International Journal of Physical Sciences Vol. 4 (11), pp. 663-671, November, 2009
Available online at http://www.academicjournals.org/ijps
ISSN 1992 - 1950 © 2009 Academic Journals

Full Length Research Paper

Analysis of non-newtonian fluid flow in a stenosed artery

Sapna S

Department of Mathematics H. B. Technological Institute, Kanpur- 208002. E-mail: sapna1980jan@rediffmail.com.

Accepted 14 September 2009

The numerical illustration presented at the end of the paper provides the results for the resistance to flow, apparent viscosity and the wall shear stress through their graphical representations. It has been shown that the resistance to flow, apparent viscosity and wall shear stress increases with the size of the stenosis but these increases is comparatively small due to non-Newtonian behavior of the blood indicating the usefulness of its rheological character in the functioning of the diseased arterial circulation. Few comparisons with the existing results have been made in order to validate the applicability of the present model.

Key words: Non-Newtonian fluid, apparent viscosity, stenosis, rheological character, resistance to flow.

INTRODUCTION

A Newtonian fluid, by definition, is one in which the coefficient of viscosity is constant at all rates of shear. Homogeneous liquids may behave closely like Newtonian fluids. However, there are fluids that do not obey the linear relationship between shear stress and shear strain rate. Fluids that exhibit a non-linear relationship between the shear stress and the rate of shear strain are called non-Newtonian fluids. Blood behaviour is referred to as non-Newtonian properties. These properties are of two types as follows: (a) at low shear rates, the apparent viscosity increases markedly – Sometimes even a certain “yield stress” is required for flow. (b) In small tubes, the apparent viscosity at higher rates of shear is smaller than it is in larger tubes. These two types of anomalies are often referred to as “low shear” and “high shear” effects respectively. It is thus concluded that the behaviour of blood is almost Newtonian at high shear rate, while at low shear rate the blood exhibits yield stress and non-Newtonian behaviour.

In the series of the papers, (Texon, 1957; May et al., 1963; Hershey and Cho, 1966; Young, 1968; Forrester and Young, 1970; Caro et al., 1971; Fry,1972 Young and Tsai, 1973; Lee, 1974; Richard et al., 1977) the effects on the cardiovascular system can be understood by studying the blood flow in its vicinity. In these studies the behavior of the blood has been considered as a Newtonian fluid. However, it may be noted that the blood does not behave as a Newtonian fluid under certain conditions. It is generally accepted that the blood, being a suspension of cells, behaves as a non-Newtonian fluid at low shear rate (Charm and Kurland, 1965; Hershey et at., 1964; Whitemore, 1968; Cokelet, 1972; Lih, 1975; Shukla et al., 1990).

It has been pointed out that the flow behaviour of blood in a tube of small diameter (less than 0.2 mm) and at less than 20 sec^{-1} shear rate, can be represented by a power-low fluid (Hershey et al., 19764; Charm et al., 1965). It has also been suggested that at low shear rate (0.1 sec^{-1}) the blood exhibits yield stress and behaves like a Casson-model fluid (Casson, 1959; Reiner and Scott-Blair, 1959; Charm et al., 1965). For blood flows in large arterial vessels (that is, vessel diameter ≥1 mm) (Labarbera, 1990), which can be considered as a large deformation flow, the predominant feature of the rheological behavior of blood is its shear rate dependent viscosity and its fact on the hemodynamics of large arterial vessel flows has not been understood well.

In this paper we investigated the effect of stenosis on the resistance to flow, apparent viscosity and wall shear stress in an artery by considering the blood as a power-law fluid and Casson’s-model fluids. And to examine the effect of stenosis shape parameter, we considered blood flow through an axially non-symmetrical but radially symmetric stenosis such that the axial shape of the stenosis can be change just by varying a parameter [stenosis shape parameter (m)].

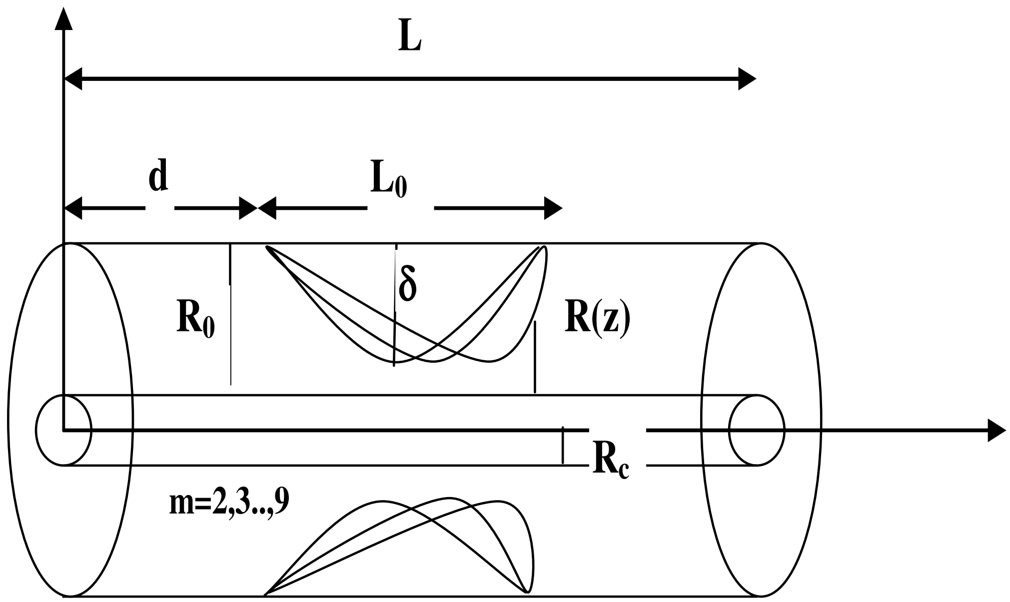

Figure1. Flow geometry of an axially non-symmetrical stenosis.

