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Full Length Research Paper

Observer design for rectangular discrete-time singular systems with time-varying delay

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In this paper, observer design problem for rectangular discrete-time singular systems with time-varying delay is discussed. Based on restricted system equivalent (RSE) transformations and by introducing new state vectors, the problem is transformed into observer design problem for time-varying delay discrete-time standard state-space systems. By Lyapunov function method and linear matrix inequality (LMI) technique, the delay-dependent sufficient condition that there exists a state observer is established. The design method of matrix K is discussed and the coefficient matrices of function observer are given. Finally, a numerical example is given showing the effectiveness of the method given in the paper.

Key words: Rectangular discrete-time singular systems, time-varying delay, observer design, linear matrix inequality (LMI), delay-dependent.

INTRODUCTION

Time-delay is frequently a source of instability and is commonly encountered in various areas such as engineering, economics, etc. Compared to systems without time-delay, time-delay systems are more complex. So, the study of time-delay systems has attracted a lot of interest. Observer design for time-delay systems has been extensively investigated (Bhat and Koivo, 1976; Fairman and Kumar, 1986; Boutayeb, 2001; Trinh and Aldeen, 1997). Commonly, the approaches for solving time-delay systems can be classified into two types: delay-dependent conditions which include information on the size of delays, and delay-independent conditions which are applicable to delays of arbitrary size. Since the stability of a system depends explicitly on the time-delay, a delay-independent condition is more conservative especially for small delays, while a delay-dependent condition is usually less conservative.

Singular systems are also referred to as descriptor systems or generalized state-space systems. It has extensive applications in many practical systems such as chemical processes, circuit boundary control systems, economy systems and other areas (Aplevich, 1991; Dai, 1989). In the last decades, many control theories based on singular systems, have been extensively studied. In recent years, much attention has been focused on time-delay singular systems. The observer design problem for time-delay singular systems was investigated in Feng et al. (2005), Ma and Cheng (2005) and Ma and Cheng (2006). Feng et al. (2005) discussed observer design problem for continuous-time singular time-delay systems. Ma and Cheng (2005, 2006) solved observer design problem for discrete time-delay singular systems with unknown inputs, the conditions in Feng et al. (2005) and Ma and Cheng (2005, 2006) are all delay-independent. However, to the best of our knowledge, the delay-dependent conditions for observer design problem for time-varying delay rectangular discrete-time singular systems have not yet appeared in the literature.

In this paper, observer design problem for time-varying delay rectangular discrete-time singular systems is discussed. First, based on restricted system equivalent (RSE) transformations and by introducing new state vectors, the problem is transformed into observer design problem for time-varying delay discrete-time standard state-space systems. Then, by Lyapunov function method and linear matrix inequality (LMI) technique, the delay-dependent sufficient condition that there exists a state observer is established. Next, the design method of matrix K is discussed and the coefficient matrices of function observer are given. Finally, a numerical example is given showing the effectiveness of the method given in the paper.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite. I is the identity matrix with appropriate dimensions. A superscript T represents transpose. $\|x\|$ refers to the Euclidean norm of the vector x, that is $\|x\|^2 = x^T x$. $*$ denotes the matrix entries implied by the symmetry of a matrix.

DESCRIPTION OF PROBLEM AND PRELIMINARIES

Consider the time-varying delay rectangular discrete-time singular system described by:

$$\begin{cases} Ex(k+1) = Ax(k) + A_d x(k-d(k)) + Bu(k), \\ y(k) = Cx(k) + C_d x(k-d(k)) + Du(k), \\ z(k) = Lx(k), \end{cases} \quad (1)$$

where, $x(k) \in \mathbb{R}^n$ is the state variable, $u(k) \in \mathbb{R}^p$ is the control input, $y(k) \in \mathbb{R}^q$ is the measurement output, $z(k) \in \mathbb{R}^l$ is the vector to be estimated, $d(k)$ is the integer time-varying delay and $0 < d(k) \le d$, $d > 0$ is a known integer. The matrix $E \in \mathbb{R}^{m\times n}$ is singular, and $rankE = r < \min\{m, n\}$, $A \in \mathbb{R}^{m\times n}$, $A_d \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times p}$, $C \in \mathbb{R}^{q\times n}$, $C_d \in \mathbb{R}^{q\times n}$, $D \in \mathbb{R}^{q\times p}$, $L \in \mathbb{R}^{l\times n}$ are real constant matrices.

The purpose of this paper is to design a function observer for system (1) as follows:

$$\begin{cases} \xi(k+1) = N_1\xi(k) + N_d\xi(k-d(k)+1) + J_1u(k) + J_2y(k), \\ \tilde{z}(k) = M_1\xi(k) + J_3y(k), \end{cases} \quad (2)$$

where the coefficient matrices have corresponding dimensions such that for any admissible initial value it satisfies that:

$$\lim_{k\to\infty}(z(k) - \tilde{z}(k)) = 0. \quad (3)$$

In order to design the functional observer above, we assume that:

Assumption 1

$$\text{rank}\begin{bmatrix} 0 & E \\ E & A \\ 0 & C \end{bmatrix} = n + r. \quad (4)$$

Assumption 2

$$\text{rank}\begin{bmatrix} 0 & E & 0 & 0 \\ E & A & A_d & B \\ 0 & C & C_d & D \\ 0 & L & 0 & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} 0 & E & 0 & 0 \\ E & A & A_d & B \\ 0 & C & C_d & D \end{bmatrix}. \quad (5)$$

Assumption 3

$$\text{rank}L = l. \quad (6)$$

Remark 1. If system (E, A) is regular, then Assumption 1 is the definition of Y-observable for system (E, A, C) (Dai, 1989), which is the necessary condition for the existence of observer.

