

Full Length Research Paper

## Flat gravity based on Hubble's law which expanded Newtonian gravity

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Hubble's law, formulated by Edwin Hubble and Milton Humason in 1929, tells us that space is expanding. However, over short distances, flat gravity caused by the expanding universe is described by the inverse square law of Newtonian gravity. This leads to heretofore unsolved gravity anomalies, such as the pioneer anomaly, which involves an abnormal slowdown relative to the Sun of the Pioneer spacecraft and the galaxy rotation problem, whereby the rotational speed of heavenly bodies reaches a constant value instead of decreasing with distance from the galactic centre. The expanding universe adds an expansion term that was divided into a strain constant  $V_0$  for the recession rate  $v = H_0 D$ , and the gravitational potential  $-GM(1/r)$  of Newtonian mechanics for a stationary universe is replaced by  $-GM(1/r)(1 + v/V_0)$ . The expansion term becomes constant ( $G_0 = GH_0/V_0$ ) at large distances because the distance  $D$  and radius  $r$  cancel. Furthermore, the total gravitational mass [ $M_0 = c^3/(2GH_0)$ ] of the observable universe affects the specific potential constant, which is multiplied by the observable gravitational mass to become  $-(G/r + G_0)M$ . Flat gravity based on Hubble's law which expanded Newtonian gravity is thus consistent with the gravity anomaly without assuming the existence of dark matter. When combined with Yukawa potential [ $\alpha e^{-(r/\lambda)}$ ], the gravity and the strong force can be unified [ $\alpha e^{-(r/\lambda)} - 1](G/r + G_0)M$ .

**Key words:** Expanding universe, inverse square law, pioneer anomaly, galaxy rotation problem, recession rate, gravitational potential, stationary universe, gravitational mass, specific potential, dark matter.

### INTRODUCTION

Several physical problems in astronomy still remain open. One is the "Pioneer anomaly", which was noticed for the Pioneer 10 and 11 spacecrafts as they left the solar system. The anomaly involves the cause of the blueshift, which indicates a reduction in speed with respect to the sun and remains unidentified (Anderson et al., 1998). Another open problem is the "galaxy rotation problem" wherein the rotational speed of galactic matter does not

decrease with distance from the galactic centre but remains constant (Zwicky, 1933, 1937). Neither of these problems can be explained by Newton's universal law of gravitation. Some theories have been proposed to revise Newton's law of gravity, such as modified Newtonian dynamics (MOND), which introduces a function that scales mass and that asymptotically approaches unity for accelerations greater than a constant acceleration defined

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in the theory to be on the order of  $10^{-10} \text{ m/s}^2$  (Milgrom, 1983). Another proposition is the modified gravitation theory (MOG), which expands the theory of general relativity and calls upon a fifth field of force to counteract gravity. However, this theory suggests that, because work becomes small at large distances, gravity must become relatively large, which would require the gravitational constant to change (Brownstein and Moffat, 2006). In addition, the dark-matter hypothesis invokes some unknown “dark” matter (that is, it does not emit radiation) that would account for the observed gravitational anomalies without requiring our current theory of gravity to be modified (Rubin et al., 1980). There is no rationale for the fifth field of force and dark matter remains undiscovered, so the debate is not settled. This paper approaches the problem by assuming an expanding universe, and that a gravitational interaction between all the observable gravitational mass of the universe is the cause of the gravity anomaly. Given this, the spatiotemporal evolution factor from Hubble’s law was first defined (Hubble, 1929) and from this the pioneer anomaly and the galaxy rotation problem was explained.

## METHODS

Given these relationships, the gravity anomalies by using the specific potential and the equivalence principle of light’s momentum (LEP) was examined.

### Definition of specific potential

Consider the equation  $v = H_0 D$ , where  $v$  is the speed (that is, recession rate) at which heavenly bodies move away from an observer and  $D$  is the distance from the observer to the heavenly bodies. The proportionality constant  $H_0$  is the Hubble constant and determines the recession rate of the current universe. As of 2013, the most accurate value for the Hubble constant, which comes from the Planck observation, is  $67.80 \pm 0.77 \text{ km/s/Mpc}$  (Ade et al., 2013). This recession rate is divided into the recession strain constant  $V_0$  (m/s) and is converted into the recession strain  $e$  for the cosmic expansion:

$$e = \frac{v}{V_0} = \frac{H_0 D}{V_0}. \quad (1)$$

The relationship between the recession strain  $e$  and the recession stretch  $\Lambda$  is

$$\Lambda = 1 + e = 1 + \frac{H_0 D}{V_0}. \quad (2)$$

These equations are expressed by using the ratio of the transformation of the initial state of the spatiotemporal evolution. In addition, we must ask if the recession of galaxies (due to cosmic expansion), and the existence of recession strain and stretch where no expansion occurs are valid before and after unification. Consider the specific potential  $G_x$  obtained by multiplying the recession stretch by the gravitational constant and dividing the product by distance:

$$G_x = \frac{\Lambda G}{D} = \frac{G}{D} + \frac{GH_0}{V_0} \text{ m}^2 \text{ s}^{-2} \text{ kg}^{-1}. \quad (3)$$

