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Full Length Research Paper

Wave-like interaction, occurring at superluminal speeds, or the same, de Broglie relationship, as imposed by the law of energy conservation: Electrically bound particles (Part 1)

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Based on the law of conservation of energy, we have shown that, the steady state electronic motion around a given nucleus in a non-circular orbit depicts a rest mass variation, though the overall relativistic energy remains constant. This is, in no way, conflicting with the usual quantum mechanical approach. On the contrary, it provides us with the possibility of bridging the special theory of relativity and quantum mechanics, to finally achieve a natural symbiosis between these two disciplines, and furthermore, elucidating the "quantum mechanical weirdness", simply based on the mere law of relativistic conservation of energy. Our theory was developed originally, vis-à-vis gravitational bodies in motion with regard to each other; hence, it is comforting to have both the atomic scale and the celestial scale described on just the same conceptual basis. One way to conceive the rest mass variation of concern is to consider a "jet effect". Accordingly, a particle on a given orbit through its journey must eject a net mass from its back to accelerate, or must pile up a net mass from its front to decelerate, while its overall relativistic energy stays constant throughout. In other words, a minimal rest mass is transformed into extra kinetic energy through an acceleration process, and kinetic energy condenses into extra rest mass through a deceleration process. The tangential speed of the jet in question, strikingly points to the de Broglie wavelength. This, on the whole, makes the jet speed ''superluminal", yet excluding any transport of energy. This conjecture coincides with the recent measurements (Kholmetskii et al., 2007; Salart et al., 2008), and provides a clue for the wave-particle duality, also insures their concurrent coexistence. An important consequence of the approach presented herein is that, either gravitationally interacting macroscopic bodies, or electrically interacting microscopic objects, sense each other, with a speed much greater than that of light, and this, in exactly the same way. In which case, though, the interaction coming into play, excludes any energy exchange; thus, we would like to call it, "wave-like interaction". The present approach ends up the existing schizophrenia, between different conceptions painting separately the micro world and the macro world, and unites these two worlds with an unequal, single and simple conception, based on just the law of relativistic energy conservation.

Key words: Special theory of relativity, electric interaction, tachyons, de Broglie relationship, superluminal or wave-like interaction, general theory of relativity, gravitation.

INTRODUCTION

Louis de Broglie has anticipated that (de Broglie, 1925), for an object of mass m_0 at rest, there should be a periodic phenomenon, depicting a frequency ν_0, such that:

$$h\nu_0 = m_0 c_0^2 \quad (1)$$

(*de Broglie's definition of the periodic phenomenon's frequency inside the object in hand, at rest*).

Here, h is the Planck's constant, and c_0 the speed of light in empty space. It is evident that, de Broglie had envisaged in extreme case, where the entire mass m_0 would be transformed into electromagnetic energy. On the other hand, it is remarkable that he considered Equation (1) at a time even when, the annihilation process of an electron with a positron remained far away from been discovered. Posing λ_0 as the wavelength, and T_0 as the period, to be associated with the electromagnetic wave coming into play, by definition we have

$$c_0 = \frac{\lambda_0}{T_0} = \lambda_0 \nu_0 \tag{2}$$

Equations (1) and (2), as usual, lead to

$$\lambda_0 = \frac{h}{m_0 c_0} \tag{3}$$

(*Wavelength of the electromagnetic radiation associated with the mass* m_0*, as originally assigned by de Broglie, to describe the periodic phenomenon inside the object in hand*).

The frequency ν_0 and the mass m_0 were transformed differently and the object is brought to a uniform translational motion (Lochak, 2005); relativistically, the frequency decreases while the mass increases, whereas according to Equation (1), mass and frequency should rather be altered in the same direction. This observation, as de Broglie mentions it, intrigued him, for a long time (de Broglie, 1925). He ended up with the introduction of a new wavelength λ_B describing the manifestation of the wave character of the object; thus for an object moving with the velocity v_0, de Broglie framed λ_B, in similarity with the RHS of Equation (3), as

$$\lambda_B = \frac{h}{m v_0} \tag{4}$$

(*de Broglie relationship written for the object in hand, brought to a translational motion*);

m is the relativistic mass of the moving object, that is,

$$m = \frac{m_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \tag{5}$$

where, λ_B is the de Broglie wavelength via Equations (3), (4) and (5). The relationship can be written in a straightforward way as:

$$\lambda_B = \lambda_0 \frac{c_0}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \quad , \; v_0 \neq 0 \tag{6-a}$$

(*de Broglie wavelength written along with Equation (1), in terms of* λ_0 *, the wavelength of the periodic phenomenon displayed by the object, at rest*)

between the two wavelengths λ_B and λ_0, in question.

Here, precaution is taken to write the de Broglie wavelength for a non-zero velocity, since ordinarily one would think that de Broglie relationship could only be defined, along with a motion. However, as will be discussed later, de Broglie relationship can be defined for a zero velocity, as well. In this case, λ_B becomes infinitely long. As we will soon give detail, this then constitutes the basis of an immediate action at a distance without any mass or energy exchange.

For this reason, in what follows, we will drop the restriction $v_0 \neq 0$. It is interesting to recall that our conjecture is well compatible with the quantum mechanical uncertainty principle, since for $v_0 = 0$, the momentum is zero too, which implies that, as strange as it may look at a first strike, the uncertainty about the location, is infinite. From our standpoint, this result alone can be considered as a clue for an "immediate action at a distance", which we propose to coin as "wave-like interaction".

Note that, λ_B decreases as v_0 increases, and for $v_0 = c_0$, λ_B becomes null (Equation (6-a)). This would mean that, the wave-like interaction ceases when the relative speed of the object, with respect to its surrounding, reaches the ceiling speed that is, the speed of light c_0. At any rate, the wave-like interaction does not involve any mass or energy exchange; any interaction involving mass or energy exchange, as usual, cannot occur with a speed above the speed of light. Note further that λ_B becomes λ_0, for $v_0 = c_0/\sqrt{2}$.

The contracted wavelength λ_1 along the direction of the translational motion (as assessed by the outside observer), is defined as usual as $\lambda_1 = \lambda_0 \sqrt{1 - v_0^2/c_0^2}$ from Equation (6-a). The original frequency ν_0, is concurrently reduced into $\nu_1 = \nu_0 \sqrt{1 - v_0^2/c_0^2}$ (pointing to the usual relativistic time dilation).

At the same time, based on Equation (1), as the object is brought to the uniform translational motion, in order to match the relativistic mass transformation, that is $m_0 / \sqrt{1 - v_0^2 / c_0^2}$); the definition of another frequency is evoked, that is, $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$.

Therefore, de Broglie had proven that, if the periodic phenomenon of frequency $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$ propagates with the velocity c_0^2 / v_0, while the periodic phenomenon of frequency $\nu_1 = \nu_0 \sqrt{1 - v_0^2 / c_0^2}$ propagates with the velocity v_0, then, the two waves, are constantly in harmony with each other (de Broglie, 1925) (Appendix A). It is in fact that the generation of these two different periodic phenomena through the uniform translational motion that confused de Broglie for a long time (according to his own statement), and led him to the formulation of Equation (4).

Thus, based on Equation (2) (that is, wave velocity = wavelength × frequency), via writing c_0^2 / v_0 instead of c_0, that is,

$$\frac{c_0^2}{v_0} = \lambda_B \nu = \lambda_B \frac{\nu_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \tag{6-b}$$

(*Set up for the de Broglie relationship, in terms of the inflated frequency* ν*, propagating with the velocity* c_0^2 / v_0*, and pointing to the wave character of the moving object*).

Louis de Boglie, could indeed very well end up with Equation (4) (or the same, Equation (6-a)). This whole idea seems to have been forgotten; de Broglie relationship takes place in all related textbooks, but how de Broglie had arrived at this idea, is practically nowhere around.

Hence, it is the velocity c_0^2 / v_0, which is associated with the de Broglie wavelength λ_B, if coupled with the frequency ν. Note that, this velocity would become $c_0^2 / v_0 \sqrt{1 - v_0^2 / c_0^2}$, still associated with the de Broglie wavelength, but then coupled with the original rest frequency ν_0 (Equation (6-b)). In any case, regardless of the fact that energy cannot be carried by a wave of such velocity, de Broglie wavelength should still be considered as a fundamental physical quantity. Electron diffraction observed already in 1927, thus long ago had been a concrete proof of this statement (Davisson and Germer, 1927). The perplexing results of neutron diffraction experiments, achieved later on, clearly display both the wave character and the particle character of diffracting neutrons (Zeilinger et al., 1988), conjointly (and not just one of these characters, at a time). In other words, neutrons lead to diffraction, although the beam is known to consist of scarce neutrons traveling in a ray, one after the other. And this is how presumably they should have gone through the diffraction slits; yet they still display a wave character. Neutrons are finally detected on the screen, as particles; hence here again, they display their particle character.

This outcome evidently seems to be profound, and should rather be considered along with the propagation velocity c_0^2 / v_0 of the "wave-like information" framed by de Broglie wavelength. He, himself, does not seem to have considered the case where $v_0 = 0$, for which the de Broglie wavelength becomes infinite. In this case, the velocity c_0^2 / v_0 too, becomes infinite.

This is nothing else, but (as will be elaborated on, throughout), the propagation velocity of an "information" associated with the wave of frequency ν_0, that is, the original frequency of the "periodic phenomenon of the internal dynamics of the object at rest", as introduced by de Broglie (Equation (1)). In other words, the internal beats of the objects, already at rest, seem to be "felt" instantaneously, everywhere in space, without any transfer of energy. This is what we mean by "propagation of information", thus without attributing any further meaning to the word, "information".

How can such propagation take place? We do not know. Perhaps it is a property of space that comes into play. Nevertheless we are, in effect, inclined to think that the internal beats of the object at rest, are somehow right away "sensed" everywhere in space, and conversely the object itself, very probably "captures", practically at once, all (atomistic, nuclear, gravitational, or other) internal beats, existing at all possible scales, in its surrounding, that is, the entire space. So in one way or the other, it must be a question of a "universal network of interaction" in between all existing substances, except transport of energy. In addition, the related "interaction speed" is a superluminal speed. Thus, though free of any energy exchange, the interaction in question occurs instantaneously between objects, at rest. Recent measurements seem to back up this arresting deduction (Kholmetskii et al., 2007; Salart et al., 2008).

On the other hand, there may be a clue for the mysterious "duality". The wave character, to us, is clearly due to the internal dynamics, the object in hand periodically delineates. This, most likely, causes a given disturbance in space, which is transmitted all over, and this at once, if the object is at rest. Thus, an object at rest, or in motion, in a way, continuously marks the signature of its being in relation to the universe (again, without however, any energy transfer). Such information would obviously go through any diffraction slits, erected on the way, and eventually, could well yield interference. The de Broglie wavelength becomes though finite, only if the object moves, making the corresponding diffraction measurement possible. Thus perhaps, it is not, the "diffracting single neutron" itself, which goes through both slits, but the "information about the periodic beatings of its internal whole activity" that it emanates. It may very well be this "piece information", which somehow digs out beforehand, the "space channels" that will host little later, moving neutrons, on the way. At any rate, the propagation velocity c_0^2 / v_0 associated with de Broglie wavelength (Equation (6-b)), is much greater than the object displacement speed. That is, in the first place, the "de Broglie wave", carrying the "wave-like information" was introduced, and the "particle" does not evidently move, at the same speed (no matter what, the wave of frequency $\nu_1 = \nu_0 \sqrt{1 - v_0^2 / c_0^2}$ corresponding to the dilated period of time displayed by the internal dynamics of the object, moves with the same speed as that of the object).

It is further interesting to note that the wave-like character would be destroyed through an energy and momentum exchange process. Indeed, it seems clear that a momentum shock received from the exterior perturbs the original beating system of the object in hand, thus most likely wiping out the anterior wave-like information together with the space channels that would have been originally framed. This may indeed constitute a clue to the classical wave-particle duality. Now, dividing the two sides of Equation (6a), by T_0 (Equation (2)), or the same, rearranging Equation (6b), yields

$$\frac{\lambda_B}{T_0} = \frac{\lambda_0}{T_0} \frac{c_0}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} = \frac{c_0^2}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \tag{7a}$$

We define the LHS as U_B:

$$U_B = \frac{\lambda_B}{T_0} \tag{7b}$$

Note that, both λ_B and T_0 are in fact, just like λ_0, defined in relation to the outside fixed observer (It is true that T_0 and λ_0 are transformed, with the motion, though the above definition stays perfectly valid.)

U_B, via Equation (2), becomes

$$U_B = \frac{c_0^2}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \tag{7c}$$

(*Velocity defined based on de Broglie relationship and the period of the periodic phenomenon of the object at rest*).

