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Full Length Research Paper

The dynamical effect of stark-shifts produced from a four-level atomic system

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We consider a two-mode quantized field described in a coherent state interacting with a four-level atom. An effective Hamiltonian is obtained by adiabatically eliminating the intermediate two levels in a cascade process. The influence of the Stark shifts on the atomic inversion is examined, as well as on the second order auto-correlation function and on the cross-correlation between the field modes. The results of the calculations are illustrated numerically.

Key word: Adiabatic elimination, Stark-shift, second order auto-correlation function, cross-correlation between the field modes.

INTRODUCTION

The interaction model between a two-level atom and a single quantized mode of a radiation field, within the rotating wave approximation (RWA), is known as the Jaynes-Cumming model (JCM) (Jaynes and Cumming, 1963). This interaction can be seen as the repeated absorption and emission by the atom of a single photon from the radiation field. These photons are the agents that can transfer energy and momentum between the atom and the radiation field. By studying the JCM we gain more insight into the atom-radiation interaction in particular, and into quantum mechanics in general. The Jaynes-Cumming model has been the subject of theoretical studies (Stenholm 1973; Shore and Knight, 1993).and also of experimental investigation (Rempe et al. 1987).

Stimulated by the success of the JCM, many people have paid special attention to extending and generalizing the model in order to explore new quantum effects (Joshi, 2000; Abdel-Aty et al., 2002; Joshi, 2004; Shore and Knight, 1993; Yoo and Eberly, 1985).Two exactly solvable generalizations of JCM have been proposed by Sukumar and Buck, (1981) one involving intensity dependent coupling and the other involving multi-photon interaction between field and atom.

Because of the given information from the Rabi frequencies which arise from such a coupling, this model presents periodic revivals, contrary to what happens in the ordinary Jaynes-Cumming model. Also, possible generalizations are a consideration of multi-mode and multi-photon instead of single mode and single photon (Abdalla et al., 1990, 1991; Abdel-Aty et al., 2002). The addition of Kerr-like medium and Stark shift have been performed in other studies (Fang and Liu, 1996; Ahmed et al., 2005; Ahmed, 2007). Extensive studies of a three-level atom with different configurations under RWA interacting with quantized fields inside an ideal cavity were carried out in detail by Yoo and Eberly, (1985). Later on several more studies on dynamical evolution and field statistical were reported on the similar type of models (Li and Peng, 1986; Zhu, 1989; Li and Niu, 2004). These models could be experimentally tested by utilizing three-level atoms in various configurations in the micromaser systems. Based on the JCM and its various extensions, an interesting non-classical effects, such as the collapse and revival of Rabi oscillations, squeezing, electromagnetic-induced transparency, etc., have been extensively studied (Shore and Knight, 1993; Yoo and Eberly, 1985; Li and Peng, 1986; Zhu, 1989; Li and Niu, 2004).

A generalization of the JCM to study a four-level atom was presented in Zait, (2005). The author has investigated the dynamics of the quasi-probability distribution Q-function of a four-level atom interacting with a single mode field in a cavity containing a Kerr-like medium. Moreover, he investigated the cavity field statistics through Mandel Q-parameter. In the present paper, we shall study the interaction between a four-level atom and two-mode quantized field. We investigate a new type of Stark

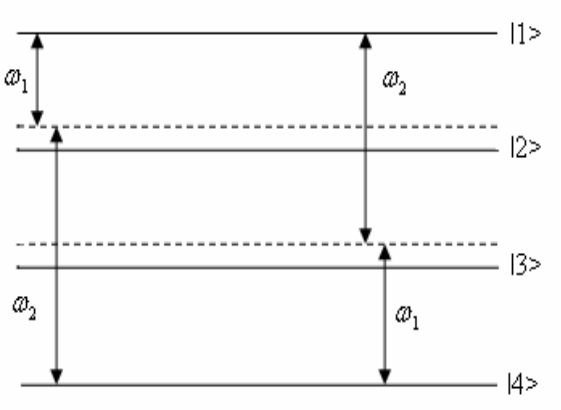

Figure 1. Four-level atom with non-equidistant, non-degenerate levels**.**

shift which differs from the one that appears in the study of three-level atomic system. In the next section we consider the equations of motion in the Heisenberg picture for the system of two modes of the electromagnetic field interacting with a four-level atom. We derive the effective Hamiltonian of the system which includes the Stark shift. In addition, the evolution operator and the wave function are obtained in section 3. Section 4 is devoted to the numerical investigations of the atomic inversion, the second order auto-correlation function and the cross-correlation function between the field modes. Finally, a conclusion is presented in section 5.