The specific potential  $G_x$  (J/kg<sup>2</sup>) multiplied by the active gravitational mass  $M_a$  (kg) gives the potential  $G_x M_a$  (J/kg). The potential  $G_x M_a$  (J/kg) multiplied by the passive gravitational mass  $m_p$  (kg) gives the potential energy  $G_x M_a m_p$  (J). To obtain the specific potential constant  $G_0$ , the Hubble constant was multiplied by the gravitational constant and the product divided by the recession-strain constant  $V_0$ :

$$G_0 = \frac{GH_0}{V_0} \text{ m}^2 \text{ s}^{-2} \text{ kg}^{-1} \left( \text{J/kg}^2 \right). \quad (4)$$

### Kepler’s 3rd law based on LEP

Centripetal force  $F$  to be constant velocity circular motion the inertial mass  $m_i$  is

$$F = m_i r \omega^2 = m_i v^2 / r. \quad (5)$$

By the active gravitational mass  $M_a$  and passive gravitational mass  $m_p$ , universal gravitation is

$$F = GM_a m_p / r^2. \quad (6)$$

By using Equations (5) and (6), the equilibrium of forces is:

$$m_i v^2 / r = GM_a m_p / r^2. \quad (7)$$

By using Equations (7) and using the equivalence principle of light’s momentum (LEP)  $\gamma = c / \omega = m_i / m_p$ , the equilibrium of potential energies is:

$$m_i v^2 = G(M_a m_i / \gamma) / r. \quad (8)$$

By using Equations (5) and (8), the equilibrium of potentials is:

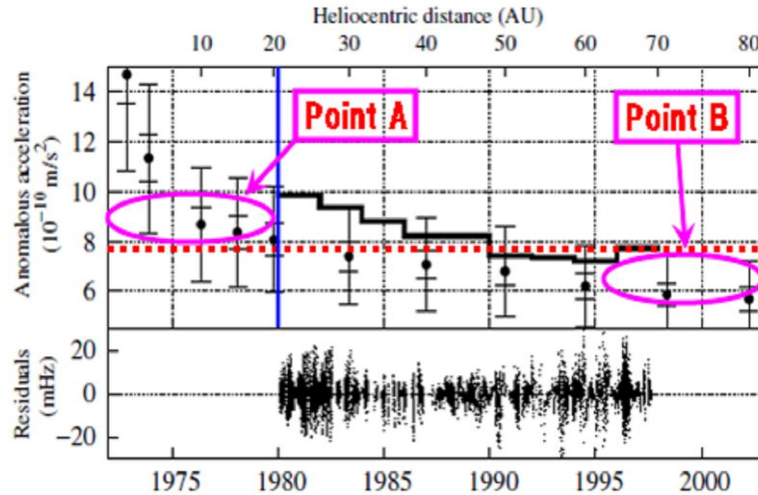
$$v^2 = \omega^2 r^2 = G(M_a / \gamma) / r. \quad (9)$$

By using Equations (2) and (9), the equilibrium of potentials in consideration of the recession stretch  $\Lambda$  is:

$$v^2 = \omega^2 r^2 = \Lambda G(M_a / \gamma) / r. \quad (10)$$

By using Equations (3) and (10) and using  $\omega = 2\pi/T$ , Kepler’s 3rd law  $r^3 = aT^2$  based on LEP is

$$\begin{aligned} r^3 &= \frac{\Lambda G(M_a / \gamma)}{\omega^2} = \frac{\Lambda G(M_a / \gamma)}{4\pi^2} T^2 \\ r^2 &= \frac{G_x(M_a / \gamma)}{4\pi^2} T^2 \\ r &= \frac{\sqrt{G_x(M_a / \gamma)}}{2\pi} T = \frac{v}{\omega}. \end{aligned} \quad (11)$$



**Figure 1.** [The red dotted line is  $7.84 \pm 0.01 \times 10^{-10} \text{ m/s}^2$  (Turyshev and Toth, 2010), and the pink circle is Discussion Point A, B. ] to [ Figure 3 (Turyshev et al., 2012): Comparison of the thermally-induced and anomalous accelerations for Pioneer 10. The estimated thermal acceleration is shown with error bars. The stochastic acceleration estimate from (Turyshev et al., 2011) appears as a step function ].

## RESULTS AND DISCUSSION

### Pioneer anomaly

Data for short distances (that is, less than 20 au from the Sun) is restored to the original state and is analysed. One report suggests that the anomaly is caused by thermal radiation (Turyshev et al., 2012). discussion to a base in Figure 1 about it include: Discussion Point A, it thus becomes difficult to explain the Pioneer anomaly by modified gravitation theories, such as MOND or MOG, where gravity or the gravitational constant changes as a function of distance. Discussion Point B, the P10 data at the furthest distance flattened and increased (but within experimental uncertainty) which is inconsistent with a declining thermal cause (Hodge, 2013; Boom, 2013). Discussion Point Other, in addition, it is difficult to envision a solar neighbourhood that does not have dark matter (Bidin et al., 2012) to explain the Pioneer anomaly by the dark-matter hypothesis. As for the major cause of the slowdown, there is no conclusive evidence that the emission of the heat is the cause of the slowdown, nor for other arguments such as the effect of expanding space on photons (Kopeikin, 2012). It is strange that there is no gravity anomaly in the heliosphere, although we have an inexplicable problem, such as the galactic rotation curve, which is based only on the matter that is visible. Furthermore, there is the discussion that insisted on “If the Pioneer anomaly has a gravitational origin, it would, according to the equivalence principle, distort the motions of the planets in the Solar System” Tangen, 2007). However it is an inherent problem of the general relativity

