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Full Length Research Paper

Fuzzy semiprime ideals in Γ-rings

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In this paper, T. K. Dutta`s and S. K. Sardar`s semiprime ideal of Γ-rings as a fuzzy semiprime ideal of a Γ-rings via its operator rings was defined. Some characterizations of fuzzy semiprime ideal of Γ-rings was obtained. That is; a characterization prove of a fuzzy semiprime ideal, the relationship between fuzzy semiprime ideal and fuzzy prime ideal was obtained. If μ is fuzzy semiprime ideal of a Γ-ring M, then μ^{+} is fuzzy semiprime ideal of L. Similarly, If μ is fuzzy semiprime ideal of a Γ-ring L left operator semi ring M, then μ^{*} is fuzzy semiprime ideal of M, Γ-ring. Lastly, it was deduced that; if $f: M \to N$ is an epimorphism of Γ-ring M, N and μ is f invariant fuzzy semiprime ideal of N(M), then, $f^{-1}(\mu)((f(\mu))$ is fuzzy semiprime ideal of M(N) .

Key words: Fuzzy ideal, fuzzy semiprime ideal, left (right) operator ring, Γ-ring.

INTRODUCTION

As it is well known Zadeh (1965), introduced the notion of a fuzzy subset μ of a nonempty set X as a function from X to unit real interval $I = [0, 1]$. Fuzzy subgroup and its important properties were defined and established by Rosenfeld (1971). Then many authors have studied about it. After this time it was necessary to define fuzzy ideal of a ring. The notion of a fuzzy ideal of a ring was introduced by Liu (1982). Malik, Mordeson and Mukherjee have studied fuzzy ideals. The notion of fuzzy ideal in Γ-ring was introduced by Jun and Lee [1992]. They studied some preliminary properties of fuzzy ideals of Γ-rings. Later Hong and Jun (1995) defined normalised fuzzy ideal and fuzzy maximal ideal in Γ-ring and studied them. Dutta and Chanda (2005), studied the structures of the set of fuzzy ideals of a Γ-ring with the help of fuzzy ideals via operator rings of Γ-ring. Jun [1995] defined fuzzy prime ideal of a Γ-ring and obtained a number of characterisations for a fuzzy ideal to be a fuzzy prime ideal.

In this paper, a characterization of a fuzzy semiprime ideal was proved and the relationship between fuzzy semiprime ideal and fuzzy prime ideal was obtained.

Let M be Γ-ring and L be its right operator semi ring M. If μ is fuzzy semiprime ideal of a Γ-ring M then μ^{+} is fuzzy semiprime ideal of L. Similarly, let M be Γ-ring and L be its left operator semi ring M. If μ is fuzzy semiprime ideal of a Γ-ring L then μ^{*} is fuzzy semiprime ideal of M. Lastly, it was obtain that; if $f: M \to N$ is an epimorphism of Γ-ring M, N and μ is f invariant fuzzy semiprime ideal of N(M), then $f^{-1}(\mu)((f(\mu))$ is fuzzy semiprime ideal of M(N).

PRELIMINARIES

Some basic definitions and theorems about gamma ring as in the paper by Dutta and Chanda (2007) was given.

From definition by Barnes (1966), Let M and Γ be two additive abelian groups. M is called a Γ –ring, if there exists a mapping f : M × Γ × M →M, where f(a, α, b) is denoted by aαb, a, b ∈ M, α ∈ Γ, satisfying the following conditions for all a, b, c ∈ M and for all α, β∈ Γ; (a + b)αc = aαc + bαc, a(α+β)b = aαb + aβb, aα(b + c) = aαb + aαc, and aα(bβc) = (aαb)βc.

By Barnes (1966), a subset A of a Γ-ring M is called a

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left (resp. right) ideal of M if A is an additive subgroup of M and mαa ∈ A (resp. aαm ∈ A) for all m∈ M, α ∈ Γ, a∈A. If A is a left and a right ideal of M, then A is called a two sided ideal of M or simply an ideal of M.