Equation (6b), or more specifically Equation (7a), or (7c) tells us that, through a motion such as the Bohr rotational motion of the electron around the proton in the hydrogen atom, each time (as assessed by the outside observer) the inherent periodic phenomenon of the electron assumed by de Broglie, and described (at rest) by λ_0 (Equations (3) beats; a "wave echo" beats, all the way through the stationary orbit of perimeter λ_B (Recall that λ_0 for the electron, based on Equation (3), turns out to be 2.45×10^{-10} cm. The de Broglie wavelength λ_B to be associated with the electron at the ground state of the hydrogen atom is about 3.33×10^{-8} cm. Thus, λ_B/λ_0 = 135. T_0, on the other hand, is 0.82×10^{-20} s).

Louis de Broglie could, in effect, show in his doctorate thesis that, in order to display a stationary motion on a given circular orbit, the wavelength λ_B to be associated with the electron motion (depicting a constant velocity) along Equation (4), should become equal to the orbit perimeter. This can be pictured somewhat as a snake with two heads, one material and the other one immaterial; each time the snake, together with the material head, makes a move forward, its immaterial head catches its queue. Recall on the other hand that, once a wave is confined, the quantization of it follows from classical physics (Appendix B). Hence, de Broglie, landed at Bohr's angular momentum quantization assumption (Bohr, 1913). It is a pity that this achievement is not cited in many textbooks.

The thing is that, the velocity U_B induced by the LHS of Equation (7) turns out to be greater than the speed of light; at the slowest, it is the speed of light; it can be infinite for a zero v_0. For this reason, de Broglie considered it very rational, as a velocity not carrying any energy, and we do well by sticking to this interpretation. We will further show that, the velocity U_B, owing to the law of conservation of energy, comes along with a rest mass ejected or received by the object, under a given interaction - where the static binding energy may vary, throughout the motion of concern – coupled with a factual speed V, and that, the ratio U_B/V is equal to c_0 / v_0.

Thus, we will see that not only can one derive Equation (6a), that is, the de Broglie wavelength, based merely on the law of conservation of energy, but also that the velocity depicted by the LHS of Equation (7c) turns out to be quite physical, if strikingly things are not considered particle-wise. But at the same time, t, wave-wise, does not involve any energy exchange with the exterior, as will be implied by the electric and gravitational interactions that we will deal with, in this work. Thus, in fact, de Broglie wavelength does not indeed carry any energy, but it certainly carries a given information about the interaction in question. Amazingly, our approach can be applied to gravitational and electric interactions, underlining the fact that both interactions work exactly the same way. In fact, our approach can even be applied to a non-linear motion, such as the motion of an object situated at the edge of a rotating disc.

It is astonishing that the "instant interaction", clearly evoked by different aspects of quantum mechanics (as will be discussed in this work), was not only thought to be against the special theory of relativity (STR) (and we will find out if it is not), but also its correlation with the simple framework of de Broglie relationship, which constitutes the basis of the wave theory of matter (or the same, quantum mechanics), following considerations (as will be elaborated on, herein), remaining within the mere frame of the STR (Appendix A), has been badly overlooked. For convenience herein, we will handle just the "electric interaction". In the subsequent part, we will handle the "gravitational interaction".

Chiefly, for this part of this work, we will deal with electrically bound particles, for a complete presentation; it would have been useful, to add to the dissertation, a discussion about how one should view the connection, between classically considered electric charges, and the bound charges, the way it will be handled in this study. As we will find out, the motion equation of bound electron that will be written soon, within the framework of the present approach, indeed, diverges no matter very little, but, conceptually speaking, still seriously, from the standard motion equation, classically coined for a bound electron. In any case, one will raise the question that, the approach we will present herein, somewhat negates the Maxwell equations. Then of course, one should be expected to write explicitly new field equations, which are compatible with the postulate, we will formulate below, in fact nothing else, but the relativistic law of energy conservation, though embodying the mass and energy equivalence of the STR. Or, even more fundamentally, one would have been expected to write a new expression for the Lagrangian density of the electromagnetic field, coming into play, and charged particles, and using the variation principle, to find new field equations, and a new force law, etc. This may even have been the topic of a separate article.

However, throughout the elapsed time since 2006, when the material we will present below, was essentially all ready, Kholmetskii et al, who framed the pure bound field theory and named it, in short, fortunately handled this problem, PBFT (Kholmetskii et al., 2010). They thus came out with new field equations and a new Lorentz force law, though in a very different means than that presented herein; nevertheless, their results back that of this present study, allowing us, now, in the first place, not to have to undertake herein the problems, just mentioned.

It is important to emphasize that the PBFT is not a controversial approach at all. In fact it has to do with the implementation of the law of momentum conservation for bound, thus non-radiating charges, based on quantum mechanics. The PBFT, briefly, stays within the framework of the standard approach, but gears it:

1. With respect to a full consistency, vis-a-vis the law of momentum conservation,
2. and for quantum mechanically bound, non-radiating electric charges. Its range of applicability, though, as mentioned, is quantum mechanically, "bound particles". PBFT, nevertheless, wipes out the long lasting quest of how to bridge the classical. Maxwell equations, and quantum mechanics, and formulate accordingly, a useful framework, essentially for non-radiating bound particles, thus filling the gap between the classical electrodynamics and the standard quantum mechanical approach.

In any case, our standpoint is that, any interaction depicts a rest mass change. Say in a free fall, in a gravitational medium, the object at hand, accelerates, due to the transformation of a minimal part of its rest mass into kinetic energy. Such an understanding brings up the question of, 'how can this take place?' The coupling of acceleration and rest mass change induces the thought that, in the example at hand, rest mass is ejected from the back of the object, to match the extra kinetic energy acquired by the object, in fact just like in a rocket. This picture, finally, as we will see, based on the relativistic energy conservation, together with the law of momentum conservation, leads to the de Broglie relationship, providing us with an invaluable bridge and symbiosis, between the STR and quantum mechanics.

Below, we consider two electrically interacting objects such as the proton and the electron. We will call the first charge, say, the proton, assumed to be at rest, the "source charge", and we will call, the other charge, say, the electron, either at rest or in motion, the "test charge".

PREVIOUS WORK: A NOVEL APPROACH TO THE EQUATION OF ELECTRIC MOTION, AND DISCUSSION ABOUT THE NOTION OF FIELD

In a previous work (Yarman, 2004), we presented a completely new approach to the derivation of the celestial equation of motion, which led to all crucial end results of the general theory of relativity (Yarman, 2006). Moreover we applied, the same approach to the atomic scale, which led to the derivation of a new relativistic quantum mechanical description well equivalent to that established by Dirac, if geared alike (Yarman, Rozanov, 2006).

It becomes interesting to note that the framework formulated so, happens to be equivalent to that of the PBFT formulated by Kholmetskii et al. (2010). In short, we had started with the following postulate, essentially, in perfect match with the "relativistic law of conservation of energy", thus embodying, in the broader sense, the concept of “mass”, though we will have to specify, accordingly, the notion of “field”, which we fundamentally reject, as will be elaborated on, below.

Postulate: *For any isolated system of particles, the rest mass of an object bound either gravitationally or electrically, amounts to less than its rest mass measured in empty space, the difference being, as much as a mass, equal to the binding energy vis-à-vis the field of concern - taking the speed of light unity*.

A mass deficiency, conversely, via quantum mechanics, yields the stretching of the size of the object in hand, as well as the weakening of its internal energy, on the basis of quantum mechanical theorems proven elsewhere (Yarman, 1999) still in full conformity with the STR. An easy way to grasp this, is to consider Equation (1). If the rest mass is decreased due to binding, so will be the frequency. Thus, one obtains, at once, the gravitational red shift. Our approach, via Equation (2), right away implies that, the size is accordingly stretched.

Such an occurrence can be experimentally checked, if say a muon is considered to be bound to a nucleus instead of the electron. The decay rate of the bound muon is indeed retarded as compared to the decay rate of a free muon (Lederman and Weinrich, 1956; Barrett et al., 1959; Herzog and Adler, 1980; Gilinsky and Mathews, 1960; Yovanovitch, 1960; Huff, 1961; Yarman, 2001, Yarman, 2005). Predictions, we made about this phenomenon, remain much better than any other available predictions. Recently, Kholmetskii et al. (2010) arrived at the same result via their pure bound field theory (PBFT).

BINDING ENERGY

Take for instance a piece of stone on Earth. We can assume that Earth is infinitely more massive than the stone. Then the static binding energy of the stone to Earth is the energy we have to furnish to the stone in order to bring it to infinity. The calculation of the gravitational static binding energy is peculiar though, since the rest mass of the stone is increased as much as the energy furnished to it, on the way. A detailed study of this problem is furnished in Yarman (2009), and is summarized in Appendix C. As the stone falls from a practically infinite distance onto Earth, the kinetic energy it would acquire at the moment it strikes Earth is equal to its static binding energy, as it comes to rest at the location it strikes Earth.

The binding energy of the electron to the proton in the hydrogen atom in its ground state (supposing first, for simplicity, that the proton is infinitely more massive as compared to the electron) is the energy one has to furnish to the electron in order to bring it from its ground state to infinity. The proton will remain at rest, throughout. Note that here we do not speak about just the static binding energy of the electron to the proton, but given that the electron is in motion around the nucleus, it is question of the overall binding energy. Yet still, one as to furnish that there is much energy in the electron, in order to bring it from its stationary orbit to infinity.

Hence, given that the proton is infinitely more massive than the electron, under the given circumstances, it is the electron which will pile up the amount of energy, in consideration (A detailed analysis on this will be presented below). In other words, the relativistic rest energy (the relativistic equivalent of the rest mass) of the free electron (thus at infinity) weighs more than the relativistic energy of the bound electron, and this as much as the electron is binding energy (in the hydrogen atom). On the other hand, when bound, and still under the given circumstances, this much energy ought to be retrieved from the relativistic rest energy of the electron. As trivial as this may sound to most readers, this is important to conservative reactions, directed to the present approach; therefore, we should insist a bit further on it. Thus, let us go back in more details.

Had we not assumed that the proton is infinitely more massive than the electron, then the overall binding energy is the energy in the non-relativistic case, and in CGS unit system $2\pi^2 e^4 \mu_0 / h^2$ (e being the charge of the electron or the proton, μ_0 the reduced mass of the proton and the electron, and h the Planck Constant). One has to furnish in order to dissociate the hydrogen atom into the electron and the proton (that is while almost all of this energy is to be delivered to the electron, a minimal part of it is to be delivered to the proton). This energy, in other words, is the “ionization energy” of the hydrogen atom, about 13.6 eV. After dissociation, the pair of electron and proton will weigh as much as compared to what the hydrogen atom weighs.

In reverse terms, the mass and energy equivalence driven by the STR, together with the law of energy conservation, requires that the total rest mass of the proton and the electron considered separately in a space free of field, shall weigh 13.6 eV less, when bound in a hydrogen atom. Thus again, the hydrogen atom weighs 13.6 eV less than the sum of the rest masses of the proton and the electron considered in a space free of field. Then, how can this mass deficiency be accounted for by the original mass of the proton and that of the electron? As explained, as a first approximation, it is the electron relativistic energy at rest, considered in free space, that will undergo, practically all of the mass deficiency in question.

When a massive charge +Ze and an electron are bound altogether at rest, the electron’s rest mass measured at infinity is decreased as much as the static binding energy coming into play

Here, we will work out the “static binding energy” of a nucleus charge, +Ze (composed of Z protons) and an electron of charge intensity, e, altogether at rest, and situated at a given distance from each other. Supposing, again for simplicity, that the nucleus in consideration is infinitely more massive than the electron, the binding energy of the nucleus and the electron, situated at rest, at a given distance from each other, is the energy one has to furnish to the electron in order to bring it from its bound location to infinity.

Had we not assumed that the nucleus is infinitely more massive than the electron, then the static binding energy in question is the energy one has to furnish in order to dissociate the pair of nucleus and electron bound at rest (situated at the given distance from each other), into the free nucleus (of charge +Ze) and the free electron (that is, while once more, almost all of this energy is to be delivered to the electron, a minimal part of it only is to be delivered to the nucleus). We call this energy static binding energy; it amounts (in CGS unit system) to Ze^2/R, where the nucleus and the electron are originally at a distance R from each other (electric charges, unlike the gravitational charges, are not affected by the energy piled up, as they are carried away from each other.)

Just like in the case of the hydrogen atom, the nucleus of charge +Ze and the electron situated at a distance R from each other (altogether at rest, owing to the mass and energy equivalence drawn by the STR, along with the law of conservation of energy) shall weigh $E_{BZe^2} = Ze^2/R$ ergs less than the sum of the rest masses, of the free nucleus and the free electron (see the footnote, at the bottom of the previous page and note that the electric charges are not affected throughout, only the rest masses are).

Let us stress that, the energy in question, ought to be retrieved practically from the electron, alone. The reason is simple. Let us assume that the electron falls from a sufficiently large distance onto the nucleus in consideration, and is originally at rest. If this nucleus is infinitely more massive than the electron, then the law of linear momentum conservation requires that the nucleus stays practically in place, while the electron keeps on falling. An outsider can intervene somehow (this point will be elaborated on below), and stop the electron, at a given distance to the nucleus. Then, the only energy he would tap would be the kinetic energy, while the electron would have piled up on the way. Thus, the system originally composed of the nucleus of charge +Ze and the electron, when bound at the given distance R, from each other (and originally at rest altogether), will lose the kinetic energy the electron would have acquired on the way. That is the energy $E_{BZe^2} = Ze^2/R$ ergs, and since the nucleus would virtually not move throughout, this energy ought to be extracted from the electron rest mass alone.