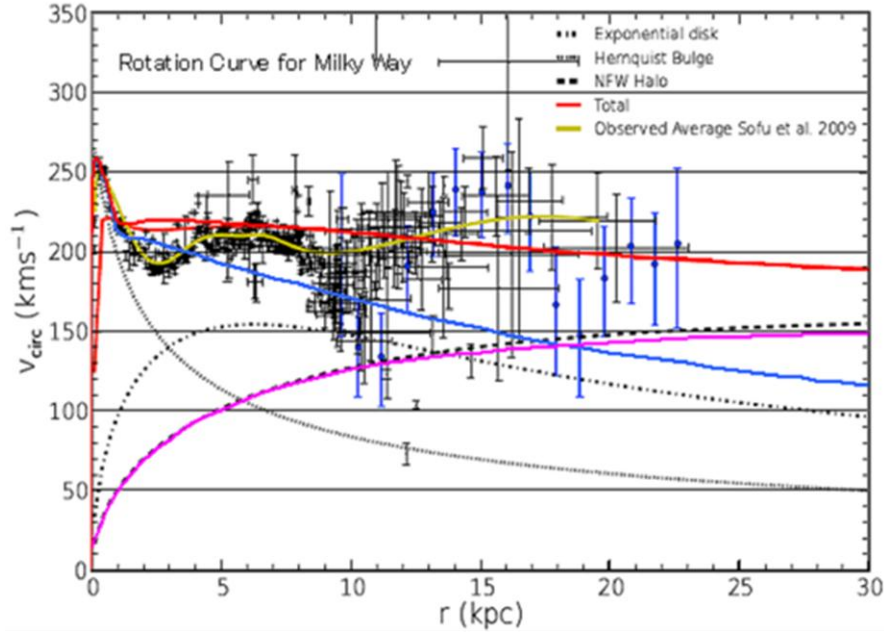
based on Weak equivalence principle (WEP) and Einstein's equivalence principle (EEP) “After an illustration by comparing the status of time in Einsteinian physics with that of the vertical direction in Newtonian physics, it was concluded that there is no pertinent notion of time in Einsteinian theories.” (Lachize, 2014, 2007; Mizony and Lachize, 2005). This paper proposes that the Doppler blueshift that revealed the reduction in Pioneer's speed relative to the Sun is due to flat gravity that is caused by cosmic expansion. The decrease  $\Delta v$  in the velocity of the receiver relative to the source with blueshift  $\Delta f$  ( $5.99 \pm 0.01 \times 10^{-9} \text{ Hz}$ ) for frequency  $f_0$  (2.29 GHz) is:

$$\Delta v = \Delta f \frac{c}{f_0} \cong 7.84 \times 10^{-10} \text{ m/s}, \quad (12)$$

which is within the error of ( $7.84 \pm 0.01 \times 10^{-8} \text{ cm/s}^2$ ) from our formal solution for the Pioneer anomaly that was obtained from the available data (Turyshev and Toth, 2010). The slowdown (escape speed) of Pioneer that is due to flat gravity is calculated from the decrease in wave speed by using Equations (1) and (10) and using the expression  $c^2 = w^2 + 2G_x M$  where  $c$  is the speed of light in vacuum and the wave speed in a gravitational field is:

$$\begin{aligned} 2eG(M_s/\gamma)/r &= 2G_0(M_s/\gamma) = c^2 - w^2 \\ &= c^2 - (c - \Delta v)^2 = 2c\Delta v - \Delta v^2 \approx 2c\Delta v. \end{aligned} \quad (13)$$

By using Equations (4), (13) and using a solar mass  $M_s = 1.989 \times 10^{30} \text{ kg}$  (Astrodynamical Constants), the specific potential constant is:



**Figure 2.** Adjusted the rotation curve (red) by Total gravity to Figure 10 (Kafle et al., 2012), and the rotation curve (blue) by Newtonian gravity and the rotation curve (pink) by Flat gravity were added.

$$G_0 = \frac{GH_0}{V_0} \approx \frac{c\Delta v}{(M_s/\gamma)} \cong 1.18 \times 10^{-31} \text{ J/kg}^2. \quad (14)$$

In Equation (14), the specific potential constant  $G_0$  is the gravitational mass and a proportionality constant for the blueshift. Transforming Equation (14) gives the recession strain constant  $V_0$ :

$$V_0 = \frac{GH_0}{G_0} \cong 1,240 \text{ m/s}. \quad (15)$$

The ratio of solar mass to total gravitational mass  $M_0 = c^3/(2GH_0)$  of the observable universe (Kragh, 1999) is:

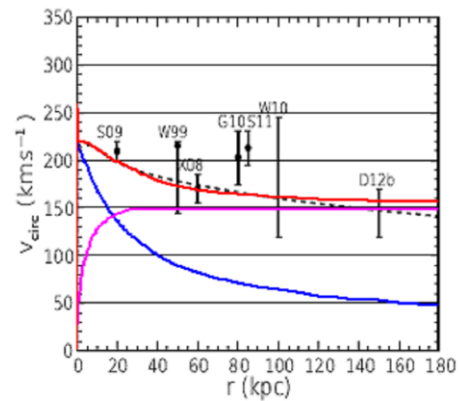
$$\frac{M_0}{(M_s/\gamma)} = \frac{c^2}{2V_0\Delta v} \cong 4.62 \times 10^{22}. \quad (16)$$

In addition, the specific potential constant  $G_0$  is a proportionality constant of flat gravity:  $G_0M = Mc^3/(2M_0V_0)$ . It acts on gravitational mass and gives the total observable gravitational mass of the universe.

