International Journal of Physical Sciences Vol. 4 (12), pp. 818-823, December, 2009
Available online at http://www.academicjournals.org/ijps
ISSN 1992 - 1950 © 2009 Academic Journals

Full Length Research Paper

Calculation of Ricci tensors by mathematica *V 5.1*

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Accepted 9 November, 2009

In solution of Einstein field equations it is necessary to contracting Riemann-Christofell tensor. The contraction of Riemann-Christofell tensor or simply the curvature tensor is called the Ricci tensor and denoted by R_{ij}. The calculation of Ricci tensor in 3 and especially in 4-dimension is not very difficult but, it is very tedious; and need more time with accuracy. In this paper, the focus has been given to construct a software programming for determination of Ricci tensors with Mathematica V 5.1. This software program is not only able to calculate for any dimension, but also has ability to add any coefficients that may be needed to assume in our line element with any coordinate systems.

Key words: Ricci tensor, Einstein field equation, mathematica software V5.1.

INTRODUCTION

An expression which expresses the distance, between two adjacent points is called a metric or line element. The simplest line element in the three dimensional spaces and Cartesian coordinates Systems is given by;

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad (1)$$

The line element with using summation convention and in terms of general curvilinear coordinates becomes;

$$ds^2 = \blacksquare\ \mathbf{EMBED\ Equation.3}\ \square\square\blacksquare du^j du^k = g_{jk} du^j du^k \qquad (2)$$

This idea was generalized by Riemann to *n*-dimensional space. Here the coefficients of g_{ik} are the components of a covariant symmetric tensor of rank two and called the metric tensor or fundamental tensor. These quantities are the functions of coordinates x^j, subject to the restriction g = determinant of g_{ik} that is $|g_{ij}| \neq 0$. The quadratic differential form of $g_{jk}\, dx^j dx^k$ is independent of the coordinate system and is called the Riemannian metric for *n*-dimensional space Gray, (1998). The space which is characterized by Riemannian metric is called Riemannian space. In the special case where the metric is represented by;

$$(\ dx^1\ )^2 + (\ dx^2\ )^2 + (\ dx^3\ )^2 + \cdots (\ dx^n\ )^2 \qquad (3)$$

or $dx^j dx^k$, the space is called *n*-dimensional Euclidean space (Landau et al., 1987). It is now obvious that Euclidean spaces are the particular cases of Riemannian space. In general theory of relativity (four dimensional spaces) and in general space (i.e. Riemannian space) the line element is given by;

$$ds^2 = g_{ij} dx^j dx^k \qquad (j, k = 1,2,3,4) \qquad (4)$$

This line element can be introduced in any coordinate systems like Cartesian, Spherical, and Ellipsoidal or in any other interesting coordinate systems. We introduce a tensor $R^{\lambda}_{\mu\nu\sigma}$ which is called Riemannian-Christofell tensor or simply curvature tensor as a tensor of rank 4, with contravariant and covariant in terms of λ and $\mu\nu\sigma$ respectively;

$$R^{\lambda}_{\mu\nu\sigma} = \left[\frac{\partial}{\partial x^{\nu}}\Gamma^{\lambda}_{\mu\sigma} - \frac{\partial}{\partial x^{\sigma}}\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\lambda}_{\alpha\sigma}\right] \qquad (5)$$

Riemannian-Christofell tensors do belong to the class of fundamental tensors and $R^{\lambda}_{\mu\nu\sigma}$ can be contracted in three ways, (a) Contraction with respect to indices λ and σ, (b) Contraction with respect to indices λ and ν, (c) Contraction with respect to indices λ, μ. But since

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$R^{\lambda}_{\mu\nu\sigma}$ are antisymmetric with respect to indices ν and, σ therefore the first and second cases give only one independent tensor. Thus there are only two ways of contracting Riemann-christofell tensor, one way leads to Ricci tensor while the other way leads to a zero tensor (Logunov, 1989). Therefore the contraction of Riemann-christofell tensor with respect to σ gives the second rank tensor called the Ricci tensor and denoted by $R_{\mu\nu}$ as,

$$R_{\mu\nu} = R^{\lambda}_{\mu\nu\lambda} = \frac{\partial}{\partial x^{\nu}}\Gamma^{\lambda}_{\mu\lambda} - \frac{\partial}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\lambda} \tag{6}$$

Ricci tensor may also be written as

$$R_{\mu\nu} = \frac{\partial(log\sqrt{g})}{\partial x^{\nu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\alpha\lambda} \tag{7}$$

With interchanging the indices μ and ν in equation (7) and comparing with equation (6), we see that Ricci tensor is symmetric $R_{\mu\nu} = R_{\nu\mu}$.