FORMULATION OF THE PROBLEM

In the present analysis, it is assumed that the stenosis develops in the arterial wall in an axially non-symmetric but radially symmetric manner and depends upon the axial distance z and the height of its growth. In such a case the radius of artery, R(z) can be written as follows (Figure 1):

$$\left.\begin{aligned} \frac{R(z)}{R_0} &= 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], && d \le z \le d + L_0 \\ &= 1, && \text{otherwise,} \end{aligned}\right\} \quad (1)$$

Where; R(z) and R_0 is the radius of the artery with and without stenosis, respectively. L_0 is the stenosis length and d indicates its location, m ≥ 2 is a parameter determining the stenosis shape and is referred to as stenosis shape parameter. Axially symmetric stenosis occurs when m = 2 and a parameter A is given by;

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$$

Where; δ denotes the maximum height of stenosis at $z = d + L_0 / m^{1/(m-1)}$. The ratio of the stenosis height to the radius of the normal artery is much less than unity.

Conservation equation and boundary condition

The equation of motion for laminar and incompressible, steady, fully-developed, one-dimensional flow of blood whose viscosity varies along the radial direction in an artery reduces to (Young, 1968):

$$\left.\begin{aligned} 0 &= -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau)}{\partial z}, \\ 0 &= -\frac{\partial P}{\partial r}, \end{aligned}\right\} \quad (2)$$

Where; (z, r) are co-ordinates with z measured along the axis and r measured normal to the axis of the artery.

Following boundary conditions are introduced to solve the above equations,

$$\left.\begin{aligned} &\partial u/\partial r = 0 && \text{at } r = 0 \\ &u = 0 && \text{at } r = R(z) \\ &\tau \text{ is finite} && \text{at } r = 0 \\ &P = P_0 && \text{at } z = 0 \\ &P = P_L && \text{at } z = L \end{aligned}\right\} \quad (3)$$

ANALYSIS OF THE PROBLEM

Case-1: Power-law fluid

Non-Newtonian fluid is that of power-law fluid which have constitutive equation,

$$\left.\begin{aligned}\left(-\frac{du}{dr}\right) &= \left(\frac{\tau}{\mu}\right)^{1/n} = f(\tau),\\ \text{where } \tau &= \left(-\frac{dp}{dz}\right)\frac{R_c}{2}\end{aligned}\right\} \tag{4}$$

Where; u is the axial velocity, μ is the viscosity of fluid, (-dp/dz) is the pressure gradient and n is the flow behaviour index of the fluid.

Solving for u from equation (2), (4) and using the boundary conditions (3), we have,

$$\frac{du}{dr} = \left(\frac{P}{2\mu}\right)^{1/n} [(r - R_C)^{1/n}], \tag{5}$$

The volumetric flow rate Q can be defined as;

$$Q = \int_0^R 2\pi u r dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr, \tag{6}$$

By the help of equations (5) and (6) we have,

$$Q = \left(\frac{P}{2\mu}\right)^{1/n} \left(\frac{n\pi}{(3n+1)}\right) (R)^{[(1/n)+1]} \tag{7}$$

From equation (7) pressure gradient is written as follows;

$$\frac{dp}{dz} = -2\mu\left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{1}{(R)^{3n+1}} \tag{8}$$

Integrating equation (8) using the condition P = P_0 at z = 0 and P = P_L at z = L. We have,

$$P_L - P_0 = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{(R_0)^{3n+1}} \int_0^L \frac{dz}{(R/R_0)^{1+3n}} \tag{9}$$

The resistance to flow (resistive impedance) is denoted by λ and defined as follows (Young, 1968):

$$\lambda = \frac{P_L - P_0}{Q} \tag{10}$$

The resistance to flow from equation (10) using equations (9) can write as:

$$\lambda_0 = \left(\frac{(3n+1)Q}{n\pi}\right)^n \frac{2\mu}{QR_0^{3n+1}} \left(\int_0^d dz + \int_0^{d+L_0} \frac{dz}{(R/R_0)^{3n+1}} + \int_{d+L_0}^{L} dz\right) \tag{11}$$

When there is no stenosis in artery then R = R_0, the resistance to flow,

$$\lambda_N = \left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{2\mu}{QR_0^{3n+1}} L \tag{12}$$

From equation (11) and (12) the ratio of (λ_0 / λ_N) is given as;

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L} \int_d^{d+L_0} \frac{dz}{(R/R_0)^{3n+1}} \tag{13}$$

Now the ratio of shearing stress at the wall can be written as;

$$\frac{\tau_R}{\tau_N} = \left(\frac{R_0}{R}\right)^{-3n} \tag{14}$$

$$\tau = \frac{\tau_R}{\tau_N} = \frac{1}{\left(1 - \frac{\delta}{R_0}\right)^{3n}} \tag{15}$$

Figure 2 reveals the variation of resistance to flow (λ) with stenosis size (δ/R_0) for different values of flow behavior index (n). It is observed that the resistance to flow (λ) increases as stenosis size (δ/R_0) increases. It is also noticed here that resistance to flow (λ) increases as flow behavior index (n) increases. It is seen from Figures 2 and 3 that the ratio is always greater than 1 and decreases as n decreases from unity.

