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Full Length Research Paper

Propagation of a pulsed laser through a plasma based waveguide with a parabolic profile of the electron density

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The paper presents a numerical study of the periodic self focusing behavior of a laser pulse propagation in a gas filled capillary plasma with a parabolic electron density profile. An implicit and inherently stable Crank-Nicholson method was used to demonstrate the periodic laser beam waist behavior inside the tube discharge. The dependence of the tube radius and beam waist on maximum intensity delivered to the waveguide and its influence on the modulated Rayleigh length was discussed.

Key words: Plasma waveguide, pulsed laser, parabolic electron density.

INTRODUCTION

The plasma-based accelerators such as laser wakefield accelerators (LWFA) have shown much interest in both theory (Cormier-Michel et al., 2011; Pathak et al., 2012; Vieira et al., 2012) and experiment (Gonsalves et al., 2011; Mo et al., 2012). In the original resonant LWFA concept (Tajima and Dawson, 1979; Sprangle et al., 1996; Esarey et al., 1996; Gorbunov and Kirsanov, 1987), a laser pulse of duration smaller than a period of plasma oscillation $\tau_p = 2\pi/\omega_{p0}$ excites a wake electron plasma wave (wakefield) with a phase velocity close to the speed of light. $\omega_{p0} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the electron plasma density, e and m_e are the charge and electron mass at rest. Electron plasma waves can be thought of as simply oscillating displacements of free electrons in the plasma from the neutralizing background of slower moving positively charged ions. These displacements of electrons give rise to large axial electric fields $E_0=m_e c\omega_{p0}/e$ (Tajima and Dawson, 1979).

In a uniform plasma, a laser beam, with a frequency of ω_{L0} can guide itself by the effect of relativistic self-focusing, provided the laser power exceeds the critical power $P_c = 17(\omega_{L0}/\omega_{p0})^2$ [GW] (Hafizi et al., 2000; Mori, 1997). After propagating for a distance known as the "dephasing length" (Hubbard et al., 2001) the electrons outrun the wake. This limits how far they can be accelerated and thus limits their energy. To increase the dephasing length requires lowering the plasma density, but at the same time the collimation of the laser beam must be maintained over the longer distance. However, when the laser power is smaller than this critical power, beam diffraction dominates relativistic self-focusing and ponderomotive self-channeling. In order to overcome the limitations imposed by diffraction and refraction, it is necessary to channel or guide the laser beam in some way. For low laser intensities this is routinely achieved using optical fibers, or related waveguide structures, in which radiation is guided through a core surrounded by a cladding layer of lower refractive index. This approach cannot be directly applied to lasers with the high intensities over 10^{18} W/cm^2, since the laser radiation would destroy the material of the core. Therefore, a preformed plasma channel, as a gradient refractive index waveguide, is usually used to guide the laser beam. A plasma waveguide suitable for guiding has a parabolic electron density profile $n_e(r) = n_e(0) + \Delta n_e\left(\frac{r}{r_{ch}}\right)^2$ (Spence et al., 2001; Bendoyro et al., 2008), where Δn_e is the difference in electron density in the middle of the channel, $n_e(0)$, and at the side of the channel, r_{ch}.

A hydrogen-filled capillary discharge is an example of such plasma waveguide (Bendoyro et al., 2008; Karsch et al., 2007). This is achieved by passing a slow-rising discharge current through a narrow capillary that is initially filled with hydrogen gas. The discharge heats and ionizes the gas to form a plasma, and an approximately parabolic radial electron density profile is formed. A refractive index profile of this form will cause the phase velocity to increase with *r* and hence the wavefronts of an initially-plane wave will become convex when viewed from the source. In other words, the channel will counteract diffraction and refractive defocusing.

In this paper we described in detail a numerical method for propagation of laser pulse valid in the intensity range ($10^{17} < I < 10^{20}\ W/cm^2$). It includes the nonlinear index of refraction in a plasma waveguide. It calculates the propagation of a Gaussian laser beam in a parabolic plasma density at any focused spot size. Intense pulse guiding through plasma channel section presented below describes the procedure for driving the modified wave equation in inhomogeneous medium, as well as the numerical techniques, while the results and discussion section describe the simulations within conditions close to experimental ones.