The basic equations

We consider a system of four-level atom with non-equidistant, non-degenerate levels $|1\rangle,|2\rangle,|3\rangle and|4\rangle$ (Figure 1) with energy levels Ω_k, (k=1, 2, 3, 4) respectively, interacting with two different modes of the electromagnetic field with frequencies ω_j, (j=1, 2).

The Hamiltonian that describes the present system within the rotating wave approximation has the following form (with ħ=1)

$$\begin{aligned} H &= \sum_{j=1}^{2}\omega_j a_j^{+}a_j + \sum_{k=1}^{4}\Omega_k S_{kk} \\ &+ \lambda_1(a_1^{+}S_{21} + h.c.) + \lambda_2(a_1^{+}S_{43} + h.c.) \\ &+ \lambda_3(a_2^{+}S_{31} + h.c.) + \lambda_4(a_2^{+}S_{42} + h.c.) \end{aligned} \quad (1)$$

Where $a_j(a_j^{+})$ is the annihilation (creation) operator for the j^{th} mode of the field and satisfy the Boson commutation relation $\left[a_i, a_j^{+}\right] = \delta_{ij}$. $\lambda_k; (k = 1,2,3,4)$ are the coupling constants and the atomic operators S_{lk} satisfy the commutation relation Yoo and Eberly, (1985)

$$\left[S_{ij}, S_{lk}\right] = S_{ik}\delta_{lj} - S_{lj}\delta_{ik} \ .$$

In order to apply the adiabatic elimination method to eliminate the two levels $|2\rangle \ and \ |3\rangle$ we need to induce the slowly varying operators (Puri and Bullough, 1988; Obada et al., 2006)

$$A_j = a_j e^{i\omega_j t}, Q_{lk} = S_{lk} e^{-i\Omega_{lk} t}$$

$\Omega_{lk} = \Omega_l - \Omega_k$, l, k =1,2,3,4, l ≠ k **(2)**

The Heisenberg equation of motion for any operator O is

$$i\frac{\partial O}{\partial t} = [O, H]$$

Therefore, we have the following equations of motion for the atomic operators

$$\begin{aligned} i\frac{dQ_{11}}{dt} &= -\lambda_1(A_1^{+}Q_{21}e^{-i\Delta t} - h.c) - \lambda_3(A_2^{+}Q_{31}e^{-i\Delta t} - h.c) \\ i\frac{dQ_{22}}{dt} &= \lambda_1(A_1^{+}Q_{21}e^{-i\Delta t} - h.c.) - \lambda_4(A_2^{+}Q_{42}e^{i\Delta t} - h.c.) \\ i\frac{dQ_{33}}{dt} &= \lambda_3(A_2^{+}Q_{31}e^{-i\Delta t} - h.c) - \lambda_2(A_1^{+}Q_{43}e^{i\Delta t} - h.c) \\ i\frac{dQ_{44}}{dt} &= \lambda_4(A_2^{+}Q_{43}e^{i\Delta t} - h.c) + \lambda_2(A_1^{+}Q_{43}e^{i\Delta t} - h.c) \end{aligned} \quad (3)$$

$$\begin{aligned} i\frac{dQ_{12}}{dt} &= \lambda_1 A_1^{+}(Q_{11} - Q_{22})e^{-i\Delta t} - \lambda_3 A_2^{+}Q_{32}e^{-i\Delta t} + \lambda_4 A_2 Q_{14}e^{-i\Delta t} \\ i\frac{dQ_{24}}{dt} &= \lambda_4 A_2^{+}(Q_{22} - Q_{44})e^{i\Delta t} + \lambda_2 A_1^{+}Q_{23}e^{i\Delta t} - \lambda_1 A_1 Q_{14}e^{i\Delta t} \\ i\frac{dQ_{13}}{dt} &= \lambda_3 A_2^{+}(Q_{11} - Q_{33})e^{-i\Delta t} - \lambda_1 A_1^{+}Q_{32}e^{-i\Delta t} + \lambda_2 A_1 Q_{14}e^{-i\Delta t} \\ i\frac{dQ_{34}}{dt} &= \lambda_2 A_1^{+}(Q_{33} - Q_{44})e^{i\Delta t} - \lambda_4 A_2^{+}Q_{32}e^{i\Delta t} - \lambda_3 A_2 Q_{14}e^{i\Delta t} \end{aligned} \quad (4)$$

