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Full Length Research Paper

Magneto-hydrodynamic (MHD) flow past an infinite vertical plate immersed in a stably stratified fluid

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The effect of transverse magnetic field on the flow past an infinite vertical plate, immersed in a stably stratified fluid has been investigated. The non-dimensional governing equations are solved by Laplace transform technique. Numerical computations for velocity, temperature, plate heat flux and skin-friction are made for different values of the physical parameters and shown in graphs. Important observations due to the effect of thermal stratification is made and compared with the case of no stratification. It is observed that due to the application of transverse magnetic field on the flow, the steady state is reached at smaller times.

Key words: MHD flow, electrically conducting fluid, stratified fluid, vertical plate, unsteady flow.

INTRODUCTION

The unsteady natural convection flow of a viscous fluid adjacent to vertical plates is a fundam ental problem in fluid mechanics and heat transfer and is useful in several physical and engineering problems. The unsteadiness in the flow field is mainly caused either by time-dependent motion of the external stream or by impulsive motion of the external stream. The first closed form solutions for *Pr* = 1, in case of a step change in wall temperature with time was derived by Illingworth (1950) and for Pr ≠1; he derived the solution in integral form. Analytical solutions to the problem of flow past a semi infinite vertical plate under step change in plate temperature or plate heat flux was obtained by Siegel (1958), Menold and Yang (1962), Schetz and Eichhorn (1962) and Das et al. (1999). Goldstein and Briggs (1964) analytically studied the problem of transient free convection flow past an infinite vertical plate and introduced the idea of leading edge effect. In all these studies, pressure work and ambient thermal stratifications was not taken into consideration.

Park and Hyun (1998) and Park (2001) studied the one- dimensional natural convection of viscous stratified fluid. Shapiro and Fedorovich (2004) studied the unsteady convectively driven flow of a stably stratified fluid along a plate, taking pressure work and ambient thermal stratifications into account, while Magyari et al. (2006) restudied for a porous medium. In their study, the pressure work term is included in the thermodynamic energy equation and makes a provision for vertical temperature stratifications.

Magneto hydrodynamic (MHD) flow is related to engineering problems, such as plasma confinement, liquid-metal cooling of nuclear reactors, electromagnetic casting, etc. In all these applications, the presence of applied transverse magnetic field plays an important role. The effects of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating plate with constant heat flux was analyzed by Soundalgekar et al. (1997). Researchers like Revankar (1983), Anwar (1998) and Sahoo et al. (2003) worked on MHD natural convection flow past vertical surfaces with different boundary conditions. Mixed convective MHD mass transfer flow past an accelerated infinite vertical porous plate was studied by Ramana et al. (2009). Chaudhary and Jain (2009) investigated MHD heat and mass diffusion flow by natural convection past a vertical plate in porous medium. Recently, effect of thermally stratified ambient fluid on MHD convective flow along a moving

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Abbreviation: MHD, Magneto-hydrodynamic.

non-isothermal vertical plate is studied by Gurminder et al. (2010).

In this paper, we study the effect of transverse magnetic field on the flow of a convectively driven flow in a stably stratified fluid along a vertical plate. Here, in the thermodynamic energy equation, the pressure work term is included, taking the thermal stratification into account. The solutions are then obtained for Prandtl number of unity. The effects of parameters such as the magnetic parameter *M*, Grashof number *Gr*, stratification parameter γ and time *t* on velocity and temperature profiles are studied and shown on graphs. Effects of *M*, *Gr*, γ and *t* on other physical phenomenon like rate of heat transfer and the skin-friction (measure of shear stress) are also analyzed.

MATHEMATICAL ANALYSIS

We consider the flow of an electrically conducting, viscous fluid past an infinite vertical plate. The x´- axis is taken along the plate in vertically upward direction and y´- axis is taken normal to the plate in the direction of applied magnetic field. The fluid fills the region y´≥ 0. Initially at time t´ ≤ 0, the fluid and the plate are in a stationary condition, that is, the fluid is quiescent until a uniform thermal disturbance at the plate at time t´ > 0 is imposed. Since the plate is considered infinite in x´ direction, hence all the physical variables will be dependent on y´ and time t´ only. Then the vertical equation of motion and energy equation are;

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\frac{T'}{T_r} - \frac{\sigma B_0^2}{\rho} u' \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} - \gamma u' \tag{2}$$

Here the term $g\dfrac{T'}{T_r}$ is the buoyancy force per unit mass and $\gamma' \equiv \dfrac{dT'_\infty}{dx'} + \dfrac{g}{C_p}$, where $\dfrac{dT'_\infty}{dx'}$, is the vertical temperature advection termed as thermal stratification. Also, g/C_p is the rate of reversible work done on the fluid particles by compression, known as work of compression. As the work of compression is very small, the parameter γ' will be termed as thermal stratification parameter in our study. The work of compression is retained as additive one to thermal stratification for validating numerical models. For complete derivation related to thermal stratification and the work of compression, the readers are referred to Shapiro and Fedorovich (2004) and Ozisik (1994).

