

*Full Length Research Paper*

# Energy- angular correlation of medium energy particles produced in heavy Ion collisions

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The information carried by the medium energy nucleons (gray particles) emitted in heavy ion collisions is studied. A canonical thermo dynamical model is proposed to deal with the interacting nuclei as nucleon gas system. The predictions of the model are compared with a complete set of measured data for the reactions of proton, helium, carbon and neon nuclei with the nuclear emulsion at 4.2 A GeV. It is found that the angular distribution of the knocked on nucleons is almost independent of the projectile mass. A strong correlation between the energy of the gray particles and their emission angles is found. The gray particles emitted in the forward hemisphere show linear correlation with their average kinetic energy which reflects that these particles are emitted in non-equilibrium states. On the other hand, backward emission shows uniform behavior that represents particle emission in an almost equilibrium state.

**Key words:** Heavy Ion Collision, thermodynamic model, energy- angle correlation PACS: 25.70.Pq.

## INTRODUCTION

The nuclear emulsion is a good tool in dealing with high energy nuclear reactions. It has the ability to detect and identify particles in the outlet channel of the reaction. The dynamic characteristics of the reaction can be determined by precise measurement of the angular distribution, the energy spectra, as well as the charge distribution of the produced particles all of which carry information about the mechanism of the interaction. Unfortunately, the measurements of the energy of charged particles is a tedious work and requires pursuing their path through the emulsion plates for enough long distance to get accurate results. On the other hand it is possible to measure the angular distribution to a high extent of accuracy. In this intellect we believe that finding a kind of correlation between the angular and energy distribution of the emitted particles will be good evidence in improving the performance of using the emulsion as a tool in determination of the energy spectra of the produced particles. In this work we aim to provide a scenario that discusses what happens inside the strong

interacting nuclear matter and link between the energy and angular behavior of the emitted particles in the terminology of a thermodynamic model. The predictions of the model will be compared with the experimental data of the reactions of proton, Helium, Carbon and Neon nuclei of the emulsion target at 4.2 A GeV incident energy where the energy of the gray particles are well measured.

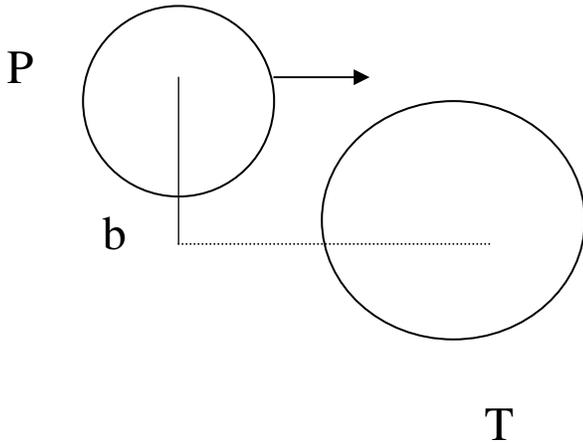
## Experimental Work

An emulsion stack of NIKFI BR-2 of size 20 cm x 10 cm x 600 microns was exposed to the beams of  $^{24}\text{Ne}$ ,  $^{12}\text{C}$ ,  $^4\text{He}$  and proton nuclei at momentum of 4.5 A. GeV/c at the Dubna Synchrophasotron (Russia). The nuclear emulsion serves as a track sensitive target consists of H, C, N, O, Ag and Br nuclei. The effective charge and the effective mass number of the emulsion are given by:

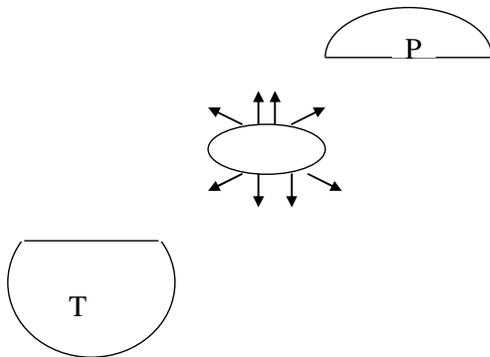
$$Z_{\text{eff}} = \frac{\sum_i n_i Z_i \sigma_i}{\sum_i n_i \sigma_i} \quad \text{and} \quad A_{\text{eff}} = \frac{\sum_i n_i A_i \sigma_i}{\sum_i n_i \sigma_i}, \quad \sigma_i \text{ is}$$

the cross section of the  $i^{\text{th}}$  atom.