**Galaxy rotation problem**

From Newton’s theory, the galactic rotational speed is:

$$v_r = \sqrt{G(M_g/\gamma)/r} \text{ m/s}. \quad (17)$$

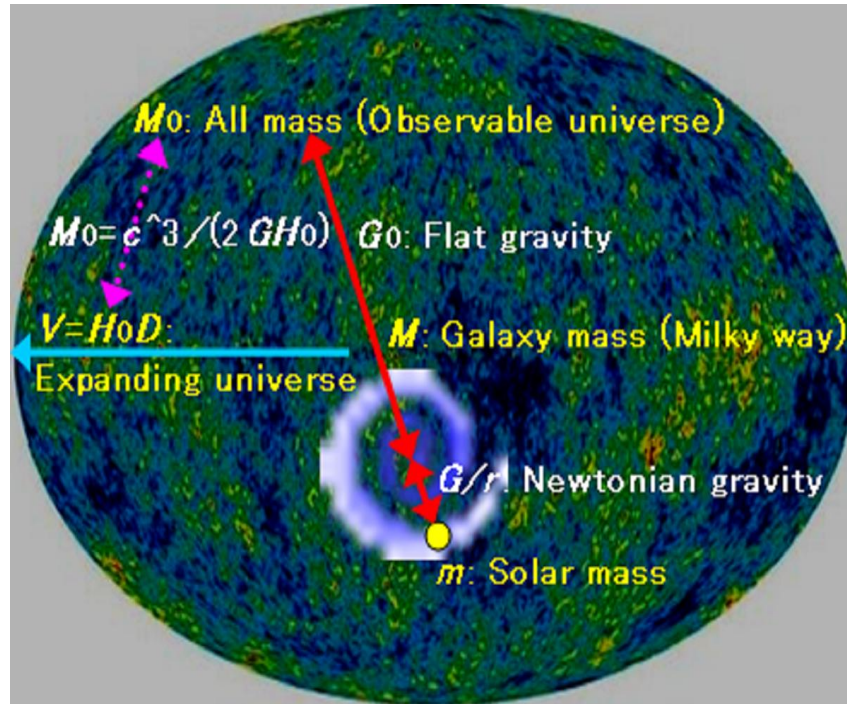


**Figure 3.** Widened Figure 11 (Kafle et al., 2012) by the setting that gravitational mass  $M_g$  (approximately 100 billion times of Solar mass) of the Milky Way galaxy did not increase from 30kpc to 180kpc.

In terms of the specific potential  $G_x$ , this is:

$$v_t = \sqrt{G_x(M_g/\gamma)} = \sqrt{(G/r + G_0)(M_g/\gamma)} \text{ m/s}. \quad (18)$$

Therefore, given the galactic rotational speed  $V_t$  and the radius  $r$ , we can determine the galactic gravitational mass  $M_g$ . this calculation was applied to Figure 2 and 3 of the STELLAR HALO model of the Milky Way galaxy (Kafle et al., 2012) [the Milky Way has an inner and outer halo that



**Figure 4.** Theory of flat gravity that is based on Hubble's law of an expanding universe and newtonian gravitation.

spreads far and wide. It is in the latter that flat gravity flattens the velocity distribution curve]. The curve for dark matter (that is, Halo) and the flat gravity curve are similar, except that a discrepancy emerges for large  $r$  (kpc). However, examples such as Abell 520 are observed and indicate that dark matter exists far from the galactic disk and bulge (Mahdavi et al., 2007). Such examples cannot be explained by the conventional dark-matter hypothesis. Because flat gravity extends to an infinite distance, if flat gravity exists, it is indifferent to dark matter (Figure 4).

