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On the effect of convective heat and mass transfer on unsteady mixed convection MHD flow through vertical porous medium

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In this paper, unsteady two-dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid which is bounded by a vertical infinite plane surface was considered. A uniform applied homogeneous magnetic field is considered in the transverse direction with first order chemical reaction. An analytical solution for two-dimensional oscillatory flow on unsteady mixed convection of an incompressible viscous fluid, through a porous medium bounded by an infinite vertical plate in the presence of chemical reaction and thermal radiation are presented. The surface absorbs the fluid with a constant suction and the free stream velocity oscillates about a constant mean value. The resulting nonlinear partial differential equations were transformed into a set of ordinary differential equations using two-term series. The closed form solutions for velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number have been obtained, using the regular perturbation technique. Numerical evaluation of the analytical solutions was performed and the results are presented in tabular and graphical form. This illustrates the influence of the various parameters involved in the problem on the solutions.

Key words: Magnetohydrodynamics (MHD) flow, heat and mass transfer, oscillatory flow, thermal radiation, chemical reaction.

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases, etc. The application of magnetohydrodynamics (MHD) flow is found in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering, and

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electronics (Saxena et al., 2014; Rabi et al., 2013).

Oscillatory flows has been known to result in higher rates of heat and mass transfer, many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors, and chemical reactors. The detailed study on fluid mechanics of oscillatory and modulated flows has been made by Cooper et al. (1993). The numerically studied influence of convective heat transfer from periodic open cavities in a channel with oscillatory flow has been studied by Fusegi (1997), Gomaa and Taweel (2005), and Abdelkader and Lounes (2007), respectively. The influence of MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity and the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel have been studied by El-Hakiem (2000), Makinde and Mhone (2005) and Mehmood and Ali (2007), respectively.

The MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source has been studied by Gholizadeh (1990). The work of Makinde (1994) is of particular interest since it demonstrated the possibility of achieving significant unsteady incompressible flow in a porous channel. The MHD mixed convection from a vertical plate embedded in porous medium with convective boundary condition has been analyzed by Makinde and Aziz (2010). Rabi et al. (2013) have investigated the influence of chemical reaction effect on MHD oscillatory flow through a porous medium bounded by two vertical porous plates with heat source Soret effect.

Chemical reactions are classified as either heterogeneous or homogeneous processes. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics and food processing (Cussler, 1998). The diffusion of chemically reactive species in a laminar boundary layer flow is analyzed by Chambre and Young (1958). The exact solution for hydrodynamic boundary layer flow and heat transfer is studied by Vajravelu (1986). Das et al. (1994), Muthucumaraswamy (2001, 2002) and Anjali Devi and Kandasamy (2002) studied the effect of a first-order chemical reaction on the flow in different cases. An analytical solution for heat and mass transfer by laminar flow of a Newtonian, viscous, and electrically conducting fluid is presented by Chamkha (2003). Kandasamy et al. (2005) studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq fluid with chemical reaction and thermal stratification effects.

The role of thermal radiation is of major importance in engineering areas occurring at high temperatures and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles. Hakeem and Sathiyanathan (2009), Srinivas and Muthuraj (2010), Pal and Talukdar (2010), Chamkha (2003), Bakr (2011) and Prakash et al. (2011) have examined the radiation effect of an oscillatory flow under different conditions. The influence of chemical reaction on unsteady MHD slip flow in a Planer channel with varying concentration has been examined by Sivaraj and Kumar (2011).

The main objective of this study is to investigate the effect of periodic heat and mass transfer on unsteady mixed convection MHD flow past an infinite vertical porous flat plate with constant suction in the presence of chemical reaction and thermal radiation when the free stream velocity is oscillating with time. The boundary layer equations governing the problem under consideration are solved by multi-parameter perturbation technique and giving more importance on analytic solution. For this study, air (*Pr=0.71*) and water (*Pr=6.2*) are the only fluids under consideration. The closed form solutions for velocity, temperature, skin friction, concentration, Nusselt number, and Sherwood number are presented. The effects of pertinent parameters on fluid flow of heat and mass transfer characteristics are studied in detail. This work is presented as follows. First, the problem is formulated and then the solution of the problem is presented. Following are the results and discussion, and finally, conclusions are summarized.

FORMULATION OF THE PROBLEM

Consider unsteady two-dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid which is bounded by a vertical infinite plane surface. A uniform applied homogeneous magnetic field is considered from the transverse direction with first order chemical reaction. It was assumed that the surface absorbs the fluid with a constant velocity and the fluid far away the surface oscillates with time and assumed to be in the form $\bar{U}(t) = U_0(1+\varepsilon e^{i\bar{\omega}t})$, where U_0 is the mean stream velocity and $\varepsilon(<<<1)$ the amplitude of the free stream variation. A Cartesian coordinate system (x', y') is assumed, where x'- axis lies along the plate and y'-axis in the normal direction. Then, under the usual Boussinesq's approximation, the following equations governing the flow field are considered.

Continuity equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' + g\beta\left(T' - T'_\infty\right) + g\beta_c\left(C' - C'_\infty\right) \tag{2}$$

Energy equation:

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} \quad (3)$$

Concentration equation:

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-\gamma\left(C'-C'_\infty\right) \quad (4)$$

where (u',v') are velocity components in x' and y' directions, g is acceleration due to gravity, T' is the temperature of the fluid, C' is the species concentration, β is the coefficient of thermal expansion, β_c is the volumetric expansion coefficient, υ is the kinematic viscosity of the fluid, k is effective thermal conductivity, ρ is the density of the fluid, c_p is the specific heat at constant pressure, D is the diffusion coefficient, B_0 is the electromagnetic induction, σ is the conductivity of the fluid, v_0 constant suction/ injection and p is non-dimensional pressure.

The corresponding boundary conditions of the problem are:

$$u'=0, \quad T'=T'_\omega+\left(T'_\omega-T'_\infty\right), \quad C'=C'_\omega+\varepsilon\left(C'_\omega-C'_\infty\right)e^{i\omega' t'} \, at\, y'=0 \quad (5)$$

$$u'=U', \quad T'=T'_\infty, \quad C'=C'_\infty \qquad at\, y'\to\infty$$

And the radiative heat flux is given by

$$\frac{\partial q_r}{\partial y'}=4\alpha^2\left(T'_0-T'\right) \quad (6)$$

where $\alpha -$ is the radiation absorption coefficient .