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**Figure (1-a)** An interaction projectile **P** incident to a Target **T** at a given impact parameter **b**



**Figure (1-b)** The formation of the hot spot (participant nucleons in the overlap region) and the spectators from both the target and the projectile.

Area scanning is carried out to specify the location of the interactions. The out going ionizing particles are carefully examined by measuring their emission angle, range, and the relative ionization density  $g^*$ . The emitted particles are classified according to the following criteria

- i. Shower tracks have relative grain density  $g^* \leq 1.4$  corresponding to velocity  $\beta \geq 0.7$ . The shower particles produced in an interaction at high energy, are mainly pions with small amount of fast protons, charged k-mesons, antiprotons and hyperons.
- ii. Grey tracks have relative grain density  $1.4 < g^* < 10$ , velocity  $0.3 < \beta < 0.7$  and range  $R > 3$  mm. Most of the grey tracks result from

recoiling protons, which have energy between 30 and 400 MeV

- iii. Black tracks have relative grain density  $g^* \geq 10$ , velocity  $\beta \leq 0.3$  and range  $R \leq 3$  mm. The black tracks are produced by the evaporation of the excited residual nucleus.

**Particle identification** is always executed by measuring a couple of parameters ( $g^*$ -momentum), (mean energy loss-momentum) and (delta ray counting-range). Momentum of the charged particles is measured by the multiple Coulomb- scattering method [1]. The energy loss of charged particle passing through specific medium can be calculated by using a special computer program called (SRIM) [2], which is a group of programs to calculate the stopping and range of ion in energy (10eV – 2GeV/amu) into matter using a full quantum mechanical treatment of ion-atom (medium) collisions [3].

**The Formulation of the Model**

It is assumed that during the collision of fast projectile nucleus **P** with a target **T** at a given impact parameter **b**, a large amount of energy is transferred from the projectile to the target and the nucleons of both nuclei diffuse through each other. It is plausible to work with a parameter that defines the fraction of the projectile nucleons in the formed nuclear gas system as:

$$\eta(b) = \frac{\rho_p(b)}{\rho_p(b) + \rho_T(b)} \tag{1}$$

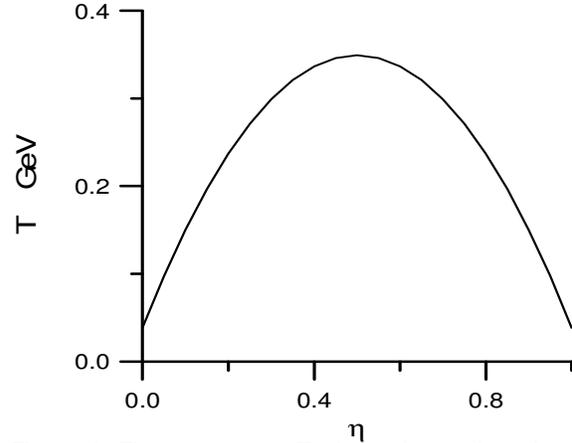
where  $\rho_p(b)$  and  $\rho_T(b)$  are the projectile and target densities at a given impact parameter **b** in the formed nuclear matter.  $\eta$  has continuous values extending from zero to 1. It is zero in the pure target region and goes to 1 as it approaches the projectile region. It is possible to imagine three separate regions without clear borders as shown in Figure(1). These are the projectile spectator, the target spectator and an overlap region. The parameter  $\eta$  plays an important role in understanding the physics inside each part of the interacting medium. The quantity of energy transferred and the activity of nuclear collisions whether it was a strong collision or even elastic or Coulomb dissociation is controlled by the value of  $\eta$ .

**The projectile spectator region** is characterized by small momentum transferred that is enough to dissociate the projectile into few fragments moving in the forward direction or scattered by relatively small angle. Simple elastic scattering [4] assuming optical potential [5], diffraction [6] and Coulomb dissociation [7] models are sufficient to describe the fragmentation process and the angular spread of the emitted fragments in this region.