The following lemmas are useful in the proof of the main results.

Lemma 1 (Xie and de Souza, 1992). Given any matrices X, Y, Z with appropriate dimensions and $Y > 0$. Then,

$$-X^T Z - Z^T X \le X^T Y X + Z^T Y^{-1} Z.$$

Lemma 2 (Xu, 2002)**.** Given a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, the following conditions are equivalent:

(1) $S < 0$;

(2) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.$

MAIN RESULTS

Since $rankE = r$, there exist two nonsingular matrices $M \in \mathbb{R}^{m\times m}$ and $N \in \mathbb{R}^{n\times n}$ such that :

$$\begin{cases} MEN=\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ MAN=\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \\ MA_dN=\begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix},\ MB=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix},\ CN=[C_1 \quad C_2], \\ C_dN=[C_{d1} \quad C_{d2}],\ LN=[L_1 \quad L_2],\ x(k)=N\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \end{cases} \quad (7)$$

where $x_1(k)\in\mathbb{R}^r$, $x_2(k)\in\mathbb{R}^{n-r}$, $A_1\in\mathbb{R}^{r\times r}$ $A_2\in\mathbb{R}^{r\times(n-r)}$, $A_3\in\mathbb{R}^{(m-r)\times r}$, $A_4\in\mathbb{R}^{(m-r)\times(n-r)}$, $A_{d1}\in\mathbb{R}^{r\times r}$, $A_{d2}\in\mathbb{R}^{r\times(n-r)}$, $A_{d3}\in\mathbb{R}^{(m-r)\times r}$, $A_{d4}\in\mathbb{R}^{(m-r)\times(n-r)}$, $B_1\in\mathbb{R}^{r\times p}$, $B_2\in\mathbb{R}^{(m-r)\times p}$, $C_1\in\mathbb{R}^{q\times r}$, $C_2\in\mathbb{R}^{q\times(n-r)}$, $C_{d1}\in\mathbb{R}^{q\times r}$, $C_{d2}\in\mathbb{R}^{q\times(n-r)}$, $L_1\in\mathbb{R}^{l\times r}$, $L_2\in\mathbb{R}^{l\times(n-r)}$. So system (1) is RSE to the following system:

$$\begin{cases} x_1(k+1)=A_1x_1(k)+A_2x_2(k)+A_{d1}x_1(k-d(k))+A_{d2}x_2(k-d(k))+B_1u(k), \\ 0=A_3x_1(k)+A_4x_2(k)+A_{d3}x_1(k-d(k))+A_{d4}x_2(k-d(k))+B_2u(k), \\ y(k)=C_1x_1(k)+C_2x_2(k)+C_{d1}x_1(k-d(k))+C_{d2}x_2(k-d(k))+Du(k), \\ z(k)=L_1x_1(k)+L_2x_2(k). \end{cases} \quad (8)$$

By Assumption 1, $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ is of full column rank (Dai, 1989). So, there exists a non-singular matrix $P\in\mathbb{R}^{(m-r+q)\times(m-r+q)}$ such that:

$$\begin{cases} P\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}=\begin{bmatrix} I_{n-r} \\ 0 \end{bmatrix},\ P\begin{bmatrix} A_3 \\ C_1 \end{bmatrix}=\begin{bmatrix} \bar{A}_3 \\ \bar{C}_1 \end{bmatrix},\ P\begin{bmatrix} A_{d3} \\ C_{d1} \end{bmatrix}=\begin{bmatrix} \bar{A}_{d3} \\ \bar{C}_{d1} \end{bmatrix}, \\ P\begin{bmatrix} A_{d4} \\ C_{d2} \end{bmatrix}=\begin{bmatrix} \bar{A}_{d4} \\ \bar{C}_{d2} \end{bmatrix},\ P\begin{bmatrix} B_2 \\ D \end{bmatrix}=\begin{bmatrix} \bar{B}_2 \\ \bar{D} \end{bmatrix},\ P\begin{bmatrix} 0 \\ y(k) \end{bmatrix}=\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}, \end{cases} \quad (9)$$

where $\bar{A}_3\in\mathbb{R}^{(n-r)\times r}$, $\bar{C}_1\in\mathbb{R}^{(m-n+q)\times r}$, $\bar{A}_{d3}\in\mathbb{R}^{(n-r)\times r}$, $\bar{C}_{d1}\in\mathbb{R}^{(m-n+q)\times r}$, $\bar{A}_{d4}\in\mathbb{R}^{(n-r)\times(n-r)}$, $\bar{C}_{d2}\in\mathbb{R}^{(m-n+q)\times(n-r)}$, $\bar{B}_2\in\mathbb{R}^{(n-r)\times p}$, $\bar{D}\in\mathbb{R}^{(m-n+q)\times p}$, $y_1(k)\in\mathbb{R}^{n-r}$, $y_2(k)\in\mathbb{R}^{m-n+q}$. Letting $P=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$,