Now, let us discuss what we mean by "an electron and a nucleus held at rest, at a given distance from each other" and how one can achieve such a pair. Here, we give an example on how this can take place. Just consider a dipole, such as a water molecule, in which, the oxygen atom (O) attracts, respectively the two binding electrons of the hydrogen (H) atoms, delineating an angle HOH of about 105º. This leads to positively charged hydrogen atoms and negatively charged oxygen atom. Thus, water molecule can indeed be described by a dipole, made of -2e situated nearby the oxygen atom, and +2e situated on the median of the triangle HOH, in between the hydrogen atoms (e is again, the electron charge intensity). In this case, r_{Dipole} is the distance between the two representative charges +2e and -2e. This then corresponds to a situation where the charges +2e and -2e are held still at a distance r_{Dipole} from each other. Thus our answer to the above question is affirmative, that is one can indeed well conceive a dipole composed of +Ze and –e, at a given distance from each other, and at rest, since this is conceptually not any different than the dipole (composed of +2e and -2e) delineated by a water molecule.

The binding energy of the water molecule (assumed at rest), or the same with that of the dipole made of +2e and -2e, is the energy one has to furnish to it, in order to carry these two charges, far away from each other. In other words, this is the energy one has to furnish to the water molecule, in order to dissociate it into its oxygen atom and its two hydrogen atoms. This energy, which we call E_{BH_2O}, is about 9.5 eV. Thus neglecting the vibrational energy along the bonds of water molecule, much of energy should be extracted from the sum of the rest masses, of respectively the hydrogen atoms, and the oxygen atom, weighed separately from each other, to get the rest mass of water molecule.

Noting that the oxygen atom is much more massive than the hydrogen atom, roughly speaking, 9.5 eV (more specifically, the mass equivalent of this much energy) must be extracted, from the hydrogen atoms. Hence, the bound hydrogen atoms, in water molecule, shall each weigh less than the mass $m_{H\infty}$ of the free hydrogen atom. How much less? Each, about half of the dissociation energy E_{BH_2O} of the molecule. (Note that, for quantities defined at infinity, normally we use the subscript "0"; but because this symbol can be confused with "O" representing the oxygen atom in the oxygen molecule, here, we prefer to use the symbol "∞", instead). Thus, the mass of the hydrogen atom bound to O, in a H_2O molecule, shall nearly weigh,

$$m_{H\infty} - E_{BH_2O} / (2c_0^2)$$

This is yet an approximation, since the mass ratio of the hydrogen atom to the oxygen atom is about 1/16. We can do much better than that. For example, the mass ratio of the hydrogen atom to the tellurium (Te) atom is about 1/128. Thus, when two H atoms are bound to a Te atom, in a H_2Te molecule, bearing the dissociation energy E_{BH_2Te}, one can with confidence affirm (though still overlooking the vibration energy of the molecule along the bonds) that, each of the H atoms will, to an acceptable precision, weigh $E_{BH_2Te}/2$ eV less, as compared to the H atom weighed at infinity; while the Te atom (owing to the law of linear momentum conservation, as explained above), practically remains untouched. Thus, the mass of the hydrogen atom bound to Te, in a H_2Te molecule, shall practically weigh,

$$m_{H\infty} - E_{BH_2Te}/(2c_0^2).$$

Furthermore, suppose that the molecule H_2Te undertakes a routine rotational motion. Since Te is much too heavy as compared to H, the rotational motion shall take place around Te atom. Let V_{Rot} be the tangential velocity of the H atoms, rotating around Te. The overall relativistic energy of such an H atom thus becomes

$$[m_{H\infty} - E_{BH_2Te}/(2c_0^2)]/\sqrt{1-V_{Rot}^2/c_0^2}$$

Thus, following our discussion we conclude that when a charge +Ze and an electron are bound altogether at rest (still supposing that Ze is very much more massive that the electron), at a distance R from each other, the electron mass m_0 measured at infinity, is decreased as much as the static binding energy $E_{BZe^2} = Ze^2/R$ ergs, coming into play, to become $m_0 - Ze^2/(Rc_0^2)$; the mass of the heavy nucleus (owing to the law of linear momentum conservation) is not virtually touched.

The energy $E_{BZe^2} = Ze^2/R$ is nothing else, but the magnitude of the classical potential energy. However, we avoid this denomination, for reasons that will become clear soon. That is within the peculiarities we have introduced, as we will see, chiefly the total energy cannot be set equal to the sum of kinetic energy and potential energy, were they classically defined. Thereby the next important question to be answered is the following: What is the overall relativistic energy of the electron quasistatically brought near the nucleus of charge Ze, if it is further set to a rotational motion of velocity v_0 around Ze? Is it

$$(m_0c_0^2 - Ze^2/R)/\sqrt{1-v_0^2/c_0^2}, \text{ or } m_0c^2/\sqrt{1-v_0^2/c_0^2} - Ze^2/R$$

Based on the "*classical potential energy concept*", all textbooks we know of, would answer the second question. However, our approach leads to the first as we will discuss in detail below. Let us think of the total relativistic energy of a H atom in the molecule of H_2Te, set to a rotational motion around Te. As discussed above, it is

$$(m_{H\infty}c_0^2 - E_{BH_2Te}/2)/\sqrt{1-V_{Rot}^2/c_0^2},$$

and not

$$m_{H\infty}c_0^2/\sqrt{1-V_{Rot}^2/c_0^2} - E_{BH_2Te}/2.$$

If so then the overall relativistic energy of the electron quasi-statically brought near the nucleus of charge Ze, if it is further set to a rotational motion of velocity v_0 around this nucleus, must be

$$(m_0c_0^2 - Ze^2/R)/\sqrt{1-v_0^2/c_0^2},$$

and not

$$m_0c_0^2/\sqrt{1-v_0^2/c_0^2} - Ze^2/R.$$

In any case, henceforth, we will solely operate on the concept of "relativistic energy" (and nothing else), which we can clearly define, and work out, with regards to a given particle, either at rest (based on the mass and energy equivalence drawn by the STR), or in motion.

Let us recall that, in order to calculate the binding energy coming into play, for electrically bound particles, we make use of the Coulomb force, yet with the restriction that, it can only be considered for static charges. As we will summarize quickly, Coulomb force well works for a pair of static charges. Beyond this, a priori, we have strictly no idea, whether it will still hold or not, were one of the charges is in motion with regards to the other, and as we will soon derive that it does not.

In effect, Coulomb force reigning between two static charges is a requirement imposed by the STR. It is that we were able to derive the $1/(\text{distance})^2$ dependency of the Coulomb force between two static charges, just based on the STR (Yarman, 2008). The underlying reason is merely that, the quantity (force) x (mass) x(distance) 3 (bearing the dimension of the square of the Planck constant) is Lorentz invariant. Thus, suppose we take a dipole into a uniform translational motion. Consider for simplicity, the case where the uniform translational motion takes place along the direction of the line joining the two poles. Let the mass in question be the mass of the dipole. Then, the quantity (mass) x (distance) is Lorentz invariant; for this case, accordingly [that is in view of the Lorentz invariance of the quantity (force) x (mass) x $(\text{distance})^3$], the quantity (force) x $(\text{distance})^2$

is Lorentz invariant. The electric charges on the other hand, are following observations, Lorentz invariant. (Otherwise, the Galilean principle of relativity would be broken, and electric charges, just like the speed of light, must remain Lorentz invariant). Thereby the force reigning between the two poles, expressed as the [product of static electric charges coming into play]/[distance]n, can only allow the exponent n=2. Therefore, the STR, exclusively implies the structure of the classic Coulomb force reigning between two static charges. Hence, the framework we set up herein, fundamentally lies on, the STR.

Note that below, just like we did above, we consider merely the closed system made of two charges (of opposite signs). This means we will continue to tackle, all the way through, with these two charges, somehow engaged with each other everlastingly. Thus, we exclude the possibility of having to deal with one charge only up to a given point of a possible process, suddenly allowing the popping out of the second charge, right next to the first one (which can, for instance, be achieved via charging at a given moment the plates of a capacitor, while the first charge is lying in the inside of it). Processes taking place in accelerators also fall in this category, which is that of an electric charge experiencing in its frame of reference the creation of an electric field on its way. All this lies outside of the scope of the present dissertation.

The equation of motion

We define m_0 as the mass of the electron, at infinity. When this is bound at rest, to a nucleus of charge +Ze, assumed for simplicity infinitely more massive as compared to m_0, this latter mass, following the discussion we have just presented, will be diminished as much as the static binding energy coming into play, through the binding process, to become $m(r_0)$, at a distance r_0 to the nucleus. So that,

$$m(r_0) = m_0\,\kappa(r_0) \tag{8}$$

(*mass of the bound electron, at rest*)

where $\kappa(r_0)$ is

$$\kappa(r_0) = 1 - \frac{Ze^2}{r_0\, m_0 c_0^2} \tag{9}$$

As is well known, and as will be specified right below, Ze2/r0 is the static binding energy of the electron, at the location r_0. In other words, this much of energy is required to bring the electron from its location, at the stage in consideration, thus from rest, on the orbit, to infinity (assuming that the effort in question leaves untouched the proton, having noted that it is practically infinitely more massive than the electron). Equation (9) thereby expresses nothing else, but a decrease in the rest mass of the electron. Indeed the rest mass of the electron at the location r_0 becomes

$$m(r_0) = m_0\kappa(r_0) = m_0 - Ze^2/(r_0\, c_0^2)$$

In the classical field theory, this latter equation, when multiplied by c_0^2, that is, in the form of $m(r_0)\,c_0^2 = m_0 c_0^2 \kappa(r_0) = m_0 c_0^2 - Ze^2/r_0$, tells us that at this location, we still have at hand, the original rest mass m0 [and, not m(r0)], while we dumped from the field, an energy of Ze^2/r_0. Whereas again, in our approach, the change does not take place in the surrounding at all, but inside the electron. We are well aware that our approach is to alter the Lorentz force law. For one thing, as we will have to face below, we will have to alter the classical electric force term Ze^2/r_0^2 written for the source charge Ze at rest, and the test charge e, regardless whether this latter charge is at rest or in motion.

Thus obviously, we must be expected to carry out the necessary discussion about the change, we are about to bring to the classical electrodynamics. Furthermore, we well realize that such a discussion must be coupled with the introduction of an appropriate magnetic force term, next to the electric force term, we are to alter. Fortunately as mentioned, Kholmetskii et al. filled this gap in their work on the pure bound field theory (Kholmetskii et al., 2010), which allows us to skip, over here the related efforts. We would like to stress indeed that, though through very different paths, Kholmetskii et al. land at exactly the same results, as those outlined by the present approach, as far as the alteration of the classical quantum electrodynamics is concerned.

Note that the distance between the electron and the nucleus, when measured by an observer bound to the electron, and when measured by the distant observer, does not point to the same quantity, but in what follows we will overlook this detail. In other words, the rest mass decreases (via Equations (1) and (3)), as we will elaborate on, a little, below, alters the metric. Fortunately this may not have to be detailed for the derivation we will now, offer. Nevertheless, it should be remembered that, in order to successfully cope with the experimental results, we should consider working in the proper frame of reference of the electron, or if the electron moves on a circle, then in the frame of an observer situated at rest in a given location on the orbit of concern. We will call the latter frame, the local frame of reference. Note further that, Equation (9) seems to allow $\kappa(r_0) = 0$, also $\kappa(r_0) < 0$.

It is that, as the electron is quasistatically brought closer and closer to the nucleus, its rest mass decreases more and more, until it comes to vanish at r_0, which, for Z=1, turns out to be the classical electron radius, that is, $e^2/(m_0c_0^2)$ = 2.8 x 10^{-13} cm.

Before proceeding further, let us consider the possible reasons why the electron, or any other similar charge, cannot fall down any further beyond a specific distance. Depending on the situation in question, one can find different explanations. For instance, the dipole representing water molecule cannot go narrower than the distance it delineates, because, the electronic structure of the atoms does not allow it; at shorter distances the atoms in consideration, would repel each other more and more.

On the other hand, if one considers an electron falling onto a nucleus, it should be remembered that the electric charge of the nucleus taking place in the expression of the force exerted by the nucleus onto the electron, decreases gradually, as the electron goes beyond the nucleus wall, assuming that it can do so, without getting absorbed, etc. In this latter case, obviously, Equation (9) should be reformulated. As we will show below, according to our approach, the electron cannot fall down into, say a proton, beyond a range making the RHS of Eq.(9) vanish, because there would be no mass left to fuel the endeavor.