$$\begin{aligned} i\frac{dQ_{14}}{dt} &= -\lambda_1 A_1^{+}Q_{24}e^{-i\Delta t} + \lambda_4 A_2^{+}Q_{12}e^{i\Delta t} \\ &- \lambda_3 A_2^{+}Q_{34}e^{-i\Delta t} + \lambda_2 A_1^{+}Q_{13}e^{i\Delta t} \end{aligned} \quad (5)$$

Similarly for the two modes of the field we have the equations

$$i\frac{dA_1}{dt} = \lambda_1 Q_{21} e^{-i\Delta t} + \lambda_2 Q_{43} e^{i\Delta t} \quad (6)$$

$$i\frac{dA_2}{dt} = \lambda_3 Q_{13} e^{-i\Delta t} + \lambda_4 Q_{42} e^{i\Delta t} \quad (7)$$

Where the detuning parameter Δ is defined as

$$\Delta = \Omega_{12} - \omega_1 = \omega_1 - \Omega_{34} = \Omega_{13} - \omega_2 = \omega_2 - \Omega_{24} \quad (8)$$

In order to integrate equation (4), we perform the slowly varying amplitudes approximation for both the atomic amplitudes and for the field envelopes. So we replace Q_{ij} (t) and A_j (t) inside the integral with their values at the upper limit of the integration and carrying out the exact integration on the remaining exponential factors from 0 to t (with initial condition Q_{ij} (0) = 0).Thus we get the following eqns.

$$Q_{12} = \frac{1}{\Delta}\left[\lambda_1 A_1^+ (Q_{11} - Q_{22}) + \lambda_4 A_2 Q_{14} - \lambda_3 A_2^+ Q_{32}\right] e^{-i\Delta t}$$

$$Q_{24} = \frac{-1}{\Delta}\left[\lambda_4 A_2^+ (Q_{22} - Q_{44}) - \lambda_1 A_1 Q_{14} + \lambda_2 A_1^+ Q_{23}\right] e^{i\Delta t} \quad (9)$$

$$Q_{13} = \frac{1}{\Delta}\left[\lambda_3 A_2^+ (Q_{11} - Q_{33}) + \lambda_4 A_1 Q_{14} - \lambda_1 A_1^+ Q_{23}\right] e^{-i\Delta t}$$

$$Q_{34} = \frac{-1}{\Delta}\left[\lambda_2 A_1^+ (Q_{33} - Q_{44}) - \lambda_3 A_2 Q_{14} + \lambda_4 A_2^+ Q_{32}\right] e^{i\Delta t},$$

By substituting eqns. (9) into (3) we get that is

$$S_{22} + S_{33} = Const. \quad (10)$$

which means that the occupation of levels $|2\rangle and |3\rangle$ is constant during the interaction. With help of eqns. (9) we can, directly, obtain from eqns. (5-7) the following equations of motion

$$i\frac{dS_{14}}{dt} = \Omega_{41} S_{14} - \lambda a_1^+ a_2^+ (S_{44} - S_{11}) - \left[\frac{1}{\Delta}(\lambda_1^2 - \lambda_2^2) a_1^+ a_1 + \frac{1}{\Delta}(\lambda_3^2 - \lambda_4^2) a_2^+ a_2\right] S_{14} \quad (11)$$

$$i\frac{da_1}{dt} = \omega_1 a_1 + \frac{1}{\Delta}(\lambda_1^2 S_{11} + \lambda_2^2 S_{44}) a_1 + \lambda a_2^+ S_{41} \quad (12)$$

$$i\frac{da_2}{dt} = \omega_2 a_2 + \frac{1}{\Delta}(\lambda_3^2 S_{11} + \lambda_4^2 S_{44}) a_2 + \lambda a_1^+ S_{41} \quad (13)$$

Where;

$$\lambda = \frac{1}{\Delta}(\lambda_1 \lambda_4 + \lambda_2 \lambda_3) \quad (14)$$