Eliminating the modified pressure gradient under the usual boundary layer approximation (Equation 2) was reduced to

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\frac{\partial U'}{\partial t'}+\upsilon\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\rho}\left(u'-U'\right)+g\beta\left(T'-T'_\infty\right)+g\beta_c\left(C'-C'_\infty\right) \quad (7)$$

introducing the following dimensionless quantities:

$$y=\frac{v_0}{\upsilon}y', \quad t=\frac{{v_0}^2}{\upsilon}t', \quad \omega=\frac{\upsilon\omega'}{v_0^2}, \quad u=\frac{u'}{U_0}, \; U=\frac{U'}{U_0},$$
$$\theta=\frac{T'-T'_\infty}{T'_w-T'}, \; \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \; k=\frac{v_0^2}{\upsilon^2}k' \quad (8)$$

With the help of the earlier non-dimensional variables, Equations 1, 3, 4 and 7 were reduced to

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}+G_r\theta+G_c\phi+M(U-u) \quad (9)$$

$$\frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2}+N\theta \quad (10)$$

$$\frac{\partial \phi}{\partial t}-\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-K_r\phi \quad (11)$$

The reduced boundary conditions are:

$$u=0, \quad \theta=1+\varepsilon e^{i\omega t} \quad \phi=1+\varepsilon e^{i\omega t} \qquad at\, y=0 \quad (12)$$

$$u=1+\varepsilon e^{i\omega t}, \quad \theta=0, \; \phi=0 \qquad at\, y\to\infty$$

where $G_r=\frac{\upsilon g\beta}{U_0 v_0^2}(T_w-T_\infty)$ is Grashof number, $G_c=\frac{\upsilon g\beta_c}{U_0 v_0^2}(C_w-C_\infty)$ is the modified Grashof number, $M=\frac{\sigma\beta_0^2\upsilon}{\rho v_0^2}$ is magnetic number, $Sc=\frac{\upsilon}{D}$ is the Schmidt number, $p_r=\frac{\rho\upsilon c_p}{k}$ is Prandtl number, $K_r=\frac{\upsilon\gamma}{v_0^2}$ is the chemical reaction parameter and $N=\frac{4\alpha^2\upsilon}{\rho c_p v_0^2}$ is radiation parameter.

METHOD OF SOLUTION

In order to solve the differential Equations 9, 10 and 11, we assume that:

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y),$$
$$\theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y), \quad (13)$$
$$\phi(y,t)=\phi_0(y)+\varepsilon e^{i\omega t}\phi_1(y)$$

where u_0 , θ_0 and ϕ_0 are respectively the mean velocity, mean temperature and mean concentration. On using Equations 13 into 9, 10 and 11 and neglecting the higher order of ε , and simplifying we get the following set of equations:

$$\left[\frac{d^2}{dy^2}+\frac{d}{dy}-M\right]u_0=-G_r\theta_0-G_c\phi_0-M \quad (14)$$

$$\left[\frac{d^2}{dy^2}+\frac{d}{dy}-M\right]\theta_0=0 \tag{15}$$

$$\left[\frac{d^2}{dy^2}+\frac{d}{dy}-M\right]\phi_0=0 \tag{16}$$

$$\left[\frac{d^2}{dy^2}+\frac{d}{dy}-M\right]u_1=-G_r\theta_1-G_c\phi_1-(M+i\omega) \tag{17}$$

$$\left[\frac{d^2}{dy^2}+p_r\frac{d}{dy}+p_r(N-i\omega)\right]\theta_1=0 \tag{18}$$

$$\left[\frac{d^2}{dy^2}+Sc\frac{d}{dy}+Sc(K_r+i\omega)\right]\phi_1=0 \tag{19}$$

The corresponding reduced boundary conditions are

$$u_0=0,\ u_1=0,\quad \theta_0=1,\ \theta_1=1,\ \phi_0=1,\ \phi_1=1\ \ at\ y=0 \tag{20}$$

$$u_0=1,\ u_1=1,\quad \theta_0=0,\ \theta_1=0,\ \phi_0=0,\ \phi_1=0\ \ at\ y\to\infty$$

Solving Equations 14 into 19 with the conditions in Equation 20, we get:

$$\begin{aligned}u=&-A_1e^{m_1y}-A_2e^{m_2y}+\left(A_1+A_2-1\right)e^{m_3y}+1+\\&\varepsilon e^{i\omega t}\left(-A_3e^{m_4y}-A_4e^{m_5y}+\left(A_3+A_4-1\right)e^{m_6y}+1\right)\end{aligned} \tag{21}$$

$$\theta=e^{m_1y}+\varepsilon e^{i\omega t}e^{m_4y}, \tag{22}$$

$$\phi=e^{m_2y}+\varepsilon e^{i\omega t}e^{m_5y}, \tag{23}$$

where

$$m_1=\frac{-pr-\sqrt{pr^2-4prN^2}}{2},$$

$$m_2=\frac{-Sc-\sqrt{Sc^2+4Sckr}}{2},$$

$$m_3=\frac{-1-\sqrt{1+4M}}{2},$$

$$m_4=\frac{-pr-\sqrt{pr^2-4pr\left(N^2-i\omega\right)}}{2},$$

$$m_5=\frac{-Sc-\sqrt{Sc^2+4Sc\left(kr+i\omega\right)}}{2},$$

$$m_6=\frac{-1-\sqrt{1+4\left(M+i\omega\right)}}{2},$$

$$A_1=\frac{G_r}{m_1^2+m_1-M},$$

$$A_2=\frac{G_c}{m_2^2+m_2-M},$$

$$A_3=\frac{G_r}{m_4^2+m_4-\left(M+i\omega\right)},$$

$$A_2=\frac{G_c}{m_5^2+m_5-\left(M+i\omega\right)},$$

The local skin friction coefficient, local Nusselt number and local Sherwood number are important physical quantities for this type of heat and mass transfer problem. They are defined as the following.

The dimensionless shearing stress on the surface of a body is written as:

$$\tau'_\omega=\mu\left.\frac{\partial u'}{\partial y'}\right|_{y'=0} \tag{24}$$

There, the local skin-friction factor is given by:

$$C_f=\frac{2\tau'_\omega}{\rho U_0V_0}=2\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{25}$$

The rate of heat transfer at the surface in terms of the local Nusselt number can be written as:

$$Nu=\frac{x}{T_\infty-T_\omega}\left.\frac{\partial T'}{\partial y'}\right|_{y=0} \tag{26}$$

$$Nu\,\mathrm{Re}_x^{-1}=-\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \tag{27}$$

where $\mathrm{Re}_x^{-1}=-\frac{xV_0}{\upsilon}$ is the local Reynolds number.

