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Full Length Research Paper

A new analytical formulation for contact stress and prediction of crack propagation path in rolling bodies and comparing with finite element model (FEM) results statically

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Rails fracture by the growth of fatigue crack or critical crack is one of the prevalent defects in railway. The rail fracture, failure and analysis of stress should be studied to prevent rail fracture and events. In this paper, new formulation of contact stress for two rolling bodies is presented, and its results are close to the hertz stress formulation. The analysis of stress is done by Finite Element Model (FEM) and it is compared with hertz stress and new stress formulation results. Then, the analysis of stress, fracture, prediction of fracture and path of crack motion in rail and wheel are studied, statically which plays an important role in this field. The methods of analysis of stress theory fracture with numerical and FEM are compared and consequently, it was proved that these approaches have acceptable results compared with other results. So we can rely on these methods and their results. The relation between maximum displacement and maximum stress is presented, and the path of crack growth and fracture is predicted. To analyze the pressure of collection of the wheel and rail instead of elliptical contact surface, quadrant of elliptical contact surface is assumed. With this assumption, acceptable results will be attained.

Key words: Rail fracture, maximum stress, Hertz's elliptic, reformation coefficients, negative discharge energy hole, crack growth.

INTRODUCTION

In order to study the fracture and the stress analysis, the factors of fracture and the critical stress are determined. One of the common defects of rails which lead to their fracture is a vertical crack on the end of the rails. This is mainly because of loading, which will make a compressive stress on the end of the rail. So, these stresses with the compressive stress will cause tensile stress on the end of rail, and these tensions lead to growth of vertical crack at the end of the rail, and finally cause fracture in it.

Significant researches were performed over the last years to investigate the rail fracture and fatigue, fatigue crack growth laws and their material parameters and life prediction of rolling contact fatigue crack initiation (Orringer and Morris, 1984; Toth, 2001; Ringsberg, 2001). A review of the damage tolerance behavior of railway rails were investigated (Zerbst et al., 2009). Fracture mechanics and failure assessment were studied by a number of researchers (Zerbst et al., 2005; Donzella and Petrogalli, 2010). Rolling contact fatigue and wear behavior of the infrastar two-material rail were studied (Hiensch et al., 2005).

In this paper, new formulation of contact stress for two rolling bodies has been presented, and the results are similar to Hertz stress formulation. Also, the analysis of stress has been done by Finite Element Model (FEM) and compared with hertz stress and new stress formulation results. Then, the analysis of stress, fracture and the prediction of fracture and the path of crack growth in rail and wheel are studied, statically which play important role in this field. In the next part we will analyze the theory through new hypothesis and then the prediction

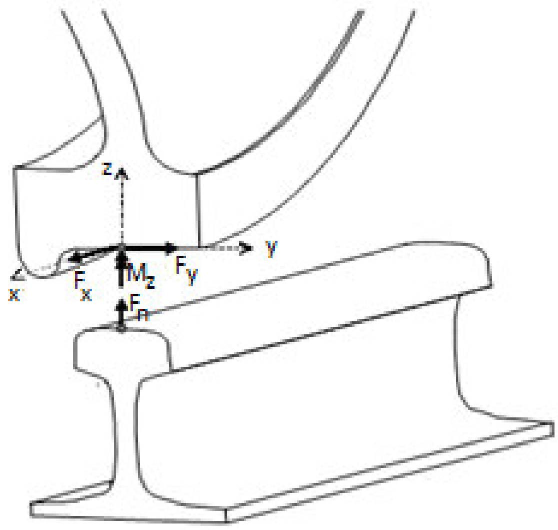

Figure 1. Contact of the wheel and rail in direct path.