where $P_{11}\in\mathbb{R}^{(n-r)\times(m-r)}$, $P_{12}\in\mathbb{R}^{(n-r)\times q}$, $P_{21}\in\mathbb{R}^{(m-n+q)\times(m-r)}$, $P_{22}\in\mathbb{R}^{(m-n+q)\times q}$, then,

$$y_1(k)=P_{12}y(k),\quad y_2(k)=P_{22}y(k). \quad (10)$$

By transformation (9), system (8) is RSE to the following system:

$$\begin{cases} x_1(k+1)=\bar{A}_1x_1(k)+\bar{A}_{d1}x_1(k-d(k))+\bar{A}_{d2}x_2(k-d(k))+\bar{B}_1u(k)+A_2y_1(k), \\ x_2(k)=-\bar{A}_3x_1(k)-\bar{A}_{d3}x_1(k-d(k))-\bar{A}_{d4}x_2(k-d(k))-\bar{B}_2u(k)+y_1(k), \\ y_2(k)=\bar{C}_1x_1(k)+\bar{C}_{d1}x_1(k-d(k))+\bar{C}_{d2}x_2(k-d(k))+\bar{D}u(k), \\ z(k)=L_1x_1(k)+L_2x_2(k), \end{cases} \quad (11)$$

where

$\bar{A}_1=A_1-A_2\bar{A}_3$, $\bar{A}_{d1}=A_{d1}-A_2\bar{A}_{d3}$, $\bar{A}_{d2}=A_{d2}-A_2\bar{A}_{d4}$, $\bar{B}_1=B_1-A_2\bar{B}_2$.

Remark 2. Since $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ is of full column rank, the solution of $x_2(k)$ is unique. So the expression of system (11) is independent of matrix P and is unique.

By introducing new state vectors as

$$\bar{x}(k+1)=\left[x_1^T(k+1) \quad x_1^T(k) \quad x_2^T(k)\right]^T, \quad (12)$$

system (11) is rewritten as:

$$\begin{cases} \bar{x}(k+1)=\bar{A}\bar{x}(k)+\bar{A}_d\bar{x}(k-d(k)+1)+\bar{B}u(k)+Gy(k), \\ y_2(k)=\bar{C}\bar{x}(k)+\bar{C}_d\bar{x}(k-d(k)+1)+\bar{D}u(k), \\ z(k)=\bar{L}\bar{x}(k+1), \end{cases} \quad (13)$$

where

$$\begin{cases} \bar{A} = \begin{bmatrix} \bar{A}_1 & 0 & 0 \\ I_r & 0 & 0 \\ -\bar{A}_3 & 0 & 0 \end{bmatrix}, \ \bar{A}_d = \begin{bmatrix} 0 & \bar{A}_{d1} & \bar{A}_{d2} \\ 0 & 0 & 0 \\ 0 & -\bar{A}_{d3} & -\bar{A}_{d4} \end{bmatrix}, \\ \bar{B} = \begin{bmatrix} \bar{B}_1 \\ 0 \\ -\bar{B}_2 \end{bmatrix}, \ G = \begin{bmatrix} A_2 P_{12} \\ 0 \\ P_{12} \end{bmatrix}, \ \bar{C} = \begin{bmatrix} \bar{C}_1 & 0 & 0 \end{bmatrix}, \\ \bar{C}_d = \begin{bmatrix} 0 & \bar{C}_{d1} & \bar{C}_{d2} \end{bmatrix}, \ \bar{L} = \begin{bmatrix} 0 & L_1 & L_2 \end{bmatrix}. \end{cases} \tag{14}$$

Remark 3. Based on the RSE transformations (7), (9) and by introducing new state vectors (12), time-varying delay discrete singular system (1) is transformed into a time-varying delay discrete standard state-space system (13). Hence, we can discuss the problem of observer design for system (13) instead of that for system (1).

First, we design a $(n+r)$-order state observer for system (13) as follows:

$$\begin{aligned} \xi(k+1) &= \bar{A}\xi(k) + \bar{A}_d \xi(k-d(k)+1) + \bar{B}u(k) + Gy(k) \\ &\quad + K(y_2(k) - \bar{C}\xi(k) - \bar{C}_d \xi(k-d(k)+1) - \bar{D}u(k)). \end{aligned} \tag{15}$$

Theorem 1. The system (15) is a state observer for system (13), if there exist matrices $X>0$, $Z>0$, $U>0$, N_1, N_2, N_3, S_1, S_2, S_3 and K satisfying the following LMI

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & (d-1)N_1 \\ * & \Phi_{22} & \Phi_{23} & (d-1)N_2 \\ * & * & \Phi_{33} & (d-1)N_3 \\ * & * & * & -(d-1)Z \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{cases} \Phi_{11} = (d+1)U + N_1 + N_1^T + S_1(\hat{A}-I) + (\hat{A}-I)^T S_1^T, \\ \Phi_{12} = -N_1 + N_2^T + S_1\hat{A}_d + (\hat{A}-I)^T S_2^T, \\ \Phi_{13} = X + N_3^T - S_1 + (\hat{A}-I)^T S_3^T, \\ \Phi_{22} = -U - N_2 - N_2^T + S_2\hat{A}_d + \hat{A}_d^T S_2^T, \\ \Phi_{23} = -N_3^T - S_2 + \hat{A}_d^T S_3^T, \\ \Phi_{33} = X + (d-1)Z - S_3 - S_3^T, \\ \hat{A} = \bar{A} - K\bar{C}, \ \hat{A}_d = \bar{A}_d - K\bar{C}_d. \end{cases}$$

