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Full Length Research Paper

A two-step Laplace decomposition method for solving nonlinear partial differential equations

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The Adomian decomposition method (ADM) is an analytical method to solve linear and nonlinear equations and gives the solution a series form. Two-step Adomian decomposition method (TSADM) is a modification on ADM and makes the calculations much simpler. In this paper we combine Laplace transform and TSADM and present a new approach for solving partial differential equations.

Key words: Two-step Adomian decomposition method, Laplace decomposition method, Adomian decomposition method.

Mathematics subject classification: 47J30, 49S05.

INTRODUCTION

Most of phenomena in nature are described by nonlinear differential equations. So scientists in different branches of science try to solve them. But because of nonlinear part of these groups of equations, finding an exact solution is not easy. Different analytical methods have been applied to find a solution to them. For example, Adomian (1986, 1988, 1989, 1990, 1991, 1994a, b) has presented and developed a so-called decomposition method for solving algebraic, differential, integro-differential, differential-delay and partial differential equations. Recently, a modification of ADM was proposed by Wazwaz (1999). The modified decomposition method needs only a slight variation from the standard ADM and has been shown to be computationally efficient. We consider the following equation:

$$u = f + L^{-1}(R \quad u) + L^{-1}(N \quad u).$$

The modified decomposition method was established based on the assumption that the function f can be divided into two parts and the success of the modified method depends on the proper choice of the parts f_1 and f_2. The TSADM (Lou, 2005) over comes this difficulty and explains how we can choose f_1 and f_2 properly without having noise term (Adomian and Race, 1992). Using Laplace transform in ADM (LDM) proposed by Khuri (2001, 2004) for the approximate solution of a class of nonlinear ordinary differential equations, other scientists have used this method for solving some important equations (Yusufoglu, 2006; Elgazery, 2008; Kiymaz 2009; Khan and Gondal, 2010; Hussain and Khan 2010).

In this work we are interested to use Laplace transform in TSADM (LTSDM). We illustrate this method with the help of several examples and compare LDM and LTSDM with each other.

DESCRIPTION OF LTSDM

Here,the purpose is to discuss the use of Laplace transform algorithm in TSADM for solving different

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equations. We consider general inhomogeneous nonlinear equation with initial conditions given below:

$$Lu + Ru + Nu = h(x,t), \tag{1}$$

where L is the highest order derivative which is assumed to be easily invertible, R is a linear differential operator of order less than L, Nu represents the non-linear terms and $h(x,t)$ is the source term. First we explain the main idea of LDM:

The methodology consists of applying Laplace transform on both sides of Equation (1)

$$\pounds[Lu(x,t)] + \pounds[Ru(x,t)] + \pounds[Nu(x,t)] = \pounds[h(x,t)]. \tag{2}$$

Using the differential property of Laplace transform and initial conditions we get

$$s^n \pounds[u(x,t)] - s^{n-1}u(x,0) - s^{n-2}u'(x,0) - \dots - u^{n-1}(x,0)$$

$$+ \pounds[Ru(x,t)] + \pounds[Nu(x,t)] = \pounds[h(x,t)]. \tag{3}$$

$$\pounds[u(x,t)] = \frac{u(x,0)}{s} + \frac{u'(x,0)}{s^2} + \dots + \frac{u^{n-1}(x,0)}{s^n}$$

$$-\frac{1}{s^n}\pounds[Ru(x,t)] - \frac{1}{s^n}\pounds[Nu(x,t)] + \frac{1}{s^n}\pounds[h(x,t)]. \tag{4}$$

The next step is representing solutions as an infinite series

$$u(x,t) = \sum_{i=0}^{\infty} u_i(x,t), \tag{5}$$

and the nonlinear operator is decomposed as

$$Nu(x,t) = \sum_{i=0}^{\infty} A_i, \tag{6}$$

where A_n is Adomian polynomial (Wazwaz, 2002a) of $u_0, u_1, \dots u_n$ and can be calculated by the formula

$$A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \qquad i = 0,1,2,\dots, \tag{7}$$

