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Modified particle swarm optimization for probabilistic slope stability analysis

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This paper develops an effective method for a probabilistic analysis of earthen slopes. Typically, the safety of a slope is assessed in terms of a factor of safety based on deterministic methods. However, these methods are unable to handle the uncertainties in the stability analysis. Therefore, the probabilistic approach is used for analyzing the safety of slopes, which is capable to take into account the uncertainties. In this study, for slope safety assessment, the reliability index defined by Hasofer and Lind is employed for estimating the reliability index or probability of failure. The performance function formulated by Spencer's method for general shape of slip surface. To calculate the minimum reliability index and corresponding critical probabilistic slip surface a modified particle swarm optimization is introduced. By this method, reliability of the nonlinear and complex performance function can be evaluated without derivation. The applicability and efficiency of the proposed algorithm are examined by considering a number of published cases and the results indicate the successful working of the new method.

Key words: Particle swarm optimization, slope stability, probabilistic analysis, uncertainty.

INTRODUCTION

Slope stability is one of the important issues in the geotechnical engineering and has been studied extensively for a long time. The stability of natural and man-made slopes has traditionally been analyzed using a deterministic method. In a deterministic procedure, variables are represented by single values. The significant variables involved in the slope stability analysis include the soil strength, soil density, and pure water pressure. Representing these variables by single values implies that the values are predicted with certainty, which the case is seldom. Slope stability problems are characterized by many uncertainties and deterministic methods are unable to account for these uncertainties. The slope may fail even though the factor of safety calculated from a deterministic model is greater than unity. This indicates a need for more objectively structured and quantitative approach toward handling uncertainties involved in the problems. The probabilistic approach is a natural choice for this type of analysis, because it allows for the direct incorporation of uncertainties into the analytical model. In recent years, several attempts have been done to develop a probabilistic slope stability analysis (Christian et al., 1994; Liang et al., 1999; Bhattacharya et al., 2003; Griffiths and Fenton, 2004; Tobutt and Richards, 2005).

The results of probabilistic analysis may be expressed as a probability of failure or reliability index. The most commonly used method to a probabilistic analysis of slope is based on computing the reliability index associated with the critical deterministic slip surface (Vanmarcke, 1977; Chowdhury et al., 1987; Christian et al., 1994). However, the critical deterministic surface with the minimum factor of safety may not be same as the surface with the lowest reliability index or the highest probability of failure (Li and Lumb, 1987; Hassan and Wolff, 1999). The common approach to estimate the reliability index of earth slope is the Mean-Value First-Order Second-Moment (MFOSM) method (Ang and Tang, 1984). In MFOSM, the performance function is expanded about the mean values of the parameters and only the first order terms are kept. Moreover, to calculate the reliability index the partial derivative of performance function is needed. Because the performance function in slope stability analysis is usually implicit, the partial

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derivatives of performance function are frequently approximated numerically (Christian et al., 1994). To overcome the problem of dependence of reliability index on performance function, Hasofer and Lind (1974) proposed an invariant definition of the reliability index. They defined the reliability index β as the minimum distance from the origin in the standard normal space to the limit state surface.

To apply the probabilistic analysis using Hasofer-Lind reliability index (β_{HL}) it is necessary to solve a constraint optimization problem to find the mini-mum reliability index or maximum probability of failure utilize the appropriate optimization technique. During the last decades, several optimization methods have been employed in field of deterministic slope stability analysis and to automate the search for the minimum factor of safety (Greco, 1996; Malkawi et al., 2001; Pham and Fredlund, 2003; Zolfaghari et al., 2005; Cheng et al., 2008; Kahatadeniya et al., 2009). However, very few studies have investigated the application of optimization algorithm to seeking the reliability index and probabilistic stability assessment of slopes (Bhattacharya et al., 2003; Xie et al., 2008).

So far, the most commonly used optimization technique is called gradient algorithm which is based on gradient information. However, the acquisition of gradient information can be costly or even altogether impossible to obtain. But another kind of optimization techniques, known as Evolutionary Algorithm (EA), is not restricted in the aforementioned manner. As a newly developed subset of EA, the particle swarm optimization has demonstrated its many advantages and robust nature in recent decades. It is derived from social psychology and the simulation of the social behavior of bird flocks in particular. Inspired by the swarm intelligence theory, Kennedy created a model which Eberhart then extended to formulate the practical optimization method known as Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1995). The PSO algorithm has some advantages compared with other optimization algorithms.

It is a simple algorithm with only a few parameters to be adjusted during the optimization process, rendering it compatible with any modern computer language. It is also a very powerful algorithm because its application is virtually unlimited. Recently, various researchers have analyzed PSO and experimented with it and many variations were created to further improve the performance of PSO.

In this paper, we propose a Modified Particle Swarm Optimization (MPSO) for minimizing the Hasofer-Lind reliability index (β_{HL}) and determine the critical probabilistic slip surface of earth slope. The proposed algorithm utilized Spencer's method for the formulation of the performance function coupled with the reliability index defined by Hasofer and Lind. The method presented herein is simple but effective to search for the critical probabilistic slip surface in slope stability analysis. Moreover, it can provide a solution to find the critical deterministic slip surface and minimum factor of safety.

DETERMINISTIC SLOPE STABILITY ANALYSIS

The generally adopted approach of deterministic analysis of slopes is the limit equilibrium method of slices. Various methods of slices have been proposed over the years such as Bishop (1955), Morgenstern and Price (1965), Spencer (1967) and Janbu (1973). The essence of these methods is to divide the sliding mass into a finite number of vertical slices to calculate the factor of safety. A number of these methods are applicable to a circular slip surface and satisfy only overall moment equilibrium, such as the ordinary and simplified Bishop's method, while others are applicable to any shape of slip surface and satisfy both moment and force equilibrium, such as Morgenstern and Price, and Spencer's method.