Proof. Let

$$e(k) = \bar{x}(k) - \xi(k). \tag{17}$$

By (13) and (15),

$$\begin{aligned} e(k+1) &= (\bar{A} - K\bar{C})e(k) + (\bar{A}_d - K\bar{C}_d)e(k-d(k)+1) \\ &= \hat{A}e(k) + \hat{A}_d e(k-d(k)+1). \end{aligned} \tag{18}$$

If system (18) is stable, then system (15) is a state observer of system (13). Next, we prove the stability of system (18). Rewrite system (18) as:

$$\begin{cases} e(k+1) = e(k) + h(k), \\ 0 = -h(k) + (\hat{A}-I)e(k) + \hat{A}_d e(k-d(k)+1). \end{cases} \tag{19}$$

Construct the discrete Lyapunov-Krasovskii functional as:

$$\begin{cases} V = V_1 + V_2 + V_3 + V_4, \\ V_1 = e^T(k)Xe(k), \ V_2 = \sum\limits_{l=k-d(k)+1}^{k-1} e^T(l)Ue(l), \\ V_3 = \sum\limits_{\theta=-d+2}^{0} \sum\limits_{l=k-1+\theta}^{k-1} h^T(l)Zh(l), \\ V_4 = \sum\limits_{\theta=-d+2}^{1} \sum\limits_{l=k+\theta}^{k-1} e^T(l)Ue(l), \end{cases}$$

where $X>0$, $Z>0$, $U>0$. Then,

$$\begin{aligned} \Delta V_1 &= e^T(k+1)Xe(k+1) - e^T(k)Xe(k) \\ &= (e(k)+h(k))^T X(e(k)+h(k)) - e^T(k)Xe(k) \\ &= e^T(k)Xh(k) + h^T(k)Xe(k) + h^T(k)Xh(k), \end{aligned} \tag{20}$$

$$\begin{aligned} \Delta V_2 &= \sum_{l=k-d(k+1)+2}^{k} e^T(l)Ue(l) - \sum_{l=k-d(k)+1}^{k-1} e^T(l)Ue(l) \\ &= e^T(k)Ue(k) + \sum_{l=k-d(k+1)+2}^{k-1} e^T(l)Ue(l) \\ &\quad - \sum_{l=k-d(k)+2}^{k-1} e^T(l)Ue(l) - e^T(k-d(k)+1)Ue(k-d(k)+1) \\ &\le e^T(k)Ue(k) - e^T(k-d(k)+1)Ue(k-d(k)+1) \\ &\quad + \sum_{l=k-d+2}^{k-1} e^T(l)Ue(l) - \sum_{l=k+2}^{k-1} e^T(l)Ue(l) \\ &= e^T(k)Ue(k) - e^T(k-d(k)+1)Ue(k-d(k)+1) \\ &\quad + \sum_{l=k-d+2}^{k+1} e^T(l)Ue(l) + \sum_{l=k+2}^{k-1} e^T(l)Ue(l) - \sum_{l=k+2}^{k-1} e^T(l)Ue(l) \\ &= e^T(k)Ue(k) - e^T(k-d(k)+1)Ue(k-d(k)+1) \\ &\quad + \sum_{l=k-d+2}^{k+1} e^T(l)Ue(l), \end{aligned} \tag{21}$$

$$\begin{aligned}\Delta V_3 &= \sum_{\theta=-d+2}^{0}\sum_{l=k+\theta}^{k} h^T(l)Zh(l) - \sum_{\theta=-d+2}^{0}\sum_{l=k-1+\theta}^{k-1} h^T(l)Zh(l) \\ &= \sum_{\theta=-d+2}^{0}\left[h^T(k)Zh(k) - h^T(k-1+\theta)Zh(k-1+\theta)\right] \\ &= (d-1)h^T(k)Zh(k) - \sum_{l=k-d+1}^{k-1} h^T(l)Zh(l) \\ &\le (d-1)h^T(k)Zh(k) - \sum_{l=k-d(k)+1}^{k-1} h^T(l)Zh(l),\end{aligned} \tag{22}$$

$$\begin{aligned}\Delta V_4 &= \sum_{\theta=-d+2}^{1}\sum_{l=k+1+\theta}^{k} e^T(l)Ue(l) - \sum_{\theta=-d+2}^{1}\sum_{l=k+\theta}^{k-1} e^T(l)Ue(l) \\ &= \sum_{\theta=-d+2}^{1}[e^T(k)Ue(k) + \sum_{l=k+1+\theta}^{k-1} e^T(l)Ue(l) \\ &\quad - \sum_{l=k+1+\theta}^{k-1} e^T(l)Ue(l) - e^T(k+\theta)Ue(k+\theta)] \\ &= \sum_{\theta=-d+2}^{1}[e^T(k)Ue(k) - e^T(k+\theta)Ue(k+\theta)] \\ &= de^T(k)Ue(k) - \sum_{l=k-d+2}^{k+1} e^T(l)Ue(l),\end{aligned} \tag{23}$$