The calculation of Ricci tensor is not difficult but is a very tedious especially for higher dimensions with assumed coefficients in most line elements and therefore is very sensitive in work. This is the main reason which we would like to apply and calculate all the process of calculation by computer, especial with Mathematica V 5.1. The Mathematica programming language allows us to construct a problem into a program simply and quickly. Mathematica contains a vast array of documentation with access in a variety of ways (Wellin, 2005). The advantage of this computer programming language is high accuracy and time saving in most of calculations (Wolfram, 1991).

ALGORITHM OF SOLUTION AND METHODOLOGY

A spatial point requires three measurements in each coordinate type. It must, however, be noted that the descriptions of a point in any of systems are equivalent. Different coordinate types are more convenience of appropriateness for a given situation. Three major coordinate systems used in the study of physics are: Rectangular, Spherical, Cylindrical (Philip M. Morse, Feshbach H, 1953). Rectangular or Cartesian coordinate system is the most convenient as it is easy to visualize and associate with our perception of motion in daily life. Spherical and cylindrical systems are specially designed to describe motions, which follow spherical or cylindrical curvatures (James Nearing 2008). For this we should select any coordinate system which is useful in our study.

The first step after running the program, is asking by the program, is selection of our interest system. The coordinate system parameters are defined as, q_1, q_2, q_3 and q_4 .

The first three parameters q_1, q_2, q_3 are in the Space coordinate system and the last parameter q_4 is in the Time. If we don't like work to use Time dimension, we can neglect q_4 with enter in computer key board.

These parameters which are coming in our windows showing our coordinate axis as input data. The main reason of this input selections are to choose our Coordinate system which we are interest in to work. This step we have different options to select as rectangular, spherical, cylindrical and or any other coordinates systems.

The chosen different coordinates systems need different coefficients too. These different coefficients are very important to our calculations (Stone M, 2008).

If any coefficients are needed to add in our line element, we are able to select automatically.

Most objects like planets and stars approximately following in ellipsoidal or spherical coordinate systems. One of the best reasons which we are interest in to use ellipsoidal coordinate system is its application in Astronomy and Astrophysics.

In the December, 2004, Mike Brown and his team discovered a new planet in outer solar system known as the Kuiper Belt which is called 2003 EL61 (Mike Brown 2008). This new discovered object which is now known as Haumea named by International Astronomy Union (IAU) and is one of the strangest known objects in the solar system. The most important matter in our study is the shape of 2003 EL61. This object is in the form of Ellipsoidal shape and for more study of this object and gravitational field of same objects, stars or planets, the metric should be use in the Ellipsoidal coordinate system.

In this program we have tested the Schwarzschild metric Solution and Kerr Solution in the regular form and also in different coordinate systems (Nikouravan, 2001) successfully to show the accuracy and validate the calculations of Ricci tensors.

Therefore, in this paper the attempt has been given to construct our programming with definition of input parameters $q_1 \cdot q_2 \cdot q_3 \cdot q_4$ in the Ellipsoidal and spherical coordinate systems. The following steps have been carried out to measure different parameters in Ellipsoidal or spherical or in any other coordinate systems for Ricci tensors.

The Input process started with running the program which is the first step for selection of $q_1 \cdot q_2 \cdot q_3 \cdot q_4$ as input parameters in terms of any coordinate system in 4D (Ellipsoidal / Spherical or any other type of coordinates).

The solution of line elements in 5D (Wesson, 1984), we need a little changes in the main body of the program. As we know in 4D we have to consider a (4×4) matrix with 16 elements whereas for 5D we have to consider an (5×5) matrix which really the solution of same matrix is very tedious and also need more time (Sajko.W.N, 1999).

The next step needed to select some necessary coefficients of the matric as e^{λ}, e^{ν}. Here λ and ν are function of r and t separately or are depend to both factors.

By selected input parameters, the program easily and automatically can be start. The procedures are by, calculation of g_{ij} in covariant form. According to this program the indexes i and j starting from 1 to 4 continually and all the g_{ij} s will produce. The same process also is going on for determinant of g_{ij} , contravariant form of g_{ij}, Christofell's symbol of first and second kinds too.

The calculation of different values of $\phi_{ij} = Log\sqrt{-g}$, with $i, j = 1,2,3,4$ have different equations. The result of this process also comes in to the final results and finally Calculating of g_{ij} as Ricci tensors will obtain respectively.

It is important to mention that in all parts of programming and the process, we are able to make print from input and output data and also all results in the process after calculation. The print ability of all parts are very helpful to consider some more contribution of our line element in probably more consideration or correction.

RESULTS AND DISCUSSION

This paper shows that the attempted test on Schwarzschild metric Solution and Kerr Solution in spherical coordinate system r, θ, φ and t with different coefficients, have been successfully validated. It shows better results than manual calculations and also it saves more times and increases the accuracy.