This result is similar with the result of (Shukla et al., 1990). In Figure 3, resistance to flow (λ) decreases as stenosis shape parameter (m) increases and maximum resistance to flow (λ) occurs at (m = 2), that is, in case of symmetric stenosis. This result is therefore consisting to the result of (Haldar, 1985). It is also seen that, for δ/R_0 = 0.1 and L_0/L = 1.0

In Figure 4 the variation of wall shear stress (τ) with stenosis length (L_0/L) for different values of flow behavior index (n) has been shown. This Figure depicts that wall shear stress (τ) increases as stenosis length (L_0/L) increases. Also it has been seen from this graph that the wall shear stress (τ) increases as value of flow behavior index (n) increases. As the stenosis grows, the wall shearing stress (τ) increases in the stenotic region. It is also noted that the shear ratio given by (15) is greater than one and decreases as n decreases (n < 1). These results are similar with the results of (Shukla et al., 1990).

Figure 5 shows the variation of wall shear stress (τ)

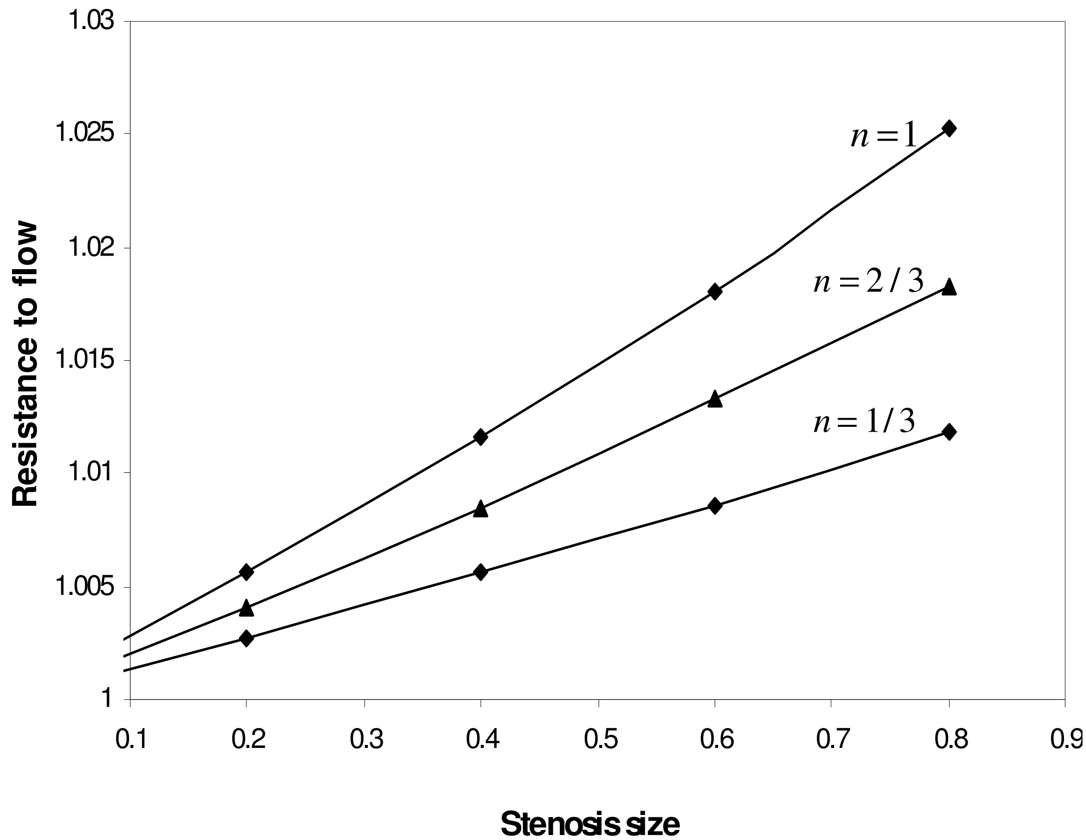

Figure 2. Variation of resistance to flow with stenosis size for different value of n.