From (20) to (23), it follows that:

$$\begin{aligned}\Delta V &\le (d+1)e^T(k)Ue(k) + e^T(k)Xh(k) + h^T(k)Xe(k) + h^T(k)Xh(k) \\ &\quad -e^T(k-d(k)+1)Ue(k-d(k)+1) + (d-1)h^T(k)Zh(k) - \sum_{l=k-d(k)+1}^{k-1} h^T(l)Zh(l).\end{aligned} \tag{24}$$

According to the first formula of (19), for appropriate dimensions $N_1,\ N_2,\ N_3$, the following is true

$$2(e^T(k)N_1 + e^T(k-d(k)+1)N_2 + h^T(k)N_3)\cdot(e(k) - e(k-d(k)+1) - \sum_{l=k-d(k)+1}^{k-1} h(l)) = 0. \tag{25}$$

Also, according to the second formula of (19), for appropriate dimensions S_1, S_2, S_3, the following is true

$$2(e^T(k)S_1 + e^T(k-d(k)+1)S_2 + h^T(k)S_3)\cdot(-h(k) + (\hat{A}-I)e(k) + \hat{A}_d e(k-d(k)+1)) = 0. \tag{26}$$

By Lemma 1, the following holds

$$\begin{aligned}&-2(e^T(k)N_1 + e^T(k-d(k)+1)N_2 + h^T(k)N_3)\sum_{l=k-d(k)+1}^{k-1} h(l) \\ &= -2\sum_{l=k-d(k)+1}^{k-1}\left[e^T(k) \quad e^T(k-d(k)+1) \quad h^T(k)\right]\left[N_1^T \quad N_2^T \quad N_3^T\right]^T h(l) \\ &\le (d-1)\left[e^T(k) \quad e^T(k-d(k)+1) \quad h^T(k)\right]Q\cdot \\ &\left[e^T(k) \quad e^T(k-d(k)+1) \quad h^T(k)\right]^T + \sum_{l=k-d(k)+1}^{k-1} h^T(l)Zh(l),\end{aligned} \tag{27}$$

where $Q=\begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}^T Z^{-1}\begin{bmatrix} N_1^T & N_2^T & N_3^T \end{bmatrix}$.

Then, adding the terms on the left of (25) and (26) to ΔV, and by (27), we obtain

$$\Delta V \le \begin{bmatrix} e^T(k) & e^T(k-d(k)+1) & h^T(k) \end{bmatrix}(\bar{\Lambda}+(d-1)Q)\cdot\begin{bmatrix} e^T(k) & e^T(k-d(k)+1) & h^T(k) \end{bmatrix}^T, \quad (28)$$

where

$$\bar{\Lambda}=\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & \Phi_{23} \\ * & * & \Phi_{33} \end{bmatrix}.$$

If $\bar{\Lambda}+(d-1)Q<0$, then $\Delta V<-\alpha\|e(k)\|^2$ for a sufficiently small $\alpha>0$, which ensures the stability of system (18). According to Lemma 2, $\bar{\Lambda}+(d-1)Q<0$, which is equivalent to (16) holds. So, (15) is a state observer for system (13).

It is notable that there is quadratic terms in LMI (16). So, it cannot be solved directly. In order to obtain observer (15), we must work out unknown matrix K. Therefore, we have the following theorem.

Theorem 2. There exists a function observer for system (1), if for given scalars $t_1, t_2, t_3,$ there exist matrices $X>0,\ Z>0,\ U>0,\ N_1,\ N_2,\ N_3, W_1,\ W_2,\ S$ and non-singular matrix R, satisfying the following conditions

$$\Psi=\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & (d-1)N_1 \\ * & \Psi_{22} & \Psi_{23} & (d-1)N_2 \\ * & * & \Psi_{33} & (d-1)N_3 \\ * & * & * & -(d-1)Z \end{bmatrix}<0, \quad (29)$$

$$S\bar{L}^{\perp}=\bar{L}^{\perp}R, \quad (30)$$

where

$$\begin{aligned}\Psi_{11}=&(d+1)U+N_1+N_1^T+t_1S\bar{A}+t_1\bar{A}^TS^T-t_1S-t_1S^T-t_1W_1\bar{C}-t_1\bar{C}^TW_1^T\\ &-t_1\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}^T\bar{L}^T\bar{L}^{+T}S^T-t_1S\bar{L}^+\bar{L}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}\\ &+t_1W_1\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}+t_1\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_1^T\\ &-t_1\bar{L}^{\perp}W_2\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}-t_1\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_2^T\bar{L}^{\perp T},\\ \Psi_{12}=&-N_1+N_2^T+t_1S\bar{A}_d-t_1W_1\bar{C}_d+t_2\bar{A}^TS^T-t_2S^T-t_2\bar{C}^TW_1^T\\ &-t_1S\bar{L}^+\bar{L}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d-t_1\bar{L}^{\perp}W_2\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d\\ &+t_1W_1\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d-t_2\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}^T\bar{L}^T\bar{L}^{+T}S^T\\ &-t_2\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_2^T\bar{L}^{\perp T}+t_2\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_1^T,\end{aligned}$$

