

Full Length Research Paper

Consistency of electrical analogy approach in the prediction of through thickness thermal conductivity of fiber reinforced plastic (FRP) composites with orientation of the square unit cell

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From the literature, it is found that two different criteria were followed for the prediction of transverse thermal conductivity (K_2) of fiber reinforced plastic (FRP) composites. In the first criterion, the internal anisotropy of the lamina is assumed negligible and K_2 is estimated using simple Fourier's law of 1-D heat conduction applied to representative volume element (RVE). Whereas in the second approach, an electrical analogy method is followed. To estimate the effect of internal anisotropy, through thickness thermal conductivity (K_3) of an FRP lamina is determined by both the approaches through finite element method for an RVE in the auxiliary plane. The problem is modeled in ANSYS 15 software. In the present paper studies are made for various volume fractions (0.1-0.75) and for various angles (20°-90°) made by the section plane with the fiber axis. It is observed that the through thickness thermal conductivity is consistent in the second approach, whereas in the first approach there is considerable variation (max 8.7%) with the orientation of the unit cell.

Key words: Through thickness thermal conductivity, finite element method (FEM), unit cell orientation.

INTRODUCTION

From the literature, it is observed that the transverse thermal conductivity (K_2) of the lamina depends on many parameters like arrangement of fibers, volume fraction, fiber angle, ratio of fiber conductivity to matrix conductivity, etc. It is found from the literature that there are two different approaches to evaluate the transverse thermal conductivity of composites. In the first approach

(A-I), the internal anisotropy of the lamina is not considered and K_2 is estimated using simple Fourier's law of 1-D heat conduction. Some of the worth mentioned studies from this criterion are Perrins et al. (1979), who had published exact analytical and experimental results for K_2 and showed very good agreement between experimental and theoretical studies. Another work using

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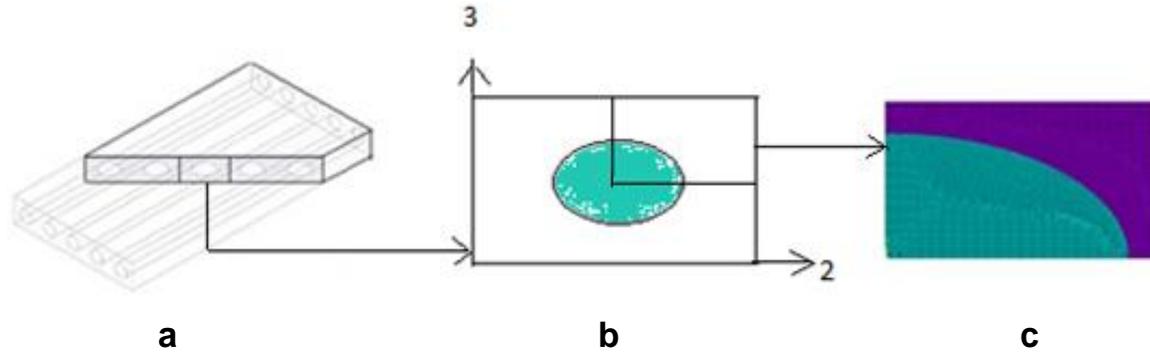


Figure 1. a. Composite b. Unit cell c. FE model.

numerical studies has been made by Lu (1994), who matched the results with Perrins et al. (1979), and stood as source of inspiration for several researches who developed finite element (FE) models for K_2 . Sambasiva Rao et al. (2008) developed a 3-D finite element model for circular fibers in square unit cell and compared the results with Perrins et al. (1979) to validate their approach.

In the second approach (A-II), Springer and Tsai (1967), Behrens (1968), Mingqing et al. (2002) considered representative volume element (RVE) as two segments, first one consisting of fiber and matrix arranged normal to the heat flow direction and the second segment being the pure matrix above the first segment, so that the two segments remain parallel to the direction of heat flow, that facilitated them to use Inverse Rule Of Mixtures (IROM) for the first segment and rule of mixtures (ROM) for the two segments together. This method allows the heat flow in considered direction only and the usage of 1-D Fourier's law of conduction is justified. Srinivasa Rao et al. (2014a) developed FE models in support of the second criterion.