The rate of heat transfer at the surface in terms of the local Sherwood number can be written as:

$$Sh=\frac{x}{C_\infty-C_\omega}\left.\frac{\partial C'}{\partial y'}\right|_{y=0}$$

$$Sh\,\mathrm{Re}_x^{-1}=-\left.\frac{\partial\phi}{\partial y}\right|_{y=0} \tag{28}$$

RESULTS AND DISCUSSION

Numerical evaluation of the analytical solutions reported in the previous section was performed and the results are presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions in plotting the results.

The values of Grashof number G_r have been chosen as they are interesting from physical point of view. The free convection currents are due to temperature difference $T_m - T_\infty$ and hence $G_r > 0$ when $T_m - T_\infty > 0$ which physically corresponding to cooling of the plate by free-convection current, $G_r < 0$ corresponds to heating of the boundary surface by free-convection currents as $T_m - T_\infty < 0$ and $G_r = 0$ corresponds to the absence of free convection currents.

Figure 1 demonstrates the variation of the velocity distribution with the chemical reaction in the cases of cooling ($G_r > 0$) and heating ($G_r < 0$) by free convection currents. It observed that the velocity in the cases of cooling and heating decreases near the boundary layer with an increases in the chemical reaction parameter K_r. Also, Figure 1 shows that the effects of heating the surface leads to a fall in the velocity inside the boundary layer while the effect of cooling by free convection current, ($G_r > 0$) is to increase the velocity more than in the case of heating the surface.

Figures 2 and 3 show the effects of chemical reaction on the concentration profiles with two values of Grashof number $G_r = 2$ (cooling of the surface) and $G_r = -2$ (heating of the surface), respectively. An increase in the values of K_r from 1, 1.5, 2, 2.5, and 3 causes a significant decrease in concentration profiles throughout the concentration boundary layer in the case of cooling and heating of the surface by free convection current.

Figure 4 illustrates the influence of the magnetic parameter M on the velocity profiles with two values of $G_r = -2$ (heating of the surface) and $G_r = 2$ (cooling of the surface). Application of a transfer magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. This is obvious from decreases in the velocity profiles. Moreover, the magnetic field is found to cause an over shoot in the velocity profiles (that is, the velocity profiles exceed the values at the edge of the momentum boundary layer) in the case of cooling of the surface. However, no overshoot is observed in the case of heating of the surface whatever the values of magnetic field parameter are. Also, the effect of the magnetic field is found to be more pronounced for the case of cooling of the surface than for the case of heating of the surface.

Figures 5 and 6 show the effects of the Grashof number G_r on the velocity profiles in the case of air ($p_r = 0.71$) and water ($p_r = 6.2$). From these figures, it was observed that the effects of cooling by free convection currents occur when $G_r > 0$. The velocity increases inside the boundary layer as G_r increases. In addition the curves show that the peak value of the velocity increases rapidly near the surface as the Grashof number increases and then decays to the free-stream velocity in the two cases air and water. Also, Figures 5 and 6 show the effect of heating by free convection currents when $G_r < 0$. An increase in greater heating of the surface leads to a fall in the velocity in the two cases of air and water. Also, it was noticed that the effects of Grashof number are found to be more pronounced for the fluid with small Prandtl number.

It was observed that the velocity and concentration decrease as the Schmidt number Sc increases, noted in the two cases of Prandtl number Pr=0.71, 6.2 (air and water) (Figures 7 to 9).

Figure 10 depicts the velocity profile for different values of radiation parameter N in two cases of Prandtl number Pr=0.71, 6.2. It was observed that the velocity decreases as the radiation parameter N decreases, velocity distribution for various values of radiation parameter N in case of the Prandtl number Pr =0.71 has a clear impact but its effect did not have a clear impact in case of the Prandtl number Pr=6.2, so we zoomed in water case to clarify influence.

The temporal development of temperature with different values of time t is elucidated in Figure 11. It is clear that the velocity of the fluid decreases as time increases, which noted in the two cases of Prandtl number Pr =0.71, 6.2, this is more evident in the case of air than in water case. It illustrated that the influence of the various parameters involved in the problem on the velocity of the air is greater than the velocity of the water under the same conditions.

Tables 1 and 2 show the effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer, respectively for $\omega t = \pi / 2$, $\varepsilon = 0.5$ in the two cases air and water, respectively. It is clear that as Grashof number, modified Grashof number, Schmidtnumber, magnetic parameter and radiation parameter increase, skin friction coefficient increases, while decreasing by increasing chemical reaction and frequency of the fluid. Nusselt number decreases as frequency of the fluid and radiation parameter increase and Sherwood number increases as the chemical reaction and Schmidt number increase, while decrease by increasing frequency of the fluid. On the other hand,

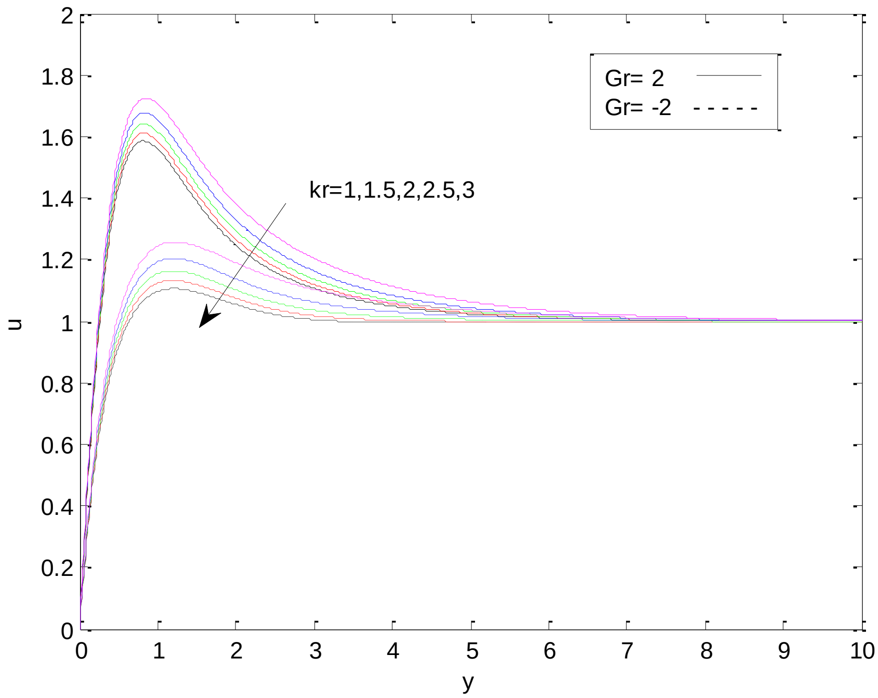

Figure 1. Velocity distribution for various values of Grashof number Gr and chemical reaction parameter k_r for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M*= 5, G_c = 5, Pr = 0.71, *Sc* = 0.22.