By using this computer programming (Appendix 1) we have found the same results for the following line elements Equations (8), (9) and (10) which have done all calculations manually and very tedious (Nikouravan, 2001).

$$ds^2_{Minkowski} = c^2dt^2 - \frac{r^2 + a^2\, Cos^2\theta}{r^2 + a^2}\, dr^2 - (r^2 + a^2\, Cos^2\theta)\, d\theta^2 - (r^2 + a^2\,)\, Sin^2\theta\, d\phi^2 \tag{8}$$

$$ds^2 = c^2dt^2 - a^2(Sinh^2u + Sin^2v)du^2 - a^2(Sinh^2u + Sin^2v)dv^2 - (a^2Cosh^2u\; Cos^2v)d\phi^2 \tag{9}$$

$$ds^2 = c^2dt^2 - a^2\left[\eta^2 + \frac{\xi^2(1-\eta^2)}{(1+\xi^2)}\right]d\xi^2 - a^2\left[\xi^2 + \frac{\eta^2(1+\xi^2)}{(1-\eta^2)}\right]d\eta^2 - \; a^2[(1+\xi^2)(1-\eta^2)]d\varphi^2 \tag{10}$$

Using this computer programming, bring more ability to consider any section and parts and printout all the outputs for more discussions, contributions and constructing of new line elements.

Conclusion

In this useful and applicable computer programming, we have tried to solve and get the Ricci tensors first, and finally work ability with any coordinate systems. Because the solution of problems in different coordinate systems are very important and interesting for astrophysical subjects and Gravitational field of objects to make contribution and also the comparison of the results gives us some new idea to do.