Substitution of Equations (5) and (6) in Equation (4) yields

$$\pounds\left[\sum_{i=0}^{\infty} u_i(x,t)\right] = \frac{u(x,0)}{s} + \frac{u'(x,0)}{s^2} + \dots + \frac{u^{n-1}(x,0)}{s^n}$$

$$-\frac{1}{s^n}\pounds[Ru(x,t)] - \frac{1}{s^n}\pounds\left[\sum_{i=0}^{\infty} A_i\right] + \frac{1}{s^n}\pounds[h(x,t)], \tag{8}$$

On comparing both sides of Equation (8) and by using standard ADM we have:

$$\pounds[u_0(x,t)] = \frac{u(x,0)}{s} + \frac{u'(x,0)}{s^2} + \dots + \frac{u^{n-1}(x,0)}{s^n} + \frac{1}{s^n}\pounds[h(x,t)] = k(x,s) \tag{9}$$

$$\pounds[u_1(x,t)] = -\frac{1}{s^n}\pounds[Ru_0(x,t)] - \frac{1}{s^n}\pounds[A_0], \tag{10}$$

$$\pounds[u_2(x,t)] = -\frac{1}{s^n}\pounds[Ru_1(x,t)] - \frac{1}{s^n}\pounds[A_1]. \tag{11}$$

In general, the recursive relation is given by

$$\pounds[u_{i+1}(x,t)] = -\frac{1}{s^n}\pounds[Ru_i(x,t)] - \frac{1}{s^n}\pounds[A_i], \qquad i \ge 0. \tag{12}$$

Applying inverse Laplace transform to Equations (9 to 12), our required recursive relation is given by

$$u_0(x,t) = G(x,t), \tag{13}$$

$$[u_{i+1}(x,t)] = -L^{-1}\left[\frac{1}{s^n}\pounds[Ru_i(x,t)] + \frac{1}{s^n}\pounds[A_i]\right] \qquad i \ge 0,$$

where $G(x,t)$ represents the term arising from source term and prescribed initial conditions.

Now we illustrate TSADM. By applying the inverse operator L^{-1} to $h(x,t)$ and using the given conditions we have:

$$\varphi = \phi + L^{-1}(h(x,t)), \tag{14}$$

where the function φ indicates the terms arising from using the given conditions, all are assumed to be prescribed. For using TSADM we set

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2 + \dots + \varphi_m, \tag{15}$$

where $\varphi_0, \varphi_1, \dots, \varphi_m$ are the terms arising from applying inverse operator on the source term $h(x,t)$ and using the given conditions. We define

$$u_0 = \varphi_k + \dots + \varphi_{k+s}, \tag{16}$$

where $k = 0,1,\dots,m$, $s = 0,1,\dots,m-k$. Then we verify that u_0 satisfies the original Equation (1) and given conditions by substituting. Once the exact solution is obtained we are done. Otherwise, we go to step two. In second step we set $u_0 = \varphi$ and continue with the standard ADM:

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(A_k), \qquad k \geq 0 \tag{17}$$

By comparison with ADM and TSADM, it is clear that TSADM may provide the solution by using one iteration only and does not have the difficulties arising in the modified method. Further, the number of terms in φ, namely *m,* is small in many practical problems. So, applying TSADM will not be time consuming. Our purpose in this paper is to combine the LDM and TSADM. So, we should divide *G*(*x,t*) into its components and check the required conditions for property choice of $u_0(x,t)$. After applying inverse transform, by TSADM criterion we can find the exact solution of our equation after one iteration.

APPLICATIONS

To illustrate LTSDM we now consider some examples.

Example 1

Consider the nonlinear partial differential equation (Wazwaz, 1999)

$$u_t + uu_x = x + xt^2, \tag{18}$$

with initial conditions

$$u(x,0) = 0.$$

Applying the Laplace transform we have:

$$s\pounds[u(x,t)] - u(x,0) = \pounds[x + xt^2] - \pounds[uu_x] \tag{19}$$

$$u(x,s) = \frac{x}{s^2} + \frac{2x}{s^4} - \frac{1}{s}\pounds[uu_x].$$

Applying inverse Laplace transform we get

$$u(x,t) = xt + \frac{xt^3}{3} - \frac{1}{s}\pounds^{-1}\left[\frac{1}{s}[uu_x]\right]. \tag{20}$$