### Small-scale crisis

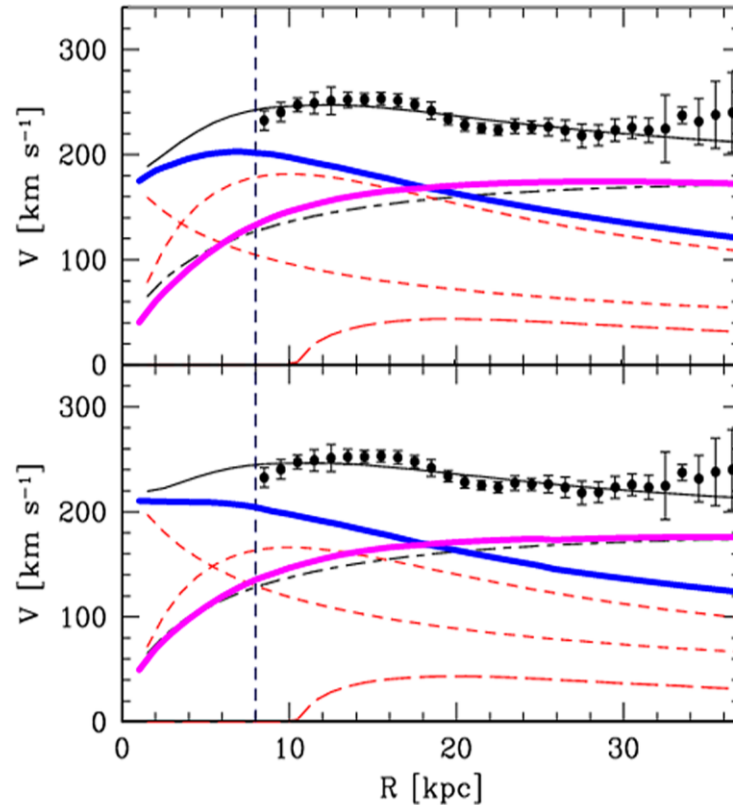
The lambda Cold Dark Matter ( $\Lambda$ -CDM) model which added the effect (cosmic constant  $\Lambda$ ) of the accelerating expansion of the universe to dark matter = Cold Dark Matter (CDM) which can disregard collisionless damping is a standard model of structure formation. However, as for the  $\Lambda$ -CDM model, disagreement with observation is pointed out in the small scale (below a Galaxy scale) (D'Onghia and Lake, 2004). Missing satellite problem (Klypin et al., 1999; Moore et al., 1999): Near the Milky Way galaxy (local group of galaxies), tens of dwarf galaxies (satellite galaxy) exist. However, N-body simulation of a  $\Lambda$ -CDM model is predicting that dark matter halo of about 500 dwarf galaxy mass exists in the same range. This suggests a possibility that the CDM model is wrong. Cuspy halo problem (Blok, 2009):

According to the N-body simulation, a center becomes high-density by the very cusp of the density profile of dark matter halo. However, if it asks for a density profile from observation of the rotation curve of a dwarf galaxy, the profile of such a cusp will not be found. There are two huge black-holes in the bulge of the Andromeda Galaxy, and the material density of the central part is not high (Corbelli et al., 2010). Flat gravity and a dark matter curve are in agreement if  $r$  (kpc) becomes large (Figure 5). Moreover, the central part does not become high-density like the density profile of CDM. Those problems seem to adjust forcibly by the dark matter hypothesis.

### Large-scale crisis

The scale of Hercules-Corona Borealis Great Wall is very huge. At the maximum of the size presumed from distribution of the gamma-ray burst, length is 10 billion light years and width is 7,200 million light years (Horvath et al., 2013). In the CDM model, the shock wave which occurred in the universe after the Big-Bang is assumed to be the base which makes the large-scale structure of the present universe. The size of the large-scale structure which arises from it should not exceed about 1,200 million light years (Yadav et al., 2010). This may show that it is unsuitable to predict the homogeneity of the actual universe and formation of large-scale structure by the CDM hypothesis. The Table 1 is the table which summarized the above.





**Figure 5.** Adjusted the rotation curve (blue) by Newtonian gravity and the rotation curve (pink) by Flat gravity to Figure 14 (Corbelli et al., 2010). The M31 rotation curve (points) and the best fitting mass model (solid line) using the NFW dark halo profile with  $C = 12$  in the frame of CDM. Also shown are the dark halo contribution (dot dashed line), the stellar disk and bulge (short dashed line) and the gas contribution (long dashed line). The bottom panel refers to the case  $((M/L)_d = (M/L)_b = 4.2 M/L)$ . The top panel refers to the best fit when the mass-to-light ratio of the disk and the bulge are two independent variables. For the best fit  $(M/L)_d = 5.0$  and an  $(M/L)_b = 2.7 M/L$ .

**Table 1.** Summary of this paper.

Open problem	Microscopic	Pioneer anomaly	Galaxy rotation	Over galaxy
MOND	Unknown	Unknown	Explanation	Unknown
MOG	Unknown	Unknown	Explanation	Unknown
Dark Matter	Unknown	Unknown	Explanation	Unknown
Flat Gravity	Unknown	Explanation	Explanation	Unknown

## Conclusion

The specific potential constant that was calculated from the abnormal acceleration of the planetary probe pioneer with respect to the Sun is  $G_0 = 1.18 \times 10^{-31} \text{ J/kg}^2$ . Given a galactic mass approximately 100 billion times that of the Sun gives a galactic rotational speed of  $v_g = (G_0 M_g / \gamma)^{1/2} =$

150 km/s. The gravitational potential is  $-GM(1/r)$  in a static universe; however, the influence of cosmic expansion should be considered in an expanding universe by using  $-GM(1/r)(1 + \text{expansion}) = -(G/r + G_0)M$  for all scales from the microscopic to the large scale of the universe. This resembles the relation between the longitudinal Doppler effect  $(1 - v/c)$  and the transverse

Doppler effect  $(1 - v^2/c^2)^{1/2}$ . Flat gravity can be named longitudinal gravity if Newtonian gravity is named transverse gravity. None of the theories (flat gravity, dark matter or modified gravitation) can explain all scales. However, flat gravity offers the possibility to explain small scales, such as the microscopic scale, which may increase correction term in the Newtonian gravitational potential, and the large scale, such as the intergalactic scale. It is important to gain experience with various scales for the expanding universe because we are familiar with Kepler's laws and Newtonian mechanics but need more experience with flat gravity. Not only an apple but all the matter in the expanding universe are in the state of free-fall.