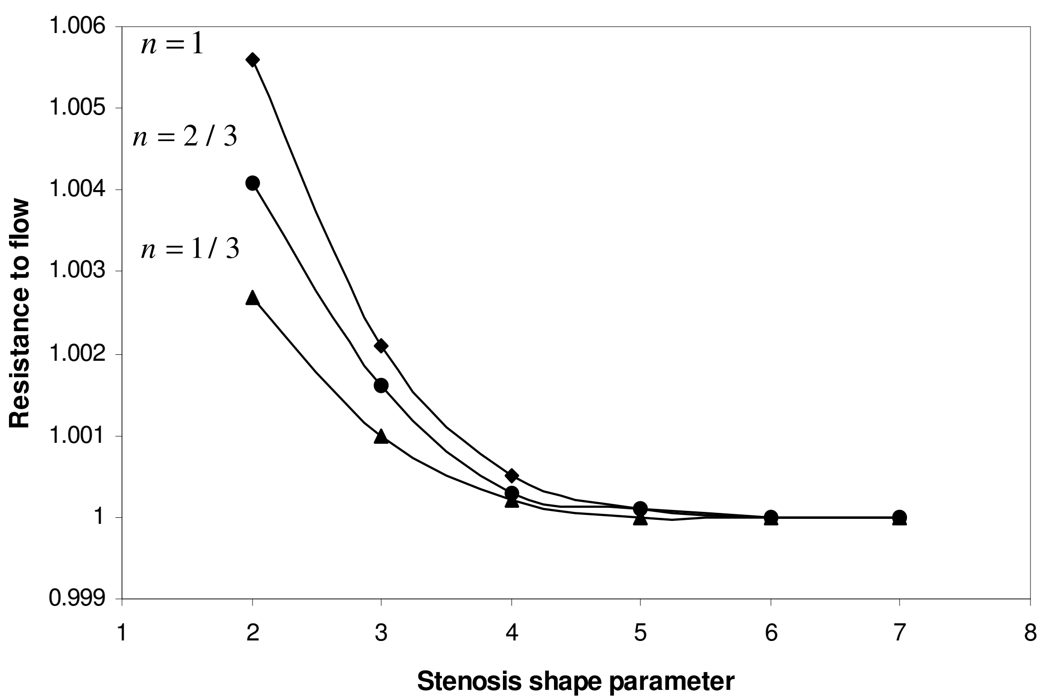

Figure 3. Variation of resistance to flow with stenosis shape parameter for different value of n.

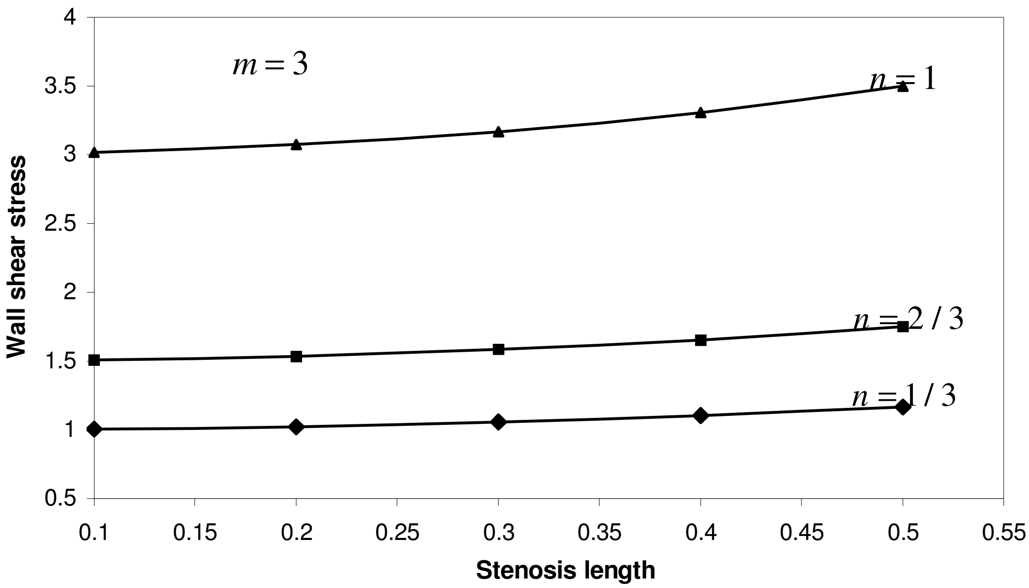

Figure 4. Variation of wall shear stress with stenosis length for different values of flow behavior index (n).

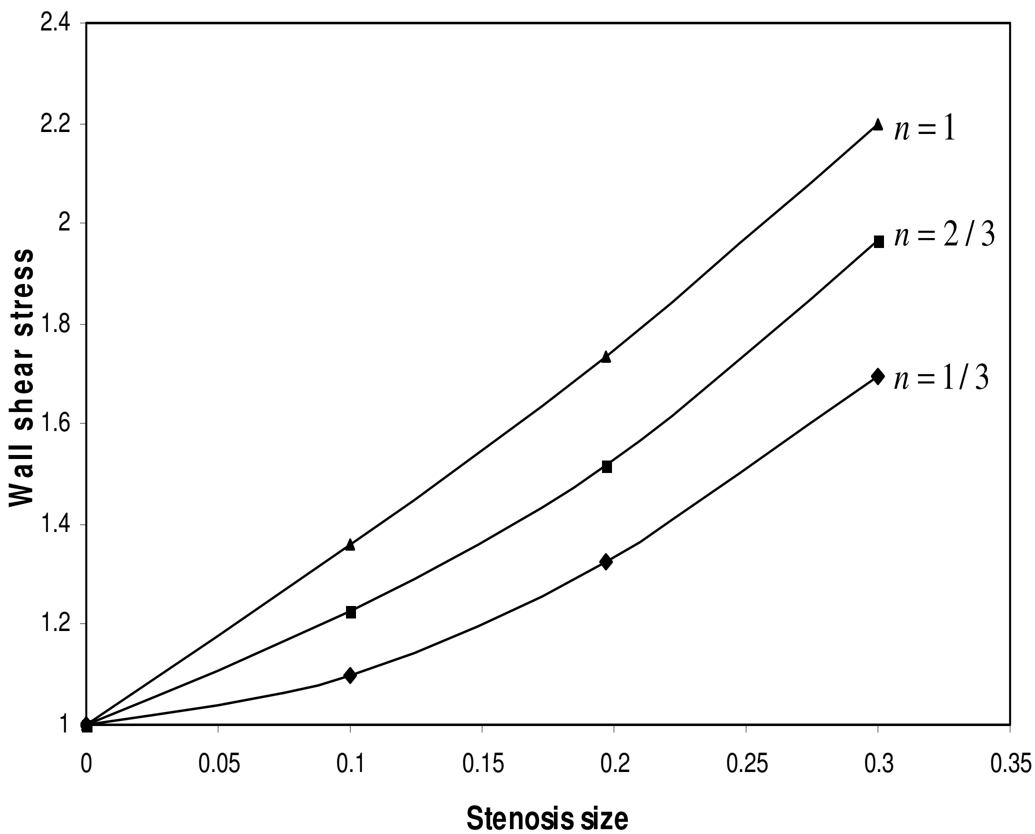

Figure 5. Variation of wall shear stress with stenosis size for different value of n.