The initial and boundary conditions are taken as:

$$\left.\begin{aligned} &u'=0,\ T'=0,\ \forall y',\ t'\le 0\\ &u'=0,\ T'=T'_o \ \text{ at } \ y'=0,\ t'>0\\ &u'=0,\ T'=0 \ \ \text{ at } \ y'\to\infty,\ t'>0\end{aligned}\right\} \tag{3}$$

Now, defining characteristic length $d = \nu^{2/3} g^{-1/3}$, we introduce the following non-dimensional quantities:

$$\left.\begin{aligned} &y=\frac{y'}{d},\ t=\frac{t'\nu}{d^2},\ \theta=\frac{T'}{T'_o},\ u=\frac{u'd}{\nu},\ \Pr=\frac{\nu}{\alpha}\\ &M=\frac{\sigma B_0^2 d^2}{\rho\nu},\ \gamma=\frac{\gamma' d}{T'_0},\ Gr=\frac{gd^3T'_0}{\nu^2 T_r}\end{aligned}\right\} \tag{4}$$

Using quantities defined in 4 above, the Equations 1 and 2 take the following forms:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta - Mu \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \gamma u \tag{6}$$

The corresponding boundary conditions are,

$$\left.\begin{aligned} &u=0,\ \theta=0\ \ \forall\ y \ \text{ and } \ t\le 0\\ &u=0,\ \theta=1 \ \text{ at } \ y=0,\ t>0\\ &u=0,\ \theta=0 \ \text{ at } \ y\to\infty,\ t>0\end{aligned}\right\} \tag{7}$$

We now solve coupled linear Equations 5 and 6 for the tractable case of Pr = 1, subject to the initial and boundary conditions (7) by Laplace transform technique. The solutions are,

$$\theta = \frac{A}{2\sqrt{M^2-4\gamma Gr}}\left[e^{-y\sqrt{B}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Bt}\right)+e^{y\sqrt{B}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Bt}\right)\right] - \frac{B}{2\sqrt{M^2-4\gamma Gr}}\left[e^{-y\sqrt{A}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{At}\right)+e^{y\sqrt{A}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{At}\right)\right] \tag{8}$$

$$u = \frac{Gr}{2\sqrt{M^2-4\gamma Gr}}\left[e^{-y\sqrt{B}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Bt}\right)+e^{y\sqrt{B}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Bt}\right)\right] - \frac{Gr}{2\sqrt{M^2-4\gamma Gr}}\left[e^{-y\sqrt{A}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{At}\right)+e^{y\sqrt{A}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{At}\right)\right] \tag{9}$$

where A $=(M+\sqrt{M^2-4\gamma Gr})/2$, B $= (M-\sqrt{M^2-4\gamma Gr})/2$ and '*erfc*' is the complementary error function, defined by $erfc(x) = 1 - erf(x)$ with $erf(x) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^x \exp(-\eta^2)\,d\eta$.

The case, when pressure work is neglected and the environment is isothermal, is termed in our study as 'Classical' case for brevity, and then the solutions can be readily obtained from Equations 5 and 6 using Equation 7 by setting γ = 0. The solutions u^*, θ^* designated for 'Classical' case are then,

$$\theta^* = erfc\left(\frac{y}{2\sqrt{t}}\right) \tag{10}$$

$$u^* = \frac{Gr}{M}erfc\left(\frac{y}{2\sqrt{t}}\right) - \frac{Gr}{2M}\left[e^{-y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Mt}\right)+e^{y\sqrt{M}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\right] \tag{11}$$