In this study, Spencer's method is used for calculation of the factor of safety and performance function. Spencer (1967) suggested a method by assuming the inter slice force inclination angles of all slices to be equal. In this method moment and force equilibrium will be satisfied simultaneously for any shape of failure surfaces. The method is one of the most accurate methods for estimation of safety factor (Duncan and Wright, 1980). The details of inter slice forces for a typical vertical slice are shown in Figure 1.

The equations of force and moment equilibrium can be respectively written as:

$$Z_R = Z_L + \frac{FS.\, W \sin\alpha - c' b \sec\alpha - W \cos\alpha \tan\varphi'}{\sin(\theta-\alpha) \tan\varphi' - FS \cos(\theta-\alpha)} + \frac{U.b \sec\alpha \tan\varphi' + W K_h (FS - \tan\varphi' \tan\alpha) \cos\alpha}{\sin(\theta-\alpha) \tan\varphi' - FS \cos(\theta-\alpha)} \quad (1)$$

$$h_R = \frac{Z_L}{Z_R} h_L - \frac{Z_L.b}{Z_R.2} \tan\alpha + \frac{Z_L.b}{Z_R.2} \tan\theta + \frac{b}{2} \tan\theta - \frac{b}{2} \tan\alpha - \frac{h_a k_h W}{Z_R \cos\theta} \quad (2)$$

Trial and error process is used to determine the factor of safety. By changing the value of *FS* and θ, the safety factor is defined through an iterative process until both force and moment equilibrium will be satisfied. The iterative procedure is completed when the difference between computed values of Z_R and h_R is within an acceptable tolerance from the known values of Z_R and h_R at the boundaries.

PROBABILISTIC SLOPE STABILITY ANALYSIS

In general, the factor of safety is not a consistent measure of risk. Slopes with the same value of *FS* may exist at different risk levels depending on the variation of soil properties. It is incomplete to quantify how much safer a slope becomes as the factor of safety is increased (Li and Lumb, 1987; Christian et al., 1994) because various uncertainties are not considered. As a result, there has been an attempt in recent years to use probabilistic techniques for analyzing the safety of soil slopes when natural variability and uncertainty inherent in soil parameters are considered. One advantage of working within a probabilistic framework is that

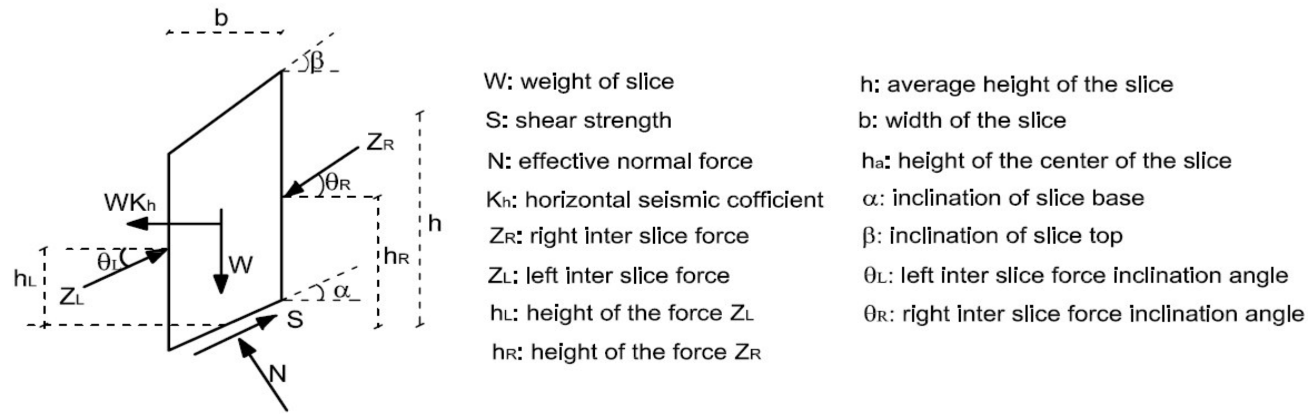

Figure 1. Forces acting on a typical slice in Spencer's method.

various soil parameter uncertainties can be considered rationally.

Two approaches can be used for probabilistic analysis of earth slope. The first approach is based on calculation of the probability of failure corresponding to the critical deterministic surface with minimum factor of safety. However, the surface of the minimum factor of safety may not be the surface of the maximum probability of failure (Hassan and Wolff, 1999). The second approach is based on the determination of the critical probabilistic surface that is associated with the highest probability of failure or the lowest reliability (Li and Lumb, 1987; Bhattacharya et al., 2003; Cho, 2007).

The problem of the probabilistic analysis is formulated by a vector, $X=[X_1,X_2,X_3,\ldots,X_n]$, representing a set of random variables. From the uncertain variables, a performance function $g(X)$ is formulated to describe the limit state in the space of X. The performance function divides the vector space X in to two distinct regions. The safety region for $g(X)>0$ and the failure region for $g(X)<0$, while the limit state surface is $g(X)=0$. The performance function for the slope stability is a function of the factor of safety (*FS*) usually defined as:

$$g(X) = FS - 1 \tag{3}$$

The probability of failure of the slope can be expressed in terms of the performance function by the following integral:

$$P_f = P[g(X \leq 0)] = \int_{g(X)\leq 0} f_X(X)\, dX \tag{4}$$

where $f_X(X)$ represents the joint probability density function of the vector of random variables and the integral is carried out over the failure domain. Consequently, the probability of safe performance, or the reliability of the slope is given by:

$$\text{Reliability} = P[g(X) > 0] = 1 - P_f \tag{5}$$

The performance function, as defined by Equation (3), is a function of several random variables. To determine the reliability (or probability of failure) the probability density function of the performance function must be evaluated. This requires multiple integration of the joint probability density function of the random variables over the entire safe (or failure) domain. The joint probability density function of the random variables is generally not well defined and the performance function is very often implicit. Hence, evaluating the probability density function of the performance function is often not possible. In addition, even if the joint probability density function of the random variables can be specified, the multi-dimensional integral in Equation (4) cannot usually be solved analytically and numerical approaches are often required to find the solution. Therefore, the most effective applications of probability theory to the analysis of slope stability have stated the uncertainties in the form of a reliability index (β). The reliability index provides more information and is a better indication of the stability of a slope than the factor of safety alone because it incorporates information of the uncertainty in the values of the performance function. It also provides a good comparative measure of safety; slopes with higher β are considered safer than slopes with lower β.