with stenosis size for different values of flow behavior index (n). This Figure depicts that wall shear stress (τ) increases as stenosis size increases. Also it has been seen from this graph that the wall shear stress (τ) increases as value of flow behavior index (n) increases. These results are consistent to the observation of (Shukla et al., 1990). It is also seen that the shear ratio is always greater than one and decreases as n decreases. For δ/R_0 = 0.1 the increases in wall shear due to stenosis is about 37% when compared to the wall shear corresponding to the normal artery in the Newtonian case (n = 1), but for n = 2/3 this increase is only 23% approximately. However, for δ/R_0 = 0.2, the corresponding increase in Newtonian (n = 1) and non-Newtonian (n = 2/3) cases are 95 and 56% respectively.

Case-2: Casson's fluid model

The Casson's relation is commonly written as:

$$\left.\begin{aligned} &\tau^{1/2} = \tau_0^{1/2} + (\mu)^{1/2}\left(-\frac{du}{dr}\right)^{1/2}, && \text{if } \tau \ge \tau_0 \\ &\left(\frac{du}{dr}\right) = 0 && \text{if } \tau < \tau_0 \end{aligned}\right\} \quad (16)$$

Where; $\tau_0 = -\frac{dp}{dz}\frac{R_c}{2}$

R_c is the radius of the plug-flow region, τ_0 is yield stress, τ is wall shear stress and μ denotes Casson's viscosity coefficient.

The Volume rate of flow using equation (16) is defined as:

$$Q = \pi \int_0^R r^2 \left(-\frac{du}{dr}\right) dr. \quad (17)$$

By integrating equation (17), using equations (16) and (3) we have,

$$Q = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right)\left[1 - \frac{16}{7}\left(\frac{R_c}{R}\right)^{1/2} + \frac{4}{3}\left(\frac{R_c}{R}\right) - \frac{1}{21}\left(\frac{R_c}{R}\right)^4\right], \quad (18)$$

Equation (18) can be rewritten as;

$$Q = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right) f(\bar{y}),$$

Where; $f(\bar{y}) = [1 - \frac{16}{7}(\bar{y})^{1/2} + \frac{4}{3}(\bar{y}) - \frac{1}{21}(\bar{y})^4],$

with $\bar{y} = \frac{R_c}{R} << 1.$

From above equation pressure gradient is written as follows;

$$\left(-\frac{dp}{dz}\right) = \frac{8\mu Q}{\pi R^4 f(\bar{y})} \quad (19)$$

Integrating equation (19) using the condition P = P_0 at z = 0 and P = P_L at z = L. We have,

$$\Delta P = P_L - P_0 = \frac{8\mu Q}{\pi R_0^4}\int_0^L \frac{dz}{\left(R(z)/R_0\right)^4 f(\bar{y}(z))} \quad (20)$$

The resistance to flow (resistive impedance) is denoted by λ and defined as follows (Young, 1964),

$$\lambda = \frac{P_L - P_0}{Q} \quad (21)$$

The resistance to flow from equation (21) using equations (20) is written as;

$$\lambda = 1 - \frac{L_0}{L} + \frac{f_0}{L}\int_d^{d+L_0} \frac{dz}{(R(z)/R_0)^4 f(\bar{y}(z))} \quad (22)$$

Where; f_0 is given by:

$$f_0 = \left[1 - \frac{16}{7}\left(\frac{R_c}{R_0}\right)^{1/2} + \frac{4}{3}\left(\frac{R_c}{R_0}\right) - \frac{1}{21}\left(\frac{R_c}{R_0}\right)^4\right].$$

Following the apparent viscosity (μ_{app}) is defined as follows;

$$\mu_{app} = \frac{1}{\left(R(z)/R_0\right)^4 f(\bar{y})} \quad (23)$$

The shearing stress at the wall can be defined as;

$$\tau_R = \left[\tau_0^{1/2} + \left(-\mu\frac{du}{dr}\right)^{1/2}_{r=R(z)}\right]^2 \quad (24)$$

Figure 6 shows the variation of resistance to flow (λ) with stenosis size (δ/R_0) for different values of stenosis shape parameter (m). It is seen from the Figure that the resistance to flow (λ) is always greater than unity and increases as stenosis size (δ/R_0) increases and decreases as the stenosis shape parameter (m) increases. Maximum resistance to flow (λ) occurs at m =2.