The equations (11-13) can be considered as the equations of the motion according to the following Hamiltonian

$$H_{eff} = \sum_{j=1}^{2} \omega_j a_j^+ a_j + \Omega_1 S_{11} + \Omega_4 S_{44} + a_1^+ a_1 \left(\frac{\lambda_1^2}{\Delta} S_{11} + \frac{\lambda_2^2}{\Delta} S_{44}\right) + a_2^+ a_2 \left(\frac{\lambda_3^2}{\Delta} S_{11} + \frac{\lambda_4^2}{\Delta} S_{44}\right) + \lambda(a_1^+ a_2^+ S_{41} + S_{14} a_1 a_2) \quad (15)$$

Since we have only two effective levels $|1\rangle \; and \; |4\rangle$ in the above Hamiltonian, we will replace $|4\rangle \; by \; |2\rangle$ just for convenience and H_{eff} can be rewritten as

$$H_{eff} = \sum_{j=1}^{2} \omega_j a_j^+ a_j + \sum_{k=1}^{2} \Omega_k S_{kk} + \frac{1}{\Delta} S_{11} (\lambda_1^2 a_1^+ a_1 + \lambda_3^2 a_2^+ a_2) + \frac{1}{\Delta} S_{22} (\lambda_2^2 a_1^+ a_1 + \lambda_4^2 a_2^+ a_2) + \lambda(a_1^+ a_2^+ S_{21} + S_{12} a_1 a_2) \quad (16)$$

In equation (16), we note that, the 3rd and the 4th terms represent the Stark-shift (Puri and Bullough, 1988) of the levels $|1\rangle \; and \; |2\rangle$ due to the field modes j=1, 2, respectively. Note the difference in the form in which it appears here compared to the single mode of the three level cases (Ahmed, 2007; Puri and Bullough, 1988). We note that the shifts are affected by the two modes with different weights depending on the original coupling constants.

The time evolution operator

The Heisenberg equations of motion for the operators

$n_j = a_j^+ a_j$, j = 1,2 and S_{11} are

$$i\frac{dn_1}{dt} = L\,,\; i\frac{dn_2}{dt} = L \text{ and } i\frac{dS_{11}}{dt} = -L \quad (17)$$

Where;

$$L = \lambda(a_1^+ a_2^+ S_{21} - S_{12} a_1 a_2) \quad (18)$$

We deduce that the following operators

$$n_1 + S_{11} = N_1 \text{ and } n_2 + S_{11} = N_2 \quad (19)$$

are constants of the motion. Thus, the Hamiltonian (16) can be written as

$$H_{eff} = N + C + \gamma I \quad (20)$$

$$N = Z_1 S_{11} + Z_2 S_{22} \quad (21)$$

with

$$Z_1(n_1,n_2) = \omega_1(n_1+1) + \omega_2(n_2+1) + \frac{1}{2\Delta}\left[(\lambda_1^2+\lambda_2^2)(n_1+1) + (\lambda_3^2+\lambda_4^2)(n_2+1)\right], \quad (22)$$

$$Z_2(n_1,n_2) = \omega_1 n_1 + \omega_2 n_2 + \frac{1}{2\Delta}\left[(\lambda_1^2+\lambda_2^2)n_1 + (\lambda_3^2+\lambda_4^2)\right], \quad (23)$$

$$\gamma = \frac{1}{2}\left[(\Omega_1 + \Omega_2 - \omega_1 - \omega_2) - \frac{1}{\Delta}(\lambda_1^2+\lambda_3^2)\right], \quad (24)$$

and *I* is the identity operator.
We note that

$$a_1 a_2 Z_2(n_1,n_2) = Z_1(n_1,n_2) a_1 a_2 \;;\; Z_2(n_1,n_2) = Z_1(n_1-1,n_2-1) \quad (25)$$

and C=Λ+L (26)

with $\Lambda = \delta_1 S_{11} - \delta_2 S_{22}$ (27)

where;

$$\delta_1(n_1,n_2) = \delta + \frac{1}{2\Delta}\left[\lambda_1^2 n_1 - \lambda_2^2(n_1+1) + \lambda_3^2 n_2 - \lambda_4^2(n_2+1)\right] \quad (28)$$

$$\delta_2(n_1,n_2) = \delta + \frac{1}{2\Delta}\left[\lambda_1^2(n_1-1) - \lambda_2^2 n_1 + \lambda_3^2(n_2-1) - \lambda_4^2 n_2\right] \quad (29)$$

represent the field-dependent Stark shift terms. The detuning parameter
δ is given by

$$\delta = (\Omega_1 - \Omega_2) - (\omega_1 + \omega_2) = 0 \quad (30)$$

where eqn.(8) is used
Also we note that

$$a_1 a_2 \delta_2(n_1,n_2) = \delta_1(n_1,n_2) a_1 a_2 \;;\; \delta_2(n_1,n_2) = \delta_1(n_1-1,n_2-1) \quad (31)$$

It is easy to show that the operators N and C commute with each other and hence with H. This means that N and C are constants of motion.