INTENSE PULSE GUIDING THROUGH PLASMA CHANNEL

The wave equation for the laser electric field $\vec{E}(r,,t) = E(r,t)e^{i(k_L z-\omega_L t)}\hat{z}$ in an inhomogeneous medium can be written:

$$\nabla^2\vec{E}(r,t) - \frac{1}{c^2}\frac{\partial^2\vec{E}(r,t)}{\partial t^2} = \frac{\omega_L^2}{c^2}\left(1-\eta(r)^2\right)\vec{E}(r,t) \tag{1}$$

Where $\eta(r) = \sqrt{1-\frac{n_e(r)}{\gamma . n_{cr}}}$ is the index of refraction of plasma, and $n_{cr} = m_e\varepsilon_0\omega_L^2/e^2$ is the critical plasma density. $\gamma = \sqrt{1+a_0^2}$ is the relativistic factor associated with the electron motion transverse to the laser propagation. It depends on the normalized vector potential $a_0 = 0.85\lambda_L\sqrt{I}$, where the unit of laser wavelength λ_L is in µm and the local laser intensity has unit 10^{18} (W cm^{-2}) (Pukhov, 2003). Assuming the slowly varying envelope approximation (SVEA) of $E(r,t)$, $\frac{\partial^2 E(r,t)}{\partial t^2} << 2i\omega_{0L}\frac{\partial E(r,t)}{\partial t}$, and paraxial approximation, $\left|\frac{\partial E(r,t)}{\partial z}\right| << \frac{\omega_{0L}}{c}E(r,t)$, the equation (1) is simplified to:

$$\nabla_r^2 E(r,t) + \frac{2i\omega_L}{c}\frac{\partial E(r,t)}{\partial z} + \frac{2i\omega_L}{c^2}\frac{\partial E(r,t)}{\partial t} = \frac{\omega_L^2}{c^2}\left(1-\eta(r)^2\right)E(r,t) \tag{2}$$

Where ∇_r^2 denotes the transverse Laplacian in cylindrical coordinate.

Using a numerical method to solve equation (2) directly require an enormous amount of memory and processing power. A conventional technique for eliminating units and normalizing magnitudes is substituting with moving-frame variables, $\zeta = \frac{z}{(c/\omega_L)}$, $\tau = \omega_L t - \zeta\frac{\omega_L}{(k_L c)}$ and $\rho = \frac{r}{(c/\omega_L)}$ (Chernev and Petrov, 1992). The final result is:

$$\nabla_\rho^2 E + 2i\frac{\partial E}{\partial \zeta} - \frac{\omega_L^2}{c^2}\left(1-\eta(\rho)^2\right)E = 0 \tag{3}$$

A Gaussian laser beam propagating through a parabolic plasma channel can be matched such that it is guided without disturbances of the beam profile. Matching means that the Gaussian beam, which is coupled into the waveguide, has a waist w_0 that equals the matched spot size $w_m = \left(\frac{r_{ch}^2}{\pi\Delta n_e r_e}\right)^{1/4}$ of the plasma channel (Spence and Hooker, 2000). Here $r_e = e^2/(4\pi\varepsilon_0 m_e c^2)$ is the classic electron radius.

Our numerical simulation utilized a linearly-polarized laser where the pulse was focused onto the entrance of the capillary with a focused spot size of different diameters. It is worth mentioning that the channel guiding method described above may be adapted to laser wakefield acceleration. However, to be in the optimal regime for a standard LWFA, the resonance ratio $a = c\tau_L/\lambda_p$ in our simulation chosen to be ~0.5 to 10. Here, λ_p is the laser pulse duration, the nominal plasma wavelength $\lambda_p = 2\pi c/\omega_{p0}$, and ω_{p0} is the on-axis plasma frequency. If a and τ_L are specified, the on-axis density $n_e(0) = \frac{\pi m}{m_e}(\frac{a}{\tau_L})^2$. For example, for $\tau_L > 35fs$, we get $n_e(0) < 2.7\times10^{24}m^{-3}$. This agrees well with the experimental values of plasma source for extending the interaction lengths, especially in LWFA experiments. To solve this equation, an implicit and inherently stable Crank-Nicholson method was used (Crank and Nicolson, 1947). This paper contains a series of simulations that the objective is to analyze and discuss the different scenario at which the laser pulse could propagate in longer distance with low dissipation in intensity. The simulation conditions and the physical parameters are based upon the recent experimental works (Wang et al., 2005; Gonsalves et al., 2007).