$$\begin{aligned}\Psi_{13}=&X+N_3^T-t_1S+t_3\bar{A}^TS^T-t_3S^T-t_3\bar{C}^TW_1^T+t_3\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_1^T\\ &-t_3\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_2^T\bar{L}^{\perp T}-t_3\bar{C}^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}^T\bar{L}^T\bar{L}^{+T}S^T,\\ \Psi_{22}=&-U-N_2-N_2^T+t_2S\bar{A}_d+t_2\bar{A}_d^TS^T-t_2W_1\bar{C}_d-t_2\bar{C}_d^TW_1^T\\ &-t_2S\bar{L}^+\bar{L}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d-t_2\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}^T\bar{L}^T\bar{L}^{+T}S^T\\ &-t_2\bar{L}^{\perp}W_2\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d-t_2\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_2^T\bar{L}^{\perp T}\\ &+t_2W_1\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\bar{C}_d+t_2\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_1^T,\\ \Psi_{23}=&-N_3^T-t_2S+t_3\bar{A}_d^TS^T-t_3\bar{C}_d^TW_1^T-t_3\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}^T\bar{L}^T\bar{L}^{+T}S^T\\ &-t_3\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_2^T\bar{L}^{\perp T}+t_3\bar{C}_d^T\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^{+T}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^TW_1^T,\\ \Psi_{33}=&X+(d-1)Z-t_3S-t_3S^T,\end{aligned}$$

$\bar{L}^+=\bar{L}^T(\bar{L}\bar{L}^T)^{-1}$ is the generalized inverse of $\bar{L}$, $\bar{L}^{\perp}\in\mathbb{R}^{(n+r)\times(n+r-l)}$ satisfies $\bar{L}\bar{L}^{\perp}=0$, and is of full column rank, $\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+$ is the generalized inverse of $\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}$. And the expression of observer is

$$\begin{cases}\xi(k+1)=\bar{A}\xi(k)+\bar{A}_d\xi(k-d+1)+\bar{B}u(k)+Gy(k)\\ \qquad +K(y_2(k)-\bar{C}\xi(k)-\bar{C}_d\xi(k-d+1)-\bar{D}u(k)),\\ \tilde{z}(k)=(\bar{L}\bar{A}-\bar{L}K\bar{C})\xi(k)+(\bar{L}G+\bar{L}KP_{22})y(k),\end{cases} \quad (31)$$

Where

$$\begin{aligned}K=&\bar{L}^+\bar{L}\begin{bmatrix}\bar{A}_d & \bar{B}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^++\bar{L}^{\perp}\bar{K}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+\\ &+\hat{K}-\hat{K}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}\begin{bmatrix}\bar{C}_d & \bar{D}\end{bmatrix}^+,\end{aligned}$$

$$\hat{K}=S^{-1}W_1,\ \bar{K}=R^{-1}W_2.$$

Proof. Let

$$\tilde{z}(k)=\bar{L}\xi(k+1). \quad (32)$$

By Theorem 1, if (16) is satisfied, then system (18) is stable. On this condition, from the third formula of (13), (17) and (32),

$$\lim_{k\to\infty}(z(k)-\tilde{z}(k))=\lim_{k\to\infty}\bar{L}e(k+1)=0.$$

According to the second formula of (10), (15) and (32),

$$\tilde{z}(k) = (\overline{LA} - \overline{L}K\overline{C})\xi(k) + (\overline{L}G + \overline{L}KP_{22})y(k) + (\overline{LA}_d - \overline{L}K\overline{C}_d)\xi(k-d+1) + (\overline{LB} - \overline{L}K\overline{D})u(k).$$

Compared with (31), we obtain

$$\overline{LA}_d - \overline{L}K\overline{C}_d = 0,\ \overline{LB} - \overline{L}K\overline{D} = 0. \tag{33}$$

By the analysis above, if we can work out K satisfying both (16) and (33), then (31) is a functional observer for system (13). First, we discuss the design of matrix K. From (33),

$$\overline{L}K\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} = \overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix} \tag{34}$$

By Assumption 3, $\overline{L}$ is of full row rank, so $\overline{L}^{+} = \overline{L}^{T}(\overline{L}\overline{L}^{T})^{-1}$ is of full column rank. Relating that $\overline{L}^{\perp} \in \mathbb{R}^{(n+r)\times(n+r-l)}$ satisfies $\overline{L}\overline{L}^{\perp} = 0$, and is of full column rank. Hence, $\begin{bmatrix}\overline{L}^{+} & \overline{L}^{\perp}\end{bmatrix}$ is a non-singular matrix. From (34), we obtain

$$K\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} = \overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix} + \overline{L}^{\perp}\overline{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}, \tag{35}$$

where $\overline{K}$ is an arbitrary matrix of appropriate dimension. By (14), we get

$$\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} = \begin{bmatrix}0 & \overline{C}_{d1} & \overline{C}_{d2} & \overline{D}\end{bmatrix}$$

And

$$\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix} = \begin{bmatrix}0 & -L_2\overline{A}_{d3} & -L_2\overline{A}_{d4} & -L_2\overline{B}_2\end{bmatrix}.$$