Prior to Srinivasa Rao et al. (2014b), there was no distinction of the two approaches and the contributors of both methods tried to convince by comparing their results with experimental results, irrespective of the approach they followed. In the present work through thickness thermal conductivity of the composite obtained using both methods are compared. Interestingly, it is observed that K_3 is consistent for all the values of theta in the second approach, whereas it is varying in the first approach.

Finite element model

A schematic diagram of the unidirectional fiber composite is shown in Figure 1a. A representative volume element (RVE) in the form of a square unit cell is adopted for the present analysis. The cross-sectional area of fiber relative to the total cross-sectional area of the unit cell is a measure of the volume of fiber relative to the total volume of the composite (Figure 1b). This fraction is an important parameter in composite materials and is called fiber volume fraction (V_f).

The 1-2-3 coordinate system shown in Figure 1b is used to study the behavior of a unit cell (Direction 1 is along the fiber axis and normal to the plane of 2D figure shown). The isolated unit cell behaves as part of a larger array of unit cells.

It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to 1-2-3 coordinate system. Therefore, a one fourth portion of the unit cell is modeled and the 2-D finite element mesh on one fourth portion of the unit cell is shown in Figure 1c. The mesh is generated using six node triangular element (PLANE-35) of ANSYS software, which is quadratic and is best suited along the curved interface between the fiber and the matrix, and has the capability of incorporating isotropic as well as orthotropic materials.

Boundary conditions

Temperature boundary conditions for one-fourth model are as follows: Sides of the unit cell is taken as '2a'.

$$T(x, 0) = T_1; T(x, a) = T_2 \quad (1)$$

The other two faces are subjected to adiabatic boundary conditions.

The effective transverse thermal conductivity is calculated using the equation:

$$q_y = -k_2 \frac{\partial T}{\partial y} \quad (2)$$

Heat flux and the temperature gradient in the above equation are obtained from the finite element solution.

RESULTS AND DISCUSSION

The variation of the normalized through thickness thermal conductivity with respect to theta is shown for both approaches (A-I and A-II) for different volume fractions (Figures 2 to 7). In all the graphs, it is observed that the through thickness thermal conductivity obtained in the second approach (A-II) is constant for all the values of theta at all the volume fractions. Whereas in the first approach (A-I), the normalized through thickness thermal

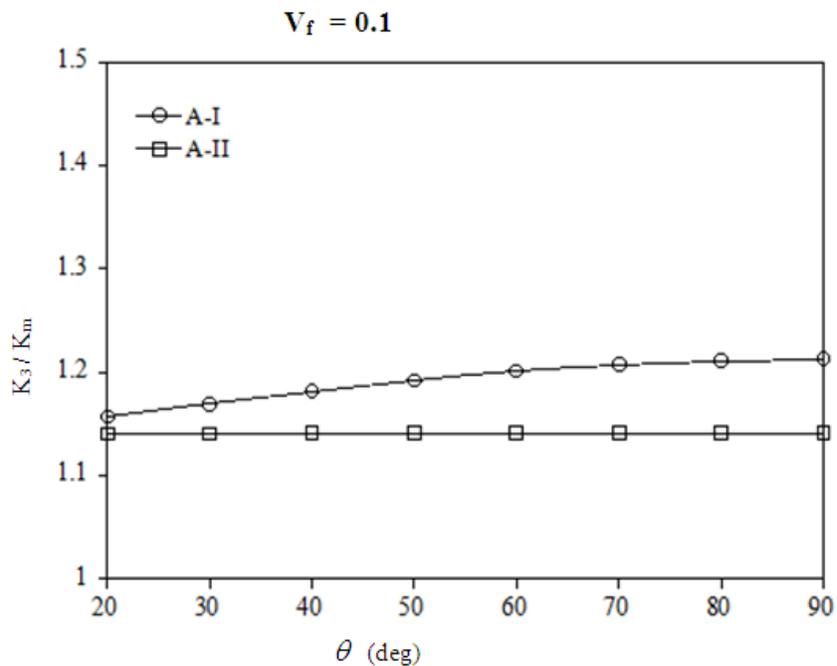


Figure 2. Effect of θ on through thickness conductivity at $V_f = 0.1$.

