified in this section by comparing the theoretical analyses and transistor level simulation. To show the flexibility of the proposed analyses, two different kinds of single stage amplifier were simulated. The inverter and cascode amplifier as shown in Figure 3 were implemented in a 0.35 µm process and used in non-inverting configuration with ideal feedback ratio of 1/8 and ½, respectively. These amplifiers were simulated in Spectre with 100 m Vp-p input signals. The output data of transient analysis was post processed by Fourier analysis to obtain the simulated distortion factors.

The curves of simulated second- and third-order distortion factors (labeled "Spectre simulation") were drawn in Figures 4 to 7. The nonlinear coefficients of these open-loop amplifiers were obtained by using Fourier analysis with low frequency input signals and reported in Tables 1 and 2. It should be noted that the nonlinear intermodulation coefficients are taken by evaluating the output current with two different frequency voltage sources at input and output of the amplifiers. Then the distortion factor curves of theoretical analyses (labeled

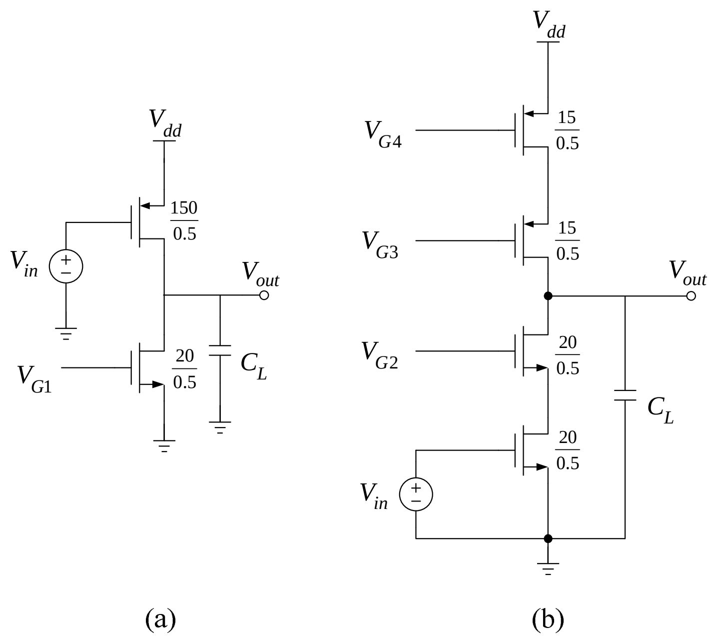

Figure 3. (a) Current-sink inverter; (b) cascade amplifier. The unit of transistor dimensions is μm.

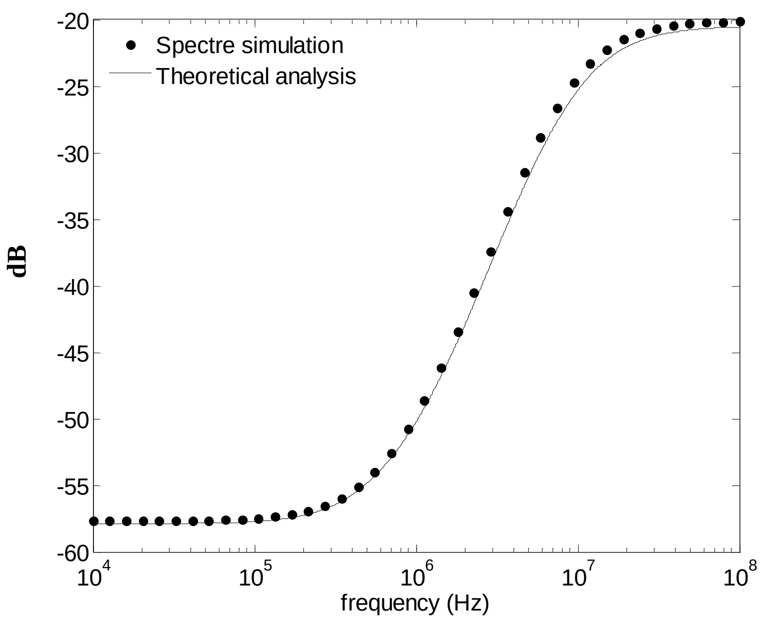

Figure 4. Second-order distortion factor of the current-sink inverter.