It is easy to prove that Assumption 2 is equivalent to

$$\operatorname{rank}\begin{bmatrix}L_2\overline{A}_{d3} & L_2\overline{A}_{d4} & L_2\overline{B}_2 \\ \overline{C}_{d1} & \overline{C}_{d2} & \overline{D}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}\overline{C}_{d1} & \overline{C}_{d2} & \overline{D}\end{bmatrix}.$$

On this condition,

$$\begin{aligned}\operatorname{rank}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} &= \operatorname{rank}\begin{bmatrix}\overline{C}_{d1} & \overline{C}_{d2} & \overline{D}\end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix}L_2\overline{A}_{d3} & L_2\overline{A}_{d4} & L_2\overline{B}_2 \\ \overline{C}_{d1} & \overline{C}_{d2} & \overline{D}\end{bmatrix} \\ &= \operatorname{rank}\begin{pmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} \\ \overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix}\end{pmatrix} = \operatorname{rank}\begin{pmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} \\ \overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix}\end{pmatrix} \\ &= \operatorname{rank}\begin{pmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix} \\ \overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix} + \overline{L}^{\perp}\overline{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\end{pmatrix}.\end{aligned}$$

Therefore, from (35),

$$K = \overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+} + \overline{L}^{\perp}\overline{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+} + \hat{K} - \hat{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+},$$

where $\hat{K}$ is an arbitrary matrix of appropriate dimension. Next, we discuss the design of matrices $\overline{K}$ and $\hat{K}$. Setting $S_1 = t_1 S,\ S_2 = t_2 S,\ S_3 = t_3 S$, from $\Phi_{33} < 0$, S is a nonsingular matrix. In Φ_{11},

$$\begin{aligned}S_1\hat{A} &= t_1 S\overline{A} - t_1 SK\overline{C} \\ &= t_1 S\overline{A} - t_1 S\overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+}\overline{C} - t_1 S\overline{L}^{\perp}\overline{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+}\overline{C} \\ &\quad - t_1 S\hat{K}\overline{C} + t_1 S\hat{K}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+}\overline{C}.\end{aligned}$$

Set $S\hat{K} = W_1$, then $\hat{K} = S^{-1}W_1$. By equality constraint (30), then $S\overline{L}^{\perp}\overline{K} = \overline{L}^{\perp}R\overline{K}$. And set $R\overline{K} = W_2$, then $\overline{K} = R^{-1}W_2$. Hence,

$$K = \overline{L}^{+}\overline{L}\begin{bmatrix}\overline{A}_d & \overline{B}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+} + \overline{L}^{\perp}R^{-1}W_2\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+} + S^{-1}W_1 - S^{-1}W_1\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}\begin{bmatrix}\overline{C}_d & \overline{D}\end{bmatrix}^{+}. \tag{36}$$

And substituting (36) into LMI (16), we obtain LMI (29). This completes the proof.

Theorem 3. If for given scalars $t_1, t_2, t_3,$ there exist matrices $X > 0,\ Z > 0,\ U > 0$, N_1, N_2, N_3, W_1, W_2, S and nonsingular matrix R, satisfying conditions (29) and (30) (Yu, 2002), then there exists a function observer such as (2) for system (1), and the coefficient matrices of observer are:

$$\begin{cases}N_1 = \overline{A} - K\overline{C},\ N_d = \overline{A}_d - K\overline{C}_d,\ J_1 = \overline{B} - K\overline{D}, \\ J_2 = G + KP_{22},\ M_1 = \overline{LA} - \overline{L}K\overline{C},\ J_3 = \overline{L}G + \overline{L}KP_{22}.\end{cases}$$

Proof. By Theorem 2, the second formula of (10) and (31), we can directly obtain the conclusion.

For the case of $L = I_n$, $\overline{L}^{\perp} = \begin{bmatrix}I_r \\ 0\end{bmatrix}$. We can set $S = \begin{bmatrix}S_1 & \\ & S_2\end{bmatrix}$, where $S_1 \in \mathbb{R}^{r\times r}$, $S_2 \in \mathbb{R}^{n\times n}$. Due to $S\overline{L}^{\perp}\overline{K} = \begin{bmatrix}S_1\overline{K} \\ 0\end{bmatrix}$, we can set $S_1\overline{K} = \overline{W}_2$, then $\overline{K} = S_1^{-1}\overline{W}_2$. Now,

$$K = \bar{L}^{+}\bar{L}\left[\bar{A}_d \quad \bar{B}\right]\left[\bar{C}_d \quad \bar{D}\right]^{+} + \bar{L}^{\perp}S_1^{-1}\bar{W}_2\left[\bar{C}_d \quad \bar{D}\right]\left[\bar{C}_d \quad \bar{D}\right]^{+}$$
$$+S^{-1}W_1 - S^{-1}W_1\left[\bar{C}_d \quad \bar{D}\right]\left[\bar{C}_d \quad \bar{D}\right]^{+}.$$

On this condition, equality constraint can be omitted. We obtain the following conclusion.

Corollary 1. If for given scalars $t_1, t_2, t_3,$ there exist matrices $X > 0,\ Z > 0,\ U > 0$, N_1, N_2, N_3, W_1, $\bar{W}_2$, S, $S = \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix}$ satisfying the following LMI

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & (d-1)N_1 \\ * & \Psi_{22} & \Psi_{23} & (d-1)N_2 \\ * & * & \Psi_{33} & (d-1)N_3 \\ * & * & * & -(d-1)Z \end{bmatrix} < 0,$$

Then, there exists a state observer (31) for system (13), where $\bar{K} = S_1^{-1}\bar{W}_2$, other parameter matrices are the same as that in Theorem 2.