RESULTS AND DISCUSSION

Figure 1 shows an example of propagation of a Gaussian laser beam in a parabolic plasma density with n_{e0} = 2.7 × 10^{24} m^{-3}, Δn_e = 1.3 × 10^{24} m^{-3} and r_{ch} = 150 µm where the matched spot size is about w_M = 37.5 µm. The laser pulse at 0.8 µm has an intensity about I = 5 × 10^{17} W cm^{-2} (a_0 ≈ 0.5).. For a waist of 50 µm, which is larger than w_M, we have the propagating Gaussian beam initially focused before defocused (red solid line), and for a reverse case of waist 25 µm the focusing take place after defocusing (blue dash line). The periodic focusing and defocusing is the result of parabolic profile of the plasma density with

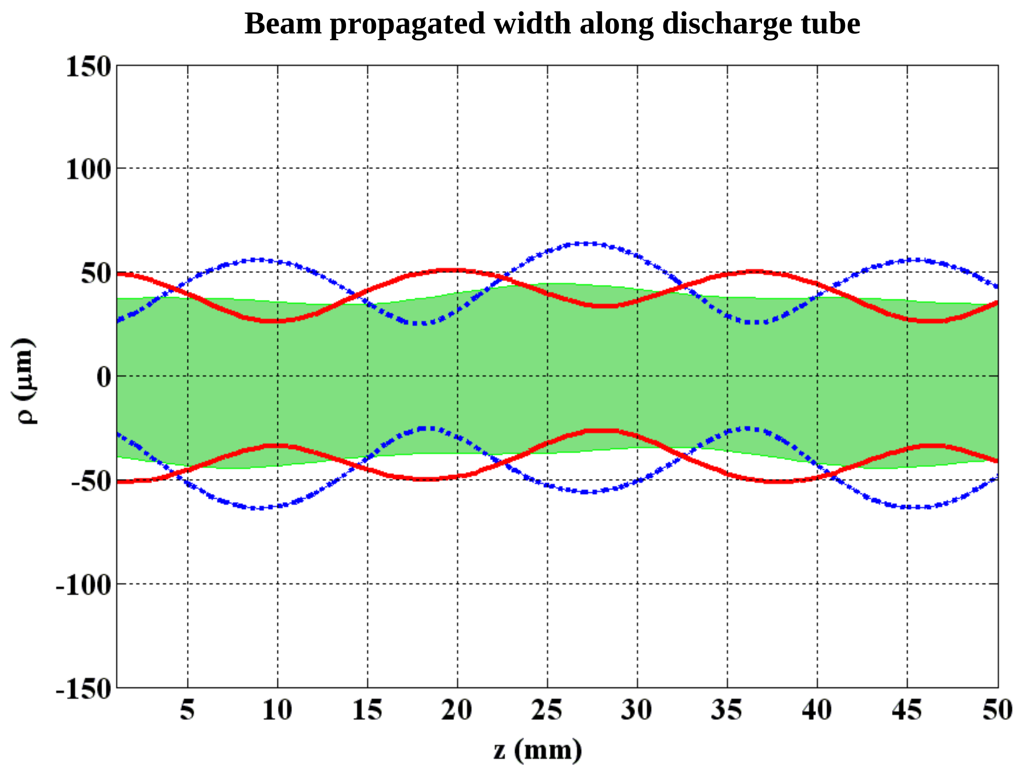

Figure 1. Simulation results for propagation of the Gaussian beam of $a_0 = 0.5$. Match spot size of w_M = 37.5 µm corresponds to the parabolic density $\eta(r) = 0.9992(1 - 1.47\times10^4 r^2)$. The green zone shows the propagation of laser pulse with spot w_0 = 37.5 µm. Dash line (blue) corresponds to w_0 = 25 µm. solid line (red) corresponds to w_0 = 50 µm.

a modulation Rayleigh length for the matched spot size λ_m = 17.3 mm where the intense laser pulse can be guided along the discharge tube without an extreme diffraction.

These simulation values confirm the experimental results of Spence et al. (2001). In the case of $w_0 \approx w_M$ (filled green space), the variation of spot size is extremely low and laser pulse intensity is almost constant along the discharge tube. In fact, in this case, the diffraction of the laser beam is compensated by the refractive properties of the channel and the beam propagates without change of its size. As is obvious from this figure, the peak of waist is smaller than the radius of discharge channel. Changing the simulation parameters could raise this peak where its maximum could be reached at side of discharge tube. All simulations have been done until the peak oscillation do not exceed the radius of discharge tube. It should be noted that the plasma channel acts as a quadratic phase modulator (Spence et al., 2003) and transforms the Gaussian beam at the entrance of the capillary into another Gaussian beam at the exit. Most experimental and simulation works of channeled LWFA with parabolic electron density waveguide have concentrated on the guiding of the laser pulse intensity at 10^{17} to 10^{18} w/cm^2 (Bobrova et al., 2001; Butler et al., 2004). Using the higher intensity can potentially improve the laser accelerator performance. Here, we seek the ways to enhance the capability of capillary discharge for guiding the higher laser intensity.