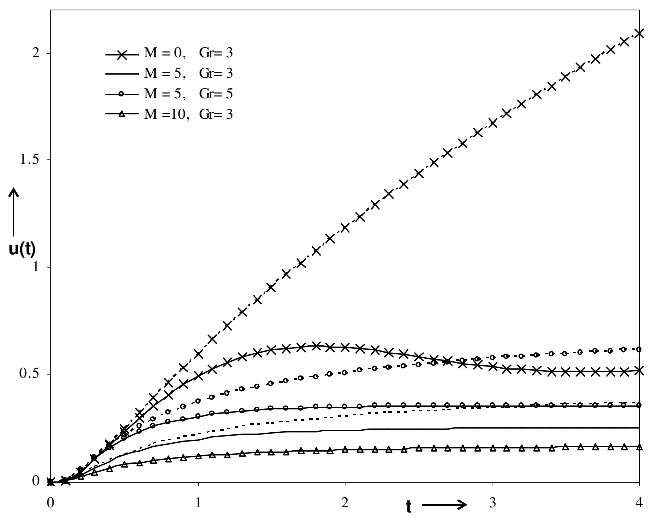

Figure 1. Variation of $u(y,t)$ at dimensionless distance y =1 from the plate. Solid lines represent solutions when γ=1 and dotted lines represent solutions for 'Classical' case (γ = 0).

Steady-state solution

In steady state, the time derivatives of *u* and *θ* are neglected. So, Equations 5 and 6 reduce to the forms,

$$\frac{\partial^2 u}{\partial y^2}+Gr\theta-Mu=0 \tag{12}$$

$$\frac{\partial^2 \theta}{\partial y^2}-\gamma u=0 \tag{13}$$

Solving the above Equations subject to Condition 7, we obtain steady-state solutions for temperature and velocity as,

$$\theta=\frac{A}{\sqrt{M^2-4\gamma Gr}}e^{-y\sqrt{B}}-\frac{B}{\sqrt{M^2-4\gamma Gr}}e^{-y\sqrt{A}} \tag{14}$$

$$u=\frac{Gr}{\sqrt{M^2-4\gamma Gr}}\left(e^{-y\sqrt{B}}-e^{-y\sqrt{A}}\right) \tag{15}$$

It can readily be seen that as $t\rightarrow\infty$, the unsteady solutions given in (8) and (9) reduce to the solutions (14) and (15) above, using the property of complementary error function viz; *erfc*(−∞) =2 and *erfc*(∞) = 0.

SKIN-FRICTION

Knowing the velocity field, we now study the effects of different parameters on the skin-friction (τ), which is the measure of shear stress on the wall. In non-dimensional form, it is given as,

$$\tau=-\left.\frac{du}{dy}\right|_{y=0}$$

Using (9) we derive it as,

$$\tau=\frac{Gr}{\sqrt{M^2-4\gamma Gr}}\left[\frac{e^{-Bt}-e^{-At}}{\sqrt{\pi t}}+\sqrt{B}erf\left(\sqrt{Bt}\right)-\sqrt{A}erf\left(\sqrt{At}\right)\right] \tag{16}$$

For 'Classical' case, skin friction is derived as,

$$\tau^*=\frac{Gr}{M}\left[\left(1-e^{-Mt}\right)\frac{1}{\sqrt{\pi t}}-\sqrt{M}\,erf\left(\sqrt{Mt}\right)\right] \tag{17}$$

The above expression shows that as $t\rightarrow\infty$, the skin-friction corresponding to 'Classical' case approaches a fixed value of $\left(-\frac{Gr}{\sqrt{M}}\right)$, while in presence of stratification the fixed value is;

$$\left\{-\frac{Gr\left(\sqrt{A}-\sqrt{B}\right)}{\sqrt{M^2-4\gamma Gr}}\right\} \text{ as } t\rightarrow\infty.$$

PLATE HEAT FLUX (NUSSELT NUMBER)

Another physical phenomenon is the rate of heat transfer (Nusselt number), which in non-dimensional form is given by,

$$Nu=-\left.\frac{d\theta}{dy}\right|_{y=0}$$

From expression 8, it is obtained as,

$$Nu=\frac{\sqrt{AB}}{\sqrt{M^2-4\gamma Gr}}\left[\sqrt{A}\,erf\left(\sqrt{Bt}\right)-\sqrt{B}\,erf\left(\sqrt{At}\right)\right]+\frac{1}{\sqrt{\pi t}\sqrt{M^2-4\gamma Gr}}\left(Ae^{-Bt}-Be^{-At}\right) \tag{18}$$

In 'Classical' case, *Nu* is given by,

$$Nu^*=\frac{1}{\sqrt{\pi t}} \tag{19}$$

which is independent of the magnetic field parameter *M* and *Gr*. It is also seen from expression 19 that, for large time as $t\rightarrow\infty$, the plate heat flux approaches zero. On the other hand, in presence of stratification, the Nusselt number approaches a fixed value of $\frac{\gamma Gr\left(\sqrt{A}-\sqrt{B}\right)}{\sqrt{M^2-4\gamma Gr}}$ as $t\rightarrow\infty$.