EXAMPLE

Consider system (1) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 4 & 1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix},\ A_d = \begin{bmatrix} 0.4 & -0.4 \\ 0.3 & 0.1 \\ -0.5 & 0.2 \end{bmatrix},\ B = \begin{bmatrix} 0.4 \\ 0.1 \\ 0.3 \end{bmatrix},$$
$$C = \begin{bmatrix} 5 & 0 \\ -2 & 0 \end{bmatrix},\ C_d = \begin{bmatrix} -0.1 & 0.4 \\ 0.2 & -0.6 \end{bmatrix},\ D = \begin{bmatrix} -0.1 \\ 0.5 \end{bmatrix},\ L = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Then, the coefficient matrices of system (13) are

$$\bar{A} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix},\ \bar{A}_d = \begin{bmatrix} 0 & 0.1 & -0.5 \\ 0 & 0 & 0 \\ 0 & -0.3 & -0.1 \end{bmatrix},\ \bar{B} = \begin{bmatrix} 0.3 \\ 0 \\ -0.1 \end{bmatrix},$$
$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \bar{C} = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix},\ \bar{C}_d = \begin{bmatrix} 0 & -0.5 & 0.2 \\ 0 & -0.1 & 0.4 \\ 0 & 0.2 & -0.6 \end{bmatrix},$$
$$\bar{L} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Let $d = 2,\ t_1 = 0.8,\ t_2 = -0.02,\ t_3 = 0.6,$ solve the LMI (29), it is obtained that

$$X = \begin{bmatrix} 95.988 & -2.7473 & -2.1658 \\ -2.7473 & 1.255 & 1.4257 \\ -2.1658 & 1.4257 & 2.8297 \end{bmatrix},\ Z = \begin{bmatrix} 5.6762 & 0.1553 & 0.6644 \\ 0.1553 & 0.0952 & 0.1139 \\ 0.6644 & 0.1139 & 0.5976 \end{bmatrix},$$

$$U = \begin{bmatrix} 3.0023 & -0.0701 & -0.1203 \\ -0.0701 & 0.1545 & 0.2081 \\ -0.1203 & 0.2081 & 0.3717 \end{bmatrix},\ N_1 = \begin{bmatrix} 7.3712 & 0.0820 & 0.3388 \\ -0.8775 & -0.0730 & -0.1077 \\ -4.0129 & -0.1242 & -0.6190 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 2.3225 & 0.0145 & 0.1043 \\ 0.1886 & 0.0803 & 0.0924 \\ 0.7109 & 0.0967 & 0.5113 \end{bmatrix},\ N_3 = \begin{bmatrix} 1.1583 & 0.0460 & 0.3148 \\ -0.1899 & 0.0040 & -0.0054 \\ -2.3134 & -0.0206 & -0.0686 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 17.166 & 203.5 & 143.03 \\ 40.479 & 525.86 & 364.07 \\ 64.194 & 831.05 & 575.8 \end{bmatrix},\ W_2 = \begin{bmatrix} 17.661 & 68.151 & 128.97 \\ -0.2840 & 1.6731 & -0.7105 \end{bmatrix},$$

$$S = \begin{bmatrix} 95.988 & -2.7473 & 1.255 \\ -2.1658 & 1.4257 & 2.8297 \\ 0 & 0 & 5.6762 \end{bmatrix}.$$

By (30), $R = \begin{bmatrix} 95.988 & -2.7473 \\ -2.1658 & 1.4257 \end{bmatrix}.$

Then,

$$\hat{K} = \begin{bmatrix} 0.2103 & 2.5566 & 1.7877 \\ 6.265 & 82.134 & 56.738 \\ 11.309 & 146.41 & 101.44 \end{bmatrix},\ \bar{K} = \begin{bmatrix} 0.1864 & 0.7774 & 1.3897 \\ 0.0839 & 2.3545 & 1.6128 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.1864 & 0.7774 & 1.3897 \\ 0.0839 & 2.3545 & 1.6128 \\ 0.6143 & -2.0143 & -0.9714 \end{bmatrix}.$$

Thus, the coefficient matrices of observer (2) are

$$N_1 = \begin{bmatrix} 0.3333 & 0 & 0 \\ -7.7985 & 0 & 0 \\ 4.2857 & 0 & 0 \end{bmatrix},\ N_d = \begin{bmatrix} 0 & -0.0070 & -0.0144 \\ 0 & -0.0451 & 0.0091 \\ 0 & 0 & 0 \end{bmatrix},$$

$$J_1 = \begin{bmatrix} -0.3730 \\ -0.5961 \\ 0 \end{bmatrix},\ J_2 = \begin{bmatrix} 0.1864 & 0.7774 & 1.3897 \\ 0.0839 & 2.3545 & 1.6128 \\ 0.6143 & -2.0143 & -0.9714 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 4.2857 & 0 & 0 \end{bmatrix},\ J_3 = \begin{bmatrix} 0.6143 & -2.0143 & -0.9714 \end{bmatrix}.$$

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