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Review

Anti-synchronization of Liénard chaotic systems via feedback control

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Based on the Lyapunov stability theory, this paper presents the feedback controllers for anti-synchronizing Liénard systems. A sufficient condition is drawn for the robust stability of the error dynamics and the design of the controllers. Numerical simulations are performed to verify and illustrate the analytical results.

Key words: Anti-synchronization, Liénard system, feedback control.
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INTRODUCTION

Tremendous efforts have been made recently in the studies of the chaotic synchronization, due to its potential applications in secure communications, biological systems, chemical reactions and so on (Pecora et al., 1990). So far, many interesting synchronization phenomenon have been observed, such as anti-synchronization (Li, 2005), phase synchronization (Wang et al., 2010), transient synchronization (Ciszak et al., 2009), projective synchronization (Mainieri et al., 1999), lag synchronization (Taherion1 et al., 1999). Among them, anti-synchronization is noticeable in periodic oscillators (Li, 2005). Many techniques and methods have been proposed to achieve anti-synchronization, such as backstepping control (Lin et al., 2009), feedback control (Li et al., 2009; Li et al., 2006), adaptive control (Al-sawalha et al., 2009; Elabbasy et al., 2009), active control (Wang et al., 2007; Njah et al., 2009) and sliding mode control (Zhang et al., 2004; Haeri et al., 2007).

Liénard system, as one of the paradigm in nonlinear dynamics, exhibits various aspects of attractors and bifurcation (Chen et al., 2005), which includes Van der Pol system, Duffing Oscillator system, Brusselator system and so on. Elabbasy et al. (2008) achieved synchronization for the stable Van der Pol oscillator and Chen chaotic dynamical system by using nonlinear control function. Fotsin et al. (2005) designed a controller to synchronize two unidirectionally coupled modified Van der Pol-Duffing oscillators via adaptive control. The aim of this article is to design feedback controllers to anti-synchronize Liénard systems.

Anti-synchronization of Liénard systems

Consider a general Liénard system described by the following nonlinear differential equation:

$$\ddot{x} + f(x)\dot{x} + g(x) = h(t), \tag{1}$$

where $f(x), g(x)$ are continuous functions in $R = (-\infty, +\infty)$, and $h(t)$ is continuous function in $R^+ = [0, +\infty)$.

The Liénard system can be rewritten as follows:

$$\begin{cases} \dot{x} = y - F(x) \\ \dot{y} = -g(x) + h(t) \end{cases}, \tag{2}$$

where $F(x) = \int_0^x f(u)du$, and we assume that $F(x), g(x)$ is Lipschitz with coefficients θ_1, θ_2 respectively, that is $|F(x)| \le \theta_1 |x|$, $|g(x)| \le \theta_2 |x|$. Therefore, one can assume $F(x) = L_1 x$, $g(x) = L_2 x$ where $\|L_1\| = \theta_1$, $\|L_2\| = \theta_2$. The

Liénard system with external disturbance can be described as the following differential equations

$$\begin{cases}\dot{\overline{x}} = \overline{y} - F(\overline{x}) \\ \dot{\overline{y}} = -g(\overline{x}) + h(t) + \delta(t)\end{cases}, \quad (3)$$

where external disturbance $\delta(t)$ is continuous function in $R^+ = [0, +\infty)$ which satisfies $\int_0^{+\infty} |\delta(t)| dt < +\infty$.

Now we design feedback controllers $U(t, x, y) = [u_1(t), u_2(t)]^T$ to anti-synchronize the Liénard systems (2) and (3), let Liénard system (2) as the master system and the response system as follows

$$\begin{cases}\dot{\overline{x}} = \overline{y} - F(\overline{x}) + u_1 \\ \dot{\overline{y}} = -g(\overline{x}) + h(t) + \delta(t) + u_2\end{cases}, \quad (4)$$

Defining the anti-synchronization error as $e = [e_1, e_2]^T$, where $e_1 = x + \overline{x}$, $e_2 = y + \overline{y}$. Our goal is to design appropriate feedback controllers $U(t, x, y) = [u_1(t), u_2(t)]^T$ such that $\lim_{t\to\infty} \|e\| = 0$, where $\|\cdot\|$ is the Euclidean norm.

Feedback controller design

From Equations (2) + (4), we get the error dynamical system as follows

$$\begin{cases}\dot{e}_1 = e_2 - F(x) - F(\overline{x}) + u_1 \\ \dot{e}_2 = -g(x) - g(\overline{x}) + 2h(t) + \delta(t) + u_2\end{cases}. \quad (5)$$

Theorem 1. The drive Liénard system (2) and the response Liénard system (4) can approach anti-synchronization asymptotically with the feedback controllers (6)

$$\begin{cases}u_1(t) = k_1 e_1 \\ u_2(t) = -2h(t) + k_2 e_2\end{cases}, \quad (6)$$

If the following inequality (7) holds

$$\begin{cases}k_1 < -\theta_1 \\ 4k_2(k_1 + \theta_1) > (1 + \theta_2)^2\end{cases}. \quad (7)$$

