

*Full Length Research Paper*

# Online inter-area oscillation monitoring in power systems using PMU data and Prony analysis

B. Mohammadi\*, A. Rabiei, M. Moradi-Dalvand and S. Mobayen

Department of Engineering, Abhar Branch, Islamic Azad University, Abhar, Iran.

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**In this paper, signals measured by phasor measurement units (PMUs) are used to monitor power system small signal stability. In the proposed method, proper signals selected based on the modes observability and Prony analysis is employed to extract signal damping information (amplitude and frequency of participated modes) in real-time. The proposed method is simulated using a test signal in ideal and noisy environment. Besides, the method is tested on a two-area four machines power system. Prony analysis results are compared with the results of modal analysis that based on the detailed model of power system. Simulation results imply the effectiveness of the proposed method in online monitoring of inter-area oscillations.**

**Key words:** Inter-area oscillations, phasor measurement unit (PMU), Prony analysis.

## INTRODUCTION

Rapid growth of communication technologies and signal processing techniques has improved both power system control and operation systems. Phasor measurement units (PMUs) introduced in early 1980s in Virginia University (Phadke, 2002). PMU has the capability of measuring and sending bus voltage and current phasors of all connected branches. Several applications of PMUs in power systems have been reported by researchers and power system operators in recent years (Phadke and de Moraes, 2008; Phadke et al., 2009). Power system stability analysis and state estimation have been studied (Nuqui and Phadke, 2005; Mohammadi-Ivatloo and Hosseini, 2008; Phadke et al., 2009). The signals measured by PMUs have been used to improve fault detection in power system (Ying-Hong et al., 2004a, 2004b), wide area power system protection (Wang et al., 2005), transient stability analysis (Chunyan et al., 2007; Hashiesh et al., 2008) and steady state angle stability (Yue et al., 2005). High data communication speed and high accuracy of PMU made it a good choice for near real-time stability analysis in power system. The maximum error of signal amplitude measurement is 0.1% and this value for phase measurement is 0.02°

(Depablos et al., 2004). The major small signal stability analysis of power systems is modal analysis method which is performed offline. In this method a linear model for the system is derived using linearizing the model equations around the operation point and the system eigenvalues are calculated using QR algorithm or other algorithms developed for large scale systems (Kundur, 1996). This method is based on the exact model of power system and as noted earlier, is performed offline. The drawback of the offline models is that these models only work for a limited range of operating points. On the other hand, power system experiences variations in both structure and operating point. Besides, the exact parameters of a power system may not be available in general.

Spectral analysis methods can be used to monitor power system small signal stability status in near real-time. Prony analysis is a powerful method that can be used to extract dynamic features of measured signal (Hauer, 1991). Other signal processing methods have been used to model identification of power system signals like minimal realization approach (Kamwa et al., 1993), Hankel matrix method (Sanchez-Gasca and Chow, 1997), matrix pencil (Guoping et al., 2007) and Hankel total least squares method (Guoping et al., 2007). Most of these methods have been employed before the introduction of PMU devices and used local

\*Corresponding author. E-mail: [behinamno@gmail.com](mailto:behinamno@gmail.com).

measurement devices to measure output signals of a single synchronous generator. Spectral transformation of Advanced Space-borne Thermal Emission and Reflection Radiometer (ASTER) remote-sensing data of a semi-desert realm has been investigated in (Beiranvand-Pour and Hashim, 2011). Dynamic structural responses in multi-storey buildings have been studied by (Saiful Islam et al., 2011). Optimal estimation of the target state is obtained using extended Kalman filter by (Jin-long Yang and Ji, 2011). Multivariate analysis techniques have been proposed to the monitoring of inter-area oscillation frequency and damping (Thambirajah et al., 2011).

This paper employs Prony analysis to monitor power system small signal stability by extracting signal damping information (amplitude and frequency of participated modes) in real-time. The signals that are used to small signal stability analysis are measured by PMUs.

**PRONY ANALYSIS**

Here, the mathematical formulation of Prony analysis is derived. Prony analysis is a signal processing technique that estimates damping features (frequency, damping, phase angle and amplitude) of measured signal (Trudnowskind et al., 1999). It is possible to measure the phase angle difference of two ending buses voltages of a tie line and extract its oscillation information using Prony analysis.