Furthermore, one should recall that Equation (8), along with Equation (9), is necessary imposed by the law of energy conservation, in the broader sense of the concept of energy, embodying the mass and energy equivalence brought by the STR. If one brings quasistatically the electron up to an obstacle, situated at a given distance R from the heavy nucleus in consideration, one must work against the attraction force. To return the electron back to infinity, one must furnish to the electron, the amount of energy equal to the work one had to spend in order to get it to R. Thus, when the electron is brought back to infinity, it must be delivered to the electron, the energy $\int_R^\infty (Ze^2/r^2)\,dr = Ze^2/R$.

This then clearly means that the electron, when separated from the nucleus, relativistically speaking, weighs more, and this as much as Ze^2/R. In other words (because, as discussed, the nucleus is supposed to be much too heavy as compared to the electron, and accordingly, owing to the law of linear momentum conservation, it will stay practically in place, through, back or forth), when statically bound, the electron will experience a mass deficit, and this is, as much as Ze^2/R. Otherwise, the law of energy conservation would be violated. Thence Equation (8), along with Equation (9), is as a must imposed by the law of energy conservation, in the broader sense of the concept of energy, embodying the mass and energy equivalence drawn by the STR. Now suppose that the electron is engaged in a given motion around the nucleus; the motion in question can be conceived as, made of two steps (Yarman, 2004):

1. Bring the electron quasistatically, from infinity to a given location r, on its orbit, but keep it still at rest.
2. Deliver to the electron at the given location, its motion on the given orbit.

The first step yields a decrease in the mass of m_0 as delineated by Equation (8) (The fact that the electron is brought to the location in consideration, quasistatically, provides us with the facility of not having to deal with the radiation problem that would arise otherwise). The second step in consideration yields the Lorentz dilation of the rest mass $m(r_0)$, at the location r_0, so that the overall relativistic energy $m_\gamma(r_0)c_0^2$, or the same, the total relativistic energy of the electron on the given orbit, becomes

$$m_\gamma(r_0)c_0^2 = m(r_0)c_0^2\frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_0c_0^2\frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \quad (10)$$

(*Overall relativistic energy of the bound electron on the given orbit*)

Where, v_0 is the magnitude of the local tangential velocity of the electron at r_0.

The velocity v0 is not to be confused with an eventual velocity V, the atom would delineate, when brought to a uniform translational motion. If so then, the overall relativistic mass $m_\gamma(r_0)$ would evidently become

$$m_\gamma(r_0)/\sqrt{1-V^2/c_0^2}$$

The total energy of the electron in orbit [that is, $m_\gamma(r_0)c_0^2$] must remain constant, so that for the motion of the object in a given orbit, one finally has

$$m_\gamma(r_0)c_0^2 = m_0c_0^2\frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = Constant \quad . \quad (11a)$$

(*total energy written by the author, for the electron in motion around the nucleus*).

This relationship is in fact the integral form of our general equation of motion, given below.

One can notice that Equation (11a) is different from what one would write classically, that is,

$$m_\gamma(r_0)c_0^2 = \frac{m_0 c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} - \frac{Ze^2}{r_0} = \textit{Constant:}\text{ Incorrect} \qquad (11b)$$

(total energy one would write classically, for the electron in motion around the nucleus). The difference in the relation given in Equation (11a) is base on our approach and that given in Equation (11b) stems from the fact that Equation (11b) delineates a violation of conservation of energy. What is incorrect in Equation (11b)? Alternatively, what, according to our approach, exactly does violate the law of energy conservation, in this equation?

It is that, if we follow our scheme made of two consecutive steps we framed right above, in order to bring the electron into its orbital motion around the proton; once we arrived at r_0, quasistatically from infinity, what is brought into the orbital motion, is not the rest mass m_0 of the electron at infinity, but the rest mass

$$m(r_0) = m_0\kappa(r_0) = m_0 - Ze^2/(r_0 c_0^2)$$

(Equations (8) and (9)); this is the rest mass of the electron at infinity, decreased as much as the static binding energy coming into play, at the location r0 on the orbit. Again, in our approach, the change, about the bound particle, does not take place outside of the particle, in the so-called "field", that surrounds it, but on the contrary in the internal dynamics of the particle, at hand. Thence, what is to be accounted for, along with the Lorentz constant coming to be associated with the particle now brought into motion, at the given location (is not the rest mass of the electron m0 at infinity) is to be the decreased rest mass $m(r_0) = m_0\kappa(r_0) = m_0 - Ze^2/(r_0 c_0^2)$ (in one single piece). And this is exactly what we wrote along with Equation (11-a).

Having failed to do this, Eq.(11-b) remains, unfortunately incorrect. It it then fails to wholly satisfy the law of relativistic energy conservation, as explained, although for almost all of us, it might still bear a certain consistency. Such an impression, most likely stems from the fact that, we are somewhat conditioned to believe that Coulomb Force Ze2/r0 reigning between the charges Ze and e, is valid, whether e is at rest or in motion. In effect, one can show that, Equation (11-b) is valid, if Coulomb force reigning between the charges Ze and e, holds in the case where Ze is at rest, but e may be in motion.

With the way we have set up our own approach, we are confident that Coulomb's force Ze2/r0 holds for both Ze and e, strictly at rest, and this, is a requirement of the STR, as we have discussed above. Based on such an occurrence, we do not a priori know that, it still holds, if Ze is at rest, but e is moving, and once Equation (11-a) is set, we will see that Coulomb force does not remain the same at all, if e is in motion with respect to Ze.

The confidence most of us have, with respect to Equation (11-b), stems further from the belief that, the energy decrease arising within the framework of Equations (8) and (9), that is, that delineated by the equation $m(r_0) = m_0\kappa(r_0) = m_0 - Ze^2/(r_0 c_0^2)$, through binding, is due to a loss of an amount of energy as much as Ze^2/r_0 in the surrounding field. At that, it is still the rest mass m0 of the electron, at infinity, which is brought in motion at r0, in the given field. Whereas as specified in our approach above that the change does not take place in the surrounding, but inside the electron itself, we will further elaborate on this below. If indeed so, the restitution of the mistake in question evidently, is to alter so very many related derivations. This may be unfortunate, but that is, the way it is. To be precise, Equation (11-a) is written, with respect to an observer at rest, situated on a given location, say where we delivered its motion to the electron on the orbit.

Conversely, Equation (11b) assumes that the total relativistic energy is composed as follows: the rest relativistic energy (that is, the rest mass multiplied by c_0^2) + the relativisitic kinetic energy + the potential energy. In addition, what is incorrect with this? To compose the total relativistic energy, that way, is what we all learned already in high school, and that is what most of us keep on teaching.

In our approach, on the other hand, we do not make use of, or refer to the concept of potential energy. To us, it is a question of the static binding energy, and this much of energy is damped from the rest energy (or, taking c0 unity, the same, rest mass of the electron), of the electron, when this is brought quasistatically at a distance r0 to the nucleus. In other words, the rest energy of the statically bound electron, is decreased as much as the static binding energy (cf. the first step we have considered, in writing Equation (10)), and it is the remaining rest energy of the electron, which is dilated by the Lorentz factor, while we deliver to it its motion on the orbit (cf. the second step we have considered in writing Equation (10)). The result we arrived at, is not any different than that we established in regards to the total relativistic energy of a H atom in the molecule of H2Te, set to a rotational motion around Te. Thus, this energy is $(m_{0H} c_0^2 - E_{BH_2Te}/2)/\sqrt{1 - V_{Rot}^2/c_0^2}$, and not $m_{0H} c_0^2/\sqrt{1 - V_{Rot}^2/c_0^2} - E_{BH_2Te}/2$!

Another reason why Equation (11b) differs from Equation (11a) is that the former assumes that Coulomb force holds between a static source charge (the nucleus) and a moving test charge (the electron). However, our approach considers that Coulomb's Law reigns in between only two static charges, which in fact, as we have shown, turns to be a requirement imposed by the STR.

Hence, Coulomb's Law does not hold in between the proton (assumed at rest, throughout) and the moving electron (the way it holds, between "the proton and the electron at rest"). In short, one cannot impose Coulomb force between a static charge and a moving charge. It is to be noted that, our conclusion as to "Coulomb's Law reigns in between, only two static charges" does not, in any way, tell us how the law of force would look, if one of the charges moves.

This is the crucial point. In other words, as will be specified below, Equation (11b) would be valid, if Coulomb's law were valid between the proton and the moving electron, the way it is written for the proton and the electron, both at rest. However, it is not, and Equation (11b) is only an approximation. This disclosure too, is to alter very many related derivations (Kholmetskii et al., 2010), but that is the way it looks. One way or the other, it is that, the mass of the bound electron, is not the same as the mass of the free electron, and as trivial as it may look at this stage, this is what essentially had been overlooked throughout the past century. Thus, although Equation (11a) looks straightforward, to our recollection, it happens to be new. The way we write it induces the need of elaborating on the concept of "field", which we avoid; this quest will be detail below.

We show elsewhere that Equation (11a) furthermore constitutes the basis of a relativistic quantum mechanical description, well equivalent to that of Dirac, if geared alike, yet established in an incomparably easier way (Yarman and Rozanov, 2006). We have to stress that the approach in question is in full harmony with all the existing quantum electrodynamical data; in fact it sheds light on the small but still measurable discrepancies between measurement and the standard approach (Kholmetskii et al., 2010).

The differentiation of Equation (11-a) leads to:

$$-\frac{Ze^2}{m_0 r_0^2}\,\frac{1-\dfrac{v_0^2}{c_0^2}}{1-\dfrac{Ze^2}{r_0 m_0 c_0^2}} = v_0\frac{dv_0}{dr_0} \tag{12a}$$

(*differential form of Equation (11a), equivalent to the equation of motion*)

One can transform Equation (12a) into a vector equation; the RHS is accordingly transformed into the acceleration (vector) of the electron on the orbit. Thus, recalling that the LHS of Equation (10) that is, $m_\gamma(r_0)c_0^2$, is constant, one can write:

$$-\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\,\frac{\underline{r}_0}{r_0} = m_\gamma(r_0)\frac{d\underline{v}_0(t_0)}{dt_0} \tag{12b}$$

(*vectorial equation written based on Equation (12a), or the same with equation of motion written by the author via the law of conservation energy, extended to cover the relativistic mass and– energy equivalence*).

Here, $\underline{r}_0$ is the radial vector of magnitude r_0, directed outward, and $\underline{v}_0$ is the "velocity vector" of the electron, at time t_0; note that $d\underline{v}_0$ and $\underline{r}_0$ lie in opposite directions; it is important to recall that this equation is written in the local frame of reference. Note that the above equation is in full conformity with what is furnished by the PBFT developed by Kholmetsii et al, if the coordinates come into play, and are transformed into those measured in the frame of the distant observer. For a small Z, a small v0, the orbit would be as customary elliptical; otherwise it is open; in other words, the perihelion of it shall be process throughout the motion. Equation (12-b) has anyway the same relationship as that proposed by Bohr, except that the Coulomb force intensity is now decreased by the factor $\sqrt{1-v_0^2/c_0^2}$, similar to what is empirically, but approximately proposed by Weber, by the end of nineteen century (Weber, 1848; Weber, 1892-1894; Phipps, 1990, 1990). Along this line, one can consult relatively recent articles (Wesley, 1992; Wesley, 1992). Note that, a realistic interpretation of Equation (12b) should consist of considering the factor $\sqrt{1-v_0^2/c_0^2}$, at the denominator of the RHS of this equation. Then, it is as if the classical force now causes a greater equivalent momentum change rate. What we do is in no way in conflict with quantum. Quite on the contrary, through our approach soon we will land at the de Broglie relationship, which is the basis of quantum mechanics. At this stage, it seems useful to draw the following table displaying the differences between our approach and the standard approach.

Discussion concerning the total dynamic energy we proposed for the electron

Since Equations (12b) and (11a) significantly differ from those derived from the conventional approach, it is worth noting and discussing some important issues related with the differences.

One apparent difference is the kinetic energy acquired by an electron freely falling onto a proton at rest. Considering the electron of mass m_0 measured in empty space, let v_1 be the electron velocity at r_1, and v_2 the electron velocity at r_2 (the proton being taken at the origin of our coordinate system). Based on Equation (11-a) one can write

$$m_0 c_0^2 \frac{1-\frac{Ze^2}{r_1 m_0 c_0^2}}{\sqrt{1-\frac{v_1^2}{c_0^2}}} = m_0 c_0^2 \frac{1-\frac{Ze^2}{r_2 m_0 c_0^2}}{\sqrt{1-\frac{v_2^2}{c_0^2}}} = m_0 c_0^2 \qquad (12c)$$

(*free fall of the electron described within the frame of our model*) which anyway makes that for a free fall

$$\frac{1-\frac{Ze^2}{r_2 m_0 c_0^2}}{\sqrt{1-\frac{v_2^2}{c_0^2}}} = 1 \qquad (12d)$$

(*basic relationship about the free fall of the electron which started with zero velocity, at a practically infinite distance from the proton, within the frame of our model*). For small velocities this yields

$$m_0\left(\frac{v_2^2}{2}-\frac{v_1^2}{2}\right) = \frac{e^2}{r_2}-\frac{e^2}{r_1}+\frac{e^2}{r_2}\frac{v_2^2}{c_0^2}-\frac{e^2}{r_1}\frac{v_1^2}{c_0^2} \qquad (12e)$$

(*difference between kinetic energies of the electron at two different altitudes, based on the present approach*).