Proof. Consider the following Lyapunov function:

$$V(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2 + 2)\exp(-\int_0^t |\delta(s)| ds), \quad (8)$$

The time derivative of $V(e_1, e_2)$ is

$$\dot{V} = (e_1\dot{e}_1 + e_2\dot{e}_2)\exp(-\int_0^t |\delta(s)| ds) + \frac{1}{2}(e_1^2 + e_2^2 + 2)\exp(-\int_0^t |\delta(s)| ds)(-|\delta(t)|),$$

$$= \exp(-\int_0^t |\delta(s)| ds)\{e_1[e_2 - F(x) - F(\overline{x}) + k_1 e_1]$$

$$+ e_2[-g(x) - g(\overline{x}) + \delta(t) + k_2 e_2]$$

$$+ [\frac{1}{2}(e_1^2 + e_2^2 + 2)(-|\delta(t)|)]\},$$

$$= \exp(-\int_0^t |\delta(s)| ds)\{[e_1 e_2 - e_1(F(x) + F(\overline{x})) + k_1 e_1^2]$$

$$+ [-e_2(g(x) + g(\overline{x})) + e_2\delta(t) + k_2 e_2^2]$$

$$+ [\frac{1}{2}(e_1^2 + e_2^2 + 2)(-|\delta(t)|)]\}$$

$$\le \exp(-\int_0^t |\delta(s)| ds)[e_1 e_2 + \theta_1 e_1^2 + k_1 e_1^2$$

$$+ \theta_2 |e_1||e_2| + |e_2||\delta(t)| + k_2 e_2^2$$

$$+ \frac{1}{2}(e_1^2 + e_2^2 + 2)(-|\delta(t)|)]$$

$$\le \exp(-\int_0^t |\delta(s)| ds)[(k_1 + \theta_1)e_1^2 + e_1 e_2 + \theta_2 |e_1||e_2| + k_2 e_2^2$$

$$- \frac{1}{2}|\delta(t)| e_1^2 - \frac{1}{2}|\delta(t)|(e_2^2 - 2|e_2| + 2)]$$

$$\le \exp(-\int_0^t |\delta(s)| ds)[(k_1 + \theta_1)e_1^2 + (1 + \theta_2)|e_1||e_2| + k_2 e_2^2$$

$$- \frac{1}{2}|\delta(t)|(e_2 - 1)^2]$$

$$\le \exp(-\int_0^t |\delta(s)| ds)[(k_1 + \theta_1)e_1^2 + (1 + \theta_2)|e_1||e_2| + k_2 e_2^2]$$

$$= \exp(-\int_0^t |\delta(s)| ds)(-\overline{e}^T A \overline{e}), \quad (9)$$

Where $\overline{e} = (|e_1|, |e_2|)^T$,

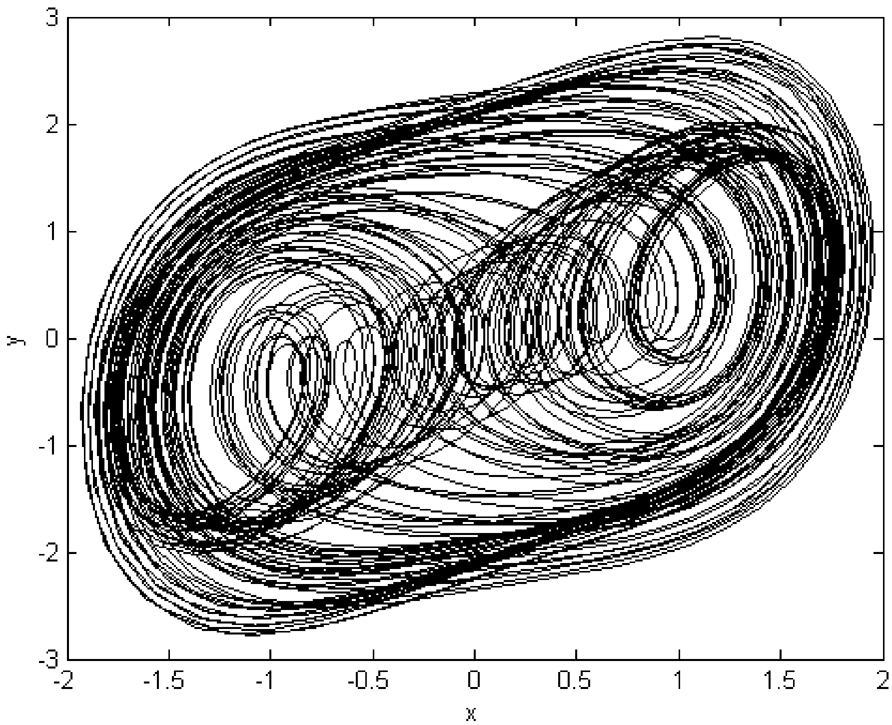

Figure 1. Chaotic attractor of Duffing oscillator system.

$$A=\begin{pmatrix} -(\theta_1+k_1) & -\dfrac{1+\theta_2}{2} \\ -\dfrac{1+\theta_2}{2} & -k_2 \end{pmatrix}. \quad (10)$$

Obviously, A is positive definite if the inequality (7) holds. Thus,

$$\dot{V}\le \exp(-\int_0^t|\delta(s)|ds)(-\bar{e}^T A\bar{e})\le 0, \quad (11)$$

and the error system described by (5) is asymptotic stable. Therefore, the response Liénard system (4) can anti-synchronize the drive Liénard system (2) asymptotically.