The output signal of a dynamic system can be expressed as sum of exponential function corresponding to its eigenvalues. The estimation of output signal  $y(t)$  can be expressed as:

$$\hat{y}(t) = \sum_{i=1}^n B_i e^{\lambda_i t} \tag{1}$$

where  $\lambda_i$  and  $B_i$  are complex values and  $B_i$  is an output residual for  $\lambda_i$  eigenvalue. The order of system is  $n$ . The estimation of observed data ( $y(t)$ ) is denoted by  $\hat{y}(t)$ . If the measured signal sampled with  $T$  sampling rate (which is more than Nyquist sampling rate), then Equation 1 could be rewritten as:

$$\hat{y}(kT) = \sum_{i=1}^n B_i z_i^k, \quad k = 0, \dots, N-1 \tag{2}$$

Where  $z_i = e^{\lambda_i T}$  is the discrete pole of the system and the objective is to estimate the  $z_i$  and  $B_i$  in a way that Equation 3 is satisfied; that is, the error between measured and estimated signals is minimized.

$$\hat{y}(kT) = y(kT), \quad k = 0, \dots, N-1 \tag{3}$$

By defining  $\hat{y}(k) = \hat{y}(kT)$  and  $y(k) = y(kT)$ , Equation 3 can be extended using Equation 2 as follows:

$$\begin{aligned} y(0) &= B_1 + B_2 + \dots + B_n \\ y(1) &= B_1 z_1 + B_2 z_2 + \dots + B_n z_n \\ y(2) &= B_1 z_1^2 + B_2 z_2^2 + \dots + B_n z_n^2 \\ &\vdots \\ y(N-1) &= B_1 z_1^{N-1} + B_2 z_2^{N-1} + \dots + B_n z_n^{N-1} \end{aligned} \tag{4}$$

The aforementioned equation can be written in matrix form as follows:

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} z_1^0 & z_2^0 & \dots & z_n^0 \\ z_1^1 & z_2^1 & \dots & z_n^1 \\ \vdots & \vdots & \vdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} \tag{5}$$

For obtaining the values of  $z_i$ , it is required that  $N$  be greater than  $2n$  ( $N \geq 2n+1$ ). It is assumed that  $z_i$ s are roots of a polynomial function  $f(z)$  known as the following characteristic polynomial.

$$\begin{aligned} f(z) &= (z-z_0)(z-z_1)\dots(z-z_n) = \\ z^n - \alpha_1 z^{n-1} - \alpha_2 z^{n-2} - \dots - \alpha_{n-1} z - \alpha_n &= 0 \end{aligned} \tag{6}$$

By multiplying first row of Equation 5 with  $-\alpha_n$ , second row with  $-\alpha_{n-1}$ , ..., and last row with 1, and the summation of all rows we obtain:

$$y(n) = \alpha_1 y(n-1) + \alpha_2 y(n-2) + \dots + \alpha_n y(0) \tag{7}$$

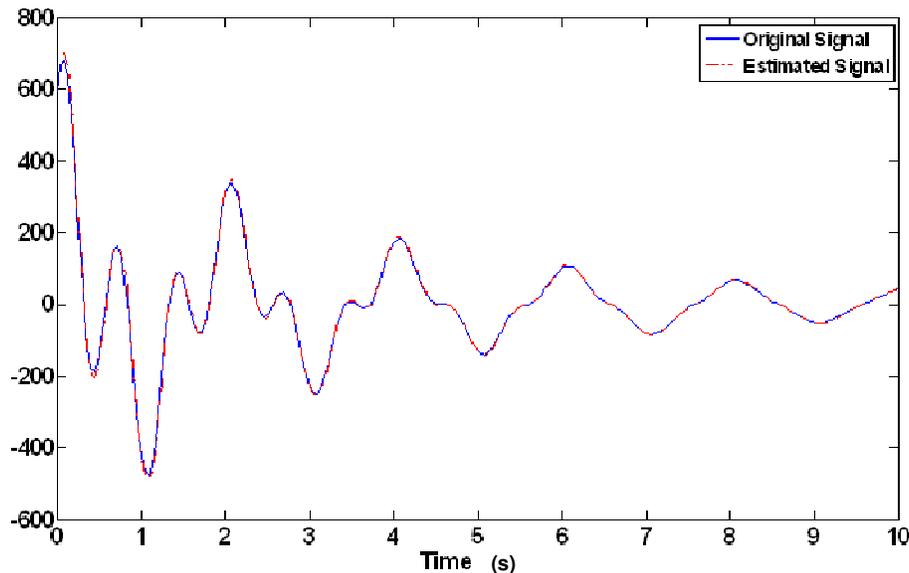
This process can be repeated with starting from second row and so on. Hence, we can obtain  $N-n-1$  linear equations for  $\alpha_i$  which can be written in matrix form as follows:

$$\begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} y(n-1) & y(n-2) & \dots & y(0) \\ y(n) & y(n-1) & \dots & y(1) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-2) & y(N-3) & \dots & y(N-n-1) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \tag{8}$$

This step results to linear prediction model. The aforementioned equation will have single solution for  $N = 2n+1$  and for  $N > 2n+1$ , values of  $\alpha_i$  will be estimated by solving the over-determined least square problem. After estimating the values of  $\alpha_i$ , the values of  $z_i$  will be determined by substituting  $\alpha_i$  values in Equation 6. After determination of  $z_i$  values and substituting them into Equation 5, the values of  $B_i$  can be calculated.