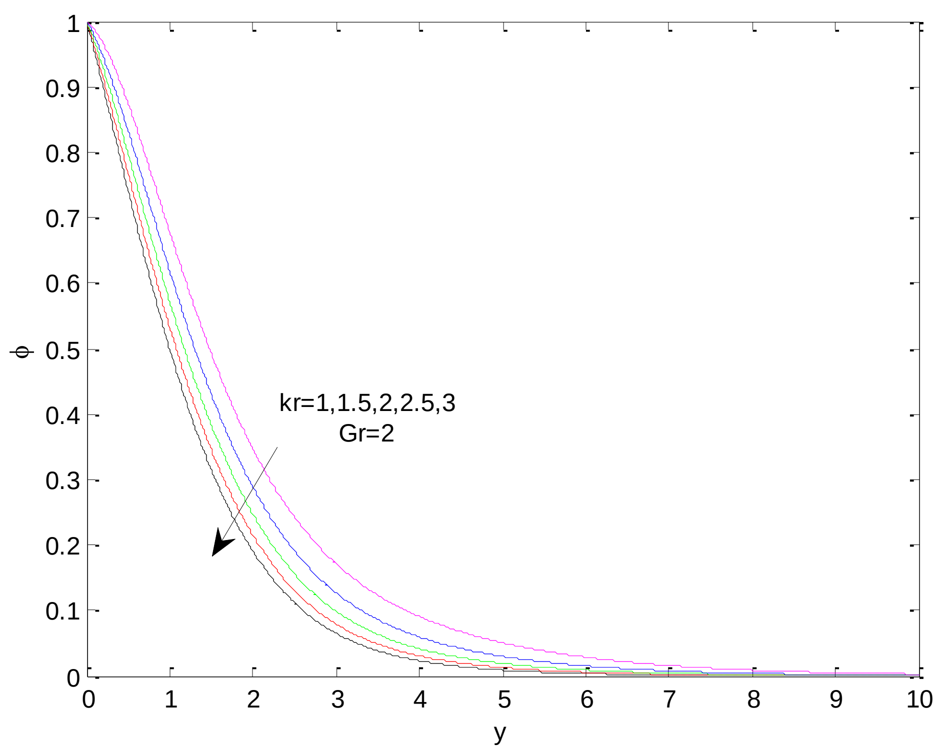

Figure 2. Concentration distribution for various values and chemical reaction parameter k_r for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M*= 5, G_c = 5, Pr = 0.71, *Sc* = 0.22.

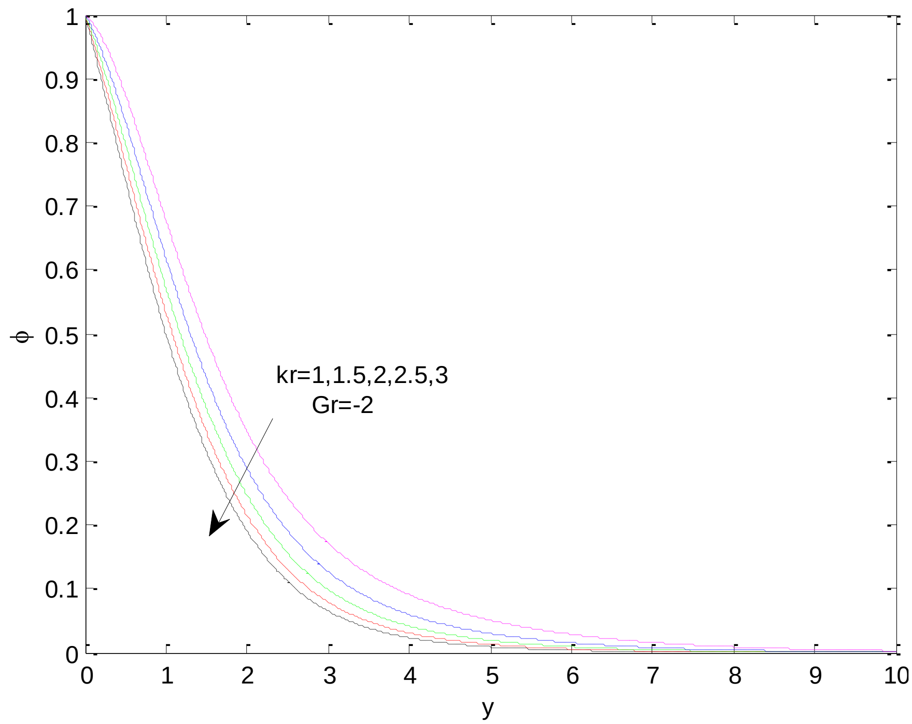

Figure 3. Concentration distribution for various values chemical reaction parameter k_r for $\omega = 10$, ωt=π/2, ε = 0.5, *N* = 0.2, *M*= 5, G_c = 5, Pr = 0.71, *Sc* = 0.22.