**The target spectator region:** The nucleons in this region are initially at rest. As the collision starts up, nucleons from the projectile defuse slowly through the target transferring a small fraction of the projectile energy. The diffusion rate depends mainly on the impact parameter. The system then behaves as perfect gas that suffers multiple of successive elastic scattering. Consequently the entropy of the system increases until it reaches equilibrium state, with an equilibrium temperature of the order of 30 MeV. At this moment the system evaporates [8] producing heavily ionizing fragments that appear as black particles with isotropic distribution in the space. In most cases it was sufficient to describe the energy distribution of the evaporated particles with a unique Maxwell distribution of classical distinguished particles.

**The hot spot region:** The overlap region between the projectile and the target that characterizes with  $\eta \approx 0.5$  is the hottest region in the space. A large amount of heat is dissipated at that point. The nuclear matter goes through different stages. In the early one a sudden compression occurs to the nuclear matter accompanied by an increase in matter density and the production of large amount of center of mass energy. The environment is now adequate for the formation of a quark-gluon plasma phase [9]. Many quarks-antiquarks are being created followed by a recombination process. The created quark pairs form what are called sea quarks. Neighboring quark-antiquark may recombine again forming meson [10]. Successive collisions go on producing more newly created particles and hence the system expands again until the collisions cease. If we treat the system thermodynamically [11], it is expected that fast light particles be produced in the early stage in the forward direction. As time goes on, the system is subjected to successive collisions each of them followed by creation of bunches of newly produced particles emitted in a wider emission angle. Finally the nucleons are emitted individually or rather in a cluster or fragment form. The singly charged fragments are emitted as knocked on nucleons with medium energy range ( $40 \text{ MeV} < E < 400 \text{ MeV}$ ) and appear in emulsion plates as gray particles [12].

In the present work we are interested in these gray particles. We treat the nuclear matter as a nonequilibrium system. Each point in the space is considered as local equilibrium subsystem behaves as a canonical ensemble that is characterized by a specific temperature and a specific projectile fraction parameter  $\eta$ . The overall distribution of the gray particles is found by the superposition of particles produced over the assembly of the different subsystems covering all the range of  $\eta$ . It is also assumed that particles are emitted isotropically in the center of mass of each ensemble showing Maxwell Boltzmann distribution for classical particles, Fermi-Dirac for fermions and Bose-Einstein for bosons [13]. On the other hand, since the center of mass itself is moving with a velocity  $\beta_{cm}$  with respect to the Lab system related to its



**Figure 2.** The temperature  $T$  of the thermo-dynamic system as a function of the projectile fraction  $\eta$ .

$\eta$  value then the emitted particles are produced with anisotropic decay. The degree of anisotropy depends on the center of mass velocity or the energy of the emitted particles. Our goal is to get a correlation between the energy and the angular spread of the emitted particles. We use Gaussian density distribution for nuclei with  $A < 20$  and Woods-Saxon for  $A > 20$  [14]. Using appropriate units where  $\hbar = c = k = 1$ , then the center of mass energy dissipated in a local position is given by [14]:

$$\zeta_{cm} = 3T + m \frac{K_1(m/T)}{K_2(m/T)} \quad (2)$$

The conservation of energy at a given location requires that:

$$[m^2 + 2\eta(1-\eta)mt_i]^{1/2} = 3T + m \frac{K_1(m/T)}{K_2(m/T)} \quad (3)$$

$m$  is the nucleon rest mass,  $t_i$  is the incident kinetic energy per nucleon in the Lab system and  $K_1$  and  $K_2$  are the McDonald's functions (modified Bessel function BesselK) of first and second orders respectively [15]. The solution of Eq.(3) results the value of the local temperature at the specified  $\eta$  value. The variation of the temperature with  $\eta$  is displayed in Figure(2). Maximum temperature corresponds to  $\eta = 0.5$ . The temperature devolves towards both the projectile and the target regions. It is assumed that at each local equilibrium point the gray particles are emitted in Maxwellian form subjected to the corresponding temperature.

$$F(E, \eta) = \frac{d^2 N}{p^2 dp d\Omega} = \frac{N}{4\pi n^3} \frac{\text{Exp}(-E/T)}{2(T/m)^2 K_1(m/T) + (T/m)K_0(m/T)} \quad (4)$$

Reaction	$\Delta$
p-Em	0.9
He-Em	0.3
C-Em	0.2
Ne-Em	0.1

**Table 1.** The diffuseness parameter  $\delta$  of the compressed nuclear matter in the hot spot region as predicted by the model.