### Conflict of Interest

The author has not declared any conflict of interest.

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### REFERENCES

- Ade PAR, Aghanim N, Alves MIR, Armitage-Caplan C, Arnaud M, Ashdown M, Atrio-Barandela F, Aumont J, Aussel H, Baccigalupi C, Banday AJ, Barreiro RB, Barrera R, Bartelmann M, Bartlett JG, Bartolo N, Basak S, Battaner E, Battye R, Benabed K, Benoît A, Benoit-Lévy A, Bernard JP, Bersanelli M, Bertinocourt B, Bethermin M, Bielewicz P, Bikmaev I, Blanchard A, Bobin J, Bock JJ, Böhringer H, Bonaldi A, Bonavera L, Bond JR, Borrill J, Bouchet FR, Boulanger F, Bourdin H, Bowyer JW, Bridges M, Brown ML, Bucher M, Burenin R, Burigana C, Butler RC, Calabrese E, Cappellini B, Cardoso JF, Carr R, Carvalho P, Casale M, Castex G, Catalano A, Challinor A, Chamballu A, Chary RR, Chen X, Chiang HC, Chiang LY, Chon G, Christensen PR, Churazov E, Church S, Clemens M, Clements DL, Colombi S, Colombo LPL, Combet C, Comis B, Couchot F, Coulais A, Crill BP, Cruz M, Curto A, Cuttaia F, Da Silva A, Dahle H, Danese L, Davies RD, Davis RJ, Bernardis PD, Rosa AD, Zotti GD, Déchelette T, Delabrouille J, Delouis JM, Démoclès J, Désert FX, Dick J, Dickinson C, Diego JM, Dolag K, Dole H, Donzelli S, Doré O, Douspis M, Ducout A, Dunkley J, Dupac X, Efstathiou G, Elsner F, Enßlin TA, Eriksen HK, Fabre O, Falgarone E, Falvella MC, Fantaye Y, Fergusson J, Filliard C, Finelli F, Flores-Cacho I, Foley S, Fornì O, Fosalba P, Frailis M, Fraisse AA, Franceschi E, Freschi M, Fromenteau S, Frommert M, Gaier TC, Galeotta S, Gallegos J, Galli S, Gandolfo B, Ganga K, Gauthier C, Génova-Santos RT, Ghosh T, Giard M, Giardino G, Gilfanov M, Girard D, Giraud-Héraud Y, Gjerløw E, González-Nuevo J, Górski KM, Gratton S, Gregorio A, Gruppuso A, Gudmundsson JE, Haissinski J, Hamann J, Hansen FK, Hansen M, Hanson D, Harrison DL, Heavens A, Helou G, Hempel A, Henrot-Versillé S, Hernández-Monteagudo C, Herranz D, Hildebrandt SR, Hivon E, Ho S, Hobson M, Holmes WA, Hornstrup A, Hou Z, Hovest W, Huey G, Huppenberger KM, Hurier G, Ilić S, Jaffe AH, Jaffe TR, Jasche J, Jewell J, Jones WC, Juvela M, Kalberla P, Kangaslahti P, Keihänen E, Kerp J, Keskitalo R, Khamitov I, Kiiveri K, Kim J, Kisner TS, Kneissl R, Knoche J, Knox L, Kunz M, Kurki-Suonio H, Lacasa F, Lagache G, Lähteenmäki A, Lamarre JM, Langer M, Lasenby A, Lattanzi M, Laureijs RJ, Lavabre A, Lawrence CR, Le Jeune M, Leach S, Leahy JP, Leonardi R, León-Tavares J, Leroy C, Lesgourgues J, Lewis A, Li C, Liddle A, Liguori M, Lilje PB, Linden-Vørnle M, Lindholm V, López-Cañiego M, Lowe S, Lubin PM, Macías-Pérez JF, MacTavish CJ, Maffei B, Maggio G, Maino D, Mandolesi N, Mangilli A, Marcos-Caballero A, Marinucci D, Maris M, Marleau F, Marshall DJ, Martin PG, Martínez-González E, Masi S, Massardi M, Matarrese S, Matsumura T, Matthai F, Maurin L, Mazzotta P, McDonald A, McEwen JD, McGehee P, Mei S, Meinhold PR, Melchiorri A, Melin JB, Mendes L, Menegoni E, Mennella A, Migliaccio M, Mikkelsen K, Millea M, Miniscalco R, Mitra S, Miville-Deschênes MA, Molinari D, Moneti A, Montier L, Morgante G, Morisset N, Mortlock D, Moss A, Munshi D, Murphy JA, Naselsky P, Nati F, Natoli P, Negrello M, Nesvadba NPH, Netterfield CB, Nørgaard-Nielsen HU, North C, Noviello F, Novikov D, Novikov I, O'Dwyer IJ, Orioux F, Osborne S, O'Sullivan C, Oxborrow CA, Paci F, Pagano L, Pajot F, Paladini R, Pandolfi S, Paoletti D, Partridge B, Pasian F, Patanchon G, Paykari P, Pearson D, Pearson TJ, Peel M, Peiris HV, Perdureau O, Perotto L, Perrotta F, Pettorino V, Piacentini F, Piat M, Pierpaoli E, Pietrobon D, Plaszczynski S, Platania P, Pogosyan D, Pointecouteau E, Polenta G, Ponthieu N, Popa L, Poutanen T, Pratt GW, Prézeau G, Prunet S, Puget JL, Pullen AR, Rachen JP, Racine B, Rahlin A, Räh C, Reach WT, Rebolo R, Reinecke M, Remazeilles M, Renault C, Renzi A, Riazuelo A, Ricciardi S, Riller T, Ringeval C, Ristorcelli I, Robbers G, Rocha G, Roman M, Rosset C, Rossetti M, Roudier G, Rowan-Robinson M, Rubiño-Martín JA, Ruiz-Granados B, Rusholme B, Salerno E, Sandri M, Sanselme L, Santos D, Savelainen M, Savini G, Schaefer BM, Schiavon F, Scott D, Seiffert MD, Serra P, Shellard EPS, Smith K, Smoot GF, Souradeep T, Spencer LD, Starck JL, Stolyarov V, Stompor R, Sudiwala R, Sunyaev R, Sureau F, Sutter P, Sutton D, Suur-Uski AS, Sygnet JF, Tauber JA, Tavagnacco D, Taylor D, Terenzi L, Texier D, Toffolatti L, Tomasi M, Torre JP, Tristram M, Tucci M, Tuovinen J, Türlér M, Tuttlebee M, Umama G, Valenziano L, Valiviita J, Van Tent B, Varis J, Vibert L, Viel M, Vielva P, Villa F, Vittorio N, Wade LA, Wandelt BD, Watson C, Watson R, Wehus IK, Welikala N, Weller J, White M, White SDM, Wilkinson A, Winkel B, Xia JQ, Yvon D, Zacchei A, Zibin JP, Zonca A (2013). Planck 2013 results. I. Overview of products and scientific results. preprint arXiv: 1303.5062.