From definition by Coppage and Luh (1971), Let M be a Γ-ring and F be the free abelian group generated by Γ × M. Then A $=\{\sum_i n_i, \zeta_i x_i \in F : a \in M \Rightarrow \sum_i n_i, \zeta_i x_i = 0\}$ is a subgroup of F. Let R = F/A be the factor group of F by A. Let us denote the coset (ζ, x) + A by [ζ, x]. It can be verified that [α, x] + [β, x] = [α+ β, x] and [α, x] + [α, y] = [α, x + y], for all α,β∈ Γ and x, y ∈ M. Let a multiplication in R be defined by $\sum_i[\alpha_i x_i] \sum_j[\beta_j y_j] = \sum_i[\alpha_i, x_i \beta_j x_j]$, then R forms a ring. This ring R is called the right operator ring of the Γ ring M. Similarly, a left operator ring L of M can be constructed. For the subsets N ⊆M, Φ⊆ Γ, [Φ,N] is denoted by the set of all finite sums $\sum_i[\alpha_i x_i]$ in R where $\alpha_i \in \Phi$ and $x_i \in$ N, and denote [(Φ,N)] as the set of all elements [ϕ, x] in R, where ϕ∈ Φ and x ∈N. Thus in particular, R = [Γ,M] and L = [M, Γ]. If there exists an element $\sum_i[\alpha_i e_i] \in R$ such that $\sum_i x\alpha_i e_i] = x$ for every element x of M then it is called the right unity of M. It can be verified that $\sum_i[\alpha_i, e_i]$ is the unity of R. Similarly, the left unity can be define as $\sum_i[e_i, \alpha_i]$ which is the unity of the left operator ring L.

By Jun and Lee (1992), a non-empty fuzzy subset μ (that is μ(x) ≠ 0 for some $x \in M$ of a Γ ring M is called a fuzzy left (right) ideal of M if:

(i) μ(x – y) ≥min{μ(x), μ(y)},

(ii) μ(xαy) _≥μ(y) (resp. μ(xαy) ≥μ(x)) for all x,y∈M and for all $\alpha \in \Gamma$.

A non-empty fuzzy subset μ of a Γ-ring M is called a fuzzy ideal if it is a fuzzy left ideal and a fuzzy right ideal of M.

Let M be a Γ-ring and R be the right operator ring and L be the left operator ring of M, respectively.

By Dutta and Chanda (2005), for a fuzzy subset μ of R, a fuzzy subset μ* of M was define by μ*(a) = $\inf_{\Upsilon\in\Gamma}$μ([Υ, a]), where a∈ M. For a fuzzy subset σ of M, a fuzzy subset σ^* of R was define by $\sigma^*(\sum_i[\alpha_i, a_i]) = \inf_{m\in M}\sigma(\sum_i(m\alpha_i a_i)$ where $\sum_i[\alpha_i, a_i] \in R$. For a fuzzy subset δ of L, a fuzzy subset δ^+ of M was define by δ^+ (a) = $\inf_{\Upsilon\in\Gamma}$δ([a,Υ]), where a∈ M. For a fuzzy subset η of M, a fuzzy subset η^+ of L was define by $\eta^+(\sum_i[a_i, \alpha_i]) = \inf_{m\in M}\sigma(\sum_i(a_i\alpha_i m)$ where $\sum_i[a_i, \alpha_i] \in L$.

From definition by Dutta and Chanda (2005), let μ, σ be two fuzzy subsets of M. Then the sum

$$(\mu \oplus \sigma) = \begin{cases} \sup_{x=u+v}[\min[\mu(u), \sigma(v)]], & u, v \in M \\ 0 & otherwise. \end{cases}$$

and

$$(\mu o \sigma)(x) = \begin{cases} \sup[\min[\min[\mu(u_i), \sigma(v_i)]]], & 1 \le i \le n,\ x - \sum_i^n u_i\gamma_i v_i, u_i, v_i \in M, \gamma_i \in \Gamma \\ 0 & otherwise. \end{cases}$$

From definition by Jun (1995), let μ, σ be two fuzzy subsets of M. Then the product μΓσ of μ and σ is defined by (μΓσ)(x) =

$$\begin{cases} \sup_{x=u\gamma v}\min[\mu(u), \sigma(v)]], & for\ u, v \in M\ and\ \gamma \in \Gamma \\ 0 & otherwise. \end{cases}$$

From definition by Barnes (1966), Let M be a Γ-ring. A proper ideal P of M is called prime, if for all pairs of ideals S and T of M, SΓT⊆ P implies that S⊆ P or T⊆ P.