Numerical simulations

To verify and demonstrate the effectiveness of the proposed method, we discuss the anti-synchronization of the Duffing oscillator systems which are typical Liénard systems. Consider the Duffing oscillator system

$$\ddot{x}+p\dot{x}+p_1x+x^3=p_2\cos(wt). \quad (12)$$

Let $y=-\int_0^t(p_1x+x^3-p_2\cos(wt))dt$, Duffing oscillator system (12) is equivalent to the following system

$$\begin{cases}\dot{x}=y-px \\ \dot{y}=-(p_1x+x^3)+p_2\cos(wt)\end{cases}. \quad (13)$$

There is a chaotic attractor of Duffing oscillator system (13) as shown in Figure 1 when $p=0.4$, $p_1=-1.1$, $p_2=1.8$, $w=1.8$. We choose Duffing Oscillator system (13) as the drive system, and the response system as follows

$$\begin{cases}\dot{\bar{x}}=\bar{y}-p\bar{x}+u_1 \\ \dot{\bar{y}}=-(p_1\bar{x}+\bar{x}^3)+p_2\cos(wt)+\delta(t)+u_2\end{cases}, \quad (14)$$

where external disturbance $\delta(t)=\dfrac{\cos(2t)}{1+t^2}$ which satisfies $\int_0^{+\infty}|\delta(t)|dt<+\infty$.

By theorem 1, we design the feedback controllers as follows

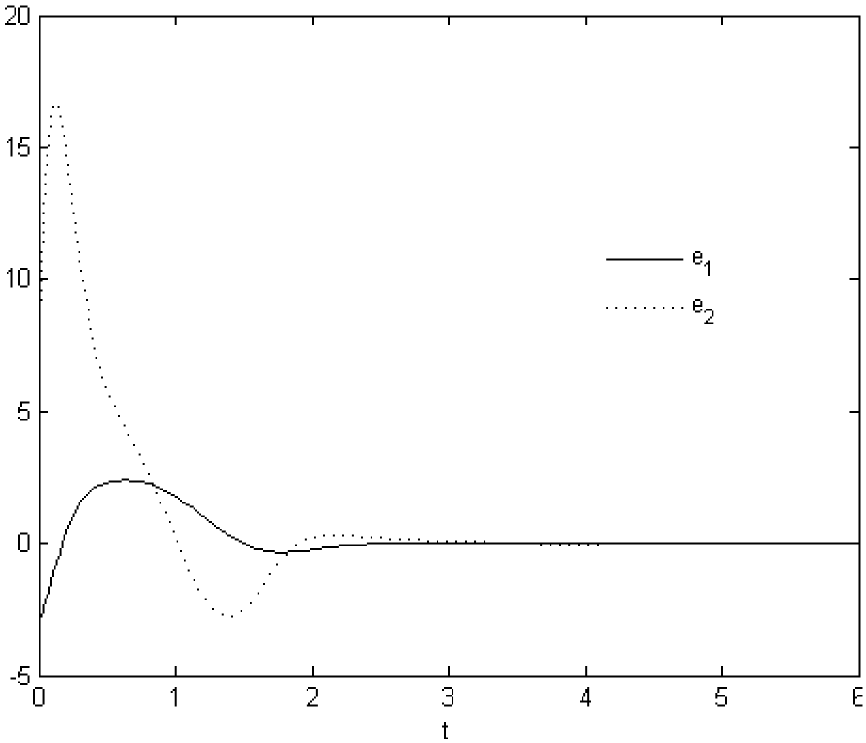

Figure 2. Anti-synchronization errors between Duffing Oscillator systems (13) and (14).

$$\begin{cases} u_1 = -1.5e_1 \\ u_2 = -2p_2\cos(wt) - 2e_2 \end{cases} \tag{15}$$

In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the Duffing oscillator systems with time step size 0.001. We employed the initial conditions $x(0)=3$, $y(0)=4$ and $\overline{x}(0)=-6$, $\overline{y}(0)=3$. Hence, the error system has the initial values $e_1(0)=-3$, $e_2(0)=7$. The simulation results are shown in Figure 2. From Figure 2, we can see that the error vector e converges to zero as $t \to \infty$. This shows that the response Duffing oscillator system (14) can anti-synchronize the drive Duffing oscillator system (13) asymptotically.

CONCLUSIONS

In conclusion, this paper presents a method to design feedback controllers for anti-synchronization of Liénard systems. Based on the Lyapunov stability theory, the feedback controllers and the selection scope of the controllers' parameters for anti-synchronization are designed. According to the simulations, the proposed method can be successfully applied to anti-synchronization problems of Liénard type systems.

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