Appendix 1. Computer programming for determination of RICC tensors.

```
\!\(\*
RowBox[{" ",
RowBox[{\(Array[g, {4, 4}];\), "\n", \(Array[r1, {4, 4, 4}];\),
"\n", \(Array[r2, {4, 4, 4}];\), "\n", \(Array[m, {3, 3}];\),
"\n", \(Array[p, {4, 4}];\), "\n", \(Array[n, {4, 4, 4}];\),
"\n", \(Array[d, {4, 4}];\), "\n", \(Array[\[Phi], {5, 5}];\),
"\n", \(Array[r, {4, 4}];\), "\n", \(Array[r22, {4, 4, 4, 4}];\),

"\[IndentingNewLine]",
"\n", \(x1[q1_, q2_, q3_, q4_] = Input["\<inter function one\>"];\),
"\n", \(x2[q1_, q2_, q3_, q4_] = Input["\<inter function two\>"];\),
"\n", \(x3[q1_, q2_, q3_, q4_] = Input["\<inter function tree\>"];\),
"\n", \(x4[q1_, q2_, q3_, q4_] = Input["\<inter function four\>"];\),
"\[IndentingNewLine]",
"\n", \(Print["\<−**Part 1 ***−\>"]\),
"\[IndentingNewLine]", "\[IndentingNewLine]", "\[IndentingNewLine]",
"\n", \(Print["\<−Minkofsky Space−\>"]\),
"\n", \(g[1, 1] =
Exp[\[Lambda][q1]]*
Simplify[\(-\[PartialD]\_q1\
x1[q1, q2, q3, q4]\)*\[PartialD]\_q1\
x1[q1, q2, q3, q4] - \[PartialD]\_q1\
x2[q1, q2, q3, q4]*\[PartialD]\_q1\
x2[q1, q2, q3, q4] - \[PartialD]\_q1\
x3[q1, q2, q3, q4]*\[PartialD]\_q1\
x3[q1, q2, q3, q4] + \[PartialD]\_q1\
x4[q1, q2, q3, q4]*\[PartialD]\_q1\
x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[1,1]=\>", g[1, 1]]\), "\[IndentingNewLine]",
"\[IndentingNewLine]",
"\n", \(g[1, 2] =
Simplify[\(-\[PartialD]\_q1\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q2\
x1[q1, q2, q3, q4] - \[PartialD]\_q1\
x2[q1, q2, q3, q4]*\[PartialD]\_q2\
x2[q1, q2, q3, q4] - \[PartialD]\_q1\
x3[q1, q2, q3, q4]*\[PartialD]\_q2\
x3[q1, q2, q3, q4] + \[PartialD]\_q1\
x4[q1, q2, q3, q4]*\[PartialD]\_q2\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[1,2]=\>", g[1, 2]]\), "\[IndentingNewLine]",
"\n", \(g[1, 3] =
Simplify[\(-\[PartialD]\_q1\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q3\
x1[q1, q2, q3, q4] - \[PartialD]\_q1\
x2[q1, q2, q3, q4]*\[PartialD]\_q3\
x2[q1, q2, q3, q4] - \[PartialD]\_q1\
x3[q1, q2, q3, q4]*\[PartialD]\_q3\
x3[q1, q2, q3, q4] + \[PartialD]\_q1\
x4[q1, q2, q3, q4]*\[PartialD]\_q3\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[1,3]=\>", g[1, 3]]\), "\[IndentingNewLine]",
"\n", \(g[1, 4] =
Simplify[\(-\[PartialD]\_q1\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q4\
x1[q1, q2, q3, q4] - \[PartialD]\_q1\
x2[q1, q2, q3, q4]*\[PartialD]\_q4\
x2[q1, q2, q3, q4] - \[PartialD]\_q1\
x3[q1, q2, q3, q4]*\[PartialD]\_q4\
x3[q1, q2, q3, q4] + \[PartialD]\_q1\
x4[q1, q2, q3, q4]*\[PartialD]\_q4\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[1,4]=\>", g[1, 4]]\), "\[IndentingNewLine]",
"\n", \(g[2, 1] =
Simplify[\(-\[PartialD]\_q2\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q1\
x1[q1, q2, q3, q4] - \[PartialD]\_q2\
x2[q1, q2, q3, q4]*\[PartialD]\_q1\
x2[q1, q2, q3, q4] - \[PartialD]\_q2\
x3[q1, q2, q3, q4]*\[PartialD]\_q1\
x3[q1, q2, q3, q4] + \[PartialD]\_q2\
x4[q1, q2, q3, q4]*\[PartialD]\_q1\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[2,1]=\>", g[2, 1]]\), "\[IndentingNewLine]",
"\n", \(g[2, 2] =
Simplify[\(-\[PartialD]\_q2\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q2\
x1[q1, q2, q3, q4] - \[PartialD]\_q2\
x2[q1, q2, q3, q4]*\[PartialD]\_q2\
x2[q1, q2, q3, q4] - \[PartialD]\_q2\