**Table 1.** Parameters of fourth order ideal signal.

Phase (radian)	Frequency (Hz)	Damping ( $\alpha$ )	Amplitude	Mode
1.32	0.5	0.2	150	1
0.86	1.5	0.5	200	2

**Figure 1.** Original fourth order ideal signal and estimated signal.

## SIMULATION RESULTS

In order to evaluate the performance of Prony analysis in estimation of signals' modes, three different cases are studied. In the first case, an ideal signal is selected for estimation purpose. In the second case, the effect of noise on measured signal is studied. In the third case, a real power system test case is studied. The sampling frequency of the Prony analysis is selected in the range of two times to three times of the largest frequency of the signal. The system order should not be higher than one third of the sampled data.

The selected ideal signal order is 4 and has two different modes. Parameters of the studied ideal signal are presented in Table 1.

Original ideal signal and estimated signal are depicted in Figure 1. It can be observed that these two signals coincide with each other, which shows the accuracy of estimation procedure.

In the second case, a Gaussian white noise is added to signal presented in Table 1 and Prony analysis is implemented. The noisy and estimated signals are shown in Figure 2. It can be observed that Prony analysis has a good performance in estimation of noisy signal.

In order to evaluate the performance of Prony analysis

in estimation of inter-area oscillations in power systems, 2-area four-machine test system is studied. The single line diagram of this system is shown in Figure 3.

Sub-transient model is used to model generators and all generators are equipped with exciter systems and governors. More detail of system model and parameters can be found (Kundur, 1996). In order to simulate data measured by PMUs, a three phase fault is implemented in line 3-101 at  $t=1$  s. The fault is removed after 0.1 s with removing the faulted line. Results of mode Observability analysis show that inter-area oscillations are more observable in tie line power (Kundur, 1996). Hence, tie line power signal is used to estimate inter area oscillations in this study. Figure 4 shows the measured signal and estimated signal for this case. It can be observed that the estimated signal detects and follows the measured signal damping characteristics. Small signal stability analysis is also done to obtain inter-area oscillation. The order of the system in this analysis is 60 and the 20th mode of the system is an inter-area mode. This can be confirmed by analysis of rotor angles mode shape. Figure 5 shows the oscillations of rotor angles in this mode.

Table 2 compares the eigenvalues of inter-area mode obtained using Prony analysis and small signal stability analysis of full order system.

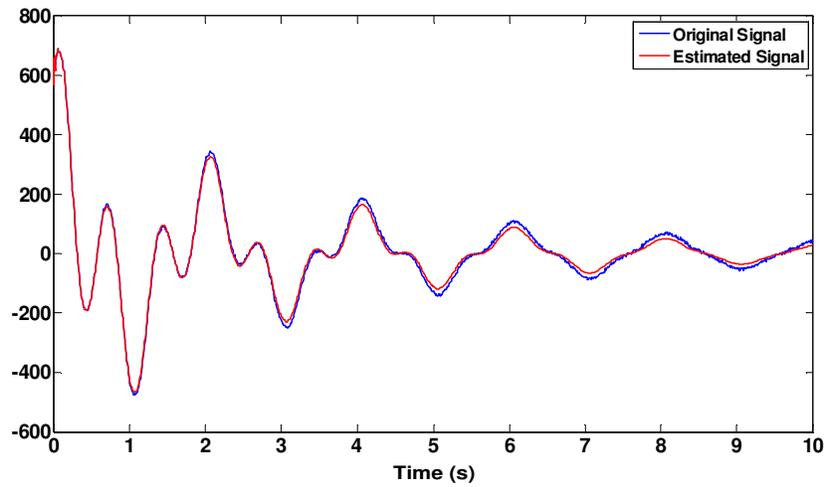


Figure 2. Noisy fourth order signal and estimated signal.