As we know the solution is in infinite series form:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{21}$$

and nonlinear term is handled with the help of Adomian polynomials given as follows:

$$uu_x = \sum_{n=0}^{\infty} A_n(u). \tag{22}$$

By substituting Equations (21) and (22) in (20) we have:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = xt + \frac{xt^3}{3} - \pounds^{-1}\left[\frac{1}{s}\pounds\left[\sum_{n=0}^{\infty} A_n(u)\right]\right]. \tag{23}$$

By using LDM we have:

$$u_0(x,t) = xt + \frac{xt^3}{3}, \tag{24}$$

$$u_1(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds[A_0(u)]\right], \tag{25}$$

$$u_{n+1}(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds[A_n(u)]\right], \qquad n \geq 1. \tag{26}$$

By this recursive relation we can find other components of the solution.

$$u_1(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds[u_0 u_{0x}]\right],$$

$$= -\frac{xt^3}{3} - \frac{xt^7}{63} - \frac{2xt^5}{15}$$

$$u_2(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds[A_1(u)]\right],$$

$$= -\pounds^{-1}\left[\frac{1}{s}\pounds[u_1 u_{0x} + u_0 u_{1x}]\right],$$

$$= -\pounds^{-1}\left[\frac{1}{s}\pounds\left[\frac{-2xt^4}{3} - \frac{2xt^6}{9} - \frac{2xt^8}{63} - \frac{2xt^20}{189} - \frac{4xt}{15} - \frac{4xt}{}\right]\right]$$

$$= \frac{2xt^5}{15} + \frac{22xt^7}{315} + \frac{38xt^9}{2835} + \frac{2xt^{11}}{2079}$$

$$\vdots$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = xt. \tag{27}$$

As we can see for finding next components large amount of computation should be done. Now using the LTSDM scheme gives:

$$\varphi = \varphi_0 + \varphi_1, \tag{28}$$

$\varphi_0 = xt \quad and \quad \varphi_1 = \frac{1}{3} xt^3$

It is obvious that φ_1 does not satisfy Equation (18). By choosing $u_0 = \varphi_0$ and by verifying that u_0 justifies Equation (18), the exact solution will be obtained immediately and we have:

$$u_0(x,t) = xt, \tag{29}$$

$$u_1(x,t) = \frac{xt^3}{3} - \pounds^{-1}\left[\frac{1}{s}\pounds[A_0(u)]\right], \tag{30}$$

$$u_{n+1}(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds\left[\sum_{n=0}^{\infty} A_n(u)\right]\right], \qquad n \geq 1 \tag{31}$$

So

$$u_1(x,t) = \frac{xt^3}{3} - \pounds^{-1}\left[\frac{1}{s}\pounds[u_0 u_{0x}]\right] = 0,$$

$$u_{n+1}(x,t) = 0 \qquad n \geq 0,$$

and the solution is

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = xt.$$

It is important to note that if we select

$$u_0(x,t) = \frac{1}{3} xt^3,$$

we obtain:

$$u_1(x,t) = xt - \pounds^{-1}\left[\frac{1}{s}\pounds[u_0 u_{0x}]\right]$$

$$= xt - \pounds^{-1}\left[\frac{1}{s}\pounds\left[\frac{1}{9} - xt^6\right]\right]$$

$$= xt - \pounds^{-1}\left[\frac{6!\,x}{9s^8}\right]$$

$$= xt - \frac{xt^7}{63},$$

$$u_2(x,t) = -\pounds^{-1}\left[\frac{1}{s}\pounds[A_1]\right]$$

$$= -\pounds^{-1}\left[\frac{1}{s}\pounds[u_1 u_{0x} + u_0 u_{1x}]\right]$$

$$= -\pounds^{-1}\left[\frac{1}{s}\pounds\left[\frac{2}{3}xt^4 - \frac{2}{189}xt^{10}\right]\right]$$

$$= -\pounds^{-1}\left[\frac{16x}{s^6} - \frac{38400x}{s^{12}}\right]$$

$$= -\frac{2xt^5}{15} + \frac{2xt^11}{2079},$$

$\vdots$

As we can see, finding the solution is not possible after one iteration and we have to calculate the components like standard ADM.