Eq.(4) describes the energy distribution of the gray particles produced in the rest frame of the hot spot nuclear matter which shows isotropic distribution there. Transforming this distribution to the Lab system, assuming that the nuclear source is moving with velocity  $\beta_{cm}$  with respect to the Lab system, hence the produced gray particles are emitted with angle  $\theta_L$  in the Lab system.

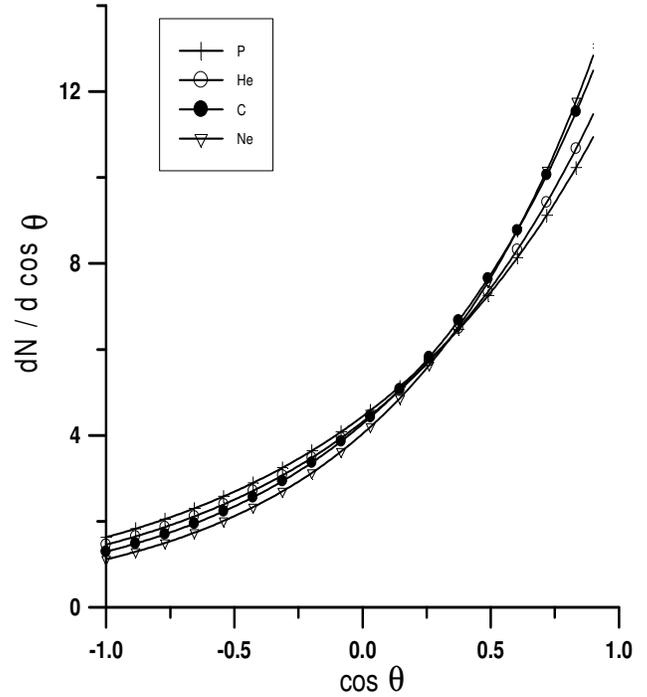
$$E = \gamma_{cm}(E_L - \beta_{cm}P_L \cos \theta_L) \tag{5}$$

$$\beta_{cm} = \frac{P_L}{E_L} = \frac{\eta[(t_i + 2m)]^{1/2}}{m + \eta t_i} \tag{6}$$

Consequently, the Lab distribution function is dependent on the emission Lab angle as well as the incident energy  $E_L$  and the projectile fraction  $\eta$ . So that  $F_L(E_L, \eta, \theta_L)$  describes the energy distribution of the emitted gray particles from a source with a specific value of  $\eta$  at a given Lab angle  $\theta_L$ . The energy distribution in the lab system is found by integration over  $\eta$  and  $\theta_L$  weighted by the corresponding statistical weight factor. The  $\eta$  weight factor depends mainly on the density distribution of the interacting nuclei, their diffuseness and their temporary compressibility at the moment of emission. The detailed formulation of this factor is much complicated. The global effects of these factors are considered in an exponential parametric form  $\chi(\eta) = \text{Exp}(-\delta\eta)$ . The parameter  $\delta$  carries information about the geometry of the system and its compressibility and the diffuseness shape of the matter density of the interacting nuclei. Data Manipulation package [15] is loaded from the *Mathematica* software to find the best values of  $\delta$  that fit the experimental data.

**RESULTS AND DISCUSSION**

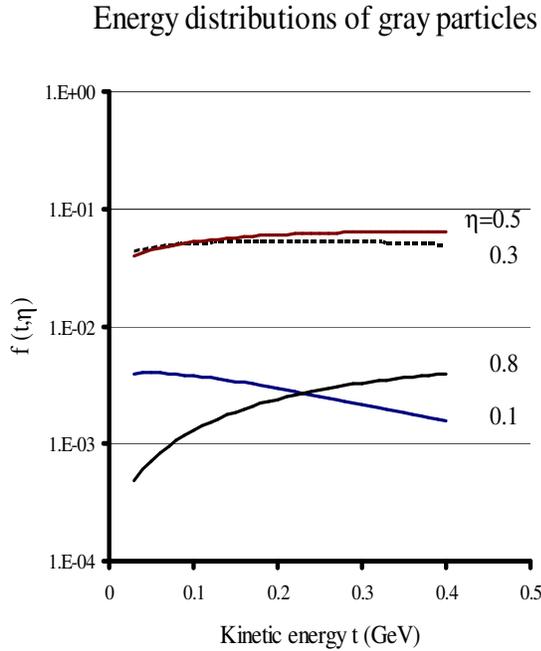
Table (1) shows that the value of the parameter  $\delta$  decreases rapidly with the projectile mass. The target