Depend on the form of the performance function several definitions of the reliability index exist. Hasofer and Lind (1974) proposed an invariant definition of the reliability index as the minimum distance from the origin in the standard normal space to the limit state surface. This distance is defined as β_{HL} and the approach is referred to as the Advanced First-Order Second-Moment (AFOSM) reliability method. The closest point on the failure surface is said to be the design point or the Most Probable Point (MPP) of failure (Figure 2). To determine the H-L reliability index (β_{HL}), all the random variables $\boldsymbol{X}$ should be transformed into a standard normal space $\boldsymbol{U}$, by an orthogonal transformation such that:

$$u_i = (x_i - \mu_i) / \sigma_i \tag{6}$$

where μ_i and σ_i represent the mean and the standard deviation of x_i, respectively. The mean and standard deviation of the standard normally distributed variable, u_i, are zero and unity, respectively. Based on the transformation of Equation (6), the mean value point in the original space ($\boldsymbol{X}$-space) is mapped into the origin of the normal space ($\boldsymbol{U}$-space). The failure surface $g(\boldsymbol{X})=0$ in $\boldsymbol{X}$-space is mapped into the corresponding failure surface $g(\boldsymbol{U})=0$ in $\boldsymbol{U}$-space, as shown in Figure 2. The reliability index is the shortest distance from the origin to the failure surface. The matrix formulation of the Hasofer-Lind reliability index (β_{HL}) is defined in the following form (Ditlevsen, 1981; Low and Tang, 1997):

$$\beta_{HL} = \min_{X\in F} \sqrt{(X-\mu)^T \boldsymbol{C}^{-1} (X-\mu)} \tag{7}$$

or, equivalently:

$$\beta_{HL} = \min_{X\in F} \sqrt{\left[\frac{x_i-\mu_i}{\sigma_i}\right]^T \left[\boldsymbol{R}\right]^{-1} \left[\frac{x_i-\mu_i}{\sigma_i}\right]} \tag{8}$$

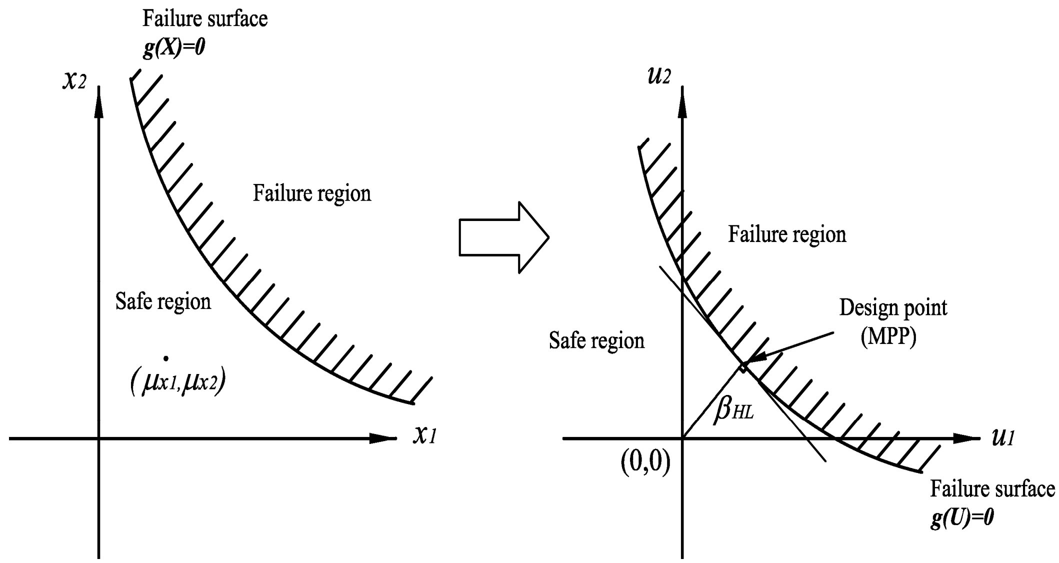

Figure 2. The geometrical representation of the definition of the reliability index.

where X is a vector representing the set of random variables x_i, F is the failure domain, μ is the vector of mean values μ_i, σ_i is the standard deviation, C is the covariance matrix and R is the correlation matrix. Mathematically, $R=[\rho_{ij}]$ $(i,j=1,2,\ldots,n)$ is a square matrix that contains the correlations among a set of n random variables. As mentioned by Low and Tang (1997) using Equation (8) is preferred to Equation (7) because the correlation matrix $\boldsymbol{R}$ is easier to set up, and conveys the correlation structure more explicitly than the covariance matrix $\boldsymbol{C}$. Although, the correlation coefficient among two random variables has a range $-1<\rho_{ij}<1$, one is not totally free in assigning any values within this range for the correlation matrix. It must be emphasized that the correlation matrix has to be positive definite (Low and Tang, 2004).