RESULT AND DISCUSSION

In order to discuss the effects of various physical parameters on velocity field and temperature field, the numerical computations of the solutions, obtained in the preceding section have been carried out and they are plotted in Figures 1 to 4.

Figure 1 represents variation of velocity (*u*) against time

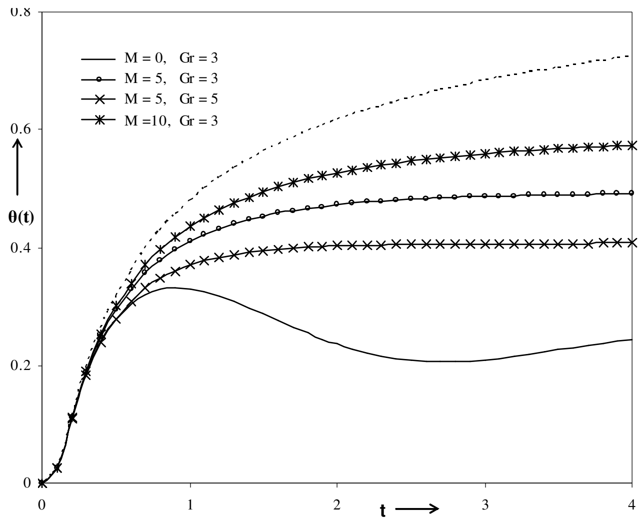

Figure 2. Variation of $\theta(y,t)$ at dimensionless distance y = 1 from the plate. Solid lines represent solutions when γ = 1 and dotted lines represent solutions for 'Classical' case (γ = 0).

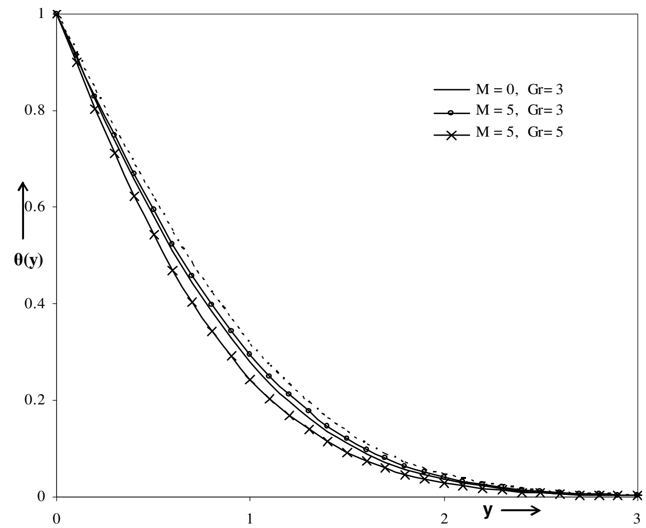

Figure 4. Temperature profiles for different values of M and Gr at time t =.5. Solid lines represent solutions for γ = 1 and dotted lines represent solutions for 'Classical' case.

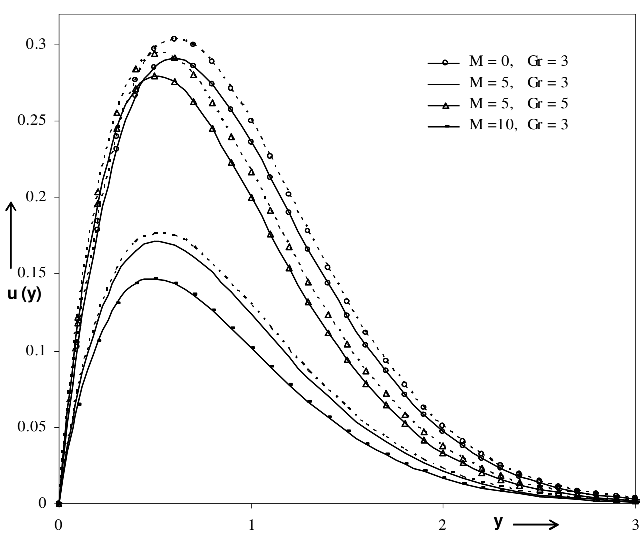

Figure 3. Velocity profiles for different values of M and Gr at time t = 0.5. Solid lines represent solutions for γ = 1 and dotted lines represent solutions for 'Classical' case.