**Figure 3.** The angular distribution of grey particles produced in (p-He-C-Ne) interactions with Emulsion nuclei at momentum 4.2 GeV/c

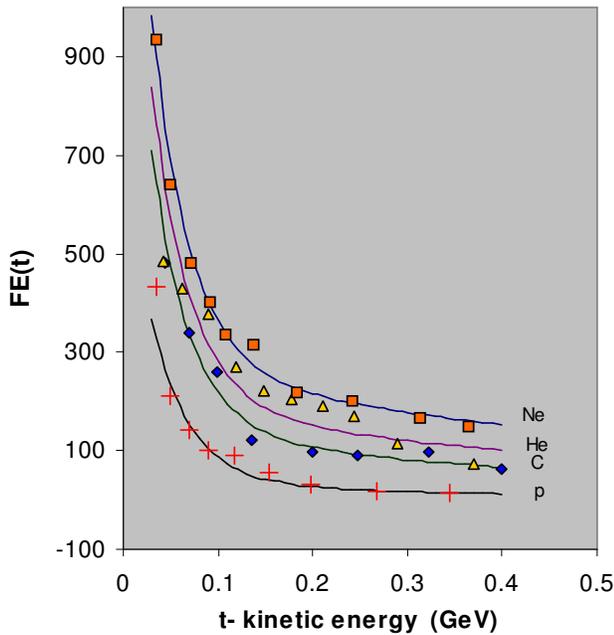
mass is fixed and considered as the average value of the composite emulsion nuclei. On the other hand, the angular weight factor is to be lending from the experimental results, since the angular distribution  $Y(\theta)$  is measured in emulsion technique to a high extent of accuracy with sufficient confidence. In Figure(3) we display the angular distribution of gray particles produced at the same incident energy 4.2A GeV, for the projectiles proton, helium, carbon and neon interaction with emulsion nuclei. The result shows that almost all the distributions come close to each other which support the idea that the target is the source of the gray particles. Figure(4) shows the energy distribution of the gray particles produced at fixed angle  $\theta = \pi/6$  at different values of  $\eta$ . In all cases the distribution is restricted for particles within the energy range  $30 < t < 400 \text{ MeV}$  at which particles appear as gray. At low  $\eta$  values the temperature is small enough so that most of the distribution area is covered within the range of the gray particles. The curves drawn at high  $\eta$  describe only the front portion of the Maxwell distribution just before recognizing the peak of the curve. The left portion corresponds to fast particles with energy greater than 400 MeV, which appear in the emulsion as shower particles. These shower particles are out of our interest.

The final form of the energy distribution of the gray particles is found by integration over the  $\eta$  and all the  $\theta$

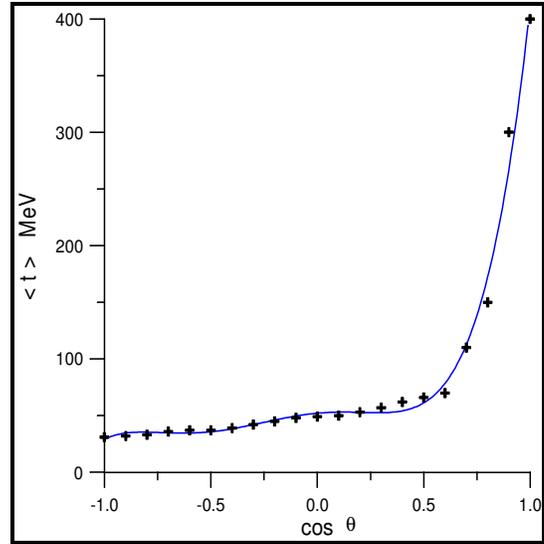


**Figure 4.** Energy distribution of medium energy protons produced at angle  $\theta=\pi/6$  at projectile fraction (a)  $\eta=0.1, 0.3$  and (b)  $\eta=0.5, 0.8$

**Energy Spectrum Of Grey Particles -Produced in (p- He- C- Ne)- with Emulsion**



**Figure 5.** The energy distribution of gray particles produced in p-Em, He-Em, C-Em and Ne- Em at 4.2 A GeV. The solid line is the prediction of the thermodynamic model which is calculated in terms of the measured angular distribution of the corresponding reaction. The nuclear matter density factor is taken as in Table (1). The symbols represent the experimental data.