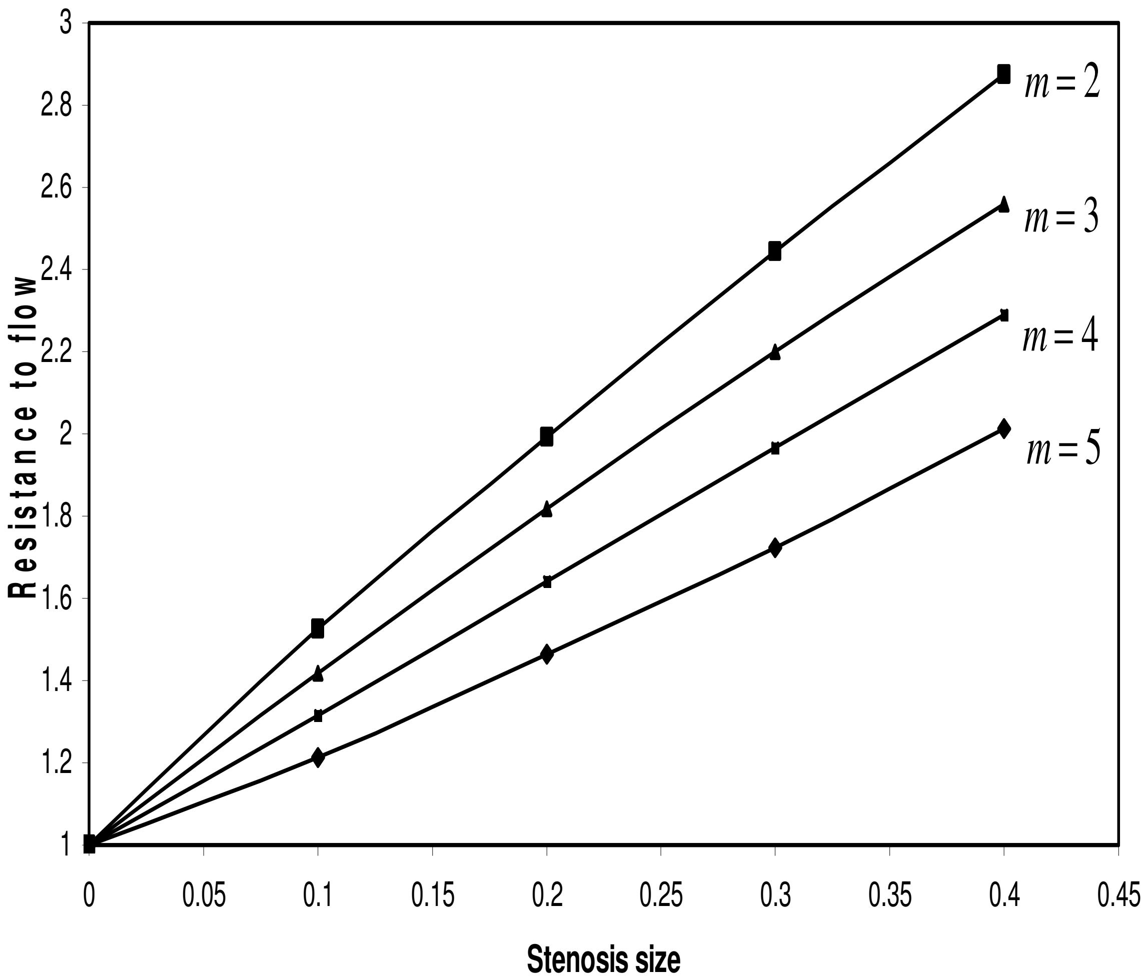

Figure 6. Variation of resistance to flow with stenosis size for different value of sterosis shape parameter.

That is, in the case of symmetric stenosis. This result is therefore consistent with the observation of (Haldar, 1985).

Figure 7 depicts the variation of resistance to flow (λ) with stenosis length (L_0/L) for different values of stenosis shape parameter (m). Figure shows that resistance to flow (λ) increases as stenosis length (L_0/L) increases and decreases as stenosis shape parameter (m) increases. This result is qualitative agreement with the observation of (Haldar, 1985).

Figure 8 represents variation of apparent viscosity (μ_{app}) with stenosis size (δ/R_0) for different values of yield stress (τ_0). Figure depicts that apparent viscosity (μ_{app}) increases as stenosis size (δ/R_0) increases but this increase is less due to non-Newtonian behaviour of the blood. In addition it may be noted from the graph that the apparent viscosity (μ_{app}) decreases as yield stress (τ_0) increases. This result is in qualitative agreement with the result of (Pontrelli, 2001). It may be observed that from these results that the apparent viscosity increases as the stenosis grows and remains constants outside from the stenotic region.

Figure 9 shows the variation of apparent viscosity (μ_{app}) with stenosis length (L_0/L) for different values of stenosis shape parameter (m). We observe that the apparent viscosity (μ_{app}) sharply increases as length of stenosis (L_0/L) increases and decreases as stenosis shape parameter (m) increases. (Tandon et al., 1991) have also noted the same results.

Conclusion

In this paper, we have studied the effects of the stenosis in an artery by considering the blood as power-law and Casson's model fluids. It has been concluded that the resistance to flow and wall shear stress increases as the size of stenosis increases for a given non-Newtonian model of the blood. These increases are however, small due to non-Newtonian behaviour of the blood. Thus it