x3[q1, q2, q3, q4]*\[PartialD]\_q2\
x3[q1, q2, q3, q4] + \[PartialD]\_q2\
x4[q1, q2, q3, q4]*\[PartialD]\_q2\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[2,2]=\>", g[2, 2]]\), "\[IndentingNewLine]",
"\n", \(g[2, 3] =
Simplify[\(-\[PartialD]\_q2\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q3\
x1[q1, q2, q3, q4] - \[PartialD]\_q2\
x2[q1, q2, q3, q4]*\[PartialD]\_q3\
x2[q1, q2, q3, q4] - \[PartialD]\_q2\
x3[q1, q2, q3, q4]*\[PartialD]\_q3\
x3[q1, q2, q3, q4] + \[PartialD]\_q2\
x4[q1, q2, q3, q4]*\[PartialD]\_q3\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[2,3]=\>", g[2, 3]]\), "\[IndentingNewLine]",
"\n", \(g[2, 4] =
Simplify[\(-\[PartialD]\_q2\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q4\
x1[q1, q2, q3, q4] - \[PartialD]\_q2\
x2[q1, q2, q3, q4]*\[PartialD]\_q4\
x2[q1, q2, q3, q4] - \[PartialD]\_q2\
x3[q1, q2, q3, q4]*\[PartialD]\_q4\
x3[q1, q2, q3, q4] + \[PartialD]\_q2\
x4[q1, q2, q3, q4]*\[PartialD]\_q4\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[2,4]=\>", g[2, 4]]\), "\[IndentingNewLine]",
"\n", \(g[3, 1] =
Simplify[\(-\[PartialD]\_q3\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q1\
x1[q1, q2, q3, q4] - \[PartialD]\_q3\
x2[q1, q2, q3, q4]*\[PartialD]\_q1\
x2[q1, q2, q3, q4] - \[PartialD]\_q3\
x3[q1, q2, q3, q4]*\[PartialD]\_q1\
x3[q1, q2, q3, q4] + \[PartialD]\_q3\
x4[q1, q2, q3, q4]*\[PartialD]\_q1\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[3,1]=\>", g[3, 1]]\), "\[IndentingNewLine]",
"\n", \(g[3, 2] =
Simplify[\(-\[PartialD]\_q3\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q2\
x1[q1, q2, q3, q4] - \[PartialD]\_q3\
x2[q1, q2, q3, q4]*\[PartialD]\_q2\
x2[q1, q2, q3, q4] - \[PartialD]\_q3\
x3[q1, q2, q3, q4]*\[PartialD]\_q2\
x3[q1, q2, q3, q4] + \[PartialD]\_q3\
x4[q1, q2, q3, q4]*\[PartialD]\_q2\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[3,2]=\>", g[3, 2]]\), "\[IndentingNewLine]",
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Simplify[\(-\[PartialD]\_q3\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q3\
x1[q1, q2, q3, q4] - \[PartialD]\_q3\
x2[q1, q2, q3, q4]*\[PartialD]\_q3\
x2[q1, q2, q3, q4] - \[PartialD]\_q3\
x3[q1, q2, q3, q4]*\[PartialD]\_q3\
x3[q1, q2, q3, q4] + \[PartialD]\_q3\
x4[q1, q2, q3, q4]*\[PartialD]\_q3\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[3,3]=\>", g[3, 3]]\), "\[IndentingNewLine]",
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Simplify[\(-\[PartialD]\_q3\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q4\
x1[q1, q2, q3, q4] - \[PartialD]\_q3\
x2[q1, q2, q3, q4]*\[PartialD]\_q4\
x2[q1, q2, q3, q4] - \[PartialD]\_q3\
x3[q1, q2, q3, q4]*\[PartialD]\_q4\
x3[q1, q2, q3, q4] + \[PartialD]\_q3\
x4[q1, q2, q3, q4]*\[PartialD]\_q4\ x4[q1, q2, q3, q4]];\),
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Simplify[\(-\[PartialD]\_q4\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q1\ x1[q1, q2, q3,
q4] - \[PartialD]\_q4\
x2[q1, q2, q3, q4]*\[PartialD]\_q1\
x2[q1, q2, q3, q4] - \[PartialD]\_q4\
x3[q1, q2, q3, q4]*\[PartialD]\_q1\
x3[q1, q2, q3, q4] + \[PartialD]\_q4\
x4[q1, q2, q3, q4]*\[PartialD]\_q1\ x4[q1, q2, q3, q4]];\),
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"\n", \(g[4, 2] =