As mentioned before, the H-L reliability index (β_{HL}) is defined as the minimum distance from the origin of the axis in the standard normal space to the limit state surface. To evaluate β_{HL} the following constrained optimization problem should be solved:

$$\begin{aligned} &Minimize\ \beta_{HL} \\ &Subject\,to\ \ g(\boldsymbol{U})=0 \end{aligned} \tag{9}$$

Solve Equation (9) is equivalent to solve the relaxed form obtained by penalty method as:

$$\text{Minimize } \beta_{HL} + r\,|\,g(\boldsymbol{U})|^{l} \tag{10}$$

The parameters r and l are problem dependent, and r should be a suitably large positive constant. In the present study, the values set for r and l were 1000 and 2, respectively. The solution of the above optimization problem is the design point or MPP in the standardized normal space. Then, the probability of failure (P_f) can be estimated from the reliability index using the established equation as:

$$P_f = 1 - \Phi\,(\beta_{HL}) = \Phi\,(-\beta_{HL}) \tag{11}$$

where Φ is the standard normal cumulative distribution function. Several algorithms have been recommended for the solution of optimization problem in Equation (10) (Liu and Kiureghian, 1991). In the current study a new approach of particle swarm optimization is proposed for the solution.

MODIFIED PARTICLE SWARM OPTIMIZATION

Particle swarm optimization is a population based stochastic optimization method. It explores for the optimal solution from a population of moving particles, based on a fitness function. Each particle represents a potential answer and has a position (X_i^k) and a velocity (V_i^k) in the problem space. Each particle keeps a record of its individual best position (P_i^k) and global best position (P_g^k). The new velocity and position of particle will be updated according to the following equations (Shi and Eberhart, 1998):

$$V_i^{k+1} = w \times V_i^k + c_1 \times r_1 \times (P_i^k - X_i^k) + c_2 \times r_2 \times (P_g^k - X_i^k) \quad i=1,2,\ldots,N \tag{12}$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{13}$$

where w is an inertia weight that controls a particle's exploration during a search, c_1 and c_2 are positive numbers illustrating the weight of the acceleration terms that guide each particle toward the individual best and the swarm best positions respectively, r_1 and r_2 are uniformly distributed random numbers in (0, 1), and N is the number of particles in the swarm. The inertia weighting function in Equation (12) is usually calculated using following equation:

$$w = w_{max} - (w_{max} - w_{min}) \times k\,/\,G \tag{14}$$

where w_{max} and w_{min} are maximum and minimum value of w, G is the maximum number of iteration and k is the current iteration number. As the PSO's equations reveal, unlike the traditional model based optimization algorithms like Newton's method, the PSO algorithm does not need a mathematical model of the problem. The only information required by the PSO to search for the optimum solution is the evaluation of fitness function.

This study proposes a MPSO to search for the minimum reliability index and probabilistic slope stability analysis. To improve the performance of basic PSO, the inertia weight (w) and acceleration

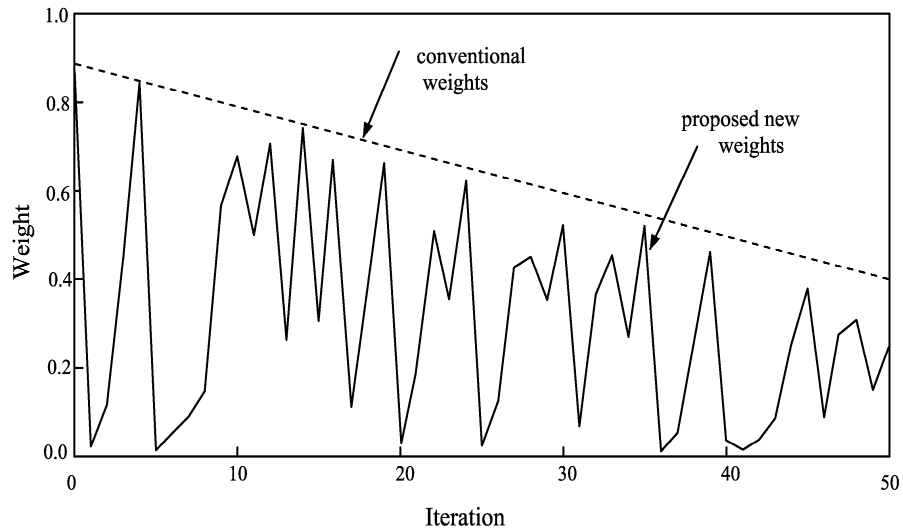

Figure 3. Comparison of the conventional and new inertia weight.

coefficients (c_1 and c_2) in Equation (12) will be defined according to the following descriptions.

In the first step, the application of chaotic sequences is introduced to improve the global seeking ability and prevent the early convergence to local minima. One of the dynamic systems showing a chaotic manner is logistic map (Caponetto et al., 2003), whose equation is described as follows:

$$f_k = \mu \,.\, f_{k-1} \,.\, (1 - f_{k-1}) \qquad (15)$$

where k is the iteration number, μ is a control parameter and has a real value in the range of [0, 4] and $0 \le f_0 \le 1$. Despite the apparent simplicity of the equation, the solution exhibits a rich variety of behaviors. The behavior of the system represented by Equation (15) is greatly changed with the variation of μ. Equation (15) is deterministic, displaying chaotic dynamics and generates chaotic evolutions when μ=4.0 and $f_0 \notin \{0, 0.25, 0.5, 0.75, 1\}$.

In the current study, the new equation for inertia weight is defined as multiplying Equation (14) by Equation (15) in order to improve the global searching capability as follows:

$$w_{new} = w \,.\, f \qquad (16)$$

As it is presented in Figure 3, although, the conventional inertia weight decreases monotonously from w_{max} to w_{min}, the new inertia weight decreases and oscillates simultaneously for total iteration when μ=4.0 and f_0=0.55.