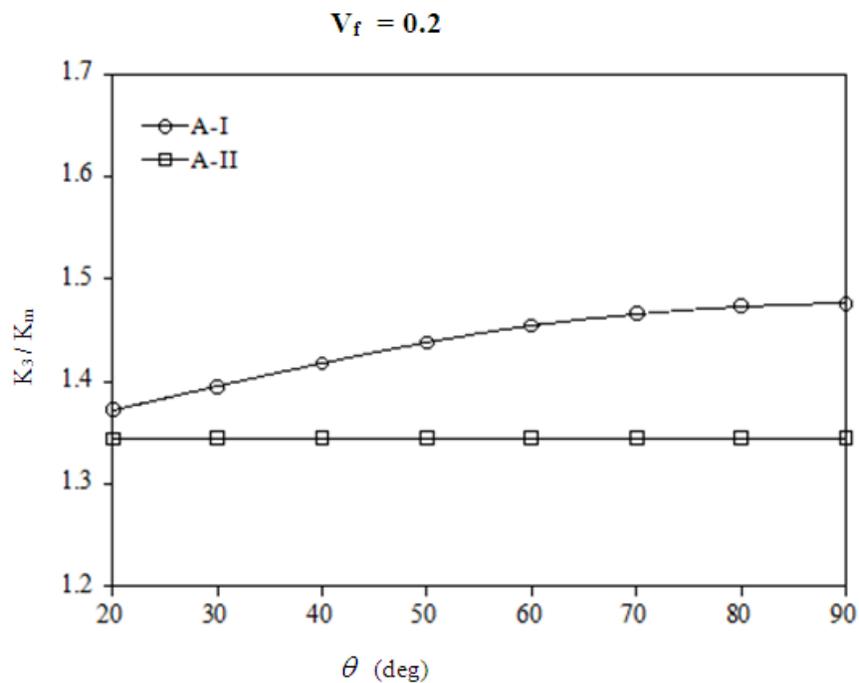


Figure 3. Effect of θ on through thickness conductivity at $V_f = 0.2$.

conductivity increases with theta for all values of V_f . It is also evident from Figures 5 to 7 that the deviation in the results of the two approaches increases with the value of

theta at all volume fractions. It is also observed that the deviation increases with V_f at any particular value of theta.