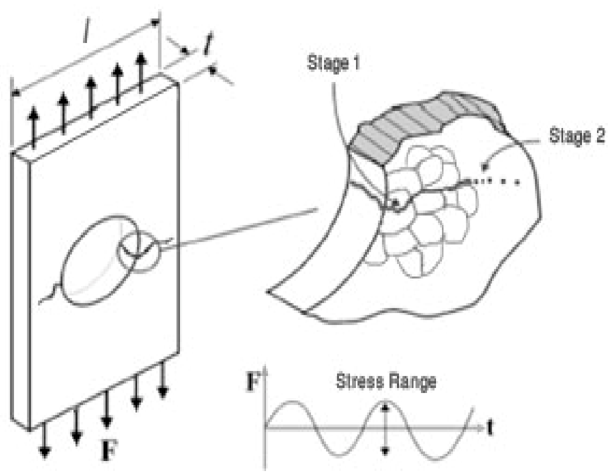

Figure 2. Illustration of stage I, II crack growth.

of the path of the cracks improvement and the correction coefficient will be presented.

First we should determine the critical surface of stress. We can determine the positions of critical tension by FEM (Figure 1). Figure 2 shows the first and the second phase of cracks progress. As we know that the Hertz stress must be almost the same as the stress in the y direction. Of course there may be some difference in the value of these stresses that we discuss in the section of stress analysis theory and the fracture theory. In the next part of this paper we will predict the path of crack growth and also the correction coefficients will be introduced.

The main work in this research is determination of the new contact stress formulation in two rolling bodies that these results are equal to the Hertz stress results. This new formulation does not necessitate any additional parameter for calculation of the contact stress. The number of parameters for determination of contact stress is lesser than the other formulations and simple calculation is the advantages. Also it is so simple and is user friendly. It is rememberable and simply can be memorized for calculation of contact stress. The new formulation results are similar to the Hertz stress formulation and are coincided to the FEM results.

INDUCED STRESSES ON THE RAIL AND ITS FRACTURE

For fast fracture, the size of the crack must be about 2.4 inches, so the area of the crack will be larger than the end surface of the rail. When crack growth happens, we can consider that thickness is crack food, that is, the crack growth requires to food (thickness). So, high thickness (high food=high energy=high thickness) provides high crack growth because of enough food (thickness=high energy). In other words, the required energy for crack growth is supplied by thickness. To prevent the crack from propagation and growth, we can plasticize the tip or the part surrounding the crack, this is like pouring hot tar on the cracks of the road and the adhesion of the tar will prevent the growth of the crack. In fact, in order to prevent the cracks growth and its propagation, we should plasticize the surrounding part of the crack or we should create some negative well for discharging the energy. So the path of crack falls into the negative well and the crack growth is stopped. This negative well can be frequent closed paths around the crack (like the cracks in the glass).

Vertical cracks will grow faster than other cracks. According to analytical and numerical results, the direction of the crack growth and the prediction of the fracture depend on the values of maximum stress and displacement or the value of multiplying of maximum stress and displacement. That is the next path of the crack growth is the maximum of the mentioned multiplying result.

THE PRESENTATION OF NEW FORMULATION FOR CONTACT STRESS FOR ROLLING BODIES

Hertz stress is comprehension, powerful and primitive formula which is used to calculate the contact stress for curved bodies, and so on. In this paper, new formulation of contact stress for two rolling bodies is presented, and its results are close to Hertz stress-formulation. This formulation is determined by the concept of contact and normal stress and numerical results. As well as constants in this method are determined by analytical and numerical results. That is, in this method, contact stress has direct proportion with square root of the force and elastic module, but it has inverse proportion with square root rectangular contact surface. That is:

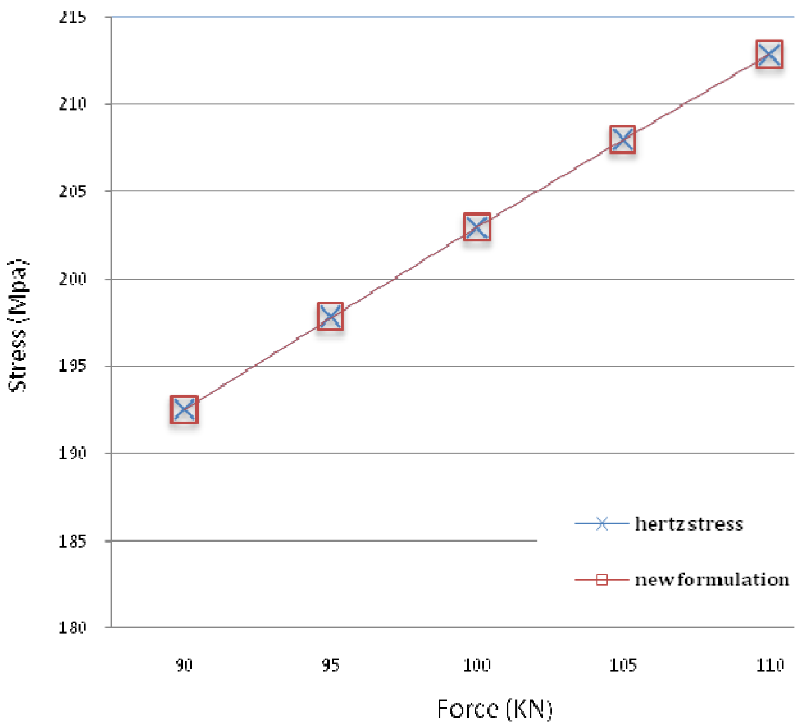

Figure 3. Comparison of hertz stress and new contact stress formulation.

Table 1. Comparison of hertz stress, new stress formulation and FEM.

Stress (MPa)	90 (KN)	95 (KN)	100 (KN)	105 (KN)	110 (KN)
Hertz stress	192.51	197.78	202.92	207.93	212.82
New formulation	192.51	197.79	202.92	207.94	212.83
FEM (a quarter of elliptic surface)	191	196	201	207	211
FEM (half of elliptic surface)	189	191	199	201	206
FEM (three fourth of elliptic surface)	184	188	197	195	200
FEM (a sixth of elliptic surface)	196	198	206	208	215
FEM (one eighth of elliptic surface)	198	202	208	209	219

$$\sigma_{max} = f(F, E, A_{rect}) \quad (1)$$

$$\sigma_{max} \propto \left(\frac{F \times E}{A_{rect}}\right)^{\frac{1}{2}} \times K \quad (2)$$

Where, *k* is constant coefficient for the proportion of equation. With attention to analytical and numerical results with FEM's considerations, the value of *k* is equal to $\sqrt{6.78} \times 10^{-7}$. Therefore, the new formulation for contact stress for rolling bodies is:

$$\sigma_{max} = \sqrt{\frac{6.78\,FE}{A_{rect}} \times 10^{-7}} \quad (3)$$

$$A_{rect} = a \times b \quad (4)$$

Where, *F* is force in Newton, elastic module $\left(\frac{N}{m^2}\right)$, and A_{rect} is rectangular contact surface (m^2). Values of hertz stress and new formulation are compared in Figure 3, and Table 1. The loading in this simulations are 90, 95, 100, 105, 110 *KN*. Table1 shows that the difference of the results is very small.

According to Figure 3 and Table 1, the results of hertz stress are the same as the new formulation for contact stress. The new formulation is simple and requires a few data for the determination of contact stress. It reveals that, the number of data for the determination of contact stress in this new formulation is lesser than that of the hertz stress.

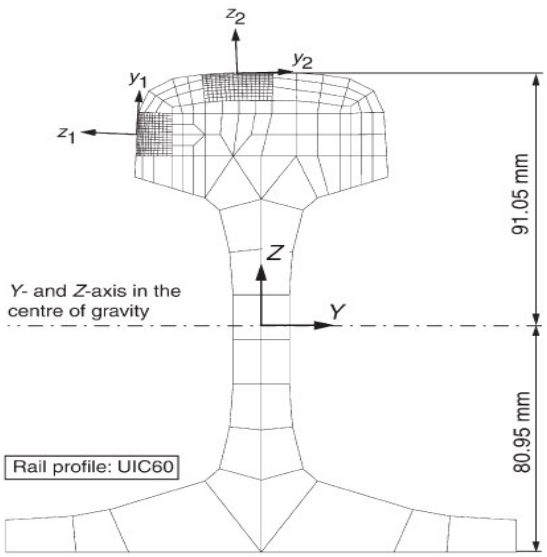

Figure 4. Position of contact for UIC60.