From definition by Kyuno (1982), If P is an ideal of a Γ-ring M, then the following conditions are equivalent:

(i) P is a prime ideal of M;

(ii) If a, b ∈ M and aΓMΓb∈ P then a∈P or b∈P.

However, from the above definition, Jun (1995) define fuzzy prime ideal of gamma ring as the following definition.

From definition by Kyuno (1982), a fuzzy ideal μ of a gamma ring R is said to be prime, if μ is a nonconstant function and for any two fuzzy ideals σ and δ of R, $\sigma\Gamma\delta \subseteq \mu$ implies that either σ⊆ μ or $\delta \subseteq \pi$.

By Kumar (1993), let f be a mapping from a Γ-ring M onto a Γ-ring N and μ∈FI(M). μ is called f-invariant, if f(x) = f(y) implies that μ(x) = μ(y), for all x, y∈M.

By Barnes (1966), a function f: M→N, where M, N are Γ-rings is said to be a Γ-homomorphism if f(a + b) = f(a) + f(b), f(aαb) = f(a)αf(b), for all a, b∈M, $\alpha \in \Gamma$.

By Kumar (1993), a fuzzy subset μ of a Γ-ring M is called a fuzzy point if μ(x)∈[0, 1] for some x∈M and μ(y) = 0 for all y∈M \ {x}. If μ(x) = β, then the fuzzy point μ is denoted by x_β.

By Jun (1995), a non-constant fuzzy ideal μ of a Γ-ring M is called a fuzzy prime ideal of **M** if for any fuzzy ideal of σ,β of M σΓβ⊆ μ implies σ⊆ μ or β⊆ μ.

FUZZY SEMIPRIME IDEAL OF GAMMA RING

Dutta and Chanda (2007) studied the fuzzy prime ideal of Gamma ring. Similarly, definition of fuzzy semi prime Ideal of gamma ring and its basic characterizations was given.

From definition, a non constant fuzzy ideal μ of a Γ-ring M is called fuzzy semiprime ideal of M, if for any fuzzy ideal of σ of M σΓσ⊆μ implies σ⊆μ.

Theorem 1

Let M be commutative Γ-ring and μ be fuzzy ideal of Γ-ring M. Then the following are equivalent:

i) $x_a\Gamma x_a \subseteq \mu \Rightarrow x_a \subseteq \mu$. Where x_a *is fuzzy point of M.*

ii) μ is fuzzy semiprime ideal of Γ-ring M

iii) σoσ⊆ μ implies that σ⊆μ.

Proof

$i) \Rightarrow ii)$ Let σΓσ⊆μ and σ$\nsubseteq \mu$. Then $\exists x \in M$ such that σ(x)> $\mu(x)$. Let σ(x) = a. By i) $x_a\Gamma x_a \subseteq \mu \Rightarrow x_a \subseteq \mu$. This shows that $x_a(x) \subseteq \mu(x) \Rightarrow a = \sigma(x) \leq \mu(x)$. This is a contradiction. $ii) \Rightarrow iii)$ Trivial. $iii) \Rightarrow i)$ Let $x_a\Gamma x_a \subseteq \mu$, where x_a *is fuzzy point of M.* Assuming that $x_a = \sigma$ is a fuzzy ideal of Γ-ring M such that σ(y) = 0 for all y∈M \ {x} and σ(x) = β. σoσ⊆ μ implies that

$$\begin{Bmatrix} \sup[\min[\min[\sigma(u_i),\sigma(v_i)]]], & 1 \leq i \leq n,\ x = \sum_i^n u_i\gamma_i v_i, u_i, v_i \in M, \gamma_i \in \Gamma \\ 0 & otherwise. \end{Bmatrix}$$

$$\subseteq$$

$$\begin{Bmatrix} \sup_{x = u_i\gamma_i v_i} \min[\sigma(u_i),\sigma(v_i)]], & for\ u_i, v_i \in M\ and\ \gamma_i \in \Gamma \\ 0 & otherwise. \end{Bmatrix}$$