Example 2

We now consider another nonlinear partial differential equation (Wazwaz, 2002b)

$$\frac{\partial^2 u(x,y)}{\partial x^2} - u_x u_{yy} = -x + u, \tag{32}$$

with initial conditions

$$u(0,y) = \sin y,$$
$$u_x(0,y) = 1.$$

Applying the Laplace transform we get

$$s^2\pounds[u(x,y)] - su(0,y) - u_x(0,y) = \pounds[-x+u] + \pounds[u_x u_{yy}], \tag{33}$$

$$u(s,y) = \frac{1}{s^2} + \frac{\sin y}{s} - \frac{1}{s^4} + \frac{1}{s^2}\pounds[u + u_x u_{yy}].$$

By applying inverse transform we get

$$u(s,y) = x + \sin y - \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}[u + u_x u_{yy}]\right]. \tag{34}$$

Likewise as in the previous example, we have

$$\sum_{n=0}^{\infty} u_n(x,y) = x + \sin y - \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds\left[\sum_{n=0}^{\infty} u_n(x,y) + \sum_{n=0}^{\infty} A_{n(u)}\right]\right]. \tag{35}$$

By using LDM:

$$u_0(x,y) = x + \sin y - \frac{x^3}{3!}, \tag{36}$$

$$u_{n+1}(x,y) = \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_n + A_n]\right], \qquad n \geq 0, \tag{37}$$

by using the previous recursive relation

$$u_1(x,y) = \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + A_0]\right]$$

$$= \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + u_{0x}u_{0yy}]\right]$$

$$= \pounds^{-1}\left[\frac{1}{S^2}\pounds\left[x + \sin y - \frac{x^3}{3!} - \sin y(1-\frac{x^2}{2})\right]\right]$$

$$= \pounds^{-1}\left[\frac{1}{S^4} + \frac{\sin y}{s^3} - \frac{1}{S^6} + \sin y(\frac{1}{3_s{}^5} - \frac{1}{S^3})\right]$$

$$= \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \frac{\sin y}{72}x^4$$

$\vdots$

and

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y) = x + \sin y \tag{38}$$

It is clear that finding the exact solution is time consuming but by using LTSDM:

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2, \tag{39}$$

$\varphi_0 = x$, $\varphi_1 = \sin y$ *and* $\varphi_2 = -\frac{1}{3!}x^3$.

By putting φ_0, φ_1 and φ_2 we can see none of them satisfies Equation (29) but $\varphi_0 + \varphi_1$ justifies the Equation (29). So, we select $u_0 = \varphi_0 + \varphi_1 = x + \sin y$ and in this case we have:

$$u_0(x,y) = x + \sin y, \tag{40}$$

$$u_1(x,y) = \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + A_0]\right], \tag{41}$$

$$u_{n+1}(x,y) = \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_n + A_n]\right], \qquad n \geq 1, \tag{42}$$

where $A_n(u)$ is Adomian polynomials and representing the nonlinear term of Equation (29).

$$u_0(x,y) = x + \sin y, \tag{43}$$

$$u_1(x,y) = -\frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + u_{0x}u_{0yy}]\right],$$

$$= -\frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds[x + \sin y - \sin y]\right],$$

$$= -\frac{x^3}{3!} + \frac{x^3}{3!}$$

$$= 0, \tag{44}$$

$u_{n+1}(x,y) = 0, \qquad n \geq 0$
(45)

and the solution will be

$$u(x,y) = \sum_{n=0}^{\infty} u_n = x + \sin y. \tag{46}$$

If we select $u_0 = x$, then

$$u_1(x,y) = \sin y - \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + A_0]\right]$$

$$= \sin y - \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_0 + u_{0x}u_{0yy}]\right],$$

$$= \sin y - \frac{x^3}{3!} + \pounds^{-1}\left[\frac{1}{s^4}\right],$$

$$= \sin y - \frac{x^3}{3!} + \frac{x^3}{3!}$$

$$= \sin y, \tag{47}$$

$$u_2(x,y) = \pounds^{-1}\left[\frac{1}{s^2}\pounds[u_1 + A_1]\right] = 0 \tag{48}$$

