International Journal of the Physical Sciences Vol. 5(15), pp. 2346-2350, 18 November, 2010
Available online at http://www.academicjournals.org/IJPS
ISSN 1992 - 1950 ©2010 Academic Journals

Full Length Research Paper

Determining economic lot size and number of deliveries for EPQ model with quality assurance using algebraic approach

Feng-Tsung Cheng[1] and Chia-Kuan Ting[2]*

[1]Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung 407, Taiwan.
[2]Department of Industrial Engineering and Management, Chaoyang University of Technology
Wufong, Taichung 413, Taiwan.

Accepted 05 November, 2010

The conventional method for determining production-shipment policy is by the use of differential calculus and Hessian matrix equations with the need of applying the first-order and second- order differentiations, on the long-run average production-inventory-delivery cost. A recent paper used conventional procedure to solve the production-shipment problem with quality assurance. This paper uses an alternative approach to re-examine such a problem without using the derivatives. As a result, optimal lot size and number of deliveries derived by the proposed algebraic method was confirmed to be identical, to what was obtained by using the conventional method. A simpler expression of cost function was also revealed.

Key words: Production-shipment policy, optimization, lot size, multiple distribution policy, algebraic approach, quality assurance, production planning.

INTRODUCTION

The most economical production lot known as the economic production quantity (EPQ) model was first introduced by Taft (1918) many decades ago. It presented an optimal replenishment quantity (lot size) to assist manufacturing firm in minimizing total production-inventory costs (Tersine, 1994). The EPQ model implicitly assumes that, all items produced are of perfect quality. However, in real world manufacturing environments, due to many different factors, generation of nonconforming items seems inevitable. For this reason, during the past decades many studies have been carried out to address the imperfect production and its related issues (Mak, 1985; Rosenblatt and Lee, 1986; Henig and Gerchak, 1990; Cheung and Hausman, 1997; Grosfeld-Nir and Gerchak, 2002; Chiu and Chiu, 2006; Wee et al., 2007; Baten and Kamil, 2009; Chiu et al., 2010a, b; Wazed et al., 2010a; b).

Another unrealistic assumption of classic EPQ model, is the continuous inventory issuing policy for satisfying the demand. However, in a vendor-buyer integrated production-shipment system, periodic delivery policy is commonly used. Schwarz (1973) first investigated a one-warehouse N-retailer deterministic inventory system, in order to determine the stocking policy which minimizes long-run average system cost. Goyal (1977) considered an integrated inventory-shipment model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems, where a product is procured by a single customer from a single supplier. An example with analysis was provided to illustrate this method. Many studies have since been conducted to address issues of various aspects of supply chain optimization (Banerjee, 1986; Goyal and Gupta, 1989; Sarker and Parija, 1994; Viswanathan, 1998; Swenseth and Godfrey, 2002; Diponegoro and Sarker, 2006; Kim et al., 2008; Chiu et al., 2009; Abolhasanpour et al., 2009; Chen et al., 2010).

Chiu et al. (2010c) jointly determined economic batch size and optimal number of deliveries for EPQ model with quality assurance issue. They used the differential calculus

*Corresponding author. Email: djk@cyut.edu.tw.

along with Hessian matrix equations to derive the optimal replenishment lot size, as well as the number of shipments for an imperfect EPQ model with failure in rework. Grubbström and Erdem (1999) presented an algebraic approach, to solve the economic order quantity (EOQ) model with backlogging, without reference to the use of derivatives. Similar studies used the same or similar method (Chiu, 2008; Lin et al., 2008). This paper applies the same alternative approach to a specific EPQ model examined by Chiu et al. (2010c). We show that, the optimal lot size, number of deliveries and the long-run average cost can all be derived without using differential calculus or the Hessian matrix equations (Rardin, 1998).

METHODS

An algebraic approach is adopted in this paper to re-examine Chiu et al.'s model (2010c), as stated earlier. Description of this production system is as follows. Consider a real world production system where process may randomly produce a portion x of defective items at a rate d. All items produced are screened and inspection cost is included in the unit production cost C. All nonconforming items produced are reworked at a rate of P1 and it starts immediately after the regular production process. A portion θ1 (where 0 < = θ1 < = 1) of reworked items, fails during the reworking and becomes scrap. In order to avoid shortage from occurring, it is assumed that the constant production rate P, has to be larger than the sum of demand rate λ and production rate of defective items d. That is: (P-d-λ) > 0; where the production rate of defective items d, can be expressed as d = Px. Let d1 denote production rate of scrap items during the rework process, then d1 can be expressed as: d1 = P1θ1.

Furthermore, this study considers a multi-delivery policy. It is assumed that the finished items can only be delivered to customers, if the whole lot is quality assured at the end of rework. Fixed-quantity, n installments of the finished batch, are delivered to the customer at a fixed interval of time, during production downtime, t3. On-hand inventory of perfect quality items of the proposed model is depicted in Figure 1. For the purpose of easing readability, this paper adopted the same notation