Whereas according to the classical relation given in Equation (11b), one finds

$$m_0\left(\frac{v_2^2}{2}-\frac{v_1^2}{2}\right) = \frac{e^2}{r_2}-\frac{e^2}{r_1}. \qquad (12f)$$

(*classical difference between kinetic energies of the electron at two different altitudes, based on the present approach*).

It is clear that the quantity given in Equation (12e) is larger than that given in Equation (12f). We attribute this difference to the decrease in the rest mass of the electron, on the way, while the overall relativistic energy stays constant (cf. Equation (12d)), since the kinetic energy acquired by the electron is (as we shall elaborate below) fueled by the mass deficiency the electron undergoes on the way. Otherwise, it is not a question of an unnatural source of an extra energy that we invent. We simply follow the law of conservation of energy (extended to embody the mass and energy equivalence drawn by the STR). And, it should be now become clear that, the law of conservation of energy is broken if one states (Equation (12f)). (Difference of kinetic energies) = (Difference of potential energies). (12g) (classical statement breaking the law of conservation of energy). Such a statement badly yields, as small as it may be, but still, the wiping out from nature, of an amount of energy as much as the difference between the right hand sides of Equations (12e) and (12f).

The correct statement instead, thus, is (cf. Equation (12d)), (Difference of kinetic energies) = (Difference of rest masses). (12 h) (*correct statement obtained within the frame of our approach*). Note further that a more precise calculation would necessitate, taking into account the change of metric, accompanying the rest mass decrease. However, this is beyond the scope of this paper.

DISCUSSION ON THE CONCEPT OF FIELD

The set up of Equations (10) and (11a) is clearly not achieved by just customarily used concepts, and mainly, the regular concept of field, since the classical field is the same whether the test charge is at rest, or in motion. The concept of field, to us, is nothing else, but a mathematical convenience. Indeed, the intensity of it cannot be measured, unless one makes use of a test charge. In other words, the field associated by a source charge at a given location is a definition drawn, based on the force exerted by this charge on a unit charge, situated at the given location; thus the customary field intensity is defined as force strength divided by intensity of the test charge, coming into play. Consequently, it is not the field intensity that one would measure, but the strength of the force developed by the source charge, on a test charge. For us, what is essential is Coulomb's force reigning in between two static charges. This is essential in two ways:

1. The electric charges are Lorentz invariant.
2. Hence the STR imposes the 1/distance2 dependency of the Coulomb force between two static charges.

Thus, Coulomb's force, as it is, reigning between only two static electric charges, is fundamentally framed by the STR. What is belief so far is that Coulomb's force holds, if the source charge is static, regardless whether the test charge is at rest or in motion; this requires the validity of Equation (11b).

However, we have shown that Equation (11b) is not correct; if the test charge is in motion, then Coulomb's force is decreased (as referred to an observer at rest on the orbit), by the factor $\sqrt{1-v_0^2/c_0^2}$ (cf. Table 1). The classical field concept can still be used by taking into account this latter correction to it. This correction becomes important only if the test charge moves at high speeds. Nonetheless, it comes straight from the fact that the rest mass of a bound charge, such as an electron, is not the same mass as that the electron delineates in empty space. This occurrence drives us to consider basically the electron (contrary to what has been done so

Table 1. Differences between the "standard approach" and "present approach", based on the Electron bound to the proton (assumed at rest throughout).

	Standard approach	Present approach
Force between the proton and the electron, altogether at rest	$\frac{Ze^2}{r_0^2}$	The same.
Total energy of the tatically bound electron	$m(r_0)c_0^2 = m_0c_0^2 - \frac{Ze^2}{r_0}$	The same; but classically the mass of the bound electron is not considered to be altered; it is the overall field energy which is believed to decrease, as much as the potential energy, coming into play.
Total dynamic energy of the electron	Rest energy + potential energy + relativistic kinetic energy	The concept of potential energy, as considered classically, is misleading. Thus, it is not considered in our approach as a separate entity. Our approach instead, considers the broad definition of relativistic energy
Mathematical expression of the total dynamic energy of the electron	$m_\gamma(r_0)c_0^2 = \frac{m_0c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} - \frac{Ze^2}{r_0}$	$m_\gamma(r_0)c_0^2 = m_0c_0^2\frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}$
Force between the proton and the moving electron	$\frac{Ze^2}{r_0^2}$	$\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}$ (*expressed in the local frame of reference*)

far) not in an extreme simplistic way. That is, we sympathize by the fact that, the electron is generally, considered as a "point-like particle". It must be obvious though, as tiny as it may be, the electron cannot be reduced to a "point", given that a point cannot be a "material being". Thus, it is pointless to consider the electron as a point-like particle. The electron must embody an "internal dynamics", just like any other particle, in fact in conformity with Equation (1). Perhaps, its "mass" is simply the "internal energy" of the "electric property", which we call "electric charge". This internal energy, is thus to be associated with (however, it may be) the internal dynamics delineated by the electric charge. When the electron is bound, say, to a proton, its internal dynamics is then (as a requirement of the law of energy conservation) slowed down, as much as the binding energy coming into play, assuming for convenience that the proton (being much more massive than the electron) is not affected by the process of binding. It is further slowed down, this time relativistically, by the usual Lorentz coefficient, due to its motion around the nucleus.

Let us emphasize that, the slowing down of the internal dynamics of the electron, due to the static binding, could not yet be checked out. But if it is a question of a bound muon instead of a bound electron, the present approach induces that the half life of the bound muon, reflecting its internal dynamics, must be retarded due to just the static binding, which is next to the usual relativistic retardation by the Lorentz coefficient. And this is what is measured indeed, and very satisfactorily intercepted by our approach (Yarman, 2005; Kholmetskii et al., 2010). Our claim regarding the weakening of the internal dynamics of the bound electron can, as elaborated above anyway be justified right away through the dissociation process.

Briefly speaking, suppose we propose to bring the bound electron back to infinity. Accordingly, we have to furnish to it, an amount of energy equal to its binding energy (still supposing that moving away the electron would not disturb the proton). The two particles, forming a closed system; furnishing energy to the electron, owing to the law of energy conservation, will increase the rest relativistic energy, thus the rest mass of the latter. This is why we are inclined to talk about the "internal dynamics" of an electron, in fact just like anything else. If the rest relativistic energy of it increases, to us, it is that its internal dynamics somehow gains an extra energy, and speeds up accordingly.

We will mention it once again, when entirely detached from the interaction domain, with the proton; the electron's rest mass would get increased as much as the energy we would have furnished to it, that is, by an amount equal to its original binding energy. Hence, the free electron is not anymore the previous bound electron, or vice versa, the bound electron is not anymore the same as the free electron. It is indeed hard to accept that it would be, given that one cannot make an omelet, and keep the eggs as they are, prior to cooking.

As a consequence of our approach, not only the expression of Coulomb's force, exerted by a source charge at rest, on a moving charged is altered, but also the expression of Lorentz Force too, exerted by a moving source charge on a moving test charge, is as well altered. We are going to leave this interesting (but, based on our approach, rather straightforward) problem, for a subsequent paper. Nonetheless, we would like to mention that, fortunately for us, we are not the first to land at such an awkward result. Weber, more than a century and half ago, arrived at a similar result, though through barely empirical means, while trying to derive from a single formula, Coulomb force and ampere force, reigning between current elements, in perfect compatibility with the law of energy conservation. Thus he introduced, the Weber Potential which is the usual Coulomb electric potential, more specifically, Qq/r, multiplied by $\left[1 - v^2/(2c_0^2)\right]$, where Q is the source charge intensity, q the test charge intensity, r the instantaneous distance between Q and q, and v the velocity of the test charge. This leads to Weber's Force, in other words, the usual Coulomb force (Qq/r^2), multiplied by $\left[1 - v^2/(2c_0^2) + rdv/(dtc_0^2)\right]$, or simply $\left[1 - v^2/(2c_0^2)\right]$ for a circular motion.

Historically Weber's electric potential had been criticized mainly because it was leading to a negative mass behavior of the charges (Helmholtz, 1872; Maxwell, 1954), and was finally discarded. This electric potential was empirically elaborated a decade and half ago, in order to surmount problems related with it. The new potential is called the Generalized Weber's potential; this turns out to be the usual Coulomb potential, divided by the familiar Lorentz dilation factor, or the same, multiplied by $\sqrt{1 - v^2/c_0^2}$, which strikingly happens to be what we have derived above, through our approach.

Recall further that, a change in the inertial mass of a charged test particle when placed inside a charged spherical shell was later confirmed; the order of magnitude of the measured change was in accordance with the mentioned theory, developed on Weber's Electric Potential (Mikhailov, 1999).

As we will soon sketch, our approach for the bound electron yields a similar result. Though, it is exciting that we arrive at this result based on Coulomb's force straight (reigning between static charges, exclusively), and no other ingredients (Recall that the extra term coming to multiply both Coulomb's potential energy and Coulomb's force is anyway the same if the motion of the test charge is circular).

Before we end this subsection, let us briefly state that, we propose to replace the effect of field, by what happens straight to the core of the interacting particle. Whether there is a field or not, what happens according to the present approach, in effect, is the alteration of the internal dynamics of the interacting particle. This is how we see the bound particles. In addition, as we have mentioned, the related changes to be brought to the standard approach are discussed in Kholmetskii et al., (2010), framing the pure bound field theory.

Let us now go back to our original approach, and question: "How does the interaction between the proton and the electron occur, if along our approach, their respective energy is not spread in the surrounding space?" We will work out the answer below.

MASS SUBLIMES INTO KINETIC ENERGY, AND KINETIC ENERGY CONDENSES INTO MASS, THROUGHOUT THE MOTION: A JET MODEL

For a closed system (thus excluding any creation of any field, on the way), according to our approach, the total relativistic energy $c_0^2 m_{0\gamma}(r_0)$ of the electron, as described by Equation (11-a), ought to remain constant, all along the electron's journey around the proton.

On an elliptic orbit, this implies an alternating decrease and increase of the static binding energy of the electron and its kinetic energy. The kinetic energy decreases, as the static binding energy increases, and vice versa. However, as elaborated above, the change in the static binding energy implies a change of the electron's rest mass. Thus, on the elliptic orbit, as the kinetic energy of the electron increases, its rest mass decreases, and vice versa.

Thereby, as the proton speeds up nearby the proton, it is that, an infinitesimal part of its rest mass somehow sublimes into extra kinetic energy (the electron acquires, as it accelerates). In other words, the extra kinetic energy in question is fueled by an equivalent rest mass. Conversely, as the electron slows down away from the proton, through its orbital motion, it is that the corresponding portion of its kinetic energy somehow condenses into rest mass", on the orbit. Note that recently a fluid model of the bound electron is proposed (Oudet, 2004), incorporating a change of the mass of the electron through an exchange of mass between the electron and the nucleus (though, in a different manner than the one proposed herein). We would like to stress that what we do is in no way in conflict with the established quantum mechanical framework. One way of conceiving the rest mass variation we disclosed, together with, say, the acceleration of the electron, is to think in terms of a jet effect. This effect, to the first strike, seems to be the only way we can think of, to account for the rest variation of the electron, causing its acceleration or deceleration (on an elliptic orbit).

Within the frame of such a modeling in order to accelerate (while keeping its overall relativistic energy constant), the electron would throw out an infinitesimal net mass from the back, just like an accelerating rocket. Conversely, in order to decelerate, it would absorb an infinitesimal net mass, from the front.

Whether in reality, the whole thing works out this way or not, we do not really know. For the present purpose, we do not need to know it, either. With this we were able to refer to a mechanism that can provide us with what we need. This mechanism is needed to take care of the variation of the kinetic energy, in relation to the variation of the electrostatic binding energy, and so in relation to the variation of the rest mass of the bound electron (imposed by the law of energy conservation). Hence, we can well base ourselves on it, to make useful predictions. Before we continue, it is worth analyzing the situation, based on the simplest motion, that is, the rectilinear motion.

STUDY OF THE RECTILINEAR MOTION IN CONJUNCTION WITH THE LAW OF CONSERVATION OF MOMENTUM

Let us assume that the electron falls onto a proton. An overall jet mass thrown by the rare of the electron insures this occurrence, according to our approach. Based on the way we have set up Equation (10), the electron, as a first step brought to the given location, quasistatically, excludes at once (an otherwise expected) radiation emission.

Note that, just like in the case of the classical approach, within the frame of the approach we summarized herein too that the "law of linear momentum conservation" holds, and this is essentially because of the "law of energy conservation", along with "Coulomb's law", leading altogether to the general equation of motion (cf. Equation (12-b)). Thus, what is essential is, as evermore, the law of energy conservation.