- Anderson JD, Laing PA, Lau EL, Liu AS, Nieto MM, Turyshev, SG (1998). Indication, from Pioneer 10/11, Galileo, and Ulysses data, of an apparent anomalous, weak, long-range acceleration. *Phys. Rev. Lett.* 81(14):2858. <http://dx.doi.org/10.1103/PhysRevLett.81.2858>
- Astrodynamic Constants, <http://ssd.jpl.nasa.gov/?constants>, Accessed 7 July 2014.
- Bidin CM, Carraro G, Méndez RA, Smith R (2012). Kinematical and chemical vertical structure of the galactic thick disk. II. A lack of dark matter in the solar neighborhood. *Astrophys. J.* 751(1):30. <http://dx.doi.org/10.1088/0004-637X/751/1/30>
- Blok WJGD (2009). The core-cusp problem. *Advances in Astronomy.* 2010.
- Boom PGT (2013). The Pioneer Anomaly: An inconvenient reality or NASA's 12 year misconception?. preprint arXiv: 1307.0537.
- Brownstein JR, Moffat JW (2006). Galaxy rotation curves without nonbaryonic dark matter. *Astrophys. J.* 636(2): 721. <http://dx.doi.org/10.1086/498208>
- Corbelli E, Lorenzoni S, Walterbos R, Braun R, Thilker D (2010). A wide-field HI mosaic of Messier 31: II. The disk warp, rotation, and the dark matter halo. *Astron Astrophys.* 511.
- D'Onghia E, Lake G (2004). Cold dark matter's small-scale crisis grows up. *Astrophys. J.* 612(2):628. <http://dx.doi.org/10.1086/422794>
- Hodge JC (2013). Comments on The Pioneer Anomaly: an inconvenient reality or NASA's 12 year misconception. L4C 3N0 Canada E-mail Web Site Series. 1.
- Horvath I, Hakkila J, Bagoly Z (2013). The largest structure of the Universe, defined by Gamma-Ray Bursts. *arXiv preprint arXiv:1311.1104*.
- Hubble E (1929). A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Natl. Acad. Sci. U.S.A.* 15(3):168-173. <http://dx.doi.org/10.1073/pnas.15.3.168>
- Kafle PR, Sharma S, Lewis GF, Bland-Hawthorn J (2012). Kinematics of the Stellar Halo and the Mass Distribution of the Milky Way Using Blue Horizontal Branch Stars. *Astrophys. J.* 761(2):98. <http://dx.doi.org/10.1088/0004-637X/761/2/98>
- Ade PAR, Aghanim N, Alves MIR, Armitage-Caplan C, Arnaud M, Ashdown M, Atrio-Barandela F, Aumont J, Aussel H, Baccigalupi C, Banday AJ, Barreiro RB, Barrera R, Bartelmann M, Bartlett JG, Bartolo N, Basak S, Battaner E, Battye R, Benabed K, Benoît A, Benoit-Lévy A, Bernard JP, Bersanelli M, Bertinocourt B, Bethermin M, Bielewicz P, Bikmaev I, Blanchard A, Bobin J, Bock JJ, Böhringer H, Bonaldi A, Bonavera L, Bond JR, Borrill J, Bouchet FR, Boulanger F, Bourdin H, Bowyer JW, Bridges M, Brown ML, Bucher M, Burenin R, Burigana C, Butler RC, Calabrese E, Cappellini B, Cardoso JF, Carr R, Carvalho P, Casale M, Castex G, Catalano A, Challinor A, Chamballu A, Chary RR, Chen X, Chiang HC, Chiang LY, Chon G, Christensen PR, Churazov E, Church S, Clemens M, Clements DL, Colombi S, Colombo LPL, Combet C, Comis B, Couchot F, Coulais A, Crill BP, Cruz M, Curto A, Cuttaia F, Da Silva A, Dahle H, Danese L, Davies RD, Davis RJ, Bernardis PD, Rosa AD, Zotti GD, Déchelette T, Delabrouille J, Delouis JM, Démoclès J, Désert FX, Dick J, Dickinson C, Diego JM, Dolag K, Dole H, Donzelli S, Doré O, Douspis M, Ducout A, Dunkley J, Dupac X, Efstathiou G, Elsner F, Enßlin TA, Eriksen HK, Fabre O, Falgarone E, Falvella MC, Fantaye Y, Fergusson J, Filliard C, Finelli F, Flores-Cacho I, Foley S, Fornì O, Fosalba P, Frailis M, Fraisse AA, Franceschi E, Freschi M, Fromenteau S, Frommert M, Gaier TC, Galeotta S, Gallegos J, Galli S, Gandolfo B, Ganga K, Gauthier C, Génova-Santos RT, Ghosh T, Giard M, Giardino G, Gilfanov M, Girard D, Giraud-Héraud Y, Gjerløw E, González-Nuevo J, Górski KM, Gratton S, Gregorio A, Gruppuso A, Gudmundsson JE, Haissinski J, Hamann J, Hansen FK, Hansen M, Hanson D, Harrison DL, Heavens A, Helou G, Hempel A, Henrot-Versillé S, Hernández-Monteagudo C, Herranz D, Hildebrandt SR, Hivon E, Ho S, Hobson M, Holmes WA, Hornstrup A, Hou Z, Hovest W, Huey G, Huppenberger KM, Hurier G, Ilić S, Jaffe AH, Jaffe TR, Jasche J, Jewell J, Jones WC, Juvela M, Kalberla P, Kangaslahti P, Keihänen E, Kerp J, Keskitalo R, Khamitov I, Kiiveri K, Kim J, Kisner TS, Kneissl R, Knoche J, Knox L, Kunz M, Kurki-Suonio H, Lacasa F, Lagache G, Lähteenmäki A, Lamarre JM, Langer M, Lasenby A, Lattanzi M, Laureijs RJ, Lavabre A, Lawrence CR, Le Jeune M, Leach S, Leahy