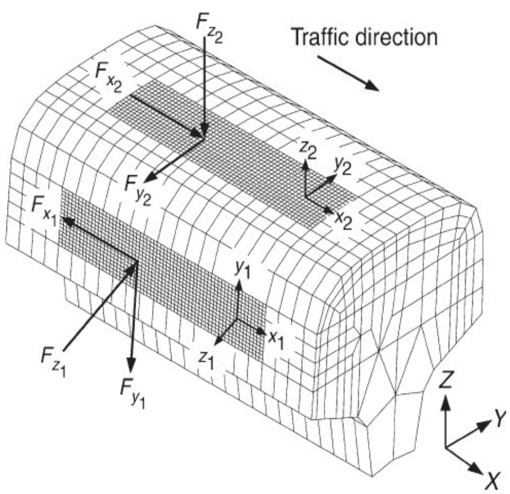

Figure 5. Direction of loading in contact surface.

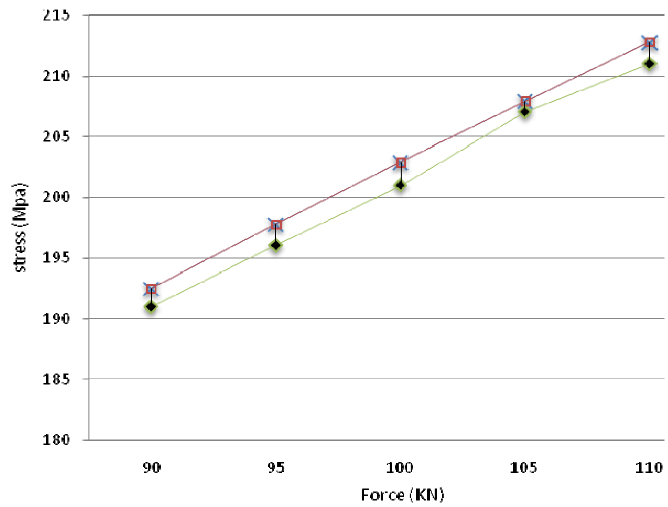

Figure 6. Comparisons of FEM (a quarter of area), hertz stress and new formulation stress.

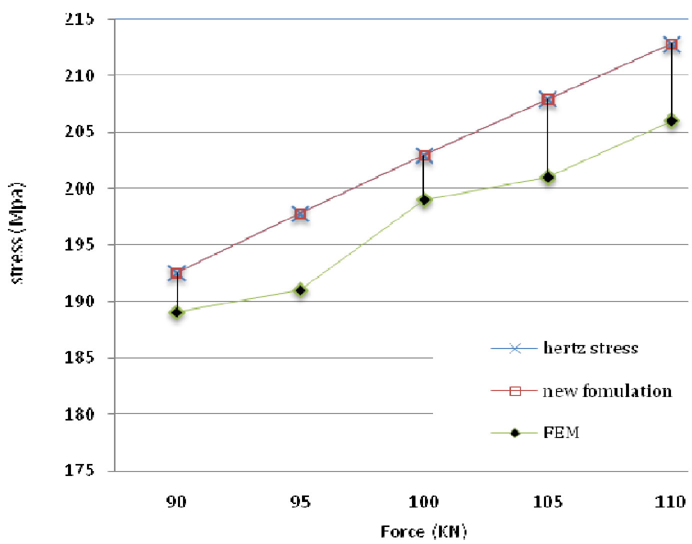

Figure 7. Comparisons of FEM (half of area), hertz stress and new formulation stress.

THEORY OF THE ANALYSIS OF THE TENSION AND FRACTURE

In order to analyze the contract stresses, Hertz stress and numerical methods are used. The induced loads in the contact area have been done by FEM analysis. Figures 4 and 5 shows the standard UIC60 and direction of forces in the contact surface.