In the second step, to reduce the cognitive component and increases the social component the time-varying acceleration coefficient is proposed. At the beginning of optimization procedure, with a large value of c_1 and a small value of c_2, particles are allowed to move around the search space, instead of moving toward p_{best}. Moreover, with a small value of c_1 and a large value of c_2 allow the particles converge to the global optima in the latter part of the optimization. Over the generations time-varying acceleration coefficients are evaluated according to the following equations:

$$c_1 = c_{1max} - (c_{1max} - c_{1min}) \times k / G \qquad (17)$$

$$c_2 = c_{2max} - (c_{2max} - c_{2min}) \times k / G \qquad (18)$$

In Equations (17) and (18), c_{1min} and c_{2min} are the minimum values of the acceleration coefficient and c_{1max} and c_{2max} are the maximum values of the acceleration coefficient, respectively. G and k were described in Equation (14). One of the major advantages of the proposed MPSO is its premature convergence, especially while handling problems with more local optima and heavily constrained.

TEST PROBLEMS AND RESULTS

This section investigates the validity and effectiveness of the proposed algorithm to probabilistic slope stability analysis. The procedure has been carried out using a computer program was developed in MATLAB. All the programs were executed on a 2.10 GHz Pentium IV processor with 2GB of Random Access Memory (RAM). The program searches for the most critical deterministic and probabilistic slip surface. The factor of safety has been defined based on the Spencer's method for general shape of slip surface. To calculate the reliability or probability of failure, the Hasofer-Lind reliability index (β_{HL}) is evaluated using modified particle swarm optimization. Based on above explanation, the implementation procedure of the proposed MPSO for the reliability analysis of the earth slope is constructed as follows:

1. Initialize a set of particles positions and velocities randomly distributed throughout the design space bounded by specified limits.
2. Evaluate the objective function values using Equation (10) for each particle in the swarm.
3. Update the optimum particle position at current iteration and global optimum particle position.
4. Update the velocity vector as specified in Equation (12) and update the position of each particle according to Equation (13).
5. Repeat steps 2–4 until the stopping criteria is met.

To verify and assess the applicability of the proposed algorithm to search for the minimum reliability index and associated probabilistic slip surface the following benchmark problems were selected from the literature. In the following problems, the random variables are supposed to be described statistically by a lognormal distribution defined by a mean μ_X and a standard devia-tion σ_X. The lognormal distribution ranges between zero and infinity, skewed to the low range, and is therefore particularly suited for parameters that cannot take on negative values. The mean and standard deviation of the underlying normal distribution of *lnX* are then given by the following equations (Fenton and Griffiths, 2008):

$$\sigma^2_{\ln X} = \ln(1+\frac{\sigma^2_X}{\mu^2_X}) \tag{19}$$

$$\mu_{\ln X} = \ln(\mu_X) - 0.5\,\sigma^2_{\ln X} \tag{20}$$

Moreover, probabilistic analysis requires estimating or assuming the correlation coefficients between random variables. The parameters that might be correlated are friction angle, unit weight and cohesion. In the following cases, the cohesion and friction angle are assumed to be negatively correlated with each other ($\rho_{c\varphi}$=-0.3) and positively correlated with unit weight ($\rho_{c\gamma}$= $\rho_{\varphi\gamma}$=0.5) in accordance with the reported values in literature (Chowdhury and Xu, 1992; Low and Tang, 1997).

To calculate the minimum value of β_{HL} using proposed MPSO the parameters of the algorithm should be adopted accurately. The parameters that may affect the performance of the algorithm include maximum and minimum values of inertia weight (w_{max} and w_{min}), maximum and minimum values of acceleration coefficient (c_{max} and c_{min}), control parameter (μ), f_0, and swarm size (N). In our study, proper fine tuning of these parameters was obtained utilizing several experimental studies examining the effect of each parameter on the final solution and convergence of the algorithm. As a result, a population of 40 individuals was used; w_{max} and w_{min} were chosen as 0.9 and 0.4 respectively; c_{1max} and c_{2max} were selected equal to 2.5 and c_{1min} and c_{2min} were selected equal to 0.5. The control parameter (μ) was set to 4.0 and f_0 is considered as 0.55. Finally, a fixed number of maximum iteration (G) of 3000 was applied. The optimization procedure was terminated when one of the following stopping criteria was met: (i) the maximum number of generations is reached; (ii) after a given number of iterations, there is no significant improvement of the solution.

Test problem 1: Application to a homogeneous slope

Figure 4 shows the geometry of a slope in homogeneous soil. The parameters considered as random variables in the probabilistic analysis are: the effective friction angle, effective cohesion, unit weight and pore water pressure ratio. Table 1 presents the mean values and standard deviation associated with each random variable.

The problem was previously solved by Li and Lumb (1987), Hassan and Wolff (1999) and Bhattacharya et al. (2003). Li and Lumb (1987) determined the reliability index using Hasofer and Lind. For the formulation of safety factor and performance function, Li and Lumb (1987) used Morgenstern and Price method with the assumption of *f(x) =1,* which is equivalent to the Spencer method. Hassan and Wolff (1999) calculated the reliability index corresponding to the searched critical probabilistic surface using the MFOSM method assuming a lognormal distribution for the factor of safety. The method was used in their study to evaluate the safety factor was Spencer's method (Spencer, 1967) for circular and non circular slip surface. Finally, Bhattacharya et al. (2003) solved the problem with the same methodology as Hassan and Wolff (1999). They used Spencer's method in conjunction with the direct search method for determination of minimum reliability index. Note that different limit equilibrium methods will give different accuracy and will usually give different factors of safety even for the same slope. Therefore, in this study for the sake of right comparison the Spencer's method is applied to calculate the factor of safety and performance function.