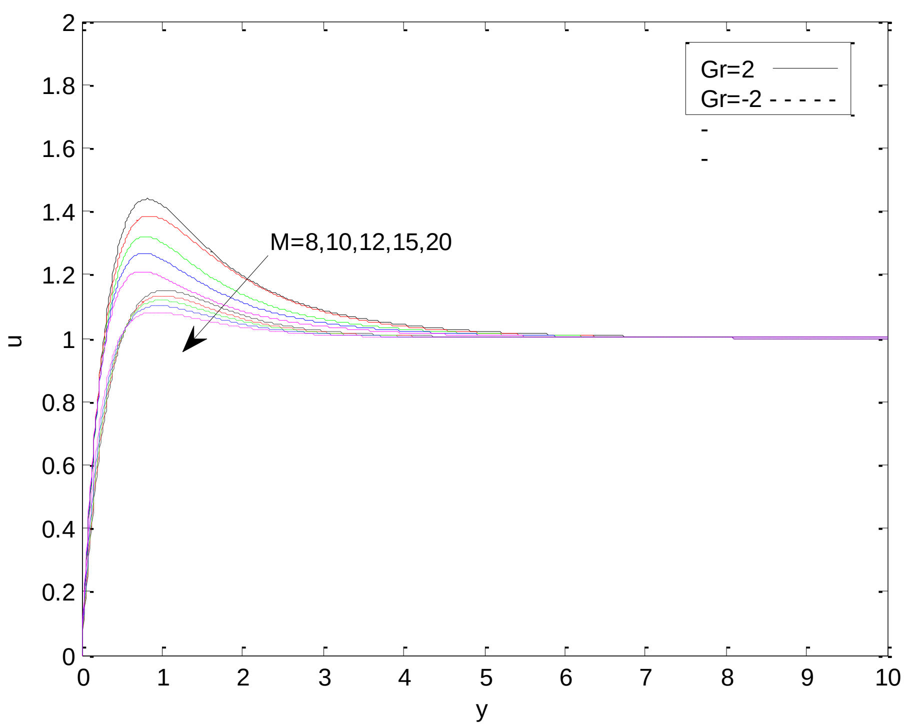

Figure 4. Velocity distribution for various values of Grashof number Gr and magnetic field parameter M for $\omega = 10$, ωt=π/2, ε = 0.5, *N* = 0.2, *kr* = 2, G_c =5, pr=0.71, *Sc* = 0.22

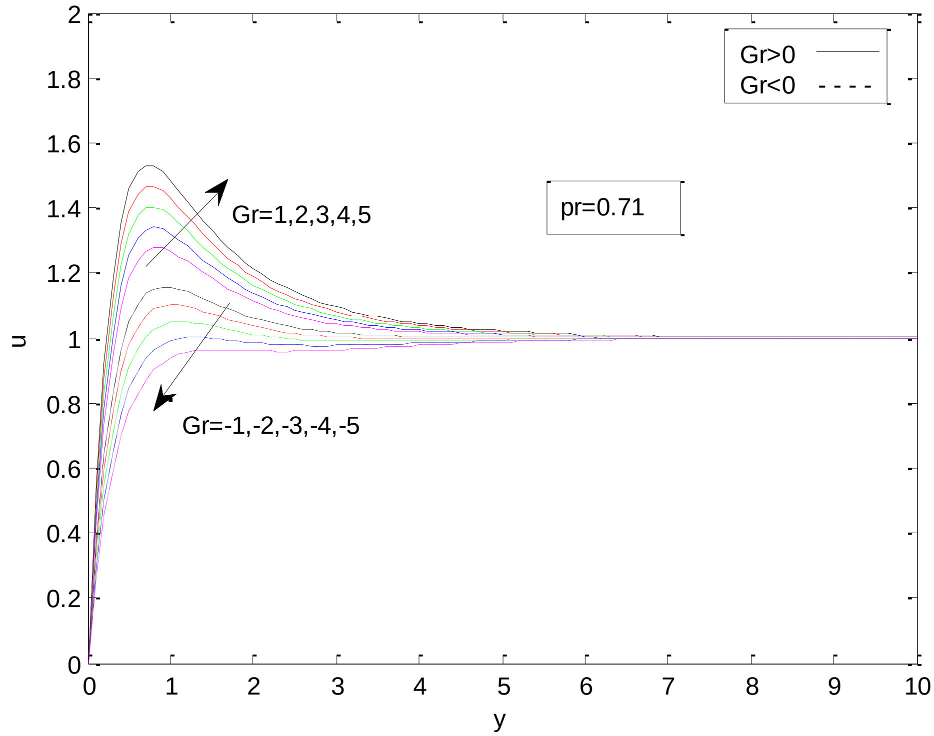

Figure 5. Velocity distribution for various values of Grashof number Gr for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M*= 10, G_c = 5, *kr* = 3, *Sc* = 0.22.

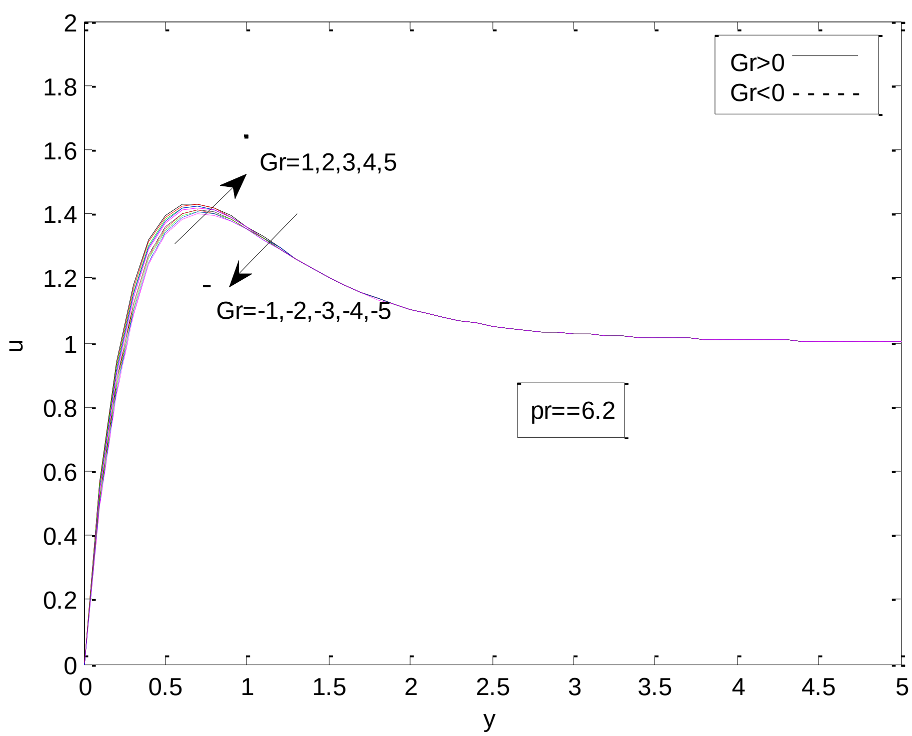

Figure 6. Velocity distribution for various values of Grash of number Gr for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M*= 10, G_c = 5, *kr* = 3, pr=6.2, *Sc* = 0.22.