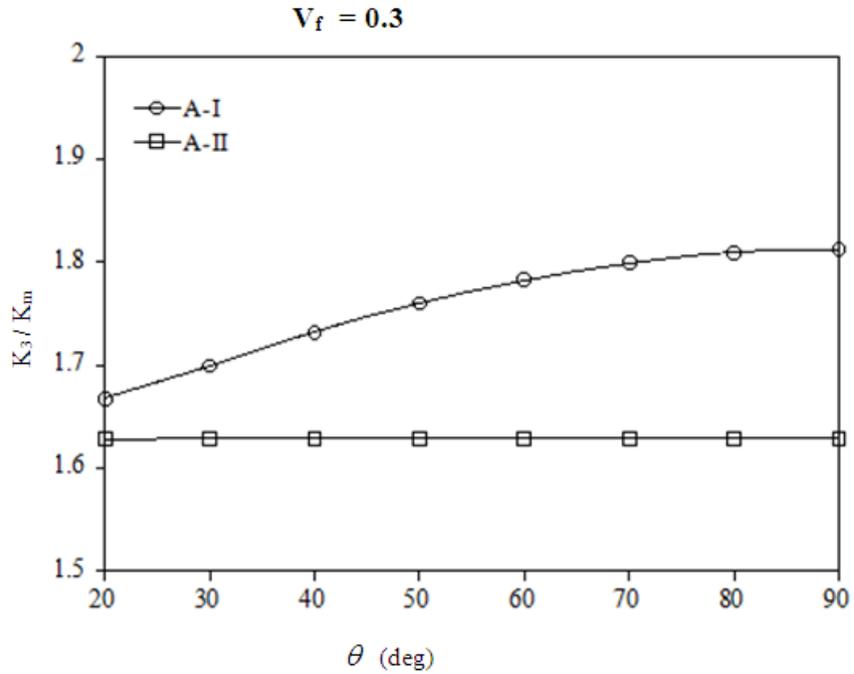


Figure 4. Effect of θ on through thickness conductivity at $V_f = 0.3$.

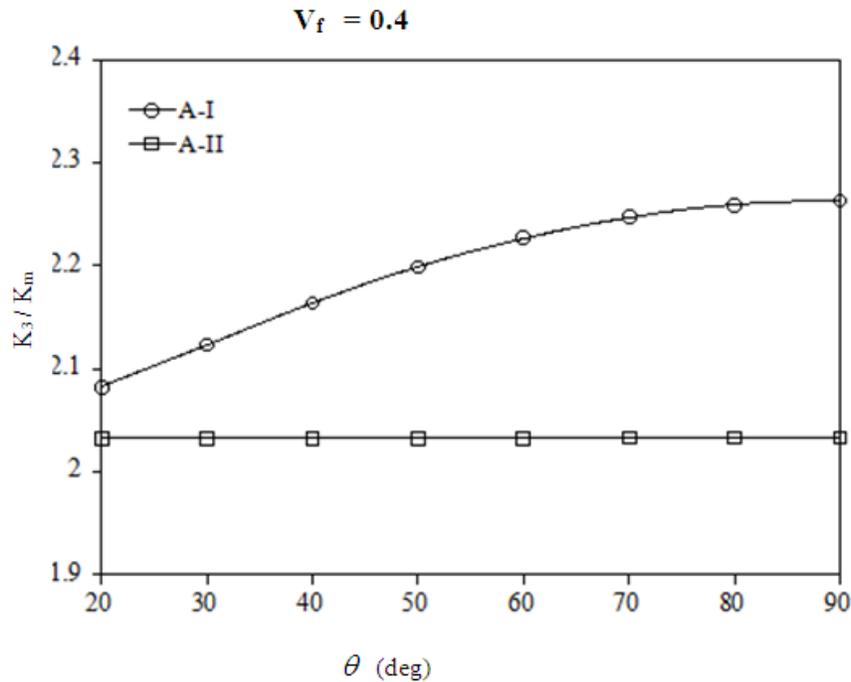


Figure 5. Effect of θ on through thickness conductivity at $V_f = 0.4$.

Figure 8 shows the percentage deviation of through thickness thermal conductivity obtained by the first

approach with theta. It is evident from the figure that the percentage deviation of through thickness thermal