Simplify[\(-\[PartialD]\_q4\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q2\
x1[q1, q2, q3, q4] - \[PartialD]\_q4\
x2[q1, q2, q3, q4]*\[PartialD]\_q2\
x2[q1, q2, q3, q4] - \[PartialD]\_q4\
x3[q1, q2, q3, q4]*\[PartialD]\_q2\
x3[q1, q2, q3, q4] + \[PartialD]\_q4\
x4[q1, q2, q3, q4]*\[PartialD]\_q2\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[4,2]=\>", g[4, 2]]\), "\[IndentingNewLine]",
"\n", \(g[4, 3] =
Simplify[\(-\[PartialD]\_q4\ x1[q1, q2, q3, q4]\)*\[PartialD]\_q3\
x1[q1, q2, q3, q4] - \[PartialD]\_q4\
x2[q1, q2, q3, q4]*\[PartialD]\_q3\
x2[q1, q2, q3, q4] - \[PartialD]\_q4\
x3[q1, q2, q3, q4]*\[PartialD]\_q3\
x3[q1, q2, q3, q4] + \[PartialD]\_q4\
x4[q1, q2, q3, q4]*\[PartialD]\_q3\ x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[4,3]=\>", g[4, 3]]\), "\[IndentingNewLine]",
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Simplify[\(-\[PartialD]\_q4\
x1[q1, q2, q3, q4]\)*\[PartialD]\_q4\
x1[q1, q2, q3, q4] - \[PartialD]\_q4\
x2[q1, q2, q3, q4]*\[PartialD]\_q4\
x2[q1, q2, q3, q4] - \[PartialD]\_q4\
x3[q1, q2, q3, q4]*\[PartialD]\_q4\
x3[q1, q2, q3, q4] + \[PartialD]\_q4\
x4[q1, q2, q3, q4]*\[PartialD]\_q4\
x4[q1, q2, q3, q4]];\),
"\n", \(Print["\<g[4,4]=\>", g[4, 4]]\), " ",
"\n", \(Print["\<—End of part 1 and part 2 started—\
\>"]\), "\n", \(Print["\<–Contravariant–Factors \–\>"]\), "\[IndentingNewLine]",
"\n", \(de =
Simplify[
Det[{{g[1, 1], g[1, 2], g[1, 3], g[1, 4]}, {g[2, 1], g[2, 2],
g[2, 3], g[2, 4]}, {g[3, 1], g[3, 2], g[3, 3],
g[3, 4]}, {g[4, 1], g[4, 2], g[4, 3], g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[1, 1] =
Simplify[
Det[{{1, 0, 0, 0}, {0, g[2, 2], g[2, 3], g[2, 4]}, {0, g[3, 2],
g[3, 3], g[3, 4]}, {0, g[4, 2], g[4, 3], g[4, 4]}}]];\),

"\[IndentingNewLine]",
"\n", \(d[1, 2] =
Simplify[
Det[{{0, 1, 0, 0}, {g[2, 1], 0, g[2, 3], g[2, 4]}, {g[3, 1], 0,
g[3, 3], g[3, 4]}, {g[4, 1], 0, g[4, 3], g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[1, 3] =
Simplify[
Det[{{0, 0, 1, 0}, {g[2, 1], g[2, 2], 0, g[2, 4]}, {g[3, 1],
g[3, 2], 0, g[3, 4]}, {g[4, 1], g[4, 2], 0, g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[1, 4] =
Simplify[
Det[{{0, 0, 0, 1}, {g[2, 1], g[2, 2], g[2, 3], 0}, {g[3, 1],
g[3, 2], g[3, 3], 0}, {g[4, 1], g[4, 2], g[4, 3], 0}}]];\),
"\[IndentingNewLine]",
"\n", \(d[2, 1] =
Simplify[
Det[{{0, g[1, 2], g[1, 3], g[1, 4]}, {1, 0, 0, 0}, {0, g[3, 2],
g[3, 3], g[3, 4]}, {0, g[4, 2], g[4, 3], g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[2, 2] =
Simplify[
Det[{{g[1, 1], 0, g[1, 3], g[1, 4]}, {0, 1, 0, 0}, {g[1, 3], 0,
g[3, 3], g[3, 4]}, {g[4, 1], 0, g[4, 3], g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[2, 3] =
Simplify[
Det[{{g[1, 1], g[1, 2], 0, g[1, 4]}, {0, 0, 1, 0}, {g[3, 1],
g[3, 2], 0, g[3, 4]}, {g[4, 1], g[4, 2], 0, g[4, 4]}}]];\),
"\[IndentingNewLine]",
"\n", \(d[2, 4] =
Simplify[
Det[{{g[1, 1], g[1, 2], g[1, 3], 0}, {0, 0, 0, 1}, {g[3, 1],
g[3, 2], g[3, 3], 0}, {g[4, 1], g[4, 2], g[4, 3], 0}}]];\),
"\[IndentingNewLine]",
"\n", \(d[3, 1] =
Simplify[
Det[{{0, g[1, 2], g[1, 3], g[1, 4]}, {0, g[2, 2], g[2, 3],
g[2, 4]}, {1, 0, 0, 0}, {0, g[4, 2], g[4, 3],
g[4, 4]}}]];\), "\[IndentingNewLine]",
"\n", \(d[3, 2] =
Simplify[
Det[{{g[1, 1], 0, g[1, 3], g[1, 4]}, {g[2, 1], 0, g[2, 3],