The time evolution operators for our system is given by

$$U(t) = \exp(-i\gamma t)\exp(-iNt)\exp(-iCt) \quad (32)$$

Where

$$\exp(-iNt) = \begin{bmatrix} e^{-iz_1 t} & 0 \\ 0 & e^{-iz_2 t} \end{bmatrix} \quad (33)$$

$$\exp(-iCt) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (34)$$

with

$$A_{11} = Cos\eta_1 t - i\delta_1 \frac{Sin\eta_1 t}{\eta_1} \quad (35)$$

$$A_{12} = -i\lambda \frac{Sin\,\eta_1 t}{\eta_1} a_1 a_2 \quad (36)$$

$$A_{21} = -i\lambda a_1^+ a_2^+ \frac{Sin\eta_1 t}{\eta_1} \quad (37)$$

$$A_{22} = Cos\eta_2 t + i\delta_2 \frac{Sin\eta_2 t}{\eta_2} \quad (38)$$

Where;

$$\eta_i^2(n_1,n_2) = \delta_i^2(n_1,n_2) + \nu_i(n_1,n_2)\;,\; i = 1,2 \quad (39)$$

With

$$\nu_1(n_1,n_2) = \lambda^2(n_1+1)(n_2+1)\,;$$
$$\nu_2(n_1,n_2) = \nu_1(n_1-1,n_2-1) \quad (40)$$

Evaluating the time evolution operator enable us to discuss the dynamical behavior of the system.

The wave function

Let us consider that, at time t = 0, the effective two-level atom is in a coherent atomic state

$$\left|\theta,\phi\right\rangle = Cos\frac{\theta}{2}\left|1\right\rangle + e^{-i\phi}Sin\frac{\theta}{2}\left|2\right\rangle \quad \textbf{(41)}$$

Where $\left|1\right\rangle$ and $\left|2\right\rangle$ stands for the excited and ground states of the atom respectively, ϕ is the relative phase of the two atomic levels. When $\theta \to 0$ the excited state is considered while when $\theta \to \pi$ then the wave function describes the atom in its ground state. Also we consider the fields to be initially in the uncorrelated coherent states $\left|\alpha_1,\alpha_2\right\rangle = \left|\alpha_1\right\rangle \otimes \left|\alpha_2\right\rangle$. Then the initial state of the fields takes the following form

$$\left|\alpha_1,\alpha_2\right\rangle = \sum_{n_1,n_2} q_{n_1,n_2}\left|n_1,n_2\right\rangle \quad \textbf{(42)}$$

Where;

$q_{n_1,n_2} = q_{n_1}q_{n_2}, q_{n_j}$ describes the amplitude of the state $\left|n_i\right\rangle$ of the ith mode, with $q_{ni} = \exp(-\frac{1}{2}\left|\alpha_i\right|^2\frac{\alpha_i^{n_i}}{\sqrt{n_i!}}$.

At time t=0 the wave function $\psi(0)$ is given by $\left|\psi(0)\right\rangle = \left|\theta,\phi\right\rangle \otimes \left|\alpha_1,\alpha_2\right\rangle$, At the time t > 0 the wave function takes the form

$$\left|\psi(t)\right\rangle = \left|S(t)\right\rangle\left|1\right\rangle + \left|T(t)\right\rangle\left|2\right\rangle \quad \textbf{(43)}$$

Where;

$$\left|S(t)\right\rangle = e^{-iz_1t}\left[\cos\left(\frac{\theta}{2}\right)A_{11} + e^{i\phi}\sin\left(\frac{\theta}{2}\right)A_{12}\right]\left|\alpha_1,\alpha_2\right\rangle \quad \textbf{(44)}$$

and

$$\left|T(t)\right\rangle = e^{-iz_2t}\left[\cos\left(\frac{\theta}{2}\right)A_{21} + e^{i\phi}\sin\left(\frac{\theta}{2}\right)A_{22}\right]\left|\alpha_1,\alpha_2\right\rangle \quad \textbf{(45)}$$