Suppose now that the electron of rest mass $m(r_0)$, and velocity v_0, through the free fall in consideration, speeds up as much as dv_0, through an infinitely small period of time dt_0, around the time t0. When we say, "the linear momentum is conserved, thorough the fall of the electron onto the proton", we mean, "the total linear momentum of the system made of the electron and the proton is conserved". It is that, after all, the center mass of the system stays in place. Within the frame of our model, where we simulate the fall of the electron through the jet effect, be it a fictitious assumption or not, we should concentrate on the electron along with the electron's jet on the one hand, and the proton (regardless of how little it moves in reality), along with the proton's jet on the other. This should be done separately from each other, instead of the overall system made of the electron and the proton.

The conservation of the linear momentum through the free fall of the closed system made of electron and the proton, under these circumstances, can be considered as the conservation of the linear momentum of the system made of the electron and its jet, on the one hand, and the conservation of the linear momentum of the system made of the proton, and its accompanying jet on the other hand.

Based on this assertion, because the proton is much too heavy as compared to the electron, and accordingly it will practically stay in place through the motion in consideration, thus displaying practically no jet effect, through the fall of the electron, we can overlook the motion of the proton, together with the jet we associate with it. Hence, we will only have to worry about the electron's motion along with its jet. Note that, the linear momentum of the system made of the jet associated with the electron and the jet associated with the proton too should be conserved throughout.

As proposed below, we will write the law of conservation of momentum, just concerning the system made of the falling electron and its jet. Thus, the magnitude of the linear momentum $P(t_0)$ of the electron, assumed to fall right onto the proton with the velocity v_0, at time t_0, at the given location r_0, is

$$P(t_0) = m_\gamma(r_0)v_0 \tag{13}$$

(*magnitude of the momentum of the electron in free fall, at the given location and the given time*).

In any case, this momentum alone is in no way conserved. What is conserved is, once again, the sum of the momentum of the electron and that of the proton, or along our model, instead, the sum of the momentum of the electron and that of the jet, given that we neglect the motion of the proton toward the electron, throughout.

Thus, through the period of time dt0, in order to accelerate, the electron should throw out from its back the net rest mass –dm (r0), with a jet speed U, which we like to call "wave-like speed", for reasons, which will become clear soon. This shall produce a kick forward, on the overall mass $m_\gamma(r_0)$, which accordingly acquires the velocity v_0+dv_0. Let us precise that this mass is taken away from the rest mass of the electron, measured at the given location. It should be stressed that, although the overall relativistic energy $m_\gamma(r_0)c_0^2$ stays constant throughout, the rest mass of the electron, at a given location and its kinetic energy vary reciprocally, and in opposite directions (cf. Equation (11-a)). Since $m_\gamma(r_0)$ remains constant, all the way through, to be brief, we propose to call it m_γ. On the other hand, through the acceleration process in consideration, the quantity dm (r0), by definition is negative; so that –dm(r0) is a positive quantity, for by definition, dm (r0) = m (r0+dr0) – m (r0). Thus dm (r0) turns out to be negative, when the electron accelerates via throwing out rest mass, m(r0) being the electron's mass at r_0).

Thus, at time t_0+dt_0, the magnitude of the net linear momentum of the system made of the jet of mass –dm (r0) and the electron becomes

$$P(t_0+dt_0)=dm(r_0)U+m_\gamma(v_0+dv_0) \tag{14}$$

(*magnitude of the momentum of the electron, an infinitely small period of time, after the given time*).

Because of the law of conservation of momentum (for the closed system made of the electron and the jet in question), we must have:

$$P(t_0)=m_\gamma v_0=P(t_0+dt_0)=dm(r_0)U+m_\gamma(v_0+dv_0) \tag{15}$$

(*Equation of conservation of momentum for the rectilinear motion concerning the fall of the electron assuming that the proton stays in place, all the way through*) which yields

$$m_\gamma dv_0=-dm(r_0)U \tag{16}$$

(*kick received by the electron due to the jet effect, on a rectilinear motion*) .

This equation tells us that, an infinitely small mass $|dm(r_0)|$ has to be thrown out by the falling electron, in the opposite direction, with a speed U (as referred to the fixed proton), in order to provide the electron with an extra speed dv_0.

Note that above, we happened to have associated the jet speed U with the rest mass variation $|dm(r_0)|$, the electron displays on the way. We did it on purpose, given that as we will see, it is the rest mass $|dm(r_0)|$ that can be determined directly, from the related "infinitesimal electrostatic binding energy change". Moreover, as we shall soon pin down, $|dm(r_0)|$ may well be zero, whereas U can still be defined. In fact, it can be shown that U may well come into play, as a specific quantity, in one piece, as such. At any rate, not to yield misinterpretations, the "jet momentum" $U|dm(r_0)|$, should better be written as

$$|dm(r_0)|U=\frac{|dm(r_0)|}{\sqrt{1-\frac{V^2}{c_0^2}}}V=\gamma_V|dm(r_0)|V=(\gamma_V|dm(r_0)|)V=|dm(r_0)|(\gamma_V V) \tag{17}$$

(*momentum of the jet expressed in different terms*)

where γ_V is:

$$\gamma_V=\frac{1}{\sqrt{1-\frac{V^2}{c_0^2}}} \tag{18}$$

and V is the jet speed of the "relativistic mass" $\gamma_V|dm(r_0)|$, so that:

$$\gamma_V V=U \tag{19}$$

The RHS of Equation (17), that is, $\gamma_V|dm(r_0)|V$, can visibly be read either as

$(\gamma_V|dm(r_0)|)V$, or as $(\gamma_V V)|dm(r_0)|$

The writing $(\gamma_V|dm(r_0)|)V$ is the customary one, given that it embodies the relativistic mass $(\gamma_V|dm(r_0)|)$ multiplying as usual the speed V, to yield the relativistic momentum of the jet in consideration. The second writing is $(\gamma_V V)|dm(r_0)|$; here, the Lorentz dilation factor, and the related mass are decoupled from each other. This writing is obviously unusual, but becomes very interesting, as we will see, for the case where $|dm(r_0)|$ zero is, pointing to an interaction with no net mass variation. In this case, the product $\gamma_V V=U$ is to be considered enbloc. In any case, the product $\gamma_V V$ comes into play, as a whole, since it turns out that, we will end up with U, as just one single quantity, and not with separately γ_V and V. Equation (17) is remarkable, because, as simple as it may look, here may be a clue for the wave-particle duality. The relativistic momentum $(\gamma_V|dm(r_0)|)V$ evidently points to the particle character of the electron, whereas $\gamma_V V = U$ as a whole, taking place in the product $|dm(r_0)|(\gamma_V V)$ (as we will soon discover) indeed works as the key of the wave-like character of the electron. This becomes particularly evident when $|dm\ (r_0)|$ vanishes.

We will discover that U, lying along the trajectory of concern, becomes

$$U = (c_0^2 / v_0)\sqrt{1 - v_0^2 / c_0^2}$$

and

$$V = c_0\sqrt{1 - v_0^2 / c_0^2}$$

For the reason we propose to call U the wave-like jet speed, or the superluminal jet speed (carrying no energy, but a given information), and V the relativistic jet speed (associated with the mass of the jet); to refer to U shortly, we may just write the "jet speed U". Incidentally note that $U/V=c_0/v_0$; what we mean by the denomination "superluminal", is "faster than the speed of light". However, when we say "superluminal", we do not mean, "Energy is carried faster than the speed of light". What we certainly mean nonetheless is that "given information somehow is conveyed with a speed, in effect, faster than the speed of light".

Now, let us assume that the falling electron is stopped at a given distance to the proton and thrown backward with a given initial kinetic energy. Let us assume that this energy is less than the escape kinetic energy. The electron would then get elevated, while it has to spend its kinetic energy as work achieved against the attraction force. What happens according to our approach is that, owing to the law of energy conservation, the electron transforms gradually its kinetic energy, into additional internal energy, or the same with additional rest mass.

In order to achieve this end, we conjecture that, somehow, it receives momentum from the outside, in conformity with Equation (16). Thus, its speed decreases, until it exhausts all its initial kinetic energy (after which it will undergo over again, the free fall). Through the elevation process, the electron receives momentum from the front. Through the fall, it throws out an overall momentum from the rare (In fact, the two sides happen to be the same side, that is, the "dark side" as referred to the proton). At the highest elevation, the direction of U (the jet velocity vector) is reversed. The direction of U, in any case, opposes the direction of the motion. One can as well conjecture that, if thrown, the elevating electron slows down, not because it receives momentum from the front, but because it throws out momentum from the front. The latter process can also well explain the slowing down of the elevating electron, but not a concurrent rest mass increase. Similarly, one can conjecture that, the falling electron accelerates not because it throws out momentum by the rare, but because it receives momentum from the rare. In this case, the latter process can evidently explain the acceleration of the falling electron, but not a concurrent rest mass loss.

Thus, as the electron falls, according to our approach, it should throw out momentum from the rare; this way its rest mass can decrease, or the same, a minimal part of its rest mass is transformed into kinetic energy. Likewise, if thrown away from the proton, while the electron elevates, it should receive momentum from the front, which slows it down; this way its rest mass increases, or a minimal part of its kinetic energy condenses into rest mass. How does the electron know when to accelerate or when to decelerate? The answer is easy; it is merely the Coulomb force, which tells the electron how to behave. The proton and the electron pull each other (or they seem doing so, that is, they may as well be pushed toward each other, by a certain property of their surrounding). Anyway, the answer we look for is not any different from the classical answer.

As we will soon see, the wave-like jet speed U depends only on the speed of the electron. The smaller the speed of the electron, the greater U is. It is infinite when the electron's speed is zero. It would be zero, if in an extreme case, the electron's speed were equal to the speed of light. At the highest elevation, where the speed of the electron is vanished, the magnitude of U is thus infinite. In fact, at this level, the momentum reception by the electron, from the outside, is switched into a momentum ejaculation from the electron to the outside, each of these processes (at the highest elevation in consideration), delineating an infinite magnitude for U. Under the given circumstances, it seems legitimate to interpret U, as an interaction speed, given that the manifestation of the electric interaction, is well taken care by the momentum reception, or the momentum ejaculation processes, we have introduced. Thus, it seems legitimate to admit that the processes somehow create the electric force, we just described (or of course, vice versa).

As one can notice right away, nothing would change, if we considered gravitationally interacting bodies, instead of electrically interacting charges. In fact, the present approach takes care of any kind of interaction, including a non-inertial interaction, such as an object would experience in a rotating frame. Now, let us analyze the problem of an orbital motion, say, that of an elliptic rotation of the electron around the proton.

Orbital motion: Equation of the kick due to the jet effect

Equation (16) can be written as a vector equation that is,

$$m_\gamma d\,\underline{v}_0 = \underline{U}\,dm(r_0) \qquad (20\text{-a})$$

(*vector equation delineating the conservation of* *momentum throughout the jet effect, for a rectilinear motion*).

Where, U is the wave-like jet velocity, and dv_0 is the vectorial increase of the velocity of the object, corresponding to the ejection or the absorption of the infinitely small mass $|dm(r_0)|$, through the period of time dt0. Note that U lies in the opposite direction to dv0, which in return is directed inward, that is, toward the proton. $dm(r_0)$ was negative for the free fall, through which U was directed outward, and dm (r0) was positive in case the electron is thrown away from the proton, through which U was directed inward. This picture makes clear the framework of Equation (20-a), along with the signs it displays. It is that U and dv_0 are directed in opposite directions. In addition, the sign of $dm(r_0)$ makes that the directions of both sides of Equation (20-a) are well aligned.

The division of Equation (20-a) by dt0 may constitute a clue about the root of the electric force:

$$m_\gamma \frac{d\underline{v}_0}{dt_0} = \underline{U}\frac{dm(r_0)}{dt_0} \tag{20-b}$$

(*equation depicting the creation of the force via the jet model)*

or via Equation (12-b),

$$-\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\frac{\underline{r}_0}{r_0} = \underline{U}\frac{dm(r_0)}{dt_0} \tag{21}$$

(*the force expression via the proposed jet model*).

The foregoing three equations should be expected to hold generally. Thus, it should well hold regarding an elliptic motion, which we have considered originally (In fact, the orbit drawn by the above equation is not exactly elliptic; instead, its perihelion precesses. However, we do not need to elaborate on this piece of detail. Thus below, we will call the resulting orbital motion, straight, "elliptic motion"). For such a motion, dv0 is directed radially, and inward. Thus, U must be directed accordingly, where

$dm(r_0)$ >0.

As the electron accelerates toward the proton, U is directed outward, since $dm(r_0)$ <0; thus, the electron ejaculates a minimal part of its rest mass, or that much of rest mass is transformed into extra kinetic energy. As the electron decelerates away from the proton, U is directed inward, since $dm(r_0)$ >0; thus the electron receives momentum from the outside, killing a fraction of its kinetic energy, or that much of energy is restored as extra rest mass.