THE METHOD OF STRESS ANALYSIS AND FRACTURE

In this section we will analyze the stress, critical stress, and the beginning of the fracture in the rail for the straight path. For this purpose we induce the static load of the wheel and rail by FEM and then analyze the stress. The induced load to one wheel is about 10 tones. Then we calculate hertz stress and compare it with stress in y direction, usually these results should be the same as well as Contact surface is elliptical area approximately. In this research the axial load is considered about 20 tones. The area of contact surface is assumed to be about 1 cm. The loads that are induced to the rail and the wheel are 90, 95, 100, 105 and 110 KN. Finally, the results of the three methods are the same if the area of contact is one third of the area. Figures 6, 7, 8, 9 and 10, show the results of three methods and their comparisons. For

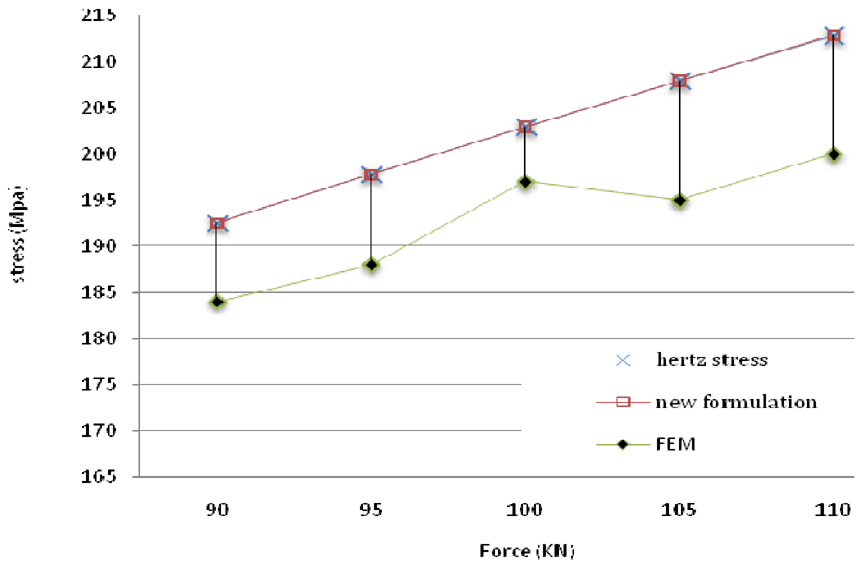

Figure 8. Comparisons of FEM (three-fourths of area), hertz stress and new formulation stress.

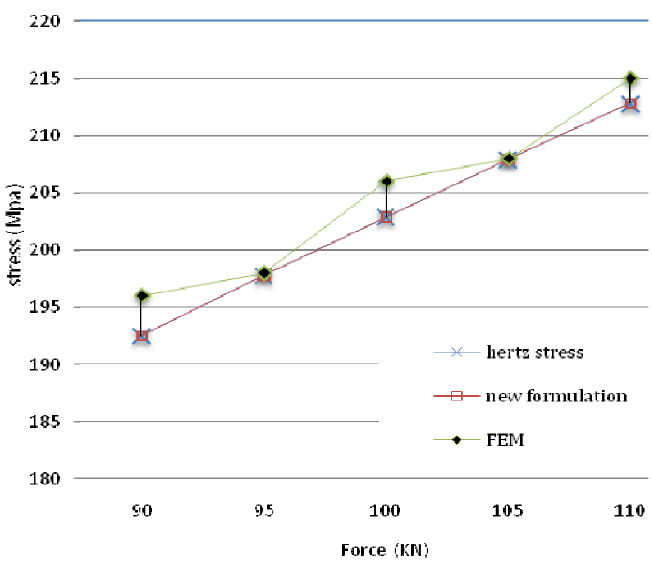

Figure 9. Comparisons of FEM (a sixth of area), hertz stress and new formulation stress.