$$u_{n+1}(x,y) = 0, \qquad n \geq 1. \tag{49}$$

By this choice for u_0 we find the solution after two iterations. So, LTSDM works better and has less amount of computation. As we can see φ_2 does not satisfy Equation (29). By choosing $u_0 = \varphi_2 = -\frac{x^3}{3!}$ we have:

$$u_0(x,y) = -\frac{x^3}{3!}, \tag{50}$$

$$u_1(x,y) = x + \sin y + \pounds^{-1}\left[\frac{1}{S^2}\pounds[u_0 + A_0]\right]$$

$$= x + \sin y + £^{-1}\left[\frac{1}{s^2} £[u_0 + u_{0x}u_{0yy}]\right],$$

$$= x + \sin y + £^{-1}\left[\frac{1}{s^2} £\left[-\frac{x^3}{3!}\right]\right],$$

$$= x + \sin y - \frac{x^5}{5!}, \tag{51}$$

$$u_2(x,y) = £^{-1}\left[\frac{1}{s^2} £[u_1 + A_1]\right]$$

$$= £^{-1}\left[\frac{1}{s^2} £[u_1 + u_{0x}u_{1yy} + u_{1x}u_{0yy}]\right],$$

$$= £^{-1}\left[\frac{1}{s^2} £\left[x + \sin y - \frac{x^5}{5!} + \frac{1}{2}x^2 \sin y\right]\right],$$

$$= \frac{x^3}{3!} + \sin y \frac{x^2}{2} - \frac{x^7}{7!} + \sin y \frac{x^4}{4!},$$

$$\vdots \tag{52}$$

As we can see it is necessary to calculate more components of $u(x,t)$ to get the exact solution.

Example 3

Consider the system of inhomogeneous partial differential equations (Wazwaz, 2003)

$$\begin{cases} u_x - uv = -1 + Exp(x+t), \\ v_x - uv_x = 1 - Exp(-x-t), \end{cases} \tag{53}$$

with initial conditions

$$u(0,t) = Exp(t),$$
$$v(0,t) = Exp(-t).$$

Applying the Laplace transform on both sides of Equation (53) we get

$$s£[u(x,t)] - u(0.t) - £[uv] = -\frac{1}{s} + Exp(t)(\frac{1}{s-1}) \tag{54}$$

$$u(s,t) = -\frac{1}{s^2} + \frac{Exp(t)}{s(s-1)} + \frac{1}{s}Exp(t) + \frac{1}{s}£[uv],$$

and by applying inverse transform

$$u(x,t) = -x + Exp(x+t) + £^{-1}\left[\frac{1}{s}£[uv]\right]. \tag{55}$$

For second equation

$$s£[v(x,t)] - v(0,t) - £[uv_x] = \frac{1}{s} - Exp(-t)(\frac{1}{S+1}) \tag{56}$$

$$v(s,t) = \frac{1}{s^2} - \frac{Exp(-t)}{s(s+1)} + \frac{1}{s}Exp(-t) + \frac{1}{s}£[uv_x].$$

By applying inverse transform we get

$$v(s,t) = x + Exp(-x-t) + £^{-1}\left[\frac{1}{s}£[uv_x]\right]. \tag{57}$$

By using LDM:

$$u_0(x,t) = -x + Exp(x+t) \tag{58}$$

$$v_0(x,t) = -x + Exp(-x-t) \tag{59}$$

$$u_{n+1}(x,t) = £^{-1}\left[\frac{1}{s}£\left[\sum_{n=0}^{\infty} A_n(u)\right]\right], \qquad n \geq 0, \tag{60}$$

$$v_{n+1}(x,t) = £^{-1}\left[\frac{1}{s}£\left[\sum_{n=0}^{\infty} A_n(u)\right]\right], \qquad n \geq 0, \tag{61}$$

by using the previous recursive relation

$$u_1(x,t) = £^{-1}\left[\frac{1}{s}£[A_0]\right]$$

$$= £^{-1}\left[\frac{1}{s}£[u_0 v_0]\right]$$

$$= £^{-1}\left[\frac{1}{s}£[(-x + Exp(x+t))(x + Exp(-x-t))]\right]$$

$$= £^{-1}\left[\frac{1}{s^2} - \frac{2}{s^4} + \frac{Exp(t)}{s(s-1)^2} - \frac{Exp(-t)}{s(s+1)^2}\right]$$