Let us now define θ, as the angle between $d\underline{v}_0$ (directed inward), and the electron's velocity $\underline{v}_0$ (tangent to the orbit). Thus, one can show that

$$dv_0 = -|d\underline{v}_0|\cos(\pi-\theta) = |d\underline{v}_0|\cos\theta \tag{22-a}$$

Based on this relationship, one can transform Equation (20-a) into

$$m_\gamma dv_0 = \cos\theta|\underline{U}dm(r_0)| = -Udm(r_0) \tag{22-b}$$

(scalar equation delineating the conservation of momentum throughout the jet effect, on a given orbit).

where , U is the quantity $|\cos\theta\underline{U}|$, that is, the magnitude of the tangential component of U.

This equation is well compatible with Equation (16), written for a rectilinear motion. Hence, it is the general equation describing the "kick", the electron receives, throughout the centripetal motion, owing to the jet effect we introduced.

Circular motion: The electric interaction can be achieved, without any energy or mass exchange, whatsoever

Here we consider the circular motion. We will show that Equation (20-a), as should be expected, still holds. In the case of a circular motion, dv_0 is null; $dm(r_0)$ too should be null, since kinetic energy is obviously not altered throughout (we have conceived the jet effect in order to take care of the variation of the kinetic energy, in relation to the variation of the electrostatic binding energy; and thus in relation to the variation of rest mass of the bound electron, throughout, as imposed by the law of energy conservation. Zero variation in the speed must indeed be coupled with zero variation in the rest mass).

Then, there seems to arise a couple of problems. First, for the circular motion, $\cos\theta$ vanishes. Already because of this occurrence, the RHS of Equation (22a) would anyway vanish. Thereby, what would it mean that the RHS of Equation (22-a) disappears, not only due to $dm(r_0)$ = 0, but also due to $\cos\theta$ = 0? This is a tricky question. Here, though, is the answer: It is that, although for a circular motion $\cos\theta$ =0, the RHS of Equation (22a) does not actually vanish, because of this latter occurrence, and the reason is that, $|\cos\theta\underline{U}|$ (that is, the tangential component of the jet velocity, as will soon be proven), turns out to be finite, even if $\cos\theta$ = 0. Thus, U in this case, must turn to be infinite. This will constitute a clue to clarify the next problem, which is the following. If $dm(r_0)$ = 0, then, the RHS of Equation (20a), at the first strike would vanish. However, the LHS of this equation does not (since the variation of the velocity vector constitutes the basis of the acceleration, and the acceleration is a finite quantity for the circular motion). Then, how is it that the RHS of Equation (20) seems to vanish and the LHS of it remains finite?

The answer to this question too, is tricky. In fact, it is that, the physical requirements we have considered regarding the rectilinear acceleration motion, and especially elliptic motion, have forced us, first to write Equation (16), then Equation (20a), and then Equation (22a), by considering either the mass ejaculation, or the mass absorption, through corresponding jets, and not the two processes, simultaneously.

However, for a circular motion, to get an infinitely small kick (momentum change) inward, through an infinitely short period, we must be allowed to consider, both the "ejaculation of an infinitesimal mass to the outside", and the "absorption of an infinitesimal mass from the outside", concurrently, given that both of these processes yield the same effect. Under the circumstances, we do not really have to account for the increase or the decrease of the speed of the object through the motion (since the speed remains the same, throughout). The only other requirement is that the mass ejaculated to secure the kick inward is balanced by the mass received from the outside; still to secure the kick inward, so that the two kicks amount to the expected overall kick. Fortunately, we will not have to formulate these anticipations.

At any rate, the result is that, for a circular motion, there is no net rest mass gain, or loss. In other words, regarding an orbital motion, one can interpret Equation (20), in two different ways. Thus, this equation can be viewed as the description of the kick due to either the absorption of an infinitesimal rest mass, or the ejaculation of an infinitesimal rest mass, coming respectively into play, along an elliptic motion. It can also be seen as the description of the "resultant kick", due to the superimposed processes of the reception of an infinitesimal momentum by the electron from the outside, and the simultaneous ejaculation of the same amount of momentum by the electron to the outside, through a circular motion.

This latter process, as expected, depicts a zero net mass change. This is important for, on the one hand, it points to the fact that, the interaction in question occurs, without any energy exchange (or the same, without any net mass exchange), with the attraction center, whatsoever. On the other hand, $dm(r_0)$ = 0, arising on the RHS of Equation (20a), along with a finite LHS taking place in this equation, can only be tolerable if U is infinite, and this is exactly what we have just established. We will see in fact that it is the quantity $|\cos\theta \underline{U}|$, we can calculate, and not U alone.

Let us further note the following, as regards to a stationary circular motion. Since, through such a motion, both (the scalar) dv and dm, are null; $\underline{U}$ must become infinite to secure a finite LHS of Equation (20a), and we have here, perhaps an expression of the Mach principle (Mach, 1906; Einstein, 1923). More specifically, the tangential component of $\underline{U}$ is $U = |\cos\theta \underline{U}|$, θ being the angle $\underline{U}$ makes with the tangent, that is, $\pi/2$. Accordingly, $\cos\theta$ is null. The magnitude of $\underline{U}$, as stated, is infinity. This, as we will disclose right below, indeed makes that, $U = |\cos\theta \underline{U}| = \infty \times 0$, a finite quantity, thus well matching Equation (16). Thence, in any case the tangential component U is finite.

It is worth stressing that the approach we have developed based on the jet effect, points to a possible mechanism of the interaction in consideration. Thus, we conjecture that, the faster the jet speed U, the quicker the interaction takes place. U, or more precisely $|\cos\theta \underline{U}|$, can be taken as the speed of the transmission of the "interactional information" assuring the electric motion. The same holds, if it were question of a gravitational motion. Now for a circular motion the overall mass change throughout is null. The information assuring the interaction must be there, though. Thus, there are reasons to believe that, even when the net mass gained or lost by the jet effect is null, whatever is the information we anticipate to be carried by the jet of speed U, this information is still transferred. In other words, whatever is the information carried by the jet speed U, this information can get transferred, along with no mass, thus no energy is involved, at all.

This is interesting since we came to say that "interactional information" can well be transferred with no need of any usage of energy. We can well call it, "wave-like information". In any case, we deduce that the charges engaged with each other, do not exchange any particles. In fact, the standard approach assumes the exchange of virtual particles. This is ironic, for no exchange particles and virtual particles may indicate the same fact, except that the former denomination is no doubt more realistic.

DERIVATION OF THE DE BROGLIE RELATIONSHIP AND SUPERLUMINAL SPEEDS

Let us now multiply Equation (16) (for a rectilinear motion), or Equation (20-a) (for an elliptic motion), by c_0^2:

$$c_0^2 m_\gamma(r_0) dv_0 = -c_0^2 U dm(r_0) \tag{23}$$

The law of conservation of energy requires that, the quantity $-c_0^2 dm(r_0)$, appearing at the RHS of this equation, must come to be equal, to the change in the corresponding kinetic energy, which in return, must be equal to the change in the corresponding electrostatic binding energy (cf. Equation (8)). Thus,

$$c_0^2 dm(r_0) = \frac{Ze^2}{r_0^2} dr_0 \tag{24}$$

(*variation of the rest mass, in terms of the static, electrostatic binding energy*)

written in CGS unit system.

Note, on the other hand that, when the electron (either through a head on free fall, or an elliptic motion) speeds up; it gets closer to the proton; in this case dr0 (just like, dm (r0)), turns out to be a negative quantity.

Equating the LHS of Equation (23), with the product of the RHS of Equation (24) by the tangential component U of the jet velocity vector U (via Equation (10)), leads to

$$c_0^2 m_\gamma dv_0 = -U \frac{Ze^2}{r_0^2} dr_0 \tag{25}$$

Here dv_0 can be replaced by the same quantity, furnished by Equation (12-a). Thence, the tangential wave-like jet speed U, as assessed by the distant observer, turns out to be

$$U = \frac{c_0^2}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \tag{26}$$

(*the wave-like jet speed as referred to the outside fixed observer*).

Note that, as mentioned, the approach allows us to determine straight, the magnitude of the tangential component of U. A vector-based derivation of this quantity will be provided in Appendix D. Equation (26) is amazingly the same as Equation (7-c), if the tangential jet speed U is taken to be same as U_B, of this latter equation. It is rigorous. It only depends on the speed of the object of concern. Equation (26) can be written as:

$$U = \frac{\lambda_0}{T_0}\frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \tag{27}$$

via the usual definition of the speed of light, that is, Equation (2). Now, if we propose to write the tangential wave-like jet speed U, in question, in terms of the period of time T_0, of the electromagnetic wave, we associate with the mass m_0, along Equation (1); we come to the expression of a wavelength λ, in terms of λ_0, that is,

$$\frac{\lambda}{T_0} = \frac{\lambda_0}{T_0}\frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \tag{28}$$

Thus, λ is nothing else, but the de Broglie wavelength (cf. Equation (6-a))

$$\lambda \equiv \lambda_B = \lambda_0 \frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \tag{29}$$

(*de Broglie wavelength obtained from the wave-like jet speed, derived in here*).

It holds generally, thus through a rectilinear motion or, as well through an elliptic motion, or as an exception, through a circular motion. Since, it is velocity dependent, it is practical to consider it, first, for a circular orbit (embodying a constant speed). Yet, under the form delineated by Equation (29), it ought to be valid only for the ground state (then leading to the original Bohr postulate).

If the motion is circular, for the nth level, one should instead write (Appendix B):

$$\lambda_{Bn} = n\lambda_0 \frac{c_0}{v_{0n}}\sqrt{1-\frac{v_{0n}^2}{c_0^2}} \tag{30}$$

(*de Broglie wavelength on the nth orbit*).

We should further determine the "relativistic jet velocity" V, appearing in Equation (19). Thus via Equations (20-a) and (26), one can readily write

$$V = c_0\sqrt{1-\frac{v_0^2}{c_0^2}} \tag{31}$$

(the relativistic jet velocity) which indeed happens to be below the velocity of light.

Thereby γ_V defined along Equation (20-a), becomes

$$\gamma_V = \frac{U_t}{V} = \frac{c_0}{v_0} \tag{32}$$

We can, on the other hand, calculate the jet speed u, with respect to the electron. As a rough approximation, one can write

$$u \approx U + v_0 \tag{33}$$

(*the wave-like jet speed as referred to the "moving electron*").

Where v_0 is the speed of the electron, with respect to the proton (assumed at rest); we will call u the superluminal jet speed, since, as clarified right below, it is always greater than the speed of light. Note that in our approach the ceiling c_0 cannot be reached, unless the photon bears an infinite amount of energy (Yarman, 2006). At this stage, we do not know the rule regarding the addition of superluminal velocities, with ordinary velocities (taking place, below the speed of light). Nonetheless, the examination of Equation (26) makes our task easy. The two interesting cases indeed occur for $v_0 = 0$ and $v_0 = c_0$. For $v_0 = 0$, $U = \infty$ thus one can right away guess that, in this case, u must be infinite. For $v_0 = c_0$, $U = 0$ thus one can guess that, in this case u must be c_0.

Hence, we can well establish that the superluminal interaction speed u (with respect to the object in question) varies between ∞ (for the object of concern, at rest), and c_0 (for the object moving with the speed of light). As tautological as it may seem, this yields the fact that, light cannot interact with anything, via a speed above the speed of light (since its "superluminal jet speed" is, at best, c_0).

It is interesting to note that, our result is somewhat in conformity with what had been established with tachyons, particles moving faster than light (Bilaniuk et al., 1962; Feinberg, 1970). However, tachyons are particles, with an imaginary mass. However "imaginary mass", may mean as well "no mass", at all, and this is indeed, what we have landed at, above. Thus, we have no energy or mass, but something moving with a superluminal speed; this appears to be somewhat interactional information needing no energy to be transferred, the transfer taking place at superluminal speeds. To us, this is like quantum mechanics, which is, next to the de Broglie relationship, based on the law of conservation of energy, but allowing well an infinite uncertainty about energy, if the uncertainty on time is null. Thus, while the STR does not allow any speed faster than that of light, amazingly, it appears to allow even an infinite speed of information transfer, if the mass or energy, involved, is missing. It seems useful to summarize different velocities we introduced, along with different values of interest they would assume. This is done in Table 2.

Conclusion

Here, based essentially on the conservation of energy, we figured out that, a motion driven by electric attraction depicts some sort of rest mass exchange, throughout. One way to conceive this phenomenon is to consider a "jet effect". Accordingly, an object on a given orbit, through its journey, must eject mass to speed up, or must pile up mass, to slow down.

The component of the speed U of the jet, tangent to the trajectory in question (as referred to the proton), strikingly delineates the de Broglie wavelength, coupled with the period of time T_0, delineated by the corresponding electromagnetic energy content of the object (as required by Equation (1)).