The results of the proposed method and previous studies are summarized in Table 2. Moreover, to verify the accuracy and efficiency of the proposed MPSO, the result of standard PSO is also presented. In Table 2, FS_{min} and β_{FS} are the minimum factor of safety and the reliability index associated with the critical deterministic slip surface, β_{min} and FS_β are the minimum reliability index and the factor of safety corresponding to the critical probabilistic slip surface.

According to analyzing the results of this table, it can be observed that, the minimum reliability index calculated using presented method is 2.205, which is lower than the values reported by Li and Lumb (1987); Hassan and Wolff (1999), Bhattacharya et al. (2003), and also standard PSO (2.214). Further, the minimum factor of safety calculated from a deterministic analysis based on the mean values of the soil properties obtained by MPSO is 1.301, which is lower than 1.326 reported by Bhattacharya et al. and slightly lower than 1.309 achieved by PSO. The values of βFS and FSβ are also comparable with the reported values. The corresponding critical deterministic and the critical probabilistic slip surfaces are also presented in Figure 4. As it can be seen, two surfaces are located reasonably close to each other as expected in a homogeneous slope. It is because of the proximity of the values of β_{FS} and β_{min} presented in Table 2. The failure surfaces reported by previous researchers are also similarly located.

To further validate the reliability of the results, statistical analysis of the data obtained from the 50 independent runs is carried out. Table 3 presents a comparison of the results evaluated by MPSO and PSO in terms of the mean (average), best (minimum), worst (maximum), standard deviation, average number of iterations and

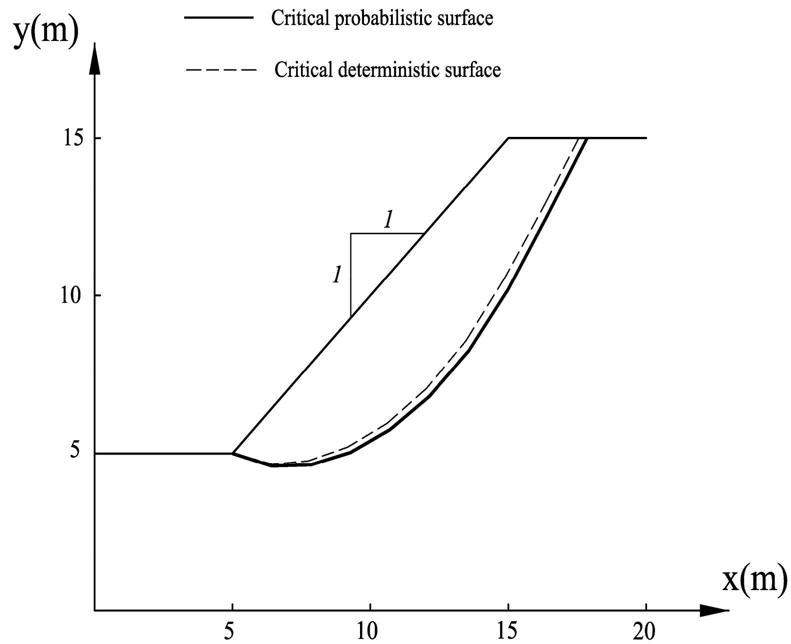

Figure 4. Cross section of homogeneous slope-test problem 1.

Table 1. Statistical properties of soil parameters-test problem 1.

Random variable	Mean	Standard deviation	Distribution
c'	18.0 kN/m^2	3.6 k N/m^2	Log-normal
$\tan \varphi'$	tan 30	0.0577	Log-normal
γ	18.0 kN/m^3	0.9 k N/m^3	Log-normal
r_u	0.2	0.02	Log-normal

Table 2. Results comparison-test problem 1.

Method	β_{FS}	β_{min}	FS_{min}	FS_{β}
Li and Lumb (1987)	-	2.5	-	-
Hassan and Wolff (1999)	2.336	2.293	-	-
Bhattacharya et al. (2003)	2.306	2.239	1.326	1.337
Present study (PSO)	2.295	2.214	1.309	1.319
Present study (MPSO)	2.286	2.205	1.301	1.309

average elapse time. Standard deviation can be used to obtain the stability of each algorithm. To compare the accuracies of the algorithms, a maximum number of iteration is considered as a stopping condition and the results obtained from the algorithms are compared. Each algorithm was run 50 times and the average elapsed time is considered as a measure of the computational time. As it can be seen from Table 3, the standard deviation for MPSO is 0.0019 which is lower than that evaluated for PSO (0.0028). The low standard deviation for the proposed method proves the consistency of the method. Moreover, Table 3 shows that the proposed algorithm requires far fewer iterations and computation time when compared with PSO. Hence, it can be concluded that the MPSO is better than PSO in terms of accuracy and convergence speed.

Test problem 2: Application to a non homogeneous slope

Figure 5 shows the cross section and geometry of a two

Table 3. Statistical comparison of the minimum reliability index obtained by PSO and MPSO.

Problem		Best	Mean	Worst	Standard deviation	Mean number of iterations	Mean time
Test problem 1	PSO	2.212	2.214	2.218	0.0028	2780	61
	MPSO	2.202	2.205	2.210	0.0019	920	20
Test problem 2	PSO	2.771	2.775	2.780	0.0036	2850	74
	MPSO	2.767	2.770	2.772	0.0014	1320	29
Test problem 3	PSO	2.657	2.660	2.664	0.0031	2230	63
	MPSO	2.654	2.657	2.664	0.0018	1240	39