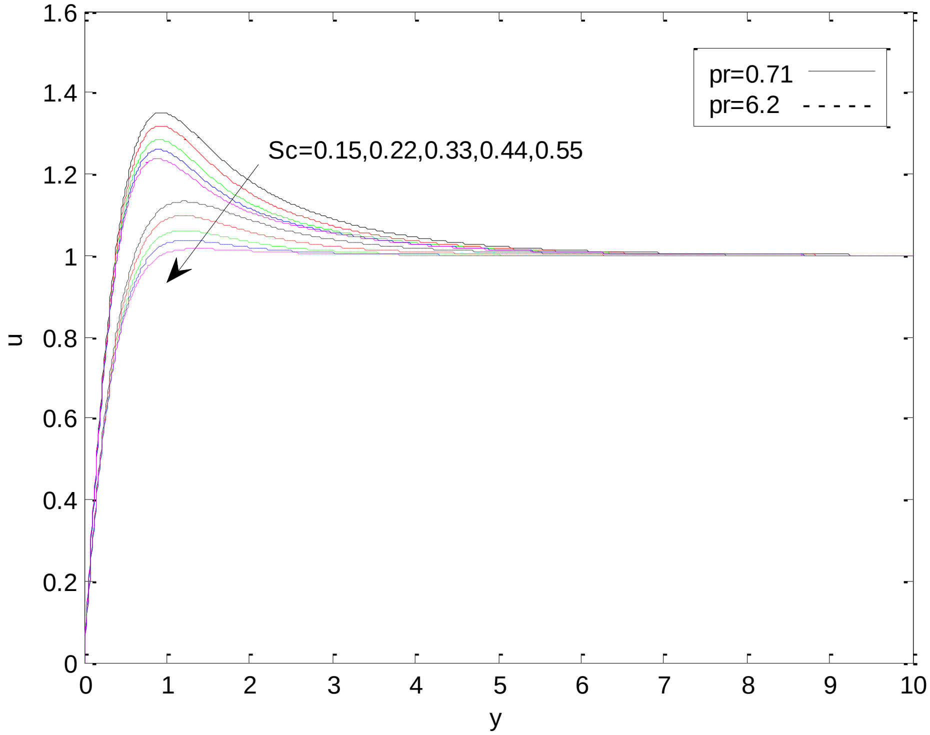

Figure 7. Velocity distribution for various values of Prandtl number Pr and the Schmidt number Sc for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M=5, kr* = 3, $G_r = G_c$ =2,

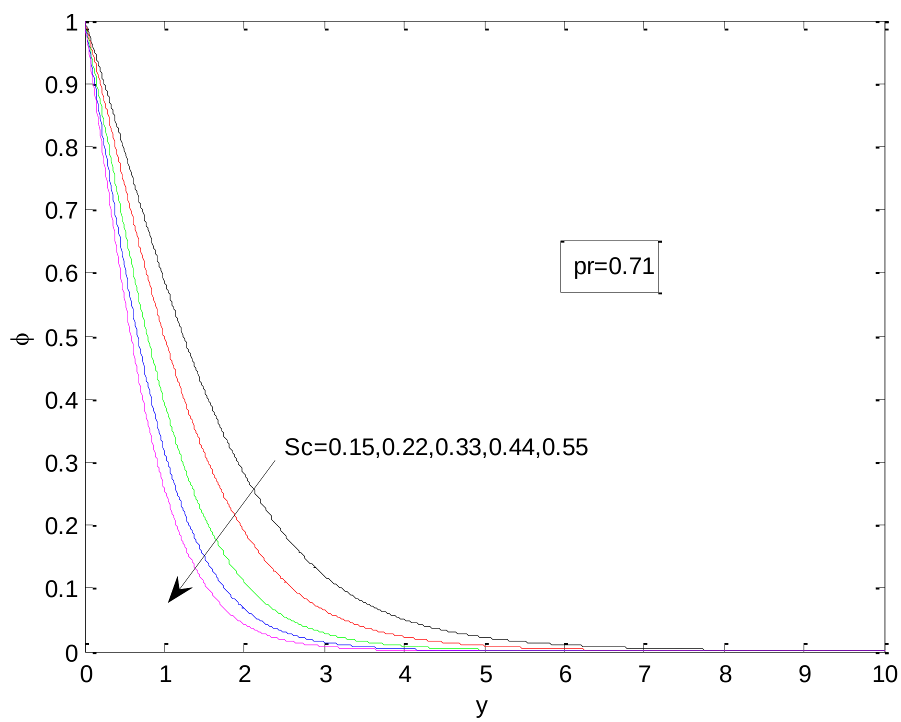

Figure 8**.** Concentration distribution for various values of the Schmidt number Sc for $\omega = 10$, ωt=π/2,*ε* = 0.5, *N* = 0.2, *M=5, kr* = 3, $G_r = G_c$ =2

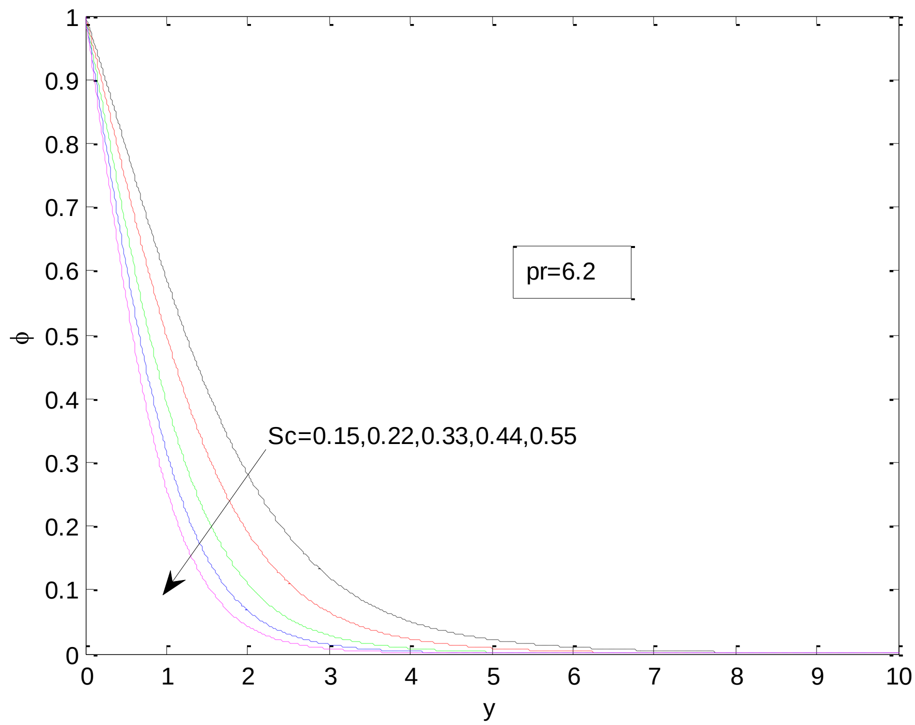

Figure 9. Concentration distribution for various values of the Schmidt number Sc for $\omega = 10$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M=5, kr* = 3, $G_r = G_c$=3, pr=6.2.