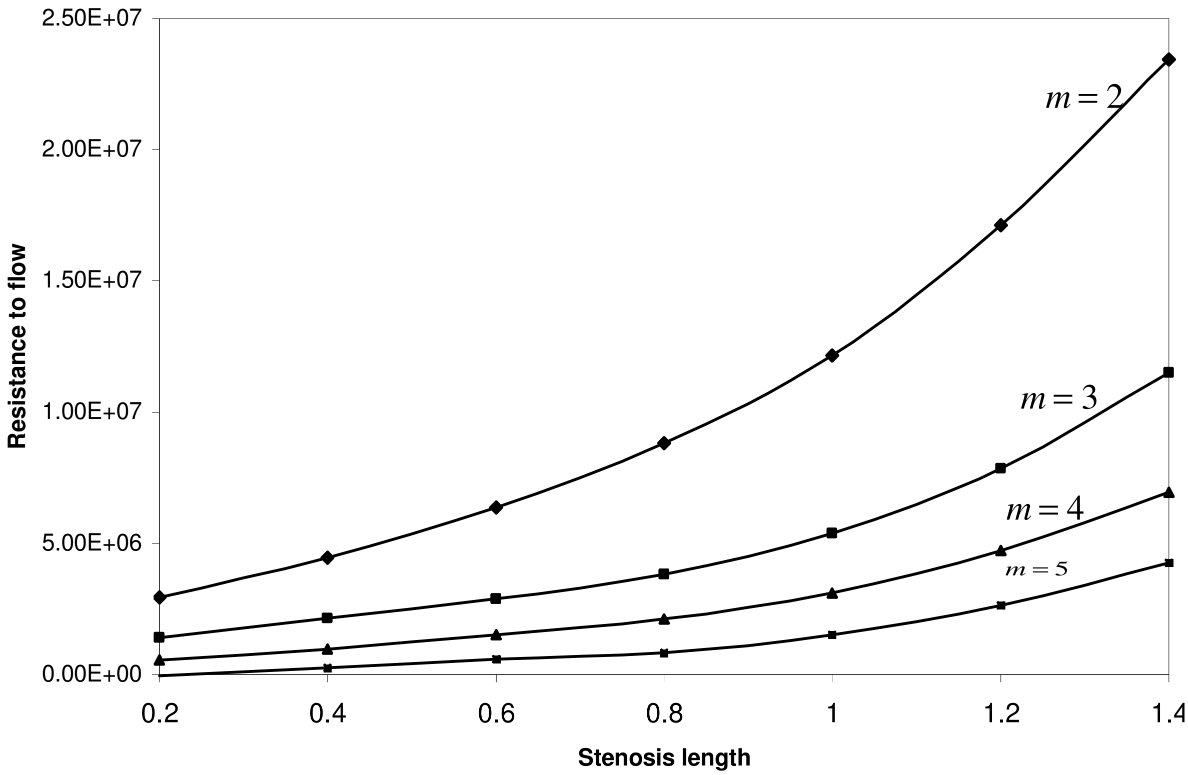

Figure 7. Variation of resistance to flow with stenosis length for different values of steriosis shape parameter.

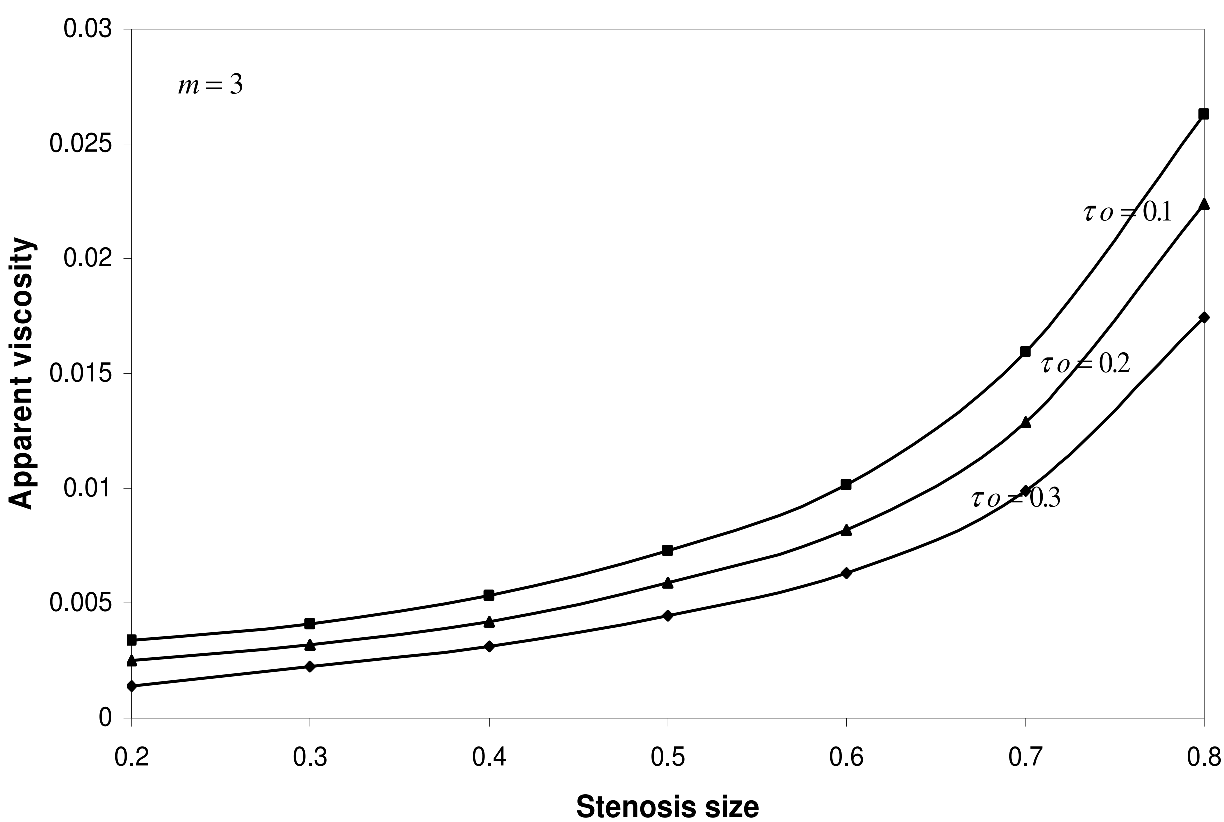

Figure 8. Variation of apparent viscosity with stenosis size for different values of yield stress.