Dependence on wavelength

As pointed out before, the refractive index of plasma is coupled to the laser intensity (a_0). For a given plasma waveguide of radius r_{ch} = 100 µm (w_M = 30.6 µm), the maximum intensity delivered to the system is limited by laser spot size. Figure 2a illustrates this dependence. For all case of wavelengths, the maximum intensity delivered to waveguide is approximately constant. These values of a_0 are about 0.1, 3.5, 8 and 13, corresponding to w_0 = 10, 20, 30 and 40 µm, respectively. But at the same time, the modulated period length for each laser spot size is diminished when the wavelength is increased. The same results were obtained for the radius of 150 and 200 µm (Figure 2b). In addition, these results can be interpreted that the different radius of tube with the same electron density profile does not change enormously the maximum laser intensity propagated along the tube.

However, the use of the larger radius enhances the

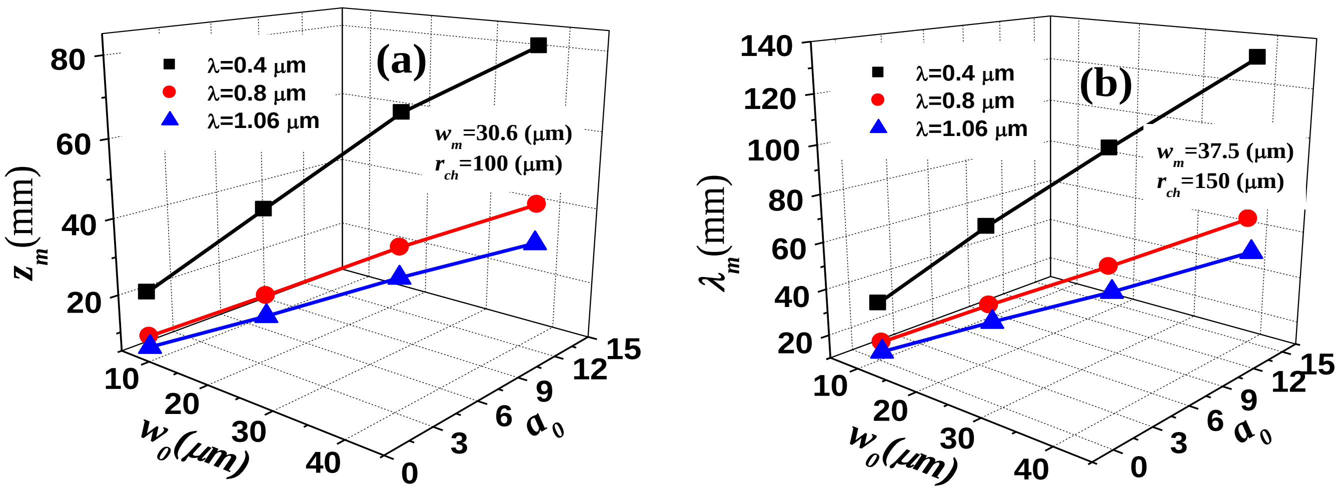

Figure 2. Dependence of maximum laser intensity guided in capillary parabolic discharge tube with laser spot size and wavelength. (■), (●) and (▲) indicate the maximum normalized intensity at wavelengths 0.4, 0.8 and 1.06 µm respectively. (a) r_{ch} = 100 µm, w_M = 30.6 µm, (b) r_{ch} = 150 µm, w_M = 37.5 µm.