g[2, 4]}, {0, 1, 0, 0}, {g[4, 1], 0, g[4, 3],
g[4, 4]}}]];\), "\[IndentingNewLine]",
"\n", \(d[3, 3] =
Simplify[
Det[{{g[1, 1], g[1, 2], 0, g[1, 4]}, {g[2, 1], g[2, 2], 0,
g[2, 4]}, {0, 0, 1, 0}, {g[4, 1], g[4, 2], 0,
g[4, 4]}}]];\), "\[IndentingNewLine]",
"\n", \(d[3, 4] =
Simplify[
Det[{{g[1, 1], g[1, 2], g[1, 3], 0}, {g[2, 1], g[2, 2], g[2, 3],
0}, {0, 0, 0, 1}, {g[4, 1], g[4, 2], g[4, 3], 0}}]];\),
"\[IndentingNewLine]",
"\n", \(d[4, 1] =
Simplify[
Det[{{0, g[1, 2], g[1, 3], g[1, 4]}, {0, g[2, 2], g[2, 3],
g[2, 4]}, {0, g[3, 2], g[3, 3], g[3, 4]}, {1, 0, 0,
0}}]];\), "\[IndentingNewLine]",
"\n", \(d[4, 2] =
Simplify[
Det[{{g[1, 1], 0, g[1, 3], g[1, 4]}, {g[2, 1], 0, g[2, 3],
g[2, 4]}, {g[3, 1], 0, g[3, 3], g[3, 4]}, {0, 1, 0,
0}}]];\), "\[IndentingNewLine]",
"\n", \(d[4, 3] =
Simplify[
Det[{{g[1, 1], g[1, 2], 0, g[1, 4]}, {g[2, 1], g[2, 2], 0,
g[2, 4]}, {g[3, 1], g[3, 2], 0, g[3, 4]}, {0, 0, 1,
0}}]];\), "\[IndentingNewLine]",
"\n", \(d[4, 4] =
Simplify[
Det[{{g[1, 1], g[1, 2], g[1, 3], 0}, {g[2, 1], g[2, 2], g[2, 3],
0}, {g[3, 1], g[3, 2], g[3, 3], 0}, {0, 0, 0, 1}}]];\),
"\[IndentingNewLine]", "\[IndentingNewLine]",
"\n", \(For[i = 1,
i <= 4, \(i++\), {For[j = 1,
j <= 4, \(j++\), {\[IndentingNewLine]p[i,
j] = \((\(-1\))\)^\((i + j)\)*d[i, j]/de;
Print["\<p[\>", i, "\<,\>", j, "\<]=\>", p[i, j]]}]}]\),
"\[IndentingNewLine]", "\[IndentingNewLine]",
"\n", \(Print["\<—End of part 2 and part 3 started—\
\>"]\), "\n", \(Print["\<—Christofell's Symboles of first \
kind–\>"]\), "\[IndentingNewLine]",
"\n", \(For[l = 1,
l <= 4, \(l++\), {For[j = 1,
j <= 4, \(j++\), {For[k = 1,
k <= 4, \(k++\), {For[z = 1,
z <= 4, \(z++\), {For[v = 1,
v <= 4, \(v++\), {n[z, v, 1] = \[PartialD]\_q1\
g[z, v]; n[z, v, 2] = \[PartialD]\_q2\ g[z, v];
n[z, v, 3] = \[PartialD]\_q3\ g[z, v];
n[z, v, 4] = \[PartialD]\_q4\
g[z, v];}]}]; \[IndentingNewLine]r1[l, j, k] =
Simplify[\((\((1/2)\)*\((n[l, j, k] + n[k, l, j] -
n[j, k, l])\))\)];
Print["\<r1[\>", l, "\<,\>", j, "\<,\>", k, "\<]=\>",
r1[l, j, k]]}]}]}]\), "\[IndentingNewLine]",
"\[IndentingNewLine]",
"\n", \(Print["\<——Christofell's Symboles of Second \
kind—\>"]\), "\[IndentingNewLine]",
"\n", \(Print["\<–End of part 3 and part 4 started–\
\>"]\), "\n", \(For[m = 1,
m <= 4, \(m++\), {For[i = 1,
i <= 4, \(i++\), {For[k = 1,
k <= 4, \(k++\), \[IndentingNewLine]{r2[m, i, k] =
Simplify[
p[m, 1]*r1[1, i, k] + p[m, 2]*r1[2, i, k] +
p[m, 3]*r1[3, i, k] + p[m, 4]*r1[4, i, k]],
Print["\<r2[\>", m, "\<,\>", i, "\<,\>", k, "\<]=\>",
r2[m, i, k]]}]}]}]\), "\[IndentingNewLine]",
"\n", \(Print["\<—End of part 4 and part 5 started—\
\>"]\), "\[IndentingNewLine]",
"\n", \(Print["\<—Calculate \
\[Phi][i,j],\[Phi][i]—\>"]\),
"\n", \(\[Phi][5, 5] =
Simplify[Log[\((\(-de\))\)^\((1/2)\)]/Log[10]];\),
"\n", \(\[Phi][5, 1] = Simplify[\[PartialD]\_q1\ \[Phi][5, 5]];
Print["\<————\[Phi][1]=\>", \[Phi][5, 1]];\),
"\n", \(\[Phi][5, 2] = Simplify[\[PartialD]\_q2\ \[Phi][5, 5]];
Print["\<————\[Phi][2]=\>", \[Phi][5, 2]];\),
"\n", \(\[Phi][5, 3] = Simplify[\[PartialD]\_q3\ \[Phi][5, 5]];
Print["\<————\[Phi][3]=\>", \[Phi][5, 3]];\),
"\n", \(\[Phi][5, 4] = Simplify[\[PartialD]\_q4\ \[Phi][5, 5]];
Print["\<————\[Phi][4]=\>", \[Phi][5, 4]];\),
"\n", \(Print["\<—\>"];\),