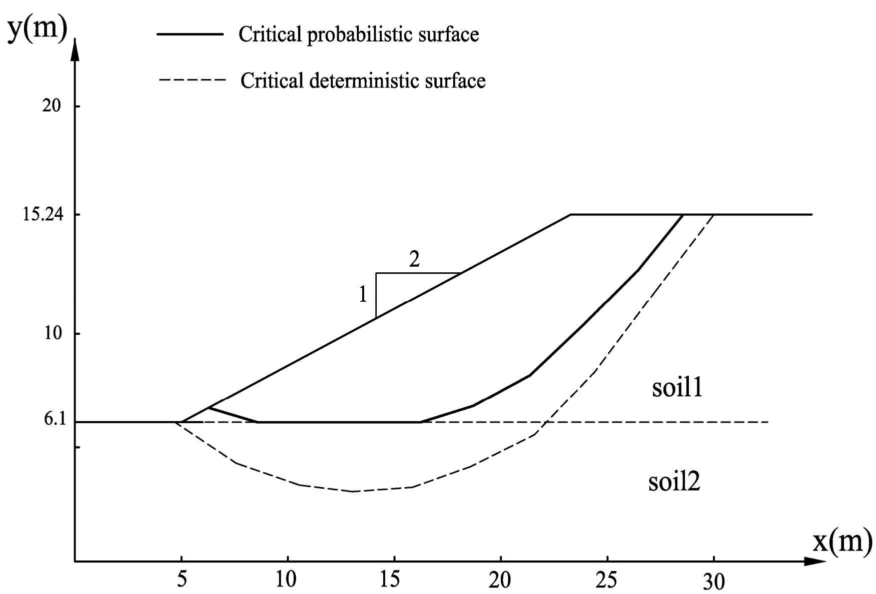

Figure 5. Cross section of non homogeneous slope-test problem 2.

layered slope in clay bounded by a hard layer below and parallel to the ground surface. The soil strength parameters that are related to the stability of slope, including friction angle φ, and cohesion c, are considered as random variables. The statistical moments (mean value and standard deviation) of the parameters are summarized in Table 4.

This example was also solved previously by Hassan and Wolff (1999) and Bhattacharya et al. (2003) in terms of FS_{min} , β_{FS} , β_{min} and FS_{β}. The methodology was used in their research illustrated in test problem 1. The results obtained from the current study together with a comparison of those reported by previous researchers are summarized in Table 5. From the results shown in this table, it can be considered that the minimum reliability index evaluated using MPSO is 2.770, which is almost lower than those reported by Hassan and Wolff (1999), Bhattacharya et al. (2003) and also PSO. The minimum factor of safety obtained by MPSO is found to be smaller than the others. The corresponding critical deterministic and the critical probabilistic slip surfaces are presented in Figure 5. In accordance with the difference in the values of β_{FS} and β_{min} presented in Table 5, the two surfaces are located significantly separate.

A statistical comparison between the results obtained by the presented method and also PSO is presented in Table 3. As it can be derived from Table 3, MPSO demonstrates the highest stability as shown in the standard deviation. Moreover, statistical comparison of the results presented in Table 3 indicates that MPSO has a very fast convergence rate in the early iterations and performed significantly better than PSO.

Test problem 3: Application to a case study of the Cannon Dam

The probabilistic analysis for the end-of-construction

Table 4. Statistical properties of soil parameters-test problem 2.

Material	Parameter	Mean	Standard deviation	Distribution
Soil 1	c_1	38.31 kN/m^2	7.662 kN/m^2	Log-normal
	φ_1	0	-	Log-normal
Soil 2	c_2	23.94 kN/m^2	4.788 kN/m^2	Log-normal
	φ_2	12	1.2	Log-normal

Table 5. Results comparison-test problem 2.

Method	β_{FS}	β_{min}	FS_{min}	FS_{β}
Hassan and Wolff (Hassan and Wolff, 1999)	4.442	2.869	1.663	-
Bhattacharya et al (Bhattacharya et al., 2003)	5.064	2.861	1.665	1.797
Present study (PSO)	4.545	2.775	1.655	1.784
Present study (MPSO)	4.536	2.770	1.649	1.780

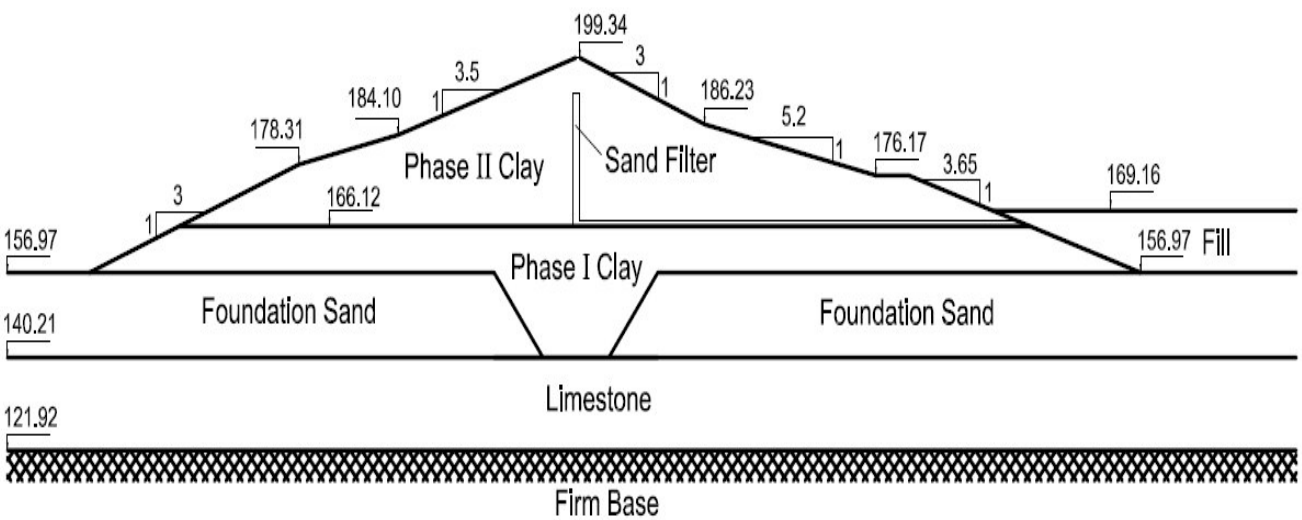

Figure 6. Cross section of Cannon Dam- test problem 3.

stage of the Cannon Dam reported in Hassan and Wolff (1999) will be investigated. A typical cross section of the dam showing the soil profile is presented in Figure 6. The structure consists of two zones of compacted clay: Phase I and Phase II fill over layers of sand and limestone. Strength parameters of the two clay layers were considered as random variables (c_1, φ_1, c_2, φ_2). Table 6 shows the mean value and standard deviation for these parameters based on UU tests of recorded samples from the embankment. Hassan and Wolff (1999) made no reductions in the variance for spatial correlation, in their study it was conservatively assumed that variance over a failure zone was potentially as large as the point variance.