t at a dimensionless distance y =1 from the plate for different values of M and Gr. It is observed that in the 'Classical' case (γ = 0), due to the application of transverse magnetic field, the velocity approaches steady state for larger time as the strength of the magnetic field decreases and Gr increases, while due to stratification (γ = 1), the steady state is reached earlier. Also, in absence of transverse magnetic field, velocity increases with time in 'Classical' case and no steady state is reached. It is to be noted that, in the 'Classical' case, the expression of temperature (θ) is independent of M and Gr (Equation 10). However, in our present study, in the presence of thermal stratification and pressure work, the temperature reaches steady state at larger time as M increases (Figure 2). On the other hand, temperature reaches a steady state at smaller time as Gr increases. Also, the temperature is more in 'Classical' case and increases gradually with time. The effect of M and Gr on the velocity profile is shown in Figure 3 for both cases of γ = 0 and γ ≠ 0. It is observed that, velocity decreases as M increases and increases as Gr increases in both cases. This is attributed to the fact that application of transverse magnetic field produces a resistive type of force (Lorentz force), similar to drag force, thereby reducing flow velocity. Also, increase in Gr means buoyancy force dominates viscous force, thereby enhances fluid velocity. Furthermore, the velocity is more in the 'Classical' case. Because of the layering effect of stratification, this acts like a resistive force, leading to reduction in fluid velocity due to the presence of stratification. Figure 4 shows the effect of M on temperature in presence of stratification. It is observed that temperature increases as M increases, while it decreases with Gr. This happens because of the fact that for higher values of Gr, buoyancy force assists the flow by increasing fluid velocity and hence the heat is convected readily, thereby reducing fluid temperature. Again, increase in magnetic parameter decreases the fluid velocity; as a result, heat will not be convected readily. This in turn increases the fluid temperature.

The effects of M, Gr and γ on the skin-friction are shown in Figure 5. It is observed that in both cases (γ = 0 and γ ≠ 0), skin-friction increases for increasing M and decreasing Gr. A smaller value of Gr implies more prominent viscous effects causing an enhanced frictional

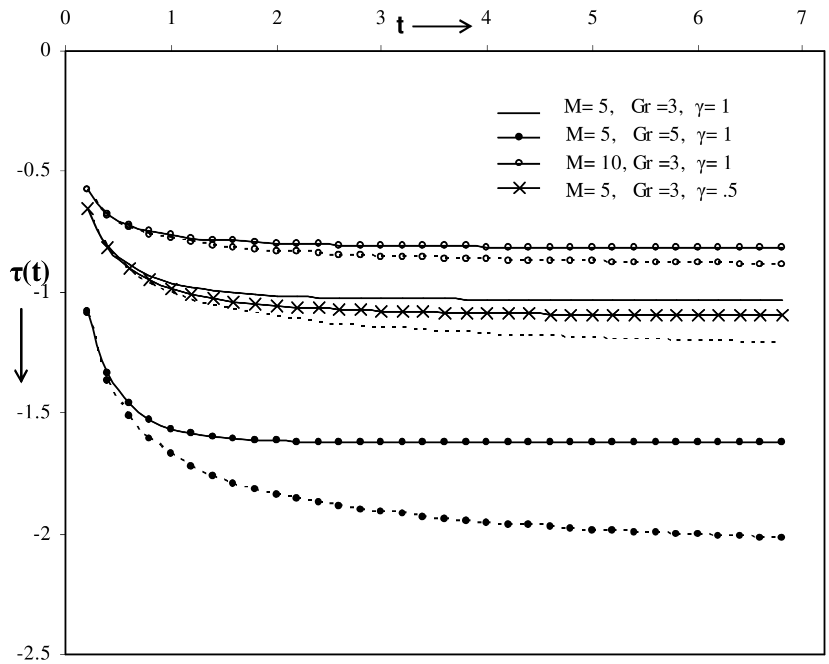

Figure 5. Skin-friction profiles for different values of M, *Gr* and γ. Dotted lines represent solutions for 'Classical' case.