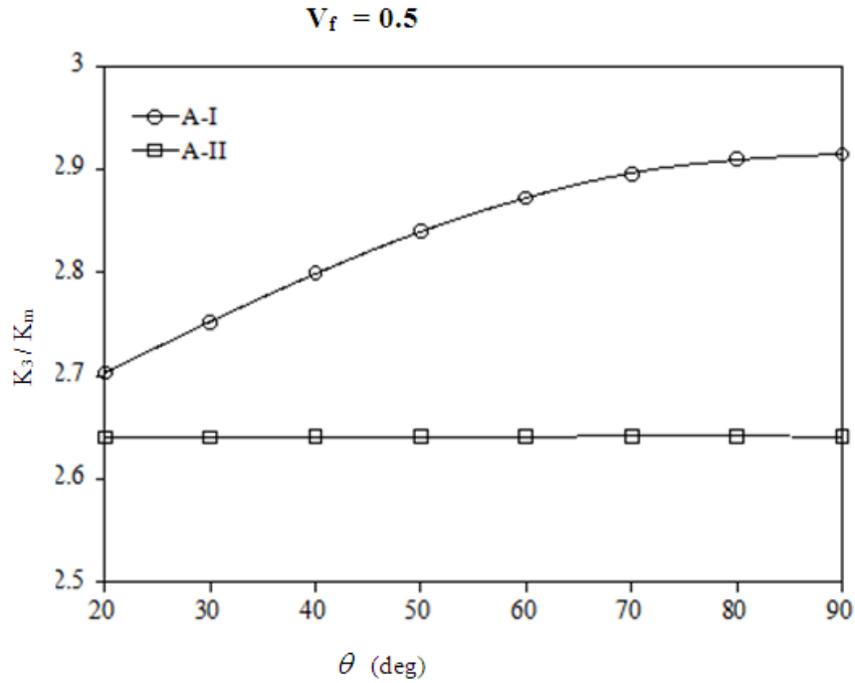


Figure 6. Effect of θ on through thickness conductivity at $V_f = 0.5$.

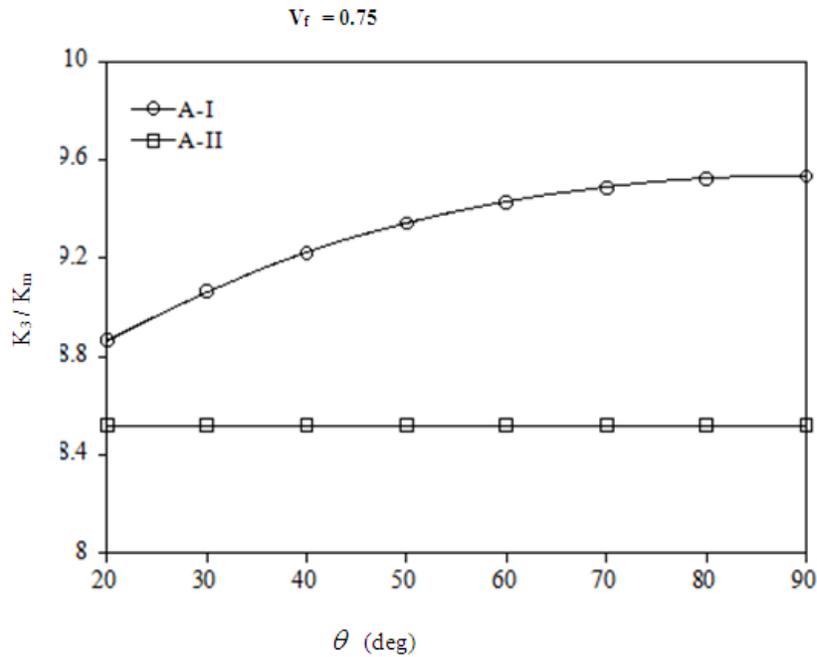


Figure 7. Effect of θ on through thickness conductivity at $V_f = 0.75$.

conductivity is varying from 4.8 to 8.7%. Up to a volume fraction of 0.35, the percentage deviation is showing increasing trend followed by a marginal decrease.

In Figure 9, percentage deviation of through thickness thermal conductivity with reference to second approach at various theta values for $K_f/K_m = 50$ is shown. It is