"\n", \(\[Phi][1, 1] = Simplify[\[PartialD]\_\(q1, q1\)\[Phi][5, 5]];
Print["\<—————\[Phi][1,1]=\>", \[Phi][1, 1]];\),
"\n", \(\[Phi][1, 2] = Simplify[\[PartialD]\_\(q1, q2\)\[Phi][5, 5]];
Print["\<—————\[Phi][1,2]=\>", \[Phi][1, 2]];\),
"\n", \(\[Phi][1, 3] = Simplify[\[PartialD]\_\(q1, q3\)\[Phi][5, 5]];
Print["\<—————\[Phi][1,3]=\>", \[Phi][1, 3]];\),
"\n", \(\[Phi][1, 4] = Simplify[\[PartialD]\_\(q1, q4\)\[Phi][5, 5]];
Print["\<—————\[Phi][1,4]=\>", \[Phi][1, 4]];\),
"\n", \(\[Phi][2, 1] = Simplify[\[PartialD]\_\(q2, q1\)\[Phi][5, 5]];
Print["\<—————\[Phi][2,1]=\>", \[Phi][2, 1]];\),
"\n", \(\[Phi][2, 2] = Simplify[\[PartialD]\_\(q2, q2\)\[Phi][5, 5]];
Print["\<—————\[Phi][2,2]=\>", \[Phi][2, 2]];\),
"\n", \(\[Phi][2, 3] = Simplify[\[PartialD]\_\(q2, q3\)\[Phi][5, 5]];
Print["\<—————\[Phi][2,3]=\>", \[Phi][2, 3]];\),
"\n", \(\[Phi][2, 4] = Simplify[\[PartialD]\_\(q2, q4\)\[Phi][5, 5]];
Print["\<—————\[Phi][2,4]=\>", \[Phi][2, 4]];\),
"\n", \(\[Phi][3, 1] = Simplify[\[PartialD]\_\(q3, q1\)\[Phi][5, 5]];
Print["\<—————\[Phi][3,1]=\>", \[Phi][3, 1]];\),
"\n", \(\[Phi][3, 2] = Simplify[\[PartialD]\_\(q3, q2\)\[Phi][5, 5]];
Print["\<—————\[Phi][3,2]=\>", \[Phi][3, 2]];\),
"\n", \(\[Phi][3, 3] = Simplify[\[PartialD]\_\(q3, q3\)\[Phi][5, 5]];
Print["\<—————\[Phi][3,3]=\>", \[Phi][3, 3]];\),
"\n", \(\[Phi][3, 4] = Simplify[\[PartialD]\_\(q3, q4\)\[Phi][5, 5]];
Print["\<—————\[Phi][3,4]=\>", \[Phi][3, 4]];\),
"\n", \(\[Phi][4, 1] = Simplify[\[PartialD]\_\(q4, q1\)\[Phi][5, 5]];
Print["\<—————\[Phi][4,1]=\>", \[Phi][4, 1]];\),
"\n", \(\[Phi][4, 2] = Simplify[\[PartialD]\_\(q4, q2\)\[Phi][5, 5]];
Print["\<—————\[Phi][4,2]=\>", \[Phi][4, 2]];\),
"\n", \(\[Phi][4, 3] = Simplify[\[PartialD]\_\(q4, q3\)\[Phi][5, 5]];
Print["\<—————\[Phi][4,3]=\>", \[Phi][4, 3]];\),
"\n", \(\[Phi][4, 4] = Simplify[\[PartialD]\_\(q4, q4\)\[Phi][5, 5]];
Print["\<—————\[Phi][4,4]=\>", \[Phi][4, 4]];\),
"\[IndentingNewLine]", "\[IndentingNewLine]",
"\n", \(Print["\<\•••••\>"];\),
"\n", \(Print["\<\•••••\>"];\),
"\n", \(Print["\<\•••••\>"];\),
"\n", \(Print["\<••••••R[i,j]=\[Phi][i,j]-\[CapitalGamma][k,i,j,k]+\
\[CapitalGamma][k,i,p]*\[CapitalGamma][p,i,k]-
\[CapitalGamma][p,i,j]*\[Phi][p]\
••••••••\>"];\),
"\n", \(Print["\<\•••••\>"];\),
"\n", \(Print["\<\•••••\>"];\),
"\n", GridBox[{
{\(Print["\<\••••••••\>"];\)}\,
{\(Print["\<\•••••••••\>"];\)}\
}], "\[IndentingNewLine]", "\[IndentingNewLine]",
"\n", \(Print["\<—End of part 5 and part 6 started—\
\>"]\), "\[IndentingNewLine]",
"\n", \(For[i = 1,
i <= 4, \(i++\), {For[j = 1,
j <= 4, \(j++\), {For[k = 1,
k <= 4, \(k++\), {r22[i, j, k, 1] = \[PartialD]\_q1\
r2[i, j, k];
r22[i, j, k, 2] = \[PartialD]\_q2\ r2[i, j, k];
r22[i, j, k, 3] = \[PartialD]\_q3\ r2[i, j, k];
r22[i, j, k, 4] = \[PartialD]\_q4\ r2[i, j, k]}]}]}];\),
"\[IndentingNewLine]",
"\n", \(For[i = 1,
i <= 4, \(i++\), {For[j = 1,
j <= 4, \(j++\), {r[i, j] =
Simplify[\((\[Phi][i,
j] - \[Sum]\+\(k = 1\)\%4 r22[k, i, j,
k] + \[Sum]\+\(k = 1\)\%4\(\[Sum]\+\(p = \
1\)\%4\((r2[k, i, p]*r2[p, j, k])\)\) - \[Sum]\+\(p = 1\)\%4 r2[p, i,
j]*\[Phi][5, p])\)];
Print["\<•••••••••R[\>", i, "\<,\>", j, "\<]=\>",
r[i, j]]}]}];\)}], "\[IndentingNewLine]",
"\[IndentingNewLine]"}]\)
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