"Theoretical analysis") were plotted in Matlab by using Equations (20) and (21). Comparing these theoretical curves with the simulated distortion factor curves, it is clear that our analyses are in accordance with the transistor level circuit simulation. The difference is less than 3 dB even up to 10 times gain bandwidth product.

Comparing the distortion curves of the two different amplifiers in Figure 4 to 7, it is clear that the distortion of cascode amplifier in low frequency range is smaller than the one of the inverter due to its higher DC gain, which

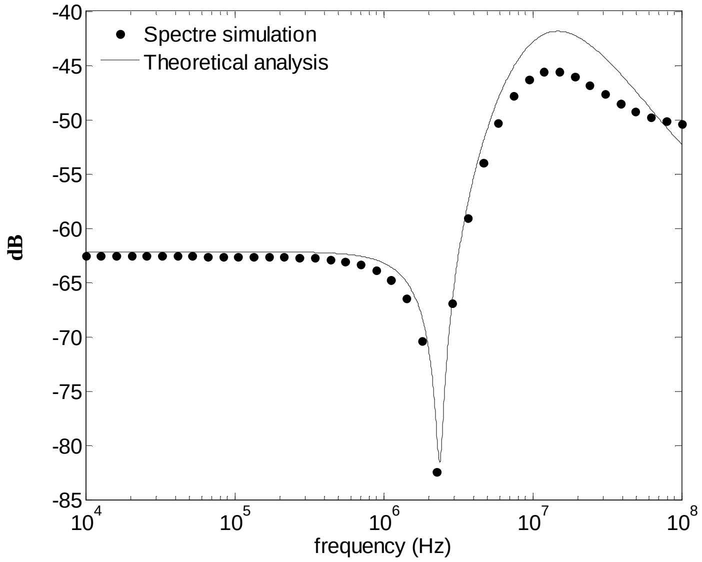

Figure 5. Third-order distortion factor of the current-sink inverter.

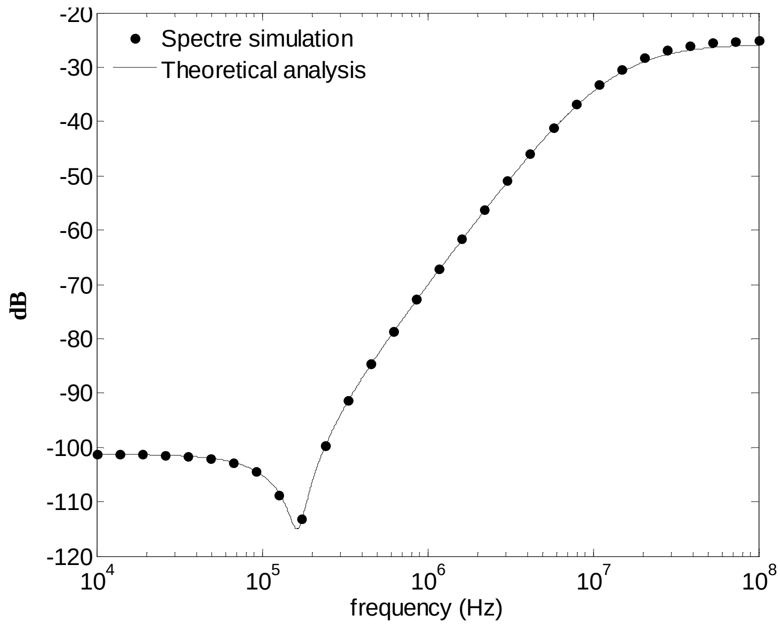

Figure 6. Second-order distortion factor of the cascode amplifier.

result in a larger closed-loop gain T_0. Observing the denominator of equations (15) and (16) or (20) and (21), it is obvious that the difference of T_0 has small impact on the distortion factor in frequencies closed to gain-band-width. Therefore, not only the second- but also the third-order distortion factor curves of the two different feedback amplifiers are drawn together respectively. In addition, the high gain and great gain-band-width of the cascode amplifier make the cancellation of second-order harmonics raised from different nonlinear sources possible. It causes the shape of approximating a negative peak in the low frequency range of the second-order distortion factor curve of the cascode amplifier as shown in Figure 6.