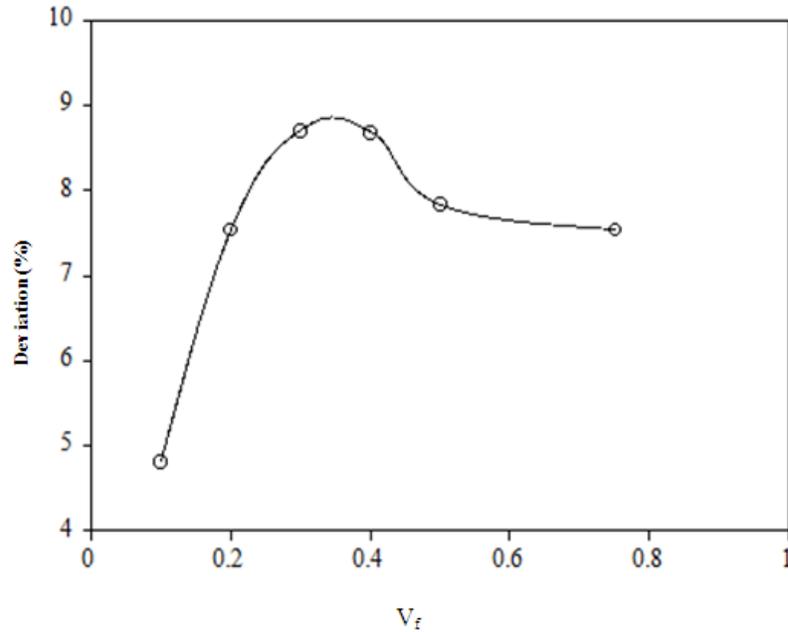


Figure 8. Deviation of K_3 (Approach-I) for $K_f/K_m = 50$ and $\theta = 90^\circ$.

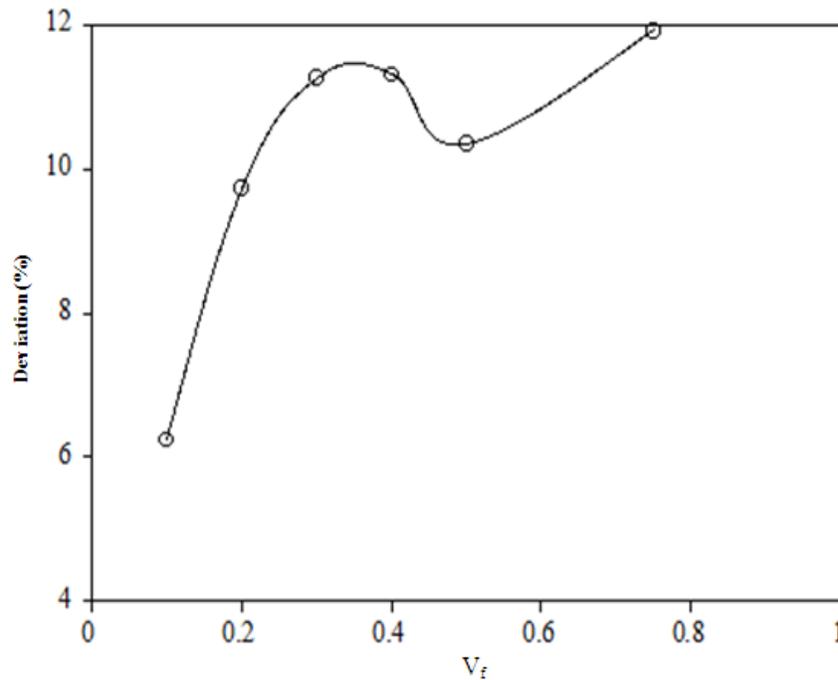


Figure 9. Deviation of K_3 (A-I&A-II) for $K_f/K_m = 50$ and $\theta = 90^\circ$.

observed that the percentage deviation is varying from 6.2 to 11.9%. Initially, the percentage deviation increases up to a volume fraction of 0.35, then decreases up to a volume fraction of 0.5 and then increases steadily beyond $V_f = 0.5$.

The variation in the percentage difference in transverse thermal conductivity with either the theta or the approach is attributed to the presence of internal anisotropy in the first approach.

Conclusions

An attempt is made to compare the through thickness conductivity of FRP composite obtained in two different approaches available in the literature. It is evident from the above results that there is considerable deviation in the results obtained from the two approaches for the range of V_f of 0.1 to 0.75 and $K_f/K_m = 50$. This difference is attributed to the assumption of negligible internal anisotropy in the first approach.

Conflict of Interest

The authors have not declared any conflict of interest.

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