Then it can be said that σoσ⊆ μ *implies* $x_a\Gamma x_a \subseteq \mu$. since σ⊆ μ, σ can be obtain as $\sigma = x_a \subseteq \mu$.

Theorem 2

Let μ be fuzzy semi prime ideal of Γ-ring M. Then For all $x \in M$, μ(x^2)=$\mu(x)$.

Proof

For all $x \in M$, , μ(x^2)=r. Then $x_r\Gamma x_r \subseteq \mu \Rightarrow x_r o x_r \subseteq \mu \Rightarrow x_r \subseteq \mu$. μ(x)≥ r. This implies that $\mu(x) \geq \mu(x^2) \geq \mu(x)$. This completes the proof.

Theorem 3

For every fuzzy prime ideal μ of Γ-ring, M is fuzzy semiprime ideal of Γ-ring M.

Proof

Let σΓσ⊆μ. Since μ is fuzzy prime ideal of Γ-ring M, then σ⊆μ.

Theorem 4

Let μ_i be fuzzy semiprime ideal of Γ-ring M. Then $\cap\, \mu_i$ is fuzzy semiprime ideal of Γ-ring M.

Proof

Let σ be fuzzy ideal of M such that σΓσ⊆ $\cap\, \mu_i$. Since μ_i is fuzzy semiprime ideal of Γ-ring M σΓσ⊆ $\cap\, \mu_i$.implies that σ⊆ μ_i. Then σ⊆ $\wedge\, \mu_i = \cap\, \mu_i$. Therefore $\cap\, \mu_i$ is fuzzy semiprime ideal of Γ-ring M.

Theorem 5

Let P be a semiprime ideal of a Γ-ring M and α≠1 an arbitrary of L. Then μ is fuzzy semiprime ideal of Γ-ring M where μ= $\begin{Bmatrix} 1 & x \in P \\ \alpha & x \notin P \end{Bmatrix}$.

Proof

For all $a, b \in M$ if μ(a)$\wedge \mu(b) = \alpha$ then $\mu(a-b) \geq \mu(a) \wedge \mu(b)$. For all $a, b \in M$ if μ(a)$\wedge \mu(b) = 1$ then $\mu(a) = \mu(b) = 1$. This implies that $a, b \in \mu_1$. Since μ_1 is an ideal of Γ-ring M then $a - b \in \mu_1$. Therefore, $1 = \mu(a-b) \geq \mu(a) \wedge \mu(b)$. Similarly, μ(xαy) _≥μ(y) (resp. μ(xαy) ≥μ(x)) for all x,y∈M and for all $\alpha \in \Gamma$. Let σΓσ⊆μ and σ$\nsubseteq \mu$. Then $\exists x \in M$ such that $\sigma(x) > \mu(x)$. This implies μ(x)=α. Since P is semiprime ideal, xΓx⊆P. Then μ(xΓx)=α. So, σΓσ(xΓx) ≥σ(x)>μ(x)=μ(xΓx). This is a contradiction. Therefore μ is fuzzy semiprime ideal of Γ-ring M.

Theorem 6

Let M be Γ-ring and L be its right operator semiring M. If μ is fuzzy semiprime ideal of a Γ-ring M then μ^{+} is fuzzy semiprime ideal of L.