- Klypin A, Kravtsov AV, Valenzuela O, Prada F (1999). Where are the missing galactic satellites?. *Astrophys. J.* 522(1): 82. <http://dx.doi.org/10.1086/307643>
- Kopeikin SM (2012). Celestial ephemerides in an expanding universe. *Phys. Rev. D.* 86(6):064004. <http://dx.doi.org/10.1103/PhysRevD.86.064004>
- Kragh H (1999). *Cosmology and controversy: The historical development of two theories of the universe*. Princeton University Press: 212.
- Lachieze-Rey M (2007). Cosmology in the solar system: The Pioneer effect is not cosmological. *Class. Quantum Grav.* 24(10):2735. <http://dx.doi.org/10.1088/0264-9381/24/10/016>
- Lachieze-Rey M (2014). In search of relativistic time. *Stud. Hist. Philos. M. P.* 46: 38-47. <http://dx.doi.org/10.1016/j.shpsb.2014.01.001>
- Mahdavi A, Hoekstra H, Babul A, Balam DD, Capak PL (2007). A dark core in Abell 520. *Astrophys. J.* 668(2):806. <http://dx.doi.org/10.1086/521383>
- Milgrom M (1983). A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys. J.* 270: 365-370. <http://dx.doi.org/10.1086/161130>
- Mizony M, Lachieze-Rey M (2005). Cosmological effects in the local static frame. *A&A* 434: 45-52.
- Moore B, Ghigna S, Governato F, Lake G, Quinn T, Stadel J, Tozzi P (1999). Dark matter substructure within galactic halos. *Astrophys. J. Lett.* 524(1): L19. <http://dx.doi.org/10.1086/312287>
- Rubin VC, Ford Jr WK, Thonnard N (1980). Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605/R= 4kpc to UGC 2885/R= 122 kpc. *Astrophys. J.* 238: 471-487. <http://dx.doi.org/10.1086/158003>
- Tangen K (2007). Could the Pioneer anomaly have a gravitational origin?. *Phys. Rev. D.* 76(4): 042005. <http://dx.doi.org/10.1103/PhysRevD.76.042005>
- Turyshev SG, Toth VT (2010). The pioneer anomaly. *Living Rev. Relat.* 13(4):9-175.
- Turyshev SG, Toth VT, Kinsella G, Lee SC, Lok SM, Ellis J (2012). Support for the thermal origin of the Pioneer anomaly. *Phys. Rev. Lett.* 108(24):241101. <http://dx.doi.org/10.1103/PhysRevLett.108.241101>
- Yadav JK, Bagla JS, Khandai N (2010). Fractal dimension as a measure of the scale of homogeneity. *Mon. Not. R. Astron. Soc.* 405(3):2009-2015.
- Zwicky F (1933). Die rotverschiebung von extragalaktischen nebeln. *Helv. Phys. Acta.* 6:110-127.
- Zwicky F (1937). On the Masses of Nebulae and of Clusters of Nebulae. *Astrophys. J.* 86: 217. <http://dx.doi.org/10.1086/143864>