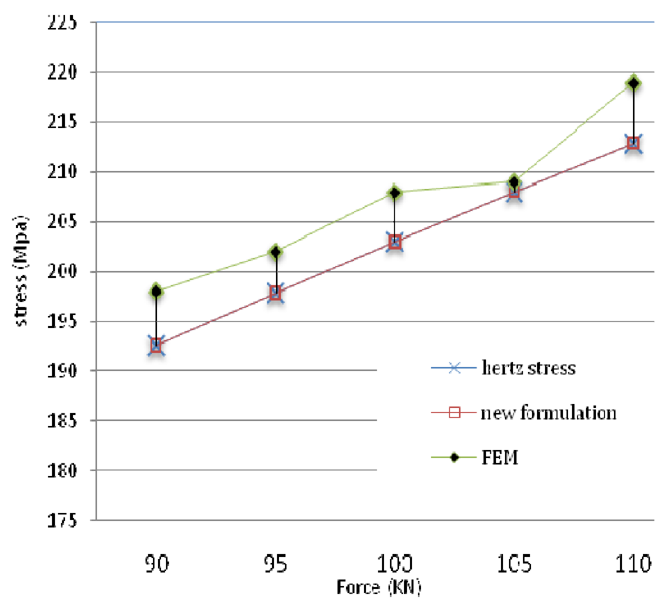

Figure 10. Comparisons of FEM (one-eighths of area), hertz stress and new formulation stress.

modeling of the wheel and the rail, we use the rail profile UIC60 and two axial wheel of bogie H665. For example, FEM (three-fourths of area) means the induced force in three-fourths of elliptical contact surface that is equal to contact stress. Therefore, the results, with assumption of induced force in one-thirds of area are coincided to the results of the hertz and new formulation methods. According to the above information, the pressure equal to 3.655 Mpa is induced at the position of contact of wheel and axel (Figure 11). Comparisons of the results of three methods are presented in Table 1.

Then we presented the reformation coefficients, that is, reformation coefficient for conversion of FEM results to hertz results is 0.0094, which means: $(201 \times 1.0094 = 202.9)$, and reformation coefficient for conversion of stress in y direction in FEM results to hertz results is 1.0352, which means $(196 \times 1.0352 = 202.9)$.

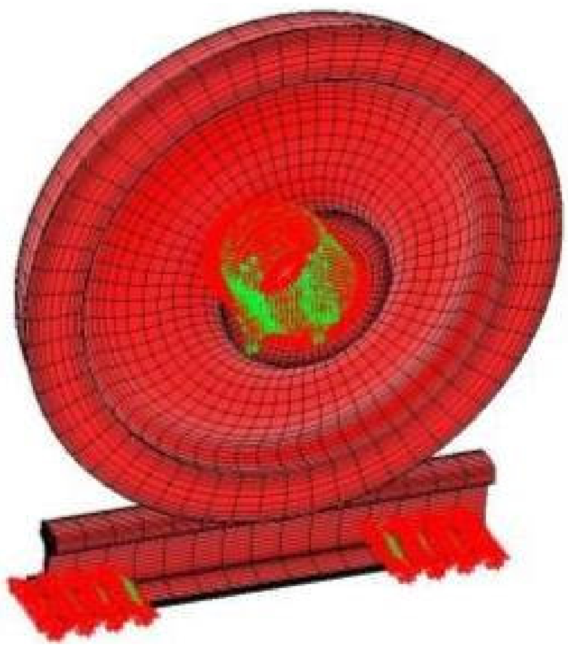

Figure 11. Induced force in position of contact of wheel and rail and constraints with FEM.

Conclusion

From the above information, we can conclude:

1. The results of new contact stress formulation for rolling bodies are coincided with the results of the hertz stress and FEM analysis.
2. With the mentioned correction or reformation coefficients, we can convert the theory results to the FEM results with suitable approximation.
3. The direction of the cracks growth and also the prediction of the fracture depend on the values of maximum stress and displacement or multiplication of two values.
4. The results, with assumption of induced force in one-thirds of area are coincided with the results of the hertz and new formulation methods.
5. In order to prevent the growth and diffusion of the crack, we can use some methods like plasticizing the surrounding part or the end of the crack, bounding the crack in some holes for discharging of the energy in some repetition or frequent paths (crack in some glasses). Creating a hole in the path of the crack will prevent the diffusion of the crack, because the crack with high energy will fall into the hole and it will lose its energy for growth and diffusion.

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