Proof

Let σ be fuzzy semiprime ideal of L such that σΓσ□μ. Then for all $a_i m \in M$ and $\alpha_i \in \Gamma$

$$\sigma\Gamma\sigma([M,\Gamma]) \subset \mu^{+}([M,\Gamma]) \rightarrow \sigma\Gamma\sigma(\sum_i[a_i,\alpha_i]) \subset \mu^{+}(\sum_i[a_i,\alpha_i])$$

$$\Rightarrow \sigma\Gamma\sigma(a_i\alpha_i m) \subseteq \mu(a_i\alpha_i m)$$

$$\Rightarrow \sigma(a_i\alpha_i m) \subseteq \mu(a_i\alpha_i m)$$

$$\Rightarrow \sigma(\textstyle\sum_i a_i\alpha_i m) \subseteq \inf\mu(\sum_i a_i\alpha_i m)$$

$$\Rightarrow \sigma(\textstyle\sum_i a_i\alpha_i m) \subseteq \mu^{+}(\sum_i a_i\alpha_i m)$$

$\Rightarrow \sigma([M,\Gamma]) \subseteq \mu^{+}([M,\Gamma])$. So μ^{*} is fuzzy semiprime ideal of L.

Theorem 7

Let M be Γ-ring and L be its left operator semiring M. If μ is fuzzy semiprime ideal of a Γ-ring L then μ^{*} is fuzzy semiprime ideal of M.

Proof

σ is fuzzy semiprime ideal of L such that σΓσ⊆μ. Then for all $a_i m \in M$ and $\alpha_i \in \Gamma$

$$\sigma\Gamma\sigma([\Gamma,M]) \subseteq \mu^{*}([\Gamma,M]) \Rightarrow \sigma\Gamma\sigma(\sum_i[\alpha_i,a_i]) \subseteq \mu^{*}(\sum_i[\alpha_i,a_i])$$

$$\Rightarrow \sigma\Gamma\sigma(m\alpha_i a_i) \subseteq \mu(m\alpha_i a_i)$$

$$\Rightarrow \sigma(m\alpha_i a_i) \subseteq \mu(m\alpha_i a_i)$$

$$\Rightarrow \sigma(\textstyle\sum_i m\alpha_i a_i) \subseteq \inf\mu(\sum_i m\alpha_i a_i)$$

$$\Rightarrow \sigma(\textstyle\sum_i m\alpha_i a_i) \subseteq \mu^{*}(\sum_i m\alpha_i a_i)$$

$\Rightarrow \sigma([\Gamma,M]) \subseteq \mu^{*}([\Gamma,M])$. So μ^{*} is fuzzy semiprime ideal of L.

Lemma

According to Kumar (1991), If f is a homomorphism from a ring M to N, then

i) f(σ)$f(\mu)$⊆ f(σμ), where σ and μ are fuzzy ideals of M.

ii) f^{-1}(σ)$f^{-1}(\mu)$⊆ f^{-1}(σμ), where σ and μ are fuzzy ideals of N.

If $f: M \rightarrow N$ is an homomorphism of Γ-ring M, N, then

i) f(σ)$\Gamma f(\mu)$⊆ f(σ$\Gamma\mu$), where σ and μ are fuzzy ideals of M.

ii) f^{-1}(σ)$\Gamma f^{-1}(\mu)$⊆ f^{-1}(σ$\Gamma\mu$), where σ and μ are fuzzy ideals of N.

Proof

i) Let $n \in N$, $t_1 =$ f(σ)$\Gamma f(\mu)$(n) and $t_2 = f$(σ$\Gamma\mu$)(n). If n can not be expressed as $n = n_1\alpha n_2$ where $n_1, n_2 \in N$ then by definition $t_1 = 0 \leq t_2$. If n can be written as $n = n_1\alpha n_2$ where $n_1, n_2 \in N$ and $t_3 > 0$ is given

$t_1 =$ f(σ)$\Gamma f(\mu)$(n)

$$- \sup\{\min(f(\sigma)(n_1)f(\mu)(n_2): \quad n - n_1\alpha n_2,\ \exists n_1,n_2 \in N, \alpha \in \Gamma\ \}$$

$$\sup\{\min(\sup(\sigma)(m_1),\sup(\mu)(m_2): \quad n = n_1\alpha n_2,\ \exists m_1 \in f^{-1}(n_1), m_2 \in f^{-1}(n_2), \alpha \in \Gamma\ \}$$