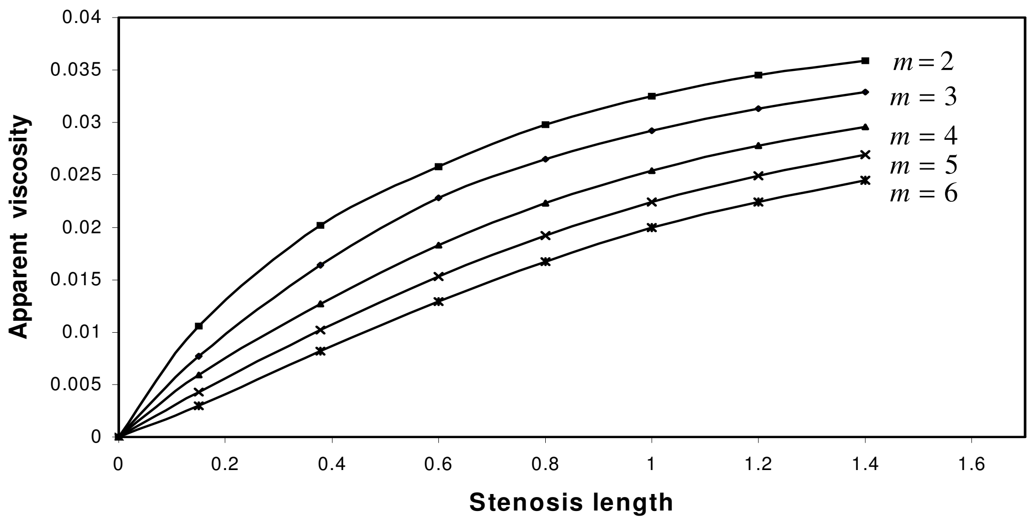

Figure 9. Variation of apparent viscosity with stenosis length for different values of steriosis shape parameter.

appears that the non-Newtonian behaviour of the blood is helpful in the functioning of diseased arterial circulation.

REFERENCES

Caro CG, Fitz-Gerald JM, Schroter RC (1971). Atheroma and arterial wall shear observation, Correlation and proposal of a shear dependent mass transfer mechanism for Atherogenesis. Proc. R. Soc. 177: 109-159.

Casson NA (1959). Flow equation for pigment oil suspensions of the printing ink type. In Rheology of disperse systems, Ed. Mill., C.C., London pp.84-102.

Charm SE, Kurland GS (1965). Viscometry of human blood for shear rate of 100,000sec^{-1}. Nature London 206: 617-618.

Cokelet GR (1972). The rheology of human blood. In Biomechanics, Ed. By YC. Fung et al, 63, Englewood Cliffs: Prentice-Hall.

Forrester JH, Young DF (1970). Flow through a converging diverging tube and its implications in occlusive vascular disease. J. Biomech. 3: 297-316.

Fry DL (1972). Localizing factor in arteriosclerosis, In atherosclerosis and coronary heart disease, New York: Grune Stratton. 85.

Hershey D, Byrnes RE, Deddens RL, Roa AM (1964). Blood rheology: Temperature dependence of the power law model. Paper presented at A.I.Ch.E. Boston.

Hershey D, Cho SJ (1966). Blood flow in ridig tubes: Thickness and slip velocity of plasma film at the wall. J. Appl. Physiolo. 21: 27. J. Eng. India. Trans. ASME. 90: 248-254. (1968).

Haldar K (1985). Effects of the shape of stenosis on the resistance to blood flow through an artery. Bull. Mathe. Bio. 47: 545-550.

Labarbera M (1997). Principles of design of fluid transport systems in zoology. Science 249: 992-1000.

Lee JS (1974). On the coupling and detection of motion between an artery with a localized lesion and its surrounding tissue. J. Biomech. 7: 403.

Lih MM (1975). Transport Phenomena in Medicine and Biology. Wiley, New York.

May AG, Deweese JA, Rob CB (1963). Hemodynamic effects of arterial stenosis. Surgery 53: 513-524.

Pontrelli G (2001). “Blood flow through an axisymmetric stenosis”, Proc. Inst Mech. Eng, Part H, Eng Med. 215: 1-10.

Reiner M, Scott Baldair GW (1959). The flow of the blood through narrow tube. Nature, London 184: 354-359.

Richard LK, Young DF, Chalvin NR (1977). Wall vibrations induced by flow through simulated stenosis in models and arteries. J. Biomech. 10-431.

Shukla JB, Parihar RS, Gupta SP (1990). Biorheological aspects of blood flow through artery with mild stenosis: Effects of peripheral layer.Biorhe.17: 403-410.

Tandon PN, Nirmala P, Tewari M, Rana US (1991). Analysis of nutritional transport through a capillary: Normal and stenosed. Comput. Math. Appl. 22(12): 3-13.

Texon M (1957). A homodynamic concept of atherosclerosis with particular reference to coronary occlusion 99: 418.

Whitmore R (1968). Rheology of the circulation, Perg New York.

Young DF, Tsai FY (1973). Flow characteristics in models of arterial stenosis–II, unsteady flow. J. Biomech. 6: 547-558.

Young DF (1968). Effects of a time-dependent stenosis on flow through a tube. J. Eng. India. Trans. ASME. 90: 248-254.