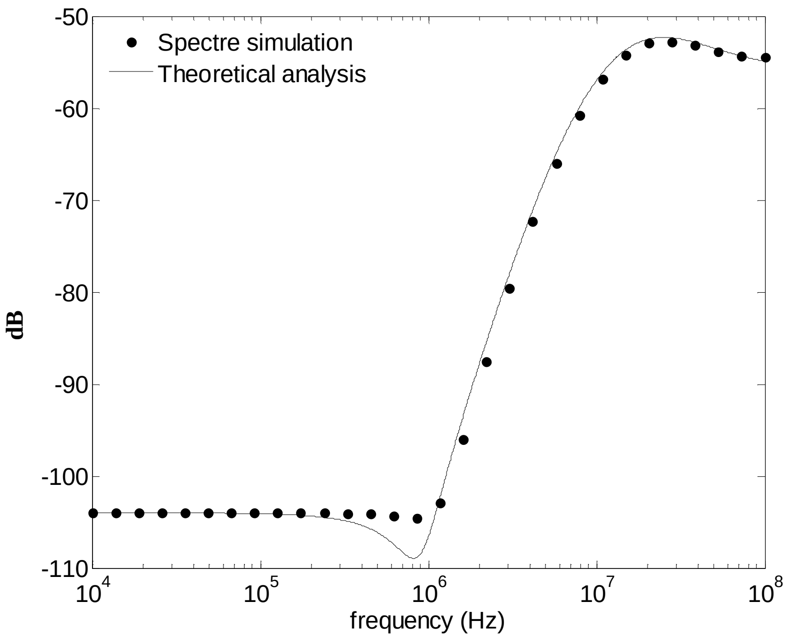

Figure 7. Third-order distortion factor of the cascode amplifier.

Table 1. Some parameters of the current-sink inverter.

Some parameters of the closed-loop amplifier			
Parameter	**Description**	**Value**	**Unit**
A_0	DC open loop gain	31	dB
f_{GBW}	Gain-band-width	9.5	MHz
C_L	Loading capacitance	5	pF
f	Feedback ratio	1/8	V/V

Small signal parameters and nonlinear coefficients					
1st order (A/V)		**2nd order(A/V^2)**		**3rd order(A/V^3)**	
g_{m1}	2.379 m	g_{m2}	8.991 m	g_{m3}	-3.412 m
g_{o1}	67.286 μ	g_{o2}	1.848 μ	g_{o3}	1.718 μ
		g_{m1o1}	0.370 m	g_{m1o2}	-16.246 μ
				g_{m2o1}	0.643 m

Conclusion

In this paper, the distortion of feedback single stage amplifier is modeled by exploring the phasor method. The expressions of second- and third-order distortion factors are derived. The theoretical analyses are in good agreements with the transistor level simulation. From these results, it is easy to investigate the frequency dependent distortion behaviors of the feedback single stage amplifiers. In addition, the equations obtained in this paper can be used in early steps of the amplifier design and the time consuming transient analysis can be avoided. More important, the analyses in this paper provide more

Table 2. Some parameters of the cascode amplifier.

Some parameters of the closed-loop amplifier			
Parameter	**Description**	**Value**	**Unit**
A_0	DC open loop gain	60	dB
f_{GBW}	Gain-band-width	17.9	MHz
C_L	Loading capacitance	5	pF
f	Feedback ratio	1/2	V/V

Small signal parameters and nonlinear coefficients					
1st order (A/V)		**2nd order(A/V^2)**		**3rd order(A/V^3)**	
g_{m1}	1.122 m	g_{m2}	2.305 m	g_{m3}	2.198 m
g_{o1}	1.193 μ	g_{o2}	49.3 n	g_{o3}	0.355 μ
		g_{m1o1}	2.733 μ	g_{m1o2}	1.616 μ
				g_{m2o1}	0.379 m

insights of the distortion behaviors of the amplifier to designers.

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