. Thus there exists $n_1, n_2 \in N$ such that

$$t_1 - t_3 < \min\{(\sigma)(m_1),(\mu)(m_2)\} \quad \exists m_1 \in f^{-1}(n_1), m_2 \in f^{-1}(n_2), \alpha \in \Gamma$$

$$\leq (\sigma\Gamma\mu)(m_1\alpha m_2)$$

$$\leq \sup(\sigma\Gamma\mu)(m) \quad where$$

$$m_1\alpha m_2 = m,\ f(m_1\alpha m_2) = n_1\alpha n_2 = n$$

=f(σ$\Gamma\mu$)(n)= t_2. This follows that f(σ)$\Gamma f(\mu)$⊆ f(σ$\Gamma\mu$).

ii) Similarly, it be can shown that f^{-1}(σ)$\Gamma f^{-1}(\mu)$⊆ f^{-1}(σ$\Gamma\mu$).

Theorem 8

$f: M \rightarrow N$ is an homomorphism of Γ-ring M, N. If μ is f invariant fuzzy semiprime ideal of M then If $f(\mu)$ is fuzzy semiprime ideal of N.

Proof

If μ is f invariant fuzzy ideal of M then $f(\mu)$ is fuzzy ideal of N. For all $n, b \in N$ and α $\in \Gamma$, σ

f(σ)Γf(σ)(nαb))⊆ f((σΓσ)(nαb))

$$= \sup\{\sigma(m) \wedge \sigma(a):\ f(m\alpha a) = n\alpha b,\ \exists m.a \in M, \alpha \in \Gamma$$

$$\subseteq \sup\{\mu(m) \wedge \mu(a):\ f(m\alpha a) = n\alpha b,\ \exists m.a \in M, \alpha \in \Gamma\ \}$$

$\subseteq f\big(\mu(n = f(m))\big) = sup\{\mu(m): \quad f(m) = n\}$.

Since μ is f invariant fuzzy semiprime ideal of M then

$sup\{\sigma(m): f(m) = \mathbf{n},\ \exists m \in M\ \} \subseteq sup\{\mu(m): \quad f(m) = n\}$. $f(\sigma)(n) \subseteq f(\mu)(n)$.

This completes the proof.

Theorem 9

This is an epimorphism of Γ-ring M, N. If μ is f invariant fuzzy semiprime ideal of N then, $f^{-1}(\mu)$ is fuzzy semiprime ideal of M.

Proof

Since $f: M \to N$ is an epimorphism of Γ-ring M, N and If μ is f invariant fuzzy ideal of N then If $f^{-1}(\mu)$ is fuzzy ideal of M. *Let* $f^{-1}(\sigma)\Gamma f^{-1}(\sigma)(m\alpha a) \subseteq f^{-1}(\mu)(m\alpha a)$. Then for all $m, a \in M$ and $\alpha \in \Gamma$,

$$f^{-1}(\sigma)\Gamma f^{-1}(\sigma)(m\alpha a) \subseteq \quad f^{-1}(\sigma\Gamma\sigma)(m\alpha a)$$

$$\Rightarrow \sigma\Gamma\sigma\big(f(m\alpha a)\big) \subseteq \mu\big(f(m\alpha a)\big)$$

$$\Rightarrow \sigma\Gamma\sigma\big(f(m)\alpha f(a)\big) \subseteq \mu\big(f(m)\alpha f(a)\big)$$

$$\Rightarrow \sup\ (\sigma\big(f(m) \wedge f(a)\big) \subseteq \sup\ (\mu(f(m) \wedge f(a))$$

$$\Rightarrow f^{-1}(\sigma) \subseteq f^{-1}(\mu).$$

Therefore $f^{-1}(\mu)$ is fuzzy semiprime ideal of M.

Conclusion

The fuzzy semiprime ideal of Gamma ring was defined and the basic theorems in fuzzy algebra were improved upon. Currently, works on the construction of a fuzzy topology at fuzzy semiprime ideals is being carried out. The purpose of this construction is that it will open up new directions for further studies.

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