$$= x - \frac{1}{3}x^3 + Exp(t) + Exp(x+t)(x-1) - Exp(-t) + Exp(-x-t)(x+1),$$

$$\vdots$$

$$v_1(x,t) = £^{-1}\left[\frac{1}{s}£[A_0]\right]$$

$$= £^{-1}\left[\frac{1}{s}£[u_0 v_{0x}]\right]$$

$$= \mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}\left[(-x+Exp(x+t))(1-Exp(-x-t))\right]\right]$$

$$= \mathfrak{L}^{-1}\left[-\frac{1}{s}-\frac{1}{s^2}+\frac{Exp(t)}{(s-1)}+\frac{Exp(-t)}{s(s+1)^2}\right]$$

$$= -1-x+Exp(x+t)+xExp(-t-x),$$

$$\vdots$$

and

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)=Exp(x+t), \tag{62}$$

$$v(x,t)=\sum_{n=0}^{\infty}u_n(x,t)=Exp(-x-t). \tag{63}$$

For finding the solution we should calculate other components of $u(x,t), v(x,t)$ and obviously it is time consuming. Now we can see by using LTSDM, getting the solution is very simple. For first equation we have

$$\varphi=\varphi_0+\varphi_1, \tag{64}$$

$$\varphi = -x \quad and \quad \varphi_1 = Exp\ (x+t),$$

and for second equation

$$\varphi'=\varphi_0'+\varphi_1', \tag{65}$$

$$\varphi_0' = x \quad and \quad \varphi_1' = Exp\,(-x-t).$$

By putting these functions in Equation (53), we can see φ_1 and φ_1' satisfy Equation (53).

$$u_0(x,t)=Exp(x+t), \tag{66}$$

$$v_0(x,t)=Exp(-x-t), \tag{67}$$

$$u_1(x,t)=-x+\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}[A_0]\right]$$

$$=-x+\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}[u_0v_0]\right]$$

$$=-x+\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}\left[(Exp(x+t))(Exp(-x-t))\right]\right]$$

$$=-x+\mathfrak{L}^{-1}\left[\frac{1}{s^2}\right]$$

$$=x-x=0 \tag{68}$$

$$u_{n+1}(x,t)=\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}\left[\sum_{n=0}^{\infty}A_n(u)\right]\right]=0, \qquad n\ge 0, \tag{69}$$

$$v_1(x,t)=x-\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}[A_0]\right]$$

$$=x-\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}[u_0v_{0x}]\right]$$

$$=x-\mathfrak{L}^{-1}\left[\frac{1}{s}\left[(Exp(x+t))(Exp(-x-t))\right]\right]$$

$$=x-\mathfrak{L}^{-1}\left[\frac{1}{s^2}\right]$$

$$=x-x=0 \tag{70}$$

$$u_{n+1}(x,t)=\mathfrak{L}^{-1}\left[\frac{1}{s}\mathfrak{L}\left[\sum_{n=0}^{\infty}A_n(u)\right]\right]=0. \qquad n\ge 0, \tag{71}$$

So

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)=Exp\,(x+t), \tag{72}$$

$$v(x,t)=\sum_{n=0}^{\infty}v_n(x,t)=Exp\,(-x-t), \tag{73}$$

Same as previous examples, by choosing $u_0=\varphi_0=-x$ and $v_0=\varphi_0'=x$ that do not justify Equation (53) we will have the same size of computation with standard ADM.

Conclusions

In this work, we have carefully combined two-step Adomian decomposition method with Laplace transform. In the three illustrated examples we showed that LTSDM consists of three steps: the first step was applying Laplace transform on our equation and then inverse transform, the second step was verifying that the zeroth component of the series solution included the exact solution. If yes, we are done. Otherwise, we should go to the third step and continue with the standard ADM. The obtained results in examples indicated that LTSDM was feasible, effective and we do not have the "noise term". The LTSDM overcomes the difficulties arising in the modified decomposition method established in (Wazwaz, 1999).The power of LTSDM depends on the proper choice of u_0 and u_1 and the occurrence of the exact solution in the zeroth term. If the exact solution exists in the zeroth component LTSDM requires much less calculation in comparison with LDM. If not, LTSDM requires only a little effort than LDM.

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