This result seems to be important, in many ways. A more detailed conclusion will be drawn at the end of Part II, where we will deal with the gravitational field. In any case one should mention a major conclusion, which is, the present work somewhat fulfills de Broglie's dream. Originally, he conceived a particle with an intrinsic periodic phenomenon (cf. Equation (1)). The particle lies along with its rest mass, thus centered inside of its wave-like behavior. When the particle moves, it still bears both of the characteristics in question, that is, the particle moves as a particle, still being centered inside of its wave-like behavior. The wave-like behavior stems from its inside periodic phenomenon, which we like to call internal dynamics. This is somewhat an oscillatory phenomenon, thus indeed displaying a wave-like behavior. The main idea over here is that, the particle character and the wave-like character coexist; one character does not destroy the other. As the neutron diffraction through the Young slits, we discussed above, in the Introduction, delineates; diffracting neutrons end up on the screen, displaying the interference, still, as single entities, as their arrivals can well be detected, one by one, in a low density beam (Zeilinger et al., 1988). The conclusion is then, they do carry, at the same time, both properties of "particle" (falling, one by one, on the screen), and "wave" (producing on the screen the interference pattern). Conceptually speaking, this is exactly what was originally conjectured by de Broglie himself, but rather forgotten, as de Broglie regrets, in a speech he gave (de Broglie, 1973), about half a century after he wrote his doctorate thesis (de Broglie, 1925). In his speech in 1973, he says that unfortunately the contemporary quantum mechanics, restraining itself to provide only a statistical view of things without disclosing the true nature of them, could not in effect, intercept the coexistence of the "particle nature" of the object, along with its wave-like nature.

We believe the present work, leading us to the de Broglie relationship, based only on the relativistic law of conservation of energy, fulfilled in effect de Broglie's dream. We have not lost a bit, the particle character of the object, from view, while we ended up, at the same time, with its wave-like character, propagating with the superluminal wave-like jet speed $U = (c_0^2/v_0)\sqrt{1 - v_0^2/c_0^2}$. This finding immediately induces a kind of interaction, without any exchange of energy but taking place with a speed greater than the speed of light, which thereby we like to call, wave-like interaction. Thence, it is well compatible with the Special Theory of Relativity (STR). In reality, it is imposed by this latter theory, coupled with the law of energy conservation.

Table 2. Different velocities Introduced throughout.

Velocity	Explanation	Expression	Special case 1 (object at rest)	Special case 2 (object moving with the speed of light)
v_0	Electron's speed as assessed by the outside fixed observer	v_0	$v_0=0$	$v_0=c_0$
c_0	Speed of light in "empty space"	c_0	c_0	c_0
U	The "magnitude of the tangential component of the superluminal jet speed", as assessed by the outside fixed observer	$U=\frac{c_0^2}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}}=\gamma_V V$	$U=\infty$	U=0
V	The factual "relativistic jet speed", as assessed by the outside fixed observer	$V=c_0\sqrt{1-\frac{v_0^2}{c_0^2}}$	$V=c_0$ (*yet null rest mass variation*)	V=0
γ_V	Lorentz Dilation Factor	$\frac{1}{\sqrt{1-\frac{V^2}{c_0^2}}}=\frac{U}{V}=\frac{c_0}{v_0}$	$\gamma_V=\infty$	$\gamma_V=1$
u	The "superluminal jet speed", as referred to the electron	$u\approx U+v_0$	$u=\infty$	$u=c_0$

Now, we understand how the particle character turns into a wave-like character, as forwarded by the contemporary quantum mechanics. It is that such an occurrence stems from the fact that each time the internal dynamics of, say, an electron revolving around a proton, beats, at locations separated by the wavelength λ_0 (cf. Equation (1)), while moving on, with the velocity v0; the "periodic information" about the beatings, propagating with the velocity c_0^2/v_0, and bearing as wavelength, the de Broglie wavelength λ_B (cf. Equations (4) and (6-a)), thus, immediately catches up with the electron's being, over the orbit of perimeter λ_B. And we obtain a perfect match of the particle character moving with the speed v0, with its wave-like character, moving with the speed c_0^2/v_0, but as an information, thereby jumping ahead just like a non-material snake to catch up his tail. This is; to our understanding, how both particle and wave-like properties fuse into each other; and at the same time, perhaps why, quantum mechanics (QM), in its mere wave-like formulation, lost, after de Broglie, from view, the particle character of the object, de Broglie originally linked with its periodic phenomenon.

In any case, our approach seems to bring a solution to the long lasting quest, since the EPR Gedanken experiment (Einstein et al., 1935). with regards to the incompatibility of STR and QM, which recently did put seriously at stake the STR (Albert and Galchen, 2009], in the light of spooky (or perhaps, to better qualify, "non-locality", that is, "quantum-mechanical-correlation") measurements (Kholmetskii et al., 2007; Salart et al., 2008) coming into play, one after the other.

The answer to the puzzle in question is that we really do not have to give up the STR for QM, given that QM is essentially based on the law of energy conservation and the de Broglie relationship, and the latter is shown in the present work to be a result of the relativistic law of energy conservation. This is our major contribution. Let us emphasize that, recent measurements back up our arresting deduction, or that they can be explained via our approach.

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APPENDIX A

THEOREM OF HARMONY OF PHASES OF DE BROGLIE, AND HIS RELATIONSHIP

Refer to the discussion we have undertaken in the Introduction, chiefly around Equation (6b) of the text. Suppose that the two periodic functions, respectively describing the two waves of frequencies $\nu = \nu_0 / \sqrt{1 - v_0^2/c_0^2}$ and $\nu_1 = \nu_0 \sqrt{1 - v_0^2/c_0^2}$ point to the same "null phase" next to the argument ϖt, at t = 0 (as assessed by the outside observer). Recall that, the angular velocity ϖ of the argument ϖt is given by

$$\varpi = 2\pi\nu \tag{A-1}$$

Where ν is the frequency in consideration?

Following de Broglie, 1925, we will show that the arguments of the periodic functions describing the waves of frequencies ν_1 and ν, are in harmony with each other, if the wave associated with ν propagates with the velocity c_0/v_0^2, the wave associated with ν_1, propagating with the usual velocity v_0.

Indeed at time t, the argument of the periodic function describing the first wave of relativistically fainted frequency $\nu_1 = \nu_0 \sqrt{1 - v_0^2 / c_0^2}$ propagating with the velocity v_0, would get increased as much as $2\pi\nu_1 t$, which at the location x, can be, via (cf. Equation (1) of the text)

$$h\nu_0 = m_0 c_0^2 \tag{A-2}$$

written as

$$2\pi\nu_1 t = 2\pi \frac{m_0 c_0^2}{h} \sqrt{1 - \frac{v_0^2}{c_0^2}} \frac{x}{v_0} \tag{A-3}$$

Now we propose to calculate the argument of the periodic function of the second wave of quantum physically increased frequency $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$, associated with the inside periodic phenomenon, and propagating with the velocity c_0^2 / v_0, thus, at time $(t - xc_0^2 / v_0)$:

$$2\pi\nu\left(t - \frac{xv_0}{c_0^2}\right) = 2\pi \frac{m_0 c_0^2}{h\sqrt{1 - \frac{v_0^2}{c_0^2}}}\left(\frac{x}{v_0} - \frac{xv_0}{c_0^2}\right) = 2\pi \frac{m_0 c_0^2}{h}\sqrt{1 - \frac{v_0^2}{c_0^2}} \frac{x}{v_0} \tag{A-4}$$

which is effectively the same as the previous result.

The equality of $2\pi\nu_1 t$ and $2\pi\nu\left(t - xv_0/c_0^2\right)$ is what de Broglie called the "Theorem of Harmony of Phases" [de Broglie, 1925]. It is that, the wave of fainted frequency ν_1 associated with the uniform translational motion and propagating with the velocity v0 , is in constant harmony with the wave of increased frequency ν associated with the internal periodic phenomenon and propagating with the velocity c_0^2 / v_0. Note that it is the frequency $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$, which is associated with de Broglie's wavelength λ_B (the wave, as mentioned, propagating with the speed c_0^2 / v_0, without though carrying any energy) (cf. Equation (6-b) of the text):

$$\frac{c_0^2}{v_0} = \lambda_B \nu = \lambda_B \frac{\nu_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \tag{A-5}$$

this equation in return, via using Equation (A-2), leads to the usual de Broglie relationship (cf. Equation (4) of the text).

$$\lambda_B = \frac{h}{mv_0} \tag{A-6}$$

along with the usual definition (cf. Equation (4) of the text).

$$m = \frac{m_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \tag{A-7}$$

Unfortunately, the above derivation looks almost totally forgotten; though, the original derivation of de Broglie relationship, appears not that clear, in de Broglie's doctorate thesis, for he happened not to have written specifically, Equation (A-5), nor the subsequent Equation (A-6).

It may be argued that, the increased frequency ν cannot be measured; it simply occurs to be an intermediary quantity. This is not so, at all, for ν, as we have just seen, is the frequency coupled with de Broglie wavelength λ_B. Thus, as long as de Broglie's wavelength, is known as a measurable quantity, so will ν be. We have further, to note, the following. We can equally affirm that, the original wave of frequency ν_0, to be associated with the overall relativistic energy of the object at hand, propagates with the superluminal wave-like velocity $U = (c_0^2 / v_0)\sqrt{1 - v_0^2 / c_0^2}$, as the objet moves with the velocity v_0.

APPENDIX B

QUANTIZATION AND DE BROGLIE RELATIONSHIP

Classically, a wave displaying a resonant standing motion of wavelength λ_n, through say, a string of length L, obeys the relationship.

$$L = \frac{n\lambda_n}{2} \quad n = 1, 2, 3 \tag{B-1}$$

If visualized for a particle in the box, along with the de Broglie relationship (cf. Equation (4) of the text), Equation (B-1), thus already classically, yields well the Schrödinger's equation solution, that is, the quantization of energy levels. The resonance condition, for a circular orbit, is

$$2\pi r_n = n\lambda_{Bn} = \frac{nh}{mv_{0n}} \tag{B-2}$$

For the de Broglie wavelength λ_{Bn}, to be associated with the nth energy level of the Bohr hydrogen atom, for which the orbital velocity of the electron is v_{0n}. This makes that, for the case in hand, Equation (4), or the same Equation (6-a) of the text, should in general, be written as

$$\lambda_{Bn} = \frac{nh}{mv_{0n}} = n\lambda_0 \frac{c_0}{v_{0n}}\sqrt{1 - \frac{v_{0n}^2}{c_0^2}} \tag{B-3}$$

which is in fact, nothing else, but [based on Equation (B-1))

$$2\pi r_{0n} v_{0n} m = nh \tag{B-4}$$

that is, the Bohr postulate, written for the nth energy level, were the orbit circular, with a radius of r_{0n}.

Hence, once we have de Broglie relationship, the quantization displayed within the frame of Equation (B-4), follows in fact from classical physics. Recall that, Equation (B-3) is used in the text.

APPENDIX C

GRAVITATIONAL BINDING ENERGY

As a first approximation, the binding energy E_{BStone} of a stone of mass m_{0Stone}, measured at infinity, bound to Earth of mass M and radius R, can be calculated as usual, to be

$$E_{BStone} = \int_R^\infty G\frac{\mathcal{M}\, m_{0Stone}}{r^2}\,dr = G\frac{\mathcal{M}\, m_{0Stone}}{r} \tag{C-1}$$

G being the gravitational constant. Otherwise, one should write for the binding energy $E_{BStone}(r)$, the stone delineates at a distance r to the center of Earth (Yarman, 2004).

$$E_{BStone}(r) = G\mathcal{M}\int_r^\infty \frac{m_{0Stone} - \frac{E_B(r')}{c_0^2}}{r'^2}\,dr' \tag{C-2}$$

Which on Earth (at R from the center of Earth), yields

$$E_{BStone}(R) = m_{0Stone}c_0^2\left[1 - \exp\left(-\frac{G\mathcal{M}}{c_0^2 R}\right)\right] \tag{C-3}$$

APPENDIX D

VECTORIAL DERIVATION OF THE SUPERLUMINAL VELOCITY

It is worth to redo the derivation of the superluminal wave-like velocity U, in a vector framework, thus starting from the vectorial equation, Equation (20-a), that is,

$$m_\gamma d\underline{v}_0 = \underline{U}dm(r_0) \tag{D-1}$$

Thus, Equation (25) of the text becomes

$$c_0^2 m_\gamma d\underline{v}_0 = \underline{U}\frac{Ze^2}{r_0^2}dr_0 \tag{D-2}$$

Let us take the absolute values of both sides, and use Equation (19):

$$c_0^2 m_\gamma dv_0 = \cos\theta\left|\underline{U}\frac{Ze^2}{r_0^2}dr_0\right| = -U\frac{Ze^2}{r_0^2}dr_0 \tag{D-3}$$

Where U is the magnitude of the tangential component of the vector U.

Finally, using Equation (12-a) of the text, to replace dv_0, one lands straight, at

$$U = \frac{c_0^2}{v_0}\sqrt{1 - \frac{v_0^2}{c_0^2}} \tag{D-4}$$

(the wave-like jet speed as referred to the outside fixed observer that is,Equation (26) of the text.