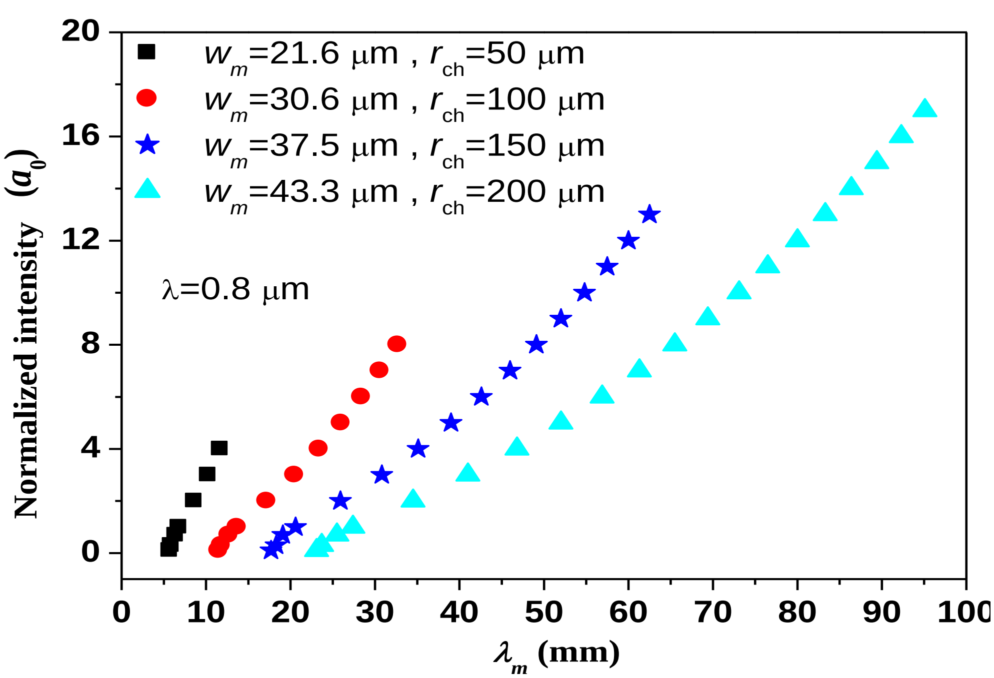

Figure 3. Dependence of maximum laser intensity guided in capillary parabolic discharge tube with respect to the tube radius and modulated Rayleigh zone. (■), (●) , (*) and (▲) indicate the maximum normalized intensity at radius 50, 100, 150 and 200 µm, respectively.

modulated length λ_m.

Dependence on geometry of the channel

The properties of the plasma established by the capillary discharge are tube geometry dependent, especially its radius. Electron density profile inside the tube can be made by adjusting the radius of channel, the electrical current and the gas pressure inside the tube. Any change of the radius of the tube discharge can affect the pulse propagation profile. The behavior of the laser pulse inside the plasma waveguide with changing the radius of tube are presented in Figure 3. The parameters used here are the same as above. For the simulation, the spots radius of the laser $w_0 \approx w_M$ was used. This figure illustrates the dependence of maximum laser intensity injected inside the waveguide with respect to modulated Rayleigh length λ_m. These are about $a_0 = 4$, 8, 13 and 17, corresponding to λ_m =11.7, 32.7, 62.5 and 95.1 µm respectively. To estimate the intensity fluctuation inside the tube, one should also consider that the maximum peak amplitude of the laser spot propagation along the discharge tube can reach the tube radius. The fluctuation ratios with respect to the initial laser intensity, $I'/I \approx (w_0 / r_{ch})^2$, are about 1.9×10^{-1}, 9.4×10^{-2}, 6.2×10^{-2} and 4.7×10^{-2} for the intensities 3.5×10^{19} ($a_0 = 4$), 1.9×10^{20} ($a_0 = 8$), 3.6×10^{20} ($a_0 = 13$) and 6.2×10^{20} ($a_0 = 17$) (w cm^{-2}), respectively in Figure 3. This means that the maximum intensity reduction happens in the case of $a_0 = 17$, but taking into account the initial intensity delivered to the waveguide, the intensity I' at waist of r_{ch} is about 2.9×10^{19} (w cm^{-2}). This results can be compared to the other values of $I' = 6.4 \times 10^{18}$, 1.3×10^{19} and 2.3×10^{19} (w cm^{-2}) for the normalized intensity values $a_0 = 4$, 8, and 13, respectively. In spite of the large difference between initial and the dissipated intensity at higher tube radius, the propagated laser pulse along the whole discharge tube remains relatively intense.

Conclusion

In this paper, several features of intense, short-pulse laser propagation in gas-filled capillary discharge waveguide are reviewed, discussed, and analyzed. When the parabolic shape of electron density is used, the natural defocusing of laser beam can be compensated by the focusing of laser beam due to the plasma refractive index profile. In this case, we can observe the periodic self focusing behavior along the discharge tube. For a different radius of tube, the maximum intensity delivered to waveguide is relatively constant for all laser beam waist. If the spot size equal to modulated spot size is used, the relative intensity fluctuation increases by increasing the tube radius. At the same time, the maximum intensity inside the tube remains high enough to use in laser wake field acceleration works.

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