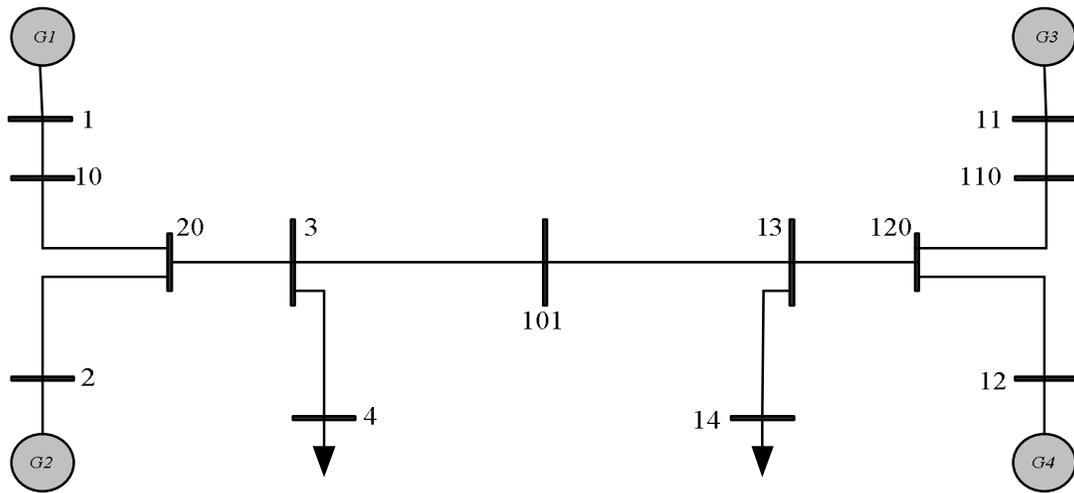


Figure 3. Single line diagram of 2-area four machine test system.

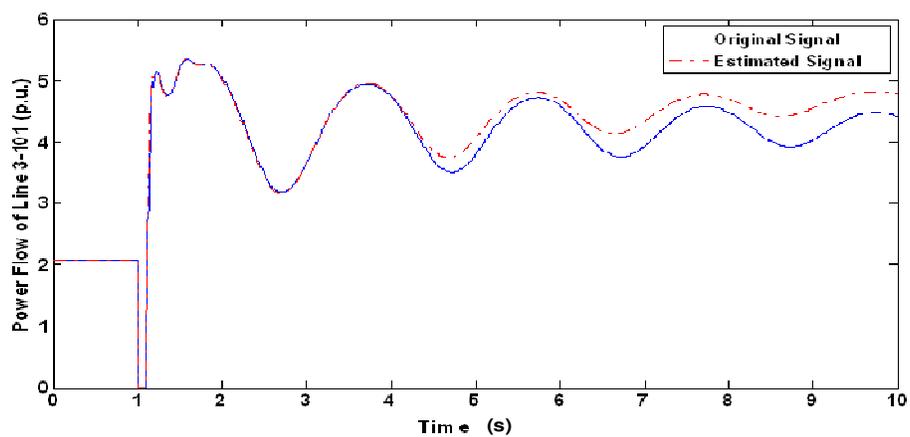
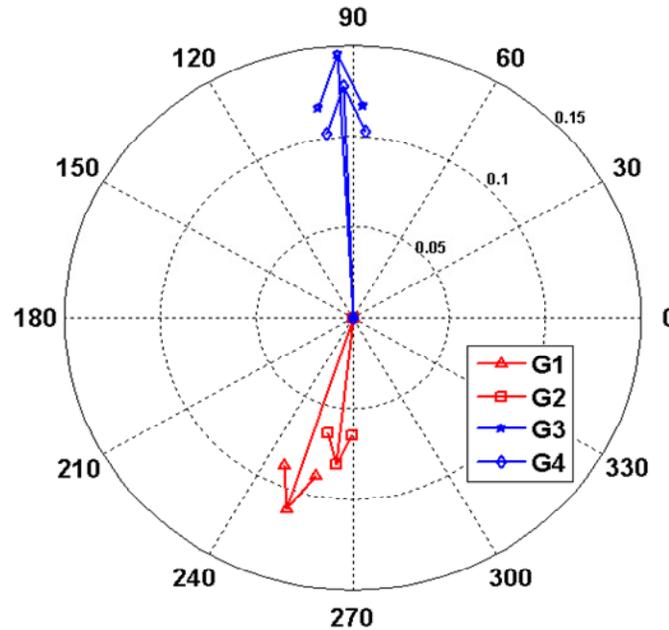


Figure 4. Measured tie line power and estimated signal.



**Figure 5.** Amplitude and phase of 20th mode oscillations in generators' rotors.

**Table 2.** Comparison of inter-area modes obtained using Prony analysis and small signal stability analysis.

Damping ( $\alpha$ )	Inter-area mode frequency (Hz)	Method
0.2802	0.5048	Prony
0.2304	0.6342	Small signal

**Conclusion**

In this paper, Prony analysis is implemented to monitor power system inter-area oscillations, which is one of the main limitations of tie line operation. The system signals are measured with high accuracy and speed by PMUs. Dynamic features of measured signals (such as damping and frequency) are extracted using Prony analysis. With online monitoring of inter-area oscillations, it is possible to obtain stability margin in real-time and hence, to use tie lines capabilities optimally. The proposed method is implemented on an ideal signal, noisy signal and two area power system test case. The results obtained by Prony analysis are compared with both original signals and small signal stability (that is, modal analysis) result. Numerical studies emphasize both suitability and effectiveness of the proposed online small signal stability monitoring approach for real-time operation of power systems.

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