**Figure 6.** The dependence of the average energy of gray particles on the emission angle for the interaction of 4.5 A GeV Ne with emulsion nuclei.

range so that:

$$F_L(t) = \int_0^{2\pi} \int_0^1 F(t, \eta, \theta) \chi(\eta) Y(\theta) d\eta d\theta \quad (7)$$

The energy  $E$  in Eq. (7) is replaced by the corresponding kinetic energy  $t, \bar{E} = m + t$ , to put the relation in an appropriate form for comparison with the available experimental data. The predictions of Eq.(7) for the reactions of p, He, C and Ne at 4.2 AGeV with emulsion target are shown in Figure(5) compared with experimental data [16,17] where fair agreement is obtained. This result gives us the confidence to apply Eq. (7) successfully to get the energy distribution of gray particles emission within an angular band by just knowing the angular distribution  $Y(\theta)$  and the compressed density factor  $\chi(\eta)$ . To demonstrate the strong energy-angle correlation among the emitted particles, the dependence of the measured average energy of grey particles on their emission angle is displayed in Figure(6). The relation shows two linear dependences. The first one has high slope and represents the forward emission. It has high resolving power i.e. it is possible to differentiate between particles with close values of energy. The most forward particles concern the emission from the very hot domains. The temperature decreases with negative gradient as the emission angel decreases. On the other hand, the backward emission shows also linear behavior with less slope that represents particle emission in approximately equilibrium state. The energy-angle relation is verified with the experimentally measured data which confirms their correlation relation.

their correlation relation. A correlation function is defined as:

$$R(\theta, t) = \frac{f(\theta)f(t)}{f(\theta, t)} - 1 \quad (8)$$

Where  $f(\theta)$  is the probability function to find a gray particle at angle  $\theta$ ,  $f(t)$  is the probability function to find a gray particle with energy  $t$ , and  $f(\theta, t)$  is the probability function to find a gray particle at angle  $\theta$  and energy  $t$  at the same time. An independent correlation requires that  $f(\theta, t) = f(\theta)f(t)$  or the correlation function  $R(\theta, t) = 0$ . Positive values of  $R(\theta, t)$  means that the production of the particle with angle  $\theta$  depends strongly on the emission of particle with energy  $t$ . On the other hand a negative value of the correlation function means that the emission at  $\theta$  prevents the production with energy  $t$ . The calculation of  $R(\theta, t)$  for different values of  $\theta$  and  $t$  pairs all over the range of  $\theta$  and  $t$  concerning the gray particles show positive correlation between high energy particles with small angles. The positive correlation decreases as the emission angle increases.

## CONCLUSIVE REMARKS

The g-particles (target recoil nucleons) play important role on revealing the dynamic mechanism of nuclear interactions among heavy ions.

There is a strong correlation between the energy of the gray particles and their emission angles. This result is predicted by both the thermodynamic model and by the direct measurement of the correlation parameter.

The thermodynamic model supports the local equilibrium hypothesis of the nucleon gas. The angular distribution of the energetic particles shows forward peaking.

The angular distribution of the gray particles is almost independent on the projectile mass. It is a target characteristic feature.

The gray particles occupy a portion of the Maxwellian distribution that depends on the value of the projectile fraction  $\eta$ . The weight factor of which depends on geometrical aspects of the hot spot domain.

The linear increase of the average kinetic energy of the grey particles emitted in the forward hemisphere means that these particles are produced in non-equilibrium states. The most forward particles concern the emission from the very hot domains. The temperature decreases with negative gradient as the angle decreases. On the other hand, backward emission shows uniform behavior that represents particle emission in an almost equilibrium state.

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