For an atom in the excited state ($\theta \to 0$) we have, by using eqns. (35-38),

$$\left|S(t)\right\rangle = e^{-iz_1t}\sum_{n_1,n_2=0}^{\infty} e^{-\frac{1}{2}\left|\tilde{\alpha}\right|^2}\frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}\left(\cos\eta_1 t - i\delta_1\frac{\sin\eta_1 t}{\eta_1}\right)\left|n_1,n_2\right\rangle \quad \textbf{(46)}$$

and

$$\left|T(t)\right\rangle = -i\lambda e^{-iz_2t}\sum_{n_1,n_2=0}^{\infty} e^{-\frac{1}{2}\left|\tilde{\alpha}\right|^2}\frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}\sqrt{(n_1+1)(n_2+1)}\frac{\sin\eta_1 t}{\eta_1}\left|n_1+1,n_2+1\right\rangle \quad \textbf{(47)}$$

Where; $\left|\tilde{\alpha}\right|^2 = \left|\alpha_1\right|^2 + \left|\alpha_2\right|^2$

Calculating the wave function enables us to investigate any phenomenon related to the atom and the field modes.

DISCUSSION OF ATOMIC AND FIELD DYNAMICS

In this section, we investigate some aspects of our system, namely, the atomic inversion, the second auto-correlation function and the cross-correlation function between the field modes.

The atomic inversion

The atomic population inversion is defined as the difference between the probabilities of finding the atom in the excited state and in the ground state. When the atom starts in its excited state, the atomic inversion takes the form

$$W(t) = \frac{1}{2}[\left\langle S_{11}\right\rangle - \left\langle S_{22}\right\rangle]$$

$$= \frac{1}{2} - \sum_{n_1,n_2}^{\infty} e^{-\left|\tilde{\alpha}\right|^2}\frac{\left|\alpha_1\right|^{2n_1}\left|\alpha_2\right|^{2n_2}}{n_1!n_2!}\nu_i\frac{Sin^2\eta_1 t}{\eta_1^2} \quad \textbf{(48)}$$

We plot the atomic population inversion W (T) against the scaled time T=λ t with the intensity of the initial coherent field equal to $\overline{n}_j = 4$, j=1,2 (Figure 2). First we ignore the terms (δ_i^2 in eq. (39)) which represent the Stark shift and the result is illustrated in Figure(2-a). It is apparent that the oscillations are around W(T)=0 and become irregular as T increases. The effect of the Stark shift ($\delta_j \neq 0$) on the atomic inversion appears clearly in Figure (2-b) where the oscillations of the atomic inversion W(T) show the collapse and revival phenomenon. The base line of W (T) is shifted upward which means more energy is stored in the atomic system.

The second order auto-correlation function

The second order auto-correlation function for the mode j is defined by Paul, (1982).