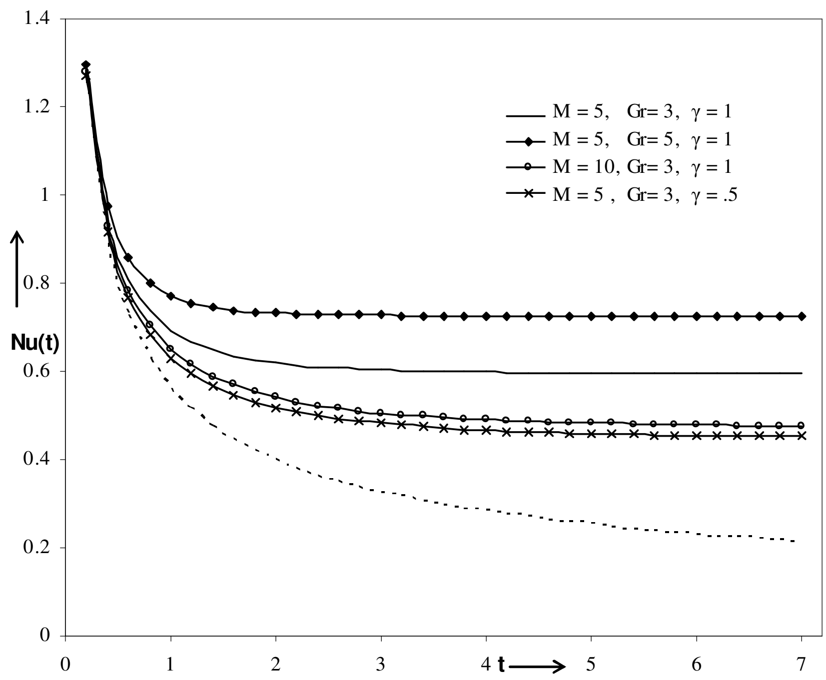

Figure 6. Nusselt number profiles for different values of *M*, *Gr* and γ. Dotted line represents solutions for 'Classical' case.

force. However, in presence of transverse magnetic field, skin-friction decreases monotonically for smaller time and then approaches a finite negative value (see Equations 16 and 17) as time progresses.

The effect of *M*, *Gr* and γ on plate heat flux (Nusselt number) against time (*t*) is shown in Figure 6. It is observed that heat flux decreases gradually as time progresses. Also, Nusselt number decreases (increases) as *M* (*Gr*) increases. In Equation 19, it has been shown that for 'Classical' case, '*Nu*' is independent of *M* and *Gr*. It is important to see that in 'Classical' case, plate heat flux decreases monotonically as time progresses.

Conclusions

We have studied the effect of the transverse magnetic field on the flow past an infinite vertical plate, immersed in a stably stratified fluid, where the energy equation is modified by introducing the work of compression term combined with thermal stratification. The solutions obtained in the present study are compared with the solutions for the 'Classical' case when the thermal stratification is absent. The conclusions of our study are as follows:

Velocity decreases as *M* increases and increases as *Gr* increases for both the cases of $\gamma = 0$ and $\gamma \neq 0$. However, velocity is more in 'Classical' case as compared to the case of thermal stratification.

Velocity reaches steady state at smaller time as *M* increases.

In presence of thermal stratification, temperature increases as *M* increases and decreases as Gr increases, while temperature is more in 'Classical' case.

Temperature reaches steady state at smaller time when *M* is small.

Skin-friction increases as time and *M* increases, but decreases as *Gr* increases. Also, skin-friction is more in presence of stratification.

In presence of thermal stratification, the plate heat flux decreases as *M* increases and increases as *Gr* increases. However, in 'Classical' case, heat flux decreases monotonically as time progresses.

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Nomenclatures: g**,** Acceleration due to gravity (LT^{-2}); ***Pr*****,** Prandtl number; ***Gr*****,** Grashof number; T', temperature of fluid (K); T_r**,** reference temperature; t'**,** dimensional time (T); u'**,** dimensional velocity (LT^{-1}); y'**,** cartesian co-ordinate normal to the plate; B_o**,** Strength of applied magnetic field; ***M*****,** non-dimensional magnetic parameter; ***u*****,** dimensionless velocity; ***t*****,** dimensionless time; ρ**,** density of the fluid (ML^{-3}); θ**,** dimensionless temperature; ν**,** kinematic viscosity (L^{2}T^{-1}); τ **,** dimensionless skin-friction; σ**,** electrical conductivity (L^{-2}T); α**,** thermal diffusivity (L^{2}T^{-1}); γ'**,** stratification parameter (KL^{-1}); **γ,** dimensionless stratification parameter; C_p**,** specific heat at constant pressure (JKg^{-1}K^{-1}).

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