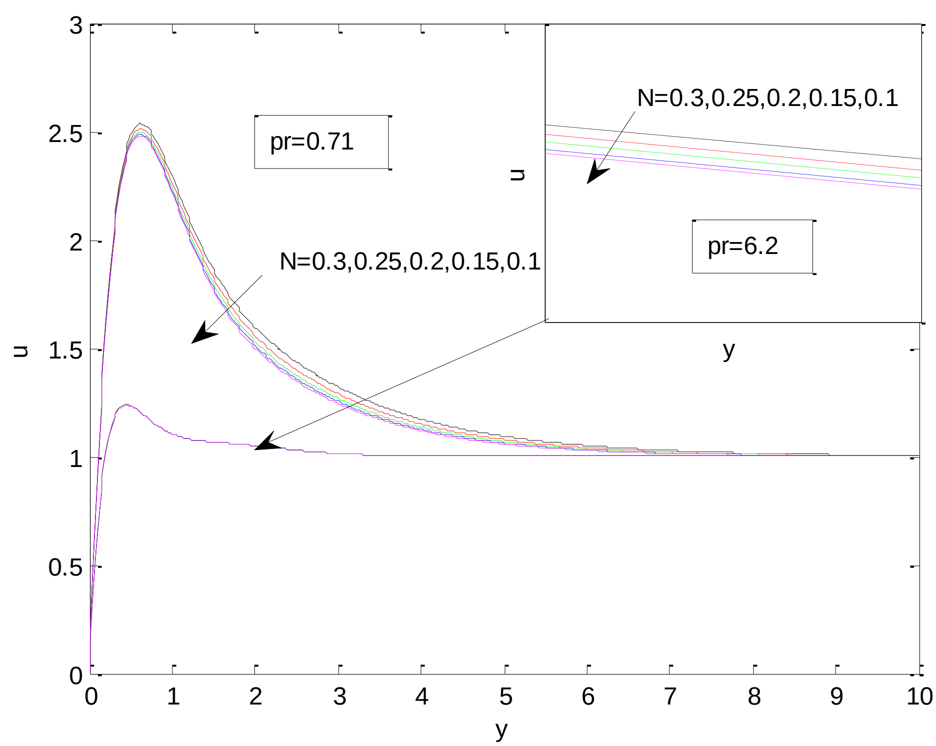

Figure 10. Velocity distribution for various values of radiation parameter N for $\omega = 10$, ωt=π/2, *ε* = 0.5, Sc = 0.66, *M=5, kr* = 3, $G_r = G_c$=5.

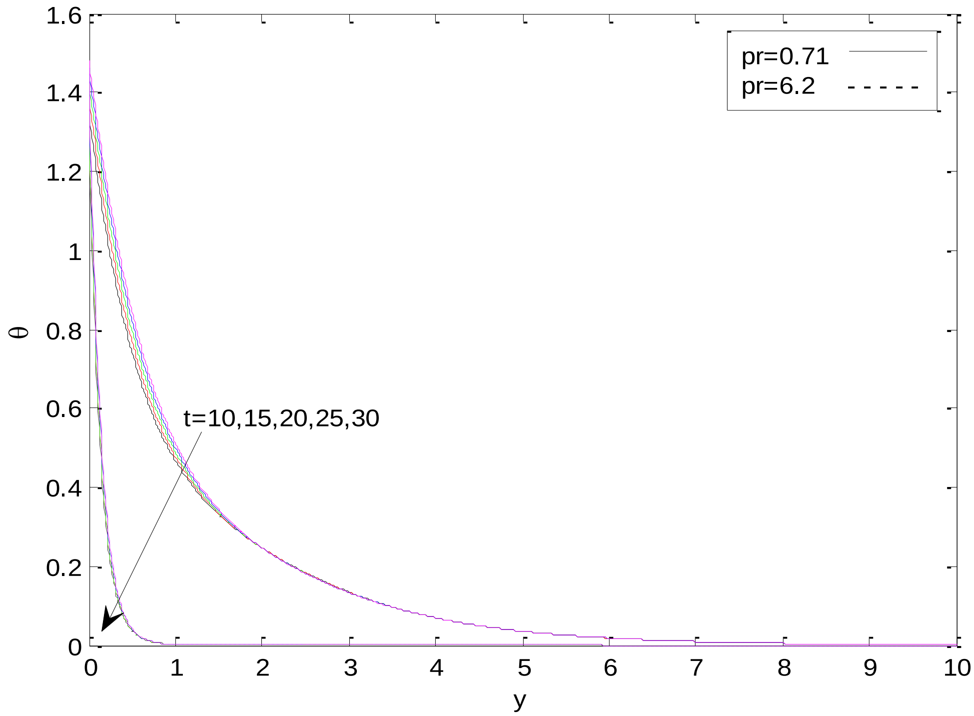

Figure 11. Temperature distribution for various values of Prandtl number Pr and the time t for $\omega = 5$, ωt=π/2, *ε* = 0.5, *N* = 0.2, *M=5, kr* = 6, $G_r = G_c$=5, $Sc = 0.22$.

Table 1. Effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer respectively for Pr=0.71, ωt=π/2 and ε=0.5.

Pr	Gr	Gc	kr	Sc	M	ω	N	C_f	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
	-2							0.9681985		
	0	-2	1	0.22	5	5	0.2	2.4504063	0.0104001	0.2278289
	2							3.9326142		
		0						6.0647835		
		2						8.1969528		
			2					8.0915512		0.4788251
			3					8.0292530		0.6521084
0.71				0.44				8.1965396		0.9988825
				0.66				8.6646388		1.2987692
					10			8.9164825		
					15			10.088027		
						10		9.5646458	-0.268168	0.9979852
						15		9.0859922	-0.481072	0.7734869
							0.3	9.1014427	-0.546189	
							0.4	9.1369066	-0.686658	

Table 2. Effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer respectively for Pr=6.2, ωt=π/2 and ε=0.5.

Pr	Gr	Gc	kr	Sc	M	w	N	C_f	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
	-2							1.9013410		
	0	-2	1	0.22	5	5	0.2	2.4504063	4.4633888	0.2578289
	2							2.9994716		
		0						5.1316409		
		2						7.2638104		
			2					7.1584088		0.4788251
			3					7.0961104		0.6521084
6.2				0.44				7.2633971		0.9988814
				0.66				7.7314962		1.2987690
					10			8.1253424		
					15			9.4666094		
						10		8.9347129	3.5775681	0.9979852
						15		8.4445792	2.9172851	0.7734869
							0.3	8.4469053	2.8608185	
							0.4	8.4502701	2.7802543	

Table 3. Effects of various values of time on the coefficients of skin friction, heat transfer and mass transfer respectively for $\omega = 5$, *ε* = 0.5, *N* = 0.2, *M*= 10, G_r=G_c=2, kr=3, Sc=0.22.