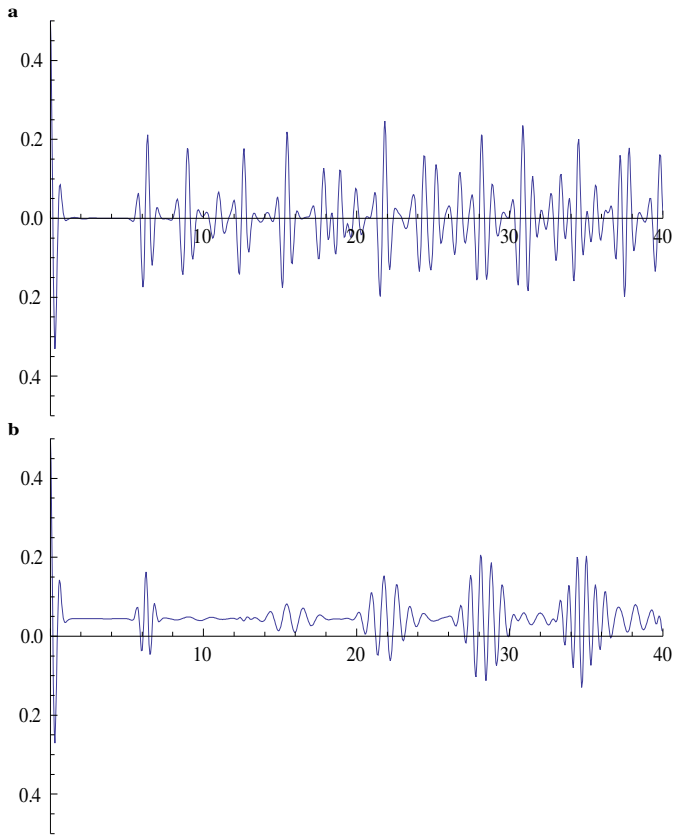

Figure 2. The atomic inversion W (T) against the scaled time T=λ t with initial average photon number $\overline{n}_j = 4$. (a) The Stark shifts are ignored. (b) the Stark shifts parameters are $\lambda_i/\sqrt{\Delta}$= 0.995 (i =1, 4) and $\lambda_i/\sqrt{\Delta}$= 0.1 (i =2, 3).

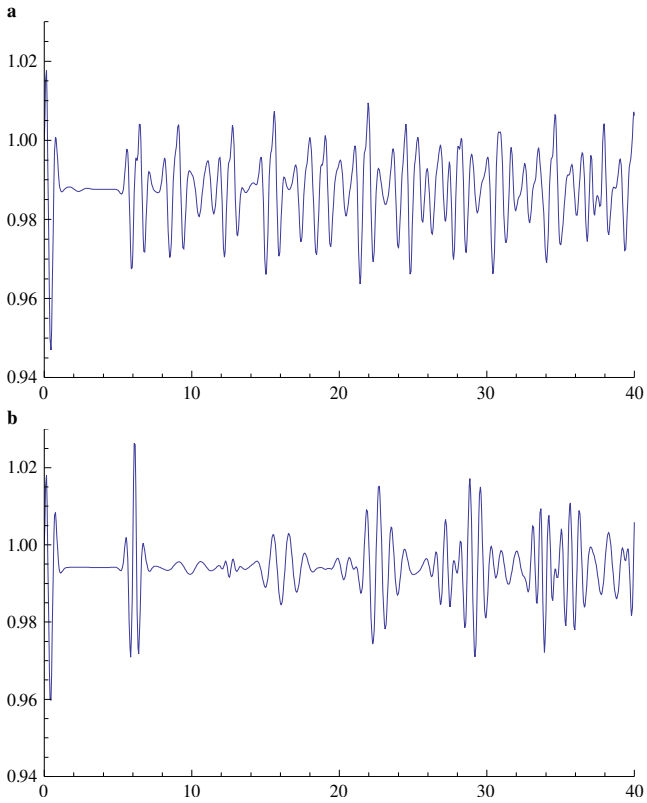

Figure 3. The second order auto-correlation function $g^{(2)}$ (T) against the scaled time T=λ t with initial average photon number $\overline{n}_j = 4$. (a) The Stark shifts are ignored. (b) The Stark shifts parameters are $\lambda_i/\sqrt{\Delta}$= 0.995 (i =1, 4) and $\lambda_i/\sqrt{\Delta}$= 0.1 (i =2, 3).

$$g_j^{(2)}(t) = \frac{\left\langle a_j^{+2}(t)a_j^2(t)\right\rangle}{\left\langle a_j^{+}(t)a_j(t)\right\rangle^2} = \frac{\left\langle n_j^2(t)\right\rangle - \left\langle n_j(t)\right\rangle}{\left\langle n_j(t)\right\rangle^2} \quad , \quad j = 1,2 \tag{49}$$

this function essentially measures the joint probability for detecting two photons at the same time t. Note that if $g^{(2)}(t) = 1$, the photons arrive at the detectors at random and the probability distribution is Poissonian. A coherent state of the radiation from a single-mode laser is an example of this case. The bunching occurs when $g^{(2)}(t) > 1$ and the distribution is super-Poissonian. This is also a classical effect (e.g., $g^{(2)}(t) = 2$ for thermal light). For the case in which $g^{(2)}(t) < 1$ the antibunching exists Carmichael, (1985) and the light field has a sub-Poissonian distribution. This means that the probability of detecting an incident pair of photons is less than it would be for a coherent field described by the Poissonian distribution. The antibunched case is a quantum mechanical manifestation and has no counterpart in classical description of radiation.