The minimum reliability index and factor of safety corresponding to critical probabilistic and deterministic slip surface obtained by different methods are shown in Table 7. Moreover, the critical probabilistic surface determined by MPSO passes through the Phase I fill, as shown in Figure 7.

As derived from Table 7, the factor of safety evaluated by the presented analysis method (2.593) is slightly lower than both the value of 2.647 achieved by Hassan and Wolff (1999) and that of 2.612 by Bhattacharya et al. (2003) and is significantly lower than that reported by Hassan and Wolff (1999) for circular slip surface. As it can be derived from Table 7, reliability index evaluated herein is lower than those reported in the previous study and also PSO. Table 3 presents a performance comparison of the algorithms for evaluating the minimum

Table 6. Statistical properties of soil parameters-test problem 3.

Material	Parameter	Mean	Standard deviation	Correlation coefficient
Phase I Fill	c_1	117.79 kN/m^2	58.89 kN/m^2	+0.10
	φ_1	8.5	8.5	
Phase II Fill	c_2	143.64 kN/m^2	79 kN/m^2	-0.55
	φ_2	15	9	

Table 7. Results comparison-test problem 3.

Method	β_{FS}	β_{min}	FS_{min}	FS_{β}
Hassan and Wolff (1999)	7.028	2.664	2.647	-
Bhattacharya et al. (2003)	3.695	2.674	2.612	2.98
Present study (PSO)	3.223	2.660	2.595	1.782
Present study (MPSO)	3.216	2.657	2.593	1.78

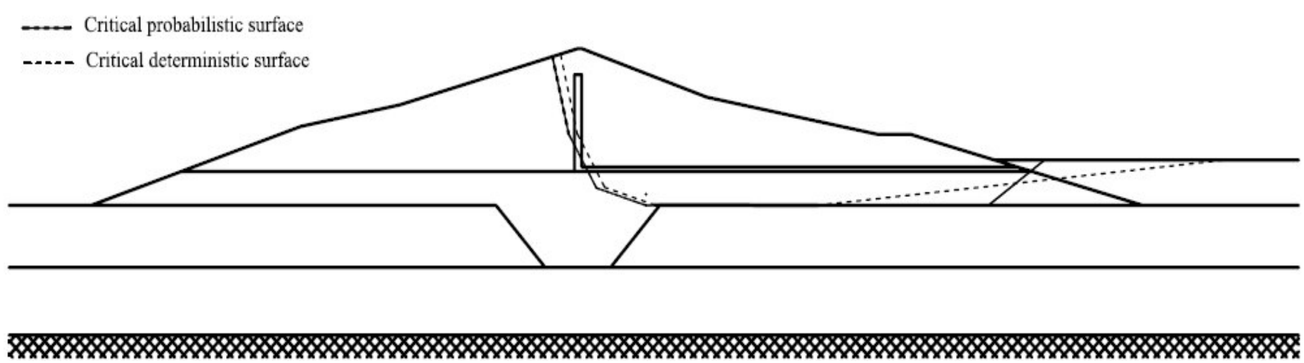

Figure 7. critical failure surface of Cannon Dam-test problem 3.

reliability index. As can be seen from Table 3, the very low value of the standard deviation for the MPSO method proves its stability and accuracy in producing the optimal solution. As well as generating superior results, the MPSO had a very fast convergence rate compared with PSO.

Conclusions

This paper outlines a procedure of probabilistic analysis of earth slope. The detailed development of the search procedure for locating the critical probabilistic failure surfaces presented herein, is based on the AFOSM method as the reliability model and the Spencer's method of slices as the slope stability model. The procedure can be applied to the analysis of the stability of a general slip surface. The Hasofer-Lind reliability index (β_{HL}) is used instead of the conventional reliability index β. The problem of searching the critical probabilistic surface with the minimum reliability index, β_{min}, can be formulated as an optimization problem and a modified particle swarm optimization is proposed for the solution. Despite the modification presented in the current study to the original PSO is not major, the advantages of the proposed modification are obviously demonstrated through some numerical problems. The described framework has been coded in MATLAB and used to carry out parametric studies for the numerical problems. The method does not make any assumptions relating to the geometry of the failure surface and can be applied to any complex slope geometry, layering and pore pressure conditions. The applicability of the proposed methodology developed herein, has been examined over a variety of slope stability problems from the literature. A comparison of the results show that in all cases the minimum reliability index obtained by MPSO is reasonably lower than those reported in the literature. Moreover, the new method has a very fast convergence rate in the early iterations and performed significantly better than standard PSO. From the probabilistic analysis point of view, the results show that, the critical failure surface which has minimum factor of safety value is not a surface at which the probability of failure is always the maximum. Therefore, the deterministic critical surface is not always an actual failure surface and factor of safety does not and cannot scale safety. Further as illustrated trough the test problems; the critical probabilistic and deterministic slip surface is almost close for slope in a homogenous soil, whereas these surfaces are located quite separate for non homogenous slopes.

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