Pr	T	C_f	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
0.71	0	14.6751855	1.5203172	1.4799403
	10	14.6269275	1.6628313	1.5335311
	20	14.2328840	1.7356010	1.5448208
	30	13.5206649	1.7335276	1.5130185
	40	12.5401740	1.6567563	1.4403525
	50	11.3601122	1.5106664	1.3319143
6.2	0	13.4775922	9.9940552	1.4799403
	10	13.4600055	10.304803	1.5335310
	20	13.1268349	10.325115	1.5448208
	30	12.5014249	10.053568	1.5130185
	40	11.6275967	9.5091880	1.4403525
	50	10.5665778	8.7301189	1.3319143

the Nusselt number increases as Pr increases.

Finally, the effects of various values of *t* on the coefficients of skin friction, heat transfer and mass transfer across the boundary layer are presented in Table 3. It can be concluded that skin friction decreases as time increases. Heat and mass transfer increase with time at values 0, 10, and 20, while decreasing at values 20, 30, 40, and 50.

Conclusion

An analytical solution for two-dimensional oscillatory flow on unsteady mixed convection of an incompressible viscous fluid, through a porous medium bounded by an infinite vertical plate in the presence of chemical reaction and thermal radiation are presented. The governing boundary layer equations for the velocity, temperature, and concentration fields were solved using the method of small perturbation approximation. Numerical evaluation of the analytical solutions was performed and the results were presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions. Two cases can be considered air ($P_r = 0.71$) and water ($P_r = 6.2$). It can be concluded that the velocity in the case of cooling and heating decreases near the boundary with an increase in the chemical reaction. Also, increasing the chemical reaction is to decrease the concentration profile throughout the boundary layer. Also, it was found that for two values of Prandtl number, under study, the concentration distribution decreases as the Schmidit number increases. Moreover, the Nusselt number decreases as the Prandtl number increases. It is interesting to note that the rate of heat mass transfer increase with increasing time for both air and water when $t \leq 20$, while the reverse behavior is observed when $t \geq 20$.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

REFERENCES

Abdelkader K, Lounes O (2007). Heat transfer enhancement in oscillatory flow in channel with periodically upper and lower walls mounted obstacles. Int. J. Heat Fluid Flow. 28:1003-1012.

Anjali Devi SP, Kandasamy R (2002). Effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction and injection. Int. Comm. Heat Mass Transf. 29:707-716.

Bakr AA (2011). Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Comm. Non. Sci. Numer. Simulat.16:698-710.

Chambre PL, Young JD (1958). On the diffusion of chemically reactive species in a laminar boundary layer flow. Phys. Fluids 1:48-54.

Chamkha A (2003). MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation absorption and a chemical reaction. Int. Comm. Heat Mass Transf. 30:413-422.

Cooper WL, Nee VW, Yang KT (1993). Fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer - a review. J. Energ. Heat Mass Transf. 15:1-19.

Cussler EL (1998). Diffusion Mass transfer in fluid Systems. 2nd Ed, Cambridge University Press. Cambridge.

Das UN, Deka RA, Soundalgekar VM (1994). Effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forsch. Ingen. 60:284-287.

El-Hakiem MA (2000). MHD oscillatory flow on free convection-radiation through a porous medium with constant suction velocity. J. Magn. Magn. Mater. 220:271-276.

Fusegi T (1997). Numerical study of convective heat transfer from periodic open cavities in a channel with oscillatory through flow. Int. J. Heat Fluid flow. 18:376-383.

Gholizadeh A (1990). MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source. Astr. Space Sci. 174:303- 310.

Gomaa H, Al Taweel AM (2005). Effect of oscillatory motion on heat transfer at vertical flat surfaces. Int. J. Heat Mass Transf. 48:1494-1504.

Hakeem AK, Sathiyanathan K (2009). An analytic solution of an oscillatory flow through a porous medium with radiation effect. Non. Analy. Hybrid Systems 3:288-295.

Kandasamy R, Periasamy K, Sivagnana Prabhu KK (2005). Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Int. J. Heat Mass Transf. 48:45-57.

Makinde OD (1994). Unsteady incompressible flow in a porous channel. Proc. Rom. Appl. Indust. Math. Oradea. pp. 47-58.

Makinde OD, Aziz A (2010). MHD mixed convection from a vertica plate embedded in a porous medium with a convective boundary condition. Int. J. Therm. Sci. 49:1813-1820.

Makinde OD, Mhone PY (2005). Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Rom. J. Phys. 50:931-938.

Mehmood A, Ali A (2007). The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Rom. J. Phys. 52:85-91.

Muthucumaraswamy R (2001). First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Act. Mech. 147:45-57.

Muthucumaraswamy R (2002). Effect of a chemical reaction on a moving isothermal vertical surface with suction. Act. Mech. 155:65-70.

Pal D, Talukdar B (2010). Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Comm. Non. Sci. Numer. Simulat. 15:1813–1830.

Prakash J, Sivaraj R, Kumar B (2011). Influence of chemical reaction on unsteady MHD mixed convective flow over a moving vertical porous plate. Int. J. fluid Mechan. 3(1):1-14.

Rabi N, Gouranga C, Arpita M (2013). Chemical reaction effect on MHD Oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and soret effect. J. Appl. Analy. Comput. 3(4):307-321.

Saxena SS, Dhiresh KP, Sandeep K (2014). MHD free convective oscillatory Rivlin-Ericksen fluid flow past a porous plate embedded in a porous medium in the presence of heat source with chemical reaction. Der Chem. Sinic. 5(4):30-45.

Sivaraj R, Kumar BR (2011). Chemically Reacting Unsteady MHD Oscillatory Slip Flow in a Planer Channel with Varying Concentration. Int. J. Math. Scientific Comput. 1(1):35-42.

Srinivas S, Muthuraj R (2010). Effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel using homotopy analysis method. Comm. Non. Sci. Numer. Simulat. 15:2098-2108.

Vajravelu K (1986). Hydrodynamic flow and heat transfer over Continuous, Moving Porous, Flat Surface. Acta Mech. 64:179-185.