For our system, we plot the second order auto-correlation function $g_j^{(2)}(T)$ against the scaled time T for $\overline{n}_j = 4$, $j = 1,2$. In the absence of Stark shifts (Figure3a), the function $g_j^{(2)}(T)$ shows irregular fluctuations. It is almost sub-Poissonian $(i.e., g^{(2)}(t) < 1)$ except at some few values of T, it be-comes greater than 1. Taking the effect of the Stark shifts into account (Figure 3-b), and however, the baseline of the fluctuation in $g^{(2)}(T)$ is pushed up, and still shows sub-Poissonian distribution. The collapses and revivals phenomenon is still pronounced.

The cross-correlation between the field modes

The cross-correlation function between the two modes of the field is given by Paul, (1982).

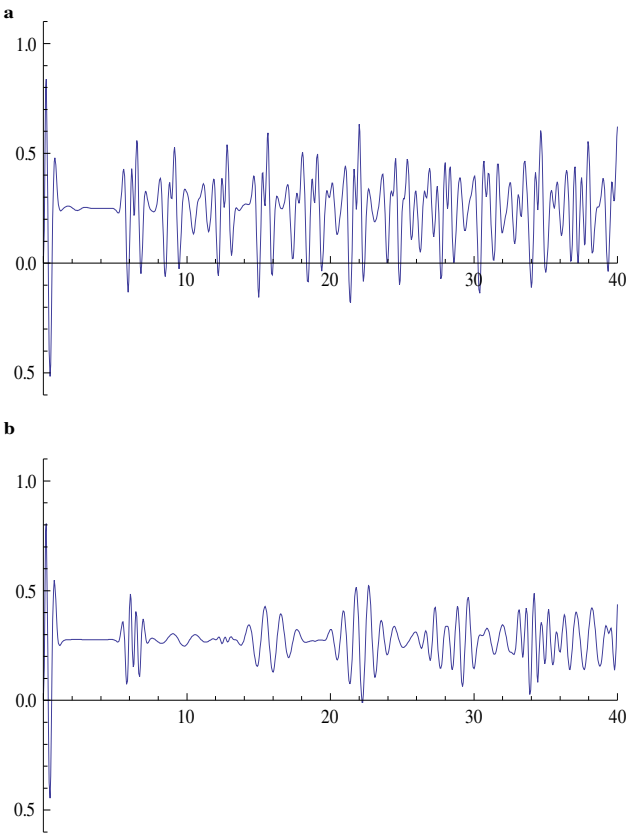

Figure 4. The cross-correlation between the field modes of the field $\Delta_{cross,}$ (T) against the scaled time T=λ t with initial average photon number $\overline{n}_j = 4$. (a) The Stark shifts are ignored. (b) The Stark shifts parameters are $\lambda_i/\sqrt{\Delta} = 0.995$ (i =1, 4) and $\lambda_i/\sqrt{\Delta} = 0.1$ (i =2, 3).

$$\Delta_{cross}(t) = \left\langle \prod_j (n_j) \right\rangle - \prod_j \langle n_j \rangle, \text{ (j=1, 2)} \qquad \textbf{(50)}$$

$$\Delta_{cross}(t) = \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle$$

This represents the correlation between the intensities of the two modes of the field. If $\Delta_{cross}(t) > 0$ then the two modes are correlated. But if $\Delta_{cross}(t) = 0$ the modes are uncorrelated and for $\Delta_{cross}(t) < 0$ the field modes are called anti-correlated. In Figure 4 we plot $\Delta_{cross}(t)$ against the scaled time T with $\overline{n}_1 = \overline{n}_2 = 4.0$. In Figure (4a) the Stark shifts are ignored and we see that most of the fluctuations are above zero. This means that the two modes of the field are correlated except at very short period of T where the two modes are anti-correlated. For non-zero the Stark shifts (Figure 4b) the modes are correlated for all time except at a very short time after the beginning of the interaction. It is noted that collapses and revivals phenomenon appears in this case also.

CONCLUSION

In this paper, we have investigated the effect of the Stark shifts in a four-level atomic system interacting with two quantized modes of the field. We used the adiabatic elimination method to get an effective two-level Hamiltonian. Our results show that the Stark shifts makes the collapse and revival phenomenon more pronounced in the atomic inversion. Both the base lines of the fluctuations in the second order auto-correlation function or the cross correlation function are pushed up due to Stark shifts.

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