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Full Length Research Paper

New variational iteration decomposition method for solving twelfth order boundary value problems

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In this paper, we introduced a new iterative method for solving twelfth order boundary value problems. The analytical results of the equations have been obtained in terms of convergent series with easily computable components. This new technique is a combination of variational iteration method and the decomposition method. We have taken two examples to illustrate the effectiveness of variational iteration decomposition method (VIDM) for solving twelfth order boundary value problems. It was concluded that VIDM is a powerful tool for solving high-order boundary value problems arising in various fields of engineering and science.

Key words: Twelfth order boundary value problems, approximate analytical solution, variational iteration method and variational iteration decomposition method, ordinary differential equations, error estimates.

INTRODUCTION

Non-linear phenomena play a crucial role in applied mathematics and physics. The results of solving nonlinear equations can guide authors to know the described process deeply. But it is difficult for us to obtain the exact solution for these problems; therefore, we hereby try to obtain such suitable method by which we find out the not exact result but very-very close to it. In this direction, we consider the general twelfth order boundary value problems of the type:

$$y^{12}(x)+f(x)y(x)=g(x),x\in[a,b], \qquad (1)$$

With boundary conditions:

$$y(a) =a_1 y(b) = b_1$$
$$y^{(1)}(a) =a_2 y^{(1)}(b) = b_2$$
$$y^{(2)}(a) =a_3 y^{(2)}(b) = b_3$$
$$y^{(3)}(a) =a_4 y^{(3)}(b) = b_4$$
$$y^{(4)}(a) =a_5 y^{(4)}(b) = b_5$$
$$y^{(5)}(a) =a_6 y^{(5)}(b) = b_6$$

Where a_i, b_j, are i, j = 1, 2, 3, 4, 5 and 6 are finite real constants and the functions f(x) and g(x) are continuous on [a, b]. This type of boundary value problems arises in the mathematical modeling of the viscoelastic flows and other branches of mathematical, physical and engineering sciences (Karageoghis et al., 1988, 1998; Caglar et al., 1999; Fyfe, 1971) and the references therein. It has been shown that this new iterative method (Noor and Noor, 2006, 2007) solves effectively, easily and accurately large classes of linear and nonlinear, ordinary, partial, deterministic or stochastic differential equations with approximate solutions which converge rapidly to accurate solutions. It has been shown that this method provides the solution in a rapid convergent series. It is observed that the new iterative method may be considered as an alternative method for solving linear and nonlinear problems to Adomian decomposition, variational iteration method and homotopy perturbation method. Variational iteration method has been favorably applied to various kinds of nonlinear problems. The main property of the method is in its flexibility and ability to solve nonlinear equations accurately and conveniently. In this paper, recent trends and developments in the use of the method are reviewed. The confluence of modem mathematics and symbol computation has posed a

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challenge to developing technologies capable of handling strongly nonlinear equations which cannot be successfully dealt with by classical methods. Variational iteration method is uniquely qualified to address this challenge. The flexibility and adaptation provided by the method have made the method a strong candidate for approximate analytical solutions. This paper outlines the basic conceptual framework of variational iteration technique with application to nonlinear problems. Variational iteration methods and homotopy perturbation methods are employed by Noor and Mohyud-Din (2007a, b, 2008) to find the series solution of such problems. It is worth mentioning that to implement the Adomian's method; one has to find the Adomian's polynomials which are itself a difficult task, this fact has motivated the development of other analytical and numerical techniques for solving boundary value problems of fifth order. To overcome these drawbacks and deficiencies, Noor and Noor (2006, 2007) have suggested and analyzed a new iterative method for solving nonlinear boundary and initial value problems. The main motivation of this paper is to use this new decomposition method for solving twelfth order boundary value problems. Variational iteration decomposition method provides the solution in a rapid convergent series. We write the correct functional for the twelfth order boundary value problem and calculate the Lagrange multiplier optimally via variational theory. The Adomain polynomials are introduced in the correct functional and evaluated; the approximants are calculated by employing the Lagrange multiplier and the Adomain polynomials scheme simultaneously. The use of Lagrange multiplier reduces the successive application of the integral operator and minimizes the computational work.

VARIATIONAL ITERATION METHOD

To illustrate the basis concept of the technique, we consider the following general differential equation:

$$Lu+Nu=g(x) \tag{2}$$

where L is a linear operator, N a non linear operator and g(x) is the in homogenous term. According to variational iteration method, we can construct a correct functional as follows:

$$u_{n+1}(x)=u_n(x)+\int_0^x \lambda(Lu_n(s)+N\widetilde{u_n}(s)) \tag{3}$$

where λ is a Lagrange multiplier (Noor and Mohyud-Din, 2009; He, 1999, 2007a, b), which can be identified optimally via variational iteration method. The subscripts n denote the nth approximation, $\widetilde{u_n}$ is considered as a restricted variation. That is, $\widetilde{u_n}$ = 0. The relation (Equation 2) is called as a correct functional. The solution of the linear problems can be solved in a single iteration step due to the exact identification of the Lagrange multiplier.

ADOMIAN VARIATIONAL ITERATION DECOMPOSITION METHOD

Now, we recall basic principles of the Adomian decomposition method (Adomian, 1990; Odibat and Momani, 2006) for solving differential equations. Consider the general equation Tu = g, where T represents a general nonlinear differential operator involving both linear and nonlinear terms. The linear term is decomposed into L + R, where L is easily invertible and R is the reminder of the linear operator. For convenience, L may be taken as the highest order derivation. Thus, the equation may be written as:

$$Lu+R(u)+Nu=g(x) \tag{4}$$

where Nu represents the nonlinear terms. From Equation 3, we have:

$$Lu=g-R(u)-Nu \tag{5}$$

Since L is invertible, the equivalent expression:

$$u=L^{-1}gL^{-1}R(u)-L^{-1}N(u) \tag{6}$$

A solution u can be expressed as in the following series:

$$u(x)=\sum_{m=0}^{\infty} u_m \tag{7}$$

with reasonable u_0 which may be identified with respect to the definition of L^{-1}, g and u_n, n > 0 is to be determined. The nonlinear term Nu will be decomposed by the infinite series of Adomian polynomials:

$$Nu=\sum_{m=0}^{\infty} B_m \tag{8}$$

where B_n's are obtained by writing:

$$v\lambda=\sum_{m=0}^{\infty} \lambda^m u_m \tag{9}$$

$$N(v\lambda)=\sum_{m=0}^{\infty} \lambda^m B_m \tag{10}$$

Here, λ is a parameter introduced for convenience. From Equations 9 and 10, we have:

$$B_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[N\sum_{m=0}^{\infty}\lambda^m u_m\right] \; n \geq 0 \tag{11}$$

The B_m's are given, since there appears to be no well-defined method for constructing a definitive set of polynomials for arbitrary F, but rather slightly different approaches are used for different specific functions. One possible set of polynomials is given by:

$B_0 = F(u_0)$,

$$B_1 = (x-x_1)\left(\frac{dy}{dx}\right)_{u=0}$$

$$B_2 = (x-x_2)\left(\frac{dy}{dx}\right)_{u=0} + \frac{(x-x_1)^2}{2!}\left(\frac{d^2y}{dx^2}\right)_{u=0}$$

$$B_3 = (x-x_3)\left(\frac{dy}{dx}\right)_{u=0} + (x-x_1)(x-x_2)\left(\frac{d^2y}{dx^2}\right)_{u=0} + \frac{(x-x_1)^2}{3!}\left(\frac{d^3y}{dx^3}\right)_{u=0} \tag{12}$$

Which can be used to construct Adomian polynomials, when $F(u_0)$ is a nonlinear function, put the value of Equations 8 and 9 in Equation 3, we get:

$$\sum_{m=0}^{\infty} u_m = u_0 + L^{-1}R\left(\sum_{m=0}^{\infty} B_m\right) - L^{-1}\sum_{m=0}^{\infty} B_m \tag{13}$$

Consequently, with a suitable u_0 we can write, put one by one j=1, 2, 3, …….. in the aforementioned expression.

$u_1(x) = -L^{-1}R\,(u_0) - L^{-1}\,B_0$

$u_2(x) = -L^{-1}R\,(u_1) - L^{-1}\,B_1$

$u_{n+1}(x) = -L^{-1}R\,(u_n) - L^{-1}B_n$

VARIATIONAL ITERATION DECOMPOSITION METHOD

To illustrate the basic concept of the variational iteration decomposition method, we consider the following general differential Equation 1. According to the variational iteration method (Odibat and Momani, 2006), we can construct a correct functional (Equation 2), and define the solution by the series:

$$u(x) = \sum_{m=0}^{\infty} u^m(x)$$

and the non linear term

$$\sum_{m=0}^{\infty} B_m(u_0, u_1, u_2, u_3, \dots\dots\dots, u_m)$$

where Bn are the Adomian polynomials and can be generated for all type of nonlinearities according to the algorithm developed by Adomian (1988), which yields the following:

$$B_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}N(u(x))l\lambda$$

$n \geq 0,$

Hence, we obtain the following iterative scheme:

$$u^{(n+1)}(x) = u^{(n)}(x) + \int_0^t \lambda\left(L\,u^{(n)}(x) + \sum_{m=0}^{\infty} B_m - g(x)\right)dx.$$

The method is called variational iterative decomposition method.

NUMERICAL EXAMPLES

Here, we apply the variational iterative decomposition method for solving twelfth order boundary value problems. We write the correct functional for the twelfth order boundary value problems and carefully select the initial values, because the approximants are heavily dependent on the initial value.

Example 1

Consider the following linear twelfth order boundary value problem:

$$y^{12}(x) = -x\,y(x) - x^3e^x - 23xe^x - 120e^x \tag{14}$$

with the following conditions:

$$y_0 = 0,\; y_0^{(1)} = 1,\; y_0^{(2)} = 0,\; y_0^{(3)} = -3,\; y_0^{(4)} = -8,\; y_0^{(5)} = -15$$

$$y_1 = 0,\; y_1^{(1)} = -e,\; y_1^{(2)} = -4e,\; y_1^{(3)} = -9e,\; y_1^{(4)} = -16e,\; y_1^{(5)} = -25e$$

Exact solution is $y(x) = x\,(1-x)\,e^x$

The correct functional for the boundary value problem is given as:

$$y_{(n+1)}(x) = y_{(n)}(x) + \int_0^x \lambda\left(\frac{d^{12}y_n}{dx^{12}} - (-x\,\widetilde{y(x)} - x^3e^x - 23xe^x - 120e^x)\right)$$

To find the optimal λs calculation variation with respect to y_n, we have the following stationary conditions:

$\delta y_n : \lambda^{(m)}(s) = 0$,

$\delta y^{(m-1)}_n : [\lambda(s)]s=x = 0$,

$\delta y^{(m-2)}_n : [\lambda'(s)]s=x = 0$,

$\delta y_n:[1-\lambda^{(m-1)}(s)]s=x=0$ (15)

But in Equation 14, the value of m is 12, putting all these values of m in Equation 15, we obtain as follows:

$\delta y_n : \lambda^{(12)}(s) = 0$,

$\delta y^{(11)}_n : [\lambda(s)]s=x = 0$,

$\delta y^{(10)}_n : [\lambda'(s)]s=x = 0$,

$\delta y_n : [1-\lambda^{(11)}(s)]s=x = 0$.

The Lagrange multiplier, therefore, can identify as follows:

$\lambda = \frac{(s-x)^{11}}{11!}$

Making the correct functional stationary, using $\lambda = \frac{1}{11!}((s-x)^{11})$ Lagrange multiplier (Noor and Mohyud-Din, 2009; He, 1999, 2007a, b) substituting the identified multiplier into the aforementioned equation, we have the following iteration formula:

$$y_{(n+1)}(x) = y_{(n)}(x) + \int_0^x \frac{1}{11!}\left((s-x)^{11}\right)\left(\frac{d^{12}y_n}{dx^{12}} - (-x\,y(x) - x^3e^x - 23xe^x - 120e^x)\right)ds$$

$$y_{n+1}(x) = x + \frac{A}{3!}x^3 + \frac{B}{4!}x^4 + \frac{C}{5!}x^5 + \frac{D}{6!}x^6 + \frac{E}{7!}x^7 + \frac{F}{8!}x^8 + \int_0^x \frac{1}{11!}((s-x)^{11})\left(\frac{d^{12}y_n}{dx^{12}} - (-x\,y(x) - x^3e^x - 23xe^x - 120e^x)\right)ds$$

where

A= $y''''''(0)$, B = $y'''''''(0)$, C= $y''''''''(0)$

D= $y'''''''''(0)$, E= $y''''''''''(0)$, F= $y'''''''\,''''(0)$

Using the variational iterative decomposition method, we get:

$$y_{n+1}(x) = x + \frac{A}{3!}x^3 + \frac{B}{4!}x^4 + \frac{C}{5!}x^5 + \frac{D}{6!}x^6 + \frac{E}{7!}x^7 + \frac{F}{8!}x^8 + \int_0^x \frac{1}{11!}((s-x)^{11})\left(\frac{d^{12}y_m}{dx^{12}} + x^3e^x + 23xe^x + 120e^x - x\sum_{m=0}^{\infty} B_m\right)ds$$

where B_m are Adomian polynomials for nonlinear operator N(y) =xy(x) and can be generated for all type of nonlinearities according to the algorithm which yields the following:

$B_0 = xy_0(x)$

$B_1 = y_1(x)N'(y_0)$

$B_2 = \left(y_0 + \frac{y_1^2}{2}\right)xy_0(x), \ldots\ldots\ldots\ldots$ (16)

From the relation (Equation 16), we find $y_0(x)$, $y_1(x)$,……..and get the series solution as follow:

$$y(x) = x - \frac{1}{2}x^3 - \frac{1}{3}x^4 - \frac{1}{8}x^5 + \frac{A}{720}x^6 + \frac{B}{5040}x^7 + \frac{C}{40320}x^8 + \frac{D}{362880}x^9 + \frac{E}{3628800}x^{10} + \frac{F}{39916800}x^{11} - \frac{1}{3991680}x^{12} - \frac{1}{43545600}x^{13} - \frac{83}{43589145600}x^{14} - \frac{1}{6706022400}x^{15} + \ldots\ldots\ldots$$

The coefficients A, B, C, D, E, F and G can be obtained using the boundary conditions at *x* = 1,

A = 23.9999985,
B =35.000057,
C= 47.998961,
D =63.0108031,
E =79.9359481,
F= 99.17376631.

The series solution can, thus, be written as:

$$y^{12}(x) = x - \frac{1}{2}x^3 - \frac{1}{3}x^4 - \frac{1}{8}x^5 + 0.0333333x^6 - 0.00694446x^7 - 0.00119045x^8 - 0.000173641x^9 - 0.0000220282x^{10} + 0.00000248451x^{11} - \frac{1}{3991680}x^{12} - \frac{1}{43545600}x^{13} - \frac{83}{43589145600}x^{14} - \frac{1}{6706022400}x^{15} + \ldots\ldots\ldots$$

Table 1. Error estimate.

x	Exact solution	Numerical solution of VIDM	Errors of VIDM
0.0	1.000000000	0.0000000000	0.00000
0.1	0.0994653826	0.0994653826	3.00×10^{-11}
0.2	0.1954244413	0.1954244413	0.00000
0.3	0.2834703497	0.2834703496	-1.00×10^{-10}
0.4	0.3580379275	0.3580379277	2.00×10^{-10}
0.5	0.4121803178	0.4121803189	1.10×10^{-9}
0.6	0.4373085120	0.4373085164	4.40×10^{-9}
0.7	0.4228880685	0.4228880820	1.35×10^{-8}
0.8	0.3560865485	0.3560865853	3.68×10^{-8}
0.9	0.2213642800	0.2213643701	9.01×10^{-8}
1.0	0.0000000000	0.0000002027	2.02700×10^{-7}

Table 1 shows the approximate solution obtained by VIDM and error obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating more iteration.

Example 2

Consider the following linear twelve-order problem

$$y^{(12)}(x)-y^{(3)}(x)=2e^{x}y^{2}(x) \qquad (17)$$

with the following boundary conditions:

$$y_1^{(0)} = y_1^{(2)} = y_1^{(4)} = y_1^{(6)} = y_1^{(8)} = y_1^{(10)} = 0.367879441$$

$$y_0^{(0)} = 1,\ y_0^{(2)} = 1,\ y_0^{(4)} = 1,\ y_0^{(6)} = 1,\ y_0^{(8)} = 1,\ y_0^{(10)} = 1$$

Exact solution is $y(x) = e^{-x}$

The correct functional for the boundary value problem is given as:

$$y_{(n+1)}(x) = y_{(n)}(x) + \int_0^x \lambda\left(\frac{d^{12}y_n}{dx^{12}} - (\tilde{y}^3(x) - 2e^x\tilde{y}^2(x))\right)$$

Making the correct functional stationary, using $\lambda = \frac{1}{11!}((s-x)^{11})$ Lagrange multiplier (Noor and Mohyud-Din, 2009; He, 1999, 2007a, b), we get following iterative formula:

$$y_{(n+1)}(x) = y_{(n)}(x) + \int_0^x \frac{1}{11!}\left((s-x)^{11}\right)\left(\frac{d^{12}y_n}{dx^{12}} - y^3(x) + 2e^x y^2(x)\right)ds$$

$$y_{n+1}(x)=x+\frac{A}{3!}x^3+\frac{B}{4!}x^4+\frac{C}{5!}x^5+\frac{D}{6!}x^6+\frac{E}{7!}x^7+\frac{F}{8!}x^8$$

$$+\int_0^x \frac{1}{11!}((s-x)^{11})\left(\frac{d^{12}y_n}{dx^{12}} - y^3(x) + 2e^x y^2(x)\right)ds$$

Using the variational iterative decomposition method, we precede the aforementioned procedure from the equations; we get series solution as follows:

$$y(x) = 1+\frac{1}{2}x^2+\frac{1}{24}x^4+\frac{1}{720}x^6+\frac{1}{40320}x^8+\frac{1}{3628800}x^{10}+\frac{1}{239500800}x^{12}$$

$$+\frac{1}{39916800}Fx^{11}+\frac{1}{362880}Ex^9+Ax+\frac{1}{120}Cx^5+\frac{1}{6}Bx^3+\frac{1}{5040}Dx^7 \ldots\ldots\ldots\ldots \qquad (18)$$

The coefficients A, B, C, D, E, F and G can be obtained using the boundary conditions at x =1,

A= 0.9999940293, B= 1.000058885, C = 0.9994190942, D =1.005725028, E= 0.9434337955, F =1.632120555.

$$y(x) = 1-0.9999940293x-0.1666764809x^3+\frac{1}{24}x^4$$

$$-0.008328492451x^5+\frac{1}{720}x^6-0.0001995486167x^7$$

$$+\frac{1}{40320}x^8-0.208710^{-9}x^9+\frac{1}{3628800}x^{10}$$

$$-0.4088806105x^{11}+\frac{1}{239500800}x^{12}+\ldots\ldots\ldots\ldots\ldots$$

Table 2 shows the approximate solution obtained by VIDM and error obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating more iteration.

Example 3

Consider the following linear twelfth order boundary value problem:

Table 2. Error estimate.

x	Exact solution	Numerical Solution of VIDM	Errors of VIDM
0.0	1.000000000	1.000000000	0.00000
0.1	0.904837418	0.904837579	-1.61×10^{-7}
0.2	0.818730753	0.818731060	-3.07×10^{-7}
0.3	0.740818221	0.740818643	-4.22×10^{-7}
0.4	0.670320046	0.670320543	-4.97×10^{-7}
0.5	0.606530659	0.606531182	-5.21×10^{-7}
0.6	0.548811636	0.548812133	-4.98×10^{-7}
0.7	0.496585304	0.496585726	-4.22×10^{-7}
0.8	0.449328964	0.449329710	-3.07×10^{-7}
0.9	0.406569659	0.406569821	-1.61×10^{-7}
1.0	0.3678794415	0.3678794412	3.00×10^{-10}

$$y^{12}(x)=-12(2x\cos(x)+11\sin(x)+y(x)), x\in[-1,1] \quad (19)$$

With following conditions:

$$y_{-1}=0,\ y_{-1}^{(1)}=2\sin(1),\ y_{-1}^{(2)}=-4\cos(1)-2\sin(1)$$

$$y_{-1}^{(3)}=6\cos(1)-6\sin(1),\ y_{-1}^{(4)}=8\cos(1)+12\sin(1),\ y_{-1}^{(5)}=-20\cos(1)+10\sin(1)$$

$$y_{1}=0,\ y_{1}^{(1)}=2\sin(1),\ y_{1}^{(2)}=-4\cos(1)-2\sin(1)$$

$$y_{1}^{(3)}=6\cos(1)-6\sin(1),\ y_{1}^{(4)}=8\cos(1)+12\sin(1),\ y_{1}^{(5)}=-20\cos(1)+10\sin(1)$$

Exact solution is $y(x) = (x^2-1)\sin(x)$.

The correct functional for the boundary value problem is given as:

$$y_{(n+1)}(x)=y_{(n)}(x)+\int_0^x \lambda\left(\frac{d^{12}y_n}{dx^{12}}-(-12(2x\cos(x)+11\sin(x))+(\widetilde{y(x)})\right)ds$$

Making the correct functional stationary, using $\lambda = \frac{1}{11!}((s-x)^{11})$ Lagrange multiplier (Noor and Mohyud-Din, 2009; He, 1999, 2007a, b) we get following iterative formula:

$$y_{(n+1)}(x)=y_{(n)}(x)+\int_0^x \frac{1}{11!}((s-x)^{11})\left(\frac{d^{12}y_n}{dx^{12}}+12(2x\cos(x))-131\sin(x)-(\widetilde{y(x)})\right)ds$$

$$y_{n+1}(x) = x+\frac{A}{3!}x^3+\frac{B}{4!}x^4+\frac{C}{5!}x^5+\frac{D}{6!}x^6+\frac{E}{7!}x^7+\frac{F}{8!}x^8+\int_0^x\frac{1}{11!}((s-x)^{11})\left(\frac{d^{12}y_n}{dx^{12}}+12(2x\cos(x))-131\sin(x)-(\widetilde{y(x)})\right)ds$$

Using the variational iterative decomposition method, we precede the aforementioned procedure from the equations; we get series solution as follows:

$$y(x)=\frac{A}{720}+\frac{B}{5040}+\frac{C}{40320}+\frac{D}{362880}+\frac{E}{3628800}+\frac{F}{39916800}+132x+\frac{A}{120}+\frac{Bx}{720}+\frac{Cx}{5040}+\frac{Dx}{40320}+\frac{Ex}{362880}+\frac{Fx}{3628800}+\frac{Ax^2}{48}+\frac{Bx^2}{240}+\frac{Cx^2}{1440}+\frac{Dx^2}{10080}+\frac{Ex^2}{80640}+\frac{Fx^2}{725760}-14x^3+\frac{Ax^3}{36}+\frac{Bx^3}{144}+\frac{Cx^3}{720}+\frac{Dx^3}{4320}+\frac{Ex^3}{30240}+\frac{Fx^3}{+241920}+\frac{Ax^4}{48}+\frac{Bx^4}{144}+\frac{Cx^4}{576}+\frac{Dx^4}{2880}+\frac{Ex^4}{17280}+\frac{Fx^4}{120960}+\frac{3x^5}{10}+\frac{Ax^5}{120}+\frac{Bx^5}{240}+\frac{Cx^5}{720}+\frac{Dx^5}{2880}+\frac{Ex^2}{14400}+\frac{Fx^2}{86400}+\frac{Ax^6}{720}+\frac{Bx^6}{720}+\frac{Cx^6}{1440}+\frac{Dx^6}{4320}+\frac{Ex^6}{17280}+\frac{Fx^6}{86400}+\frac{x^7}{420}+\frac{Bx^7}{5040}+\frac{Cx^7}{5040}+\frac{Ex^7}{30240}\ \frac{Fx^7}{120960}\ \frac{Bx^8}{40320}$$

Table 3. Error estimate.

X	Exact solution	Numerical solution of VIDM	Errors of VIDM
-1	0.000000000	-1.60000×10^{-9}	-1.60000×10^{-9}
-0.8	0.258248192	0.2582481925	-2.00000×10^{-10}
-0.6	0.361371183	0.3613711820	-1.00000×10^{-9}
-0.4	0.327111407	0.3271114041	-3.00000×10^{-9}
-0.2	0.190722557	0.1907225537	-3.90000×10^{-9}
0	0.000000000	0.000000000	0.000000000
0.2	-0.190722557	-0.1907225537	3.30000×10^{-9}
0.4	-0.327111407	-0.3271114041	2.90000×10^{-9}
0.6	-0.361371183	-0.3613711820	1.00000×10^{-9}
0.8	-0.258248192	-0.2582481925	-5.00000×10^{-10}
1.0	0.000000000	1.60000×10^{-9}	1.60000×10^{-9}

$$\frac{Cx^8}{40320}+\frac{Dx^8}{80640}+\frac{Ex^8}{241920}-\frac{x^9}{40320}+\frac{Cx^9}{+80640}+\frac{Dx^9}{241920}+\frac{Ex^9}{725760}+\frac{Fx^9}{3628800}\frac{Ex^{10}}{3628800}\frac{Ex^{11}}{39916800}-\frac{x^{13}}{39916800}+\frac{17x^{15}}{+108972864000}-\frac{439426777\cos(1)}{3326400}-+\frac{x^{10}\sin(1)}{33600}+\frac{x^{11}\sin(1)}{1663200}+O(x^{16})$$

The coefficients A, B, C, D, E and F can be obtained using the boundary conditions at x=1

A=31.7278,
B=10.9123,
C=55.7652,
D=23.7394
E=86.616,
F=41.1225.

$y(x) = -4.30162 \times 10^{-8} - x -5.92893\times 10^{-7}$

$x^2+1.16667x^3-4070511\times10^{-7}\,x^4-$

$0.175x^5+9.60877\times10^{-9}$

$x^6+0.00853175x^7+6029681\times10^{-10}x^8-$

$0.000201166x^9-7.53243\times10^{-}$

${}^{12}x^{10}+2.7799\times10^{-6}x^{11}-2.5052\times10^{-}$

${}^{8}x^{13}+1.56002\times10^{-10}x^{15}+O(x^{16})$

Comparison of the approximate solutions with exact solution is tabulated in Table 3 along with errors of VIDM, revealing the high accuracy of the results from VIDM. Once again, as stated in the examples, it is obvious that higher accuracy could be obtained without any difficulty. Table 3 shows the approximate solution obtained by (VIDM) and error obtained by comparing it with the exact solution. Higher accuracy can be obtained by evaluating more iteration.

Conclusion

Variational iteration decomposition method (VIDM) is applied to the numerical solution for solving higher order boundary value problems. Comparison of the results obtained by the present method with that obtained by exact solution reveals that the present method is very effective and convenient. The numerical results in Tables 1, 2 and 3 show that the present method provides highly accurate numerical results. It can be concluded that variational iteration decomposition method is a highly efficient method for solving high-order boundary value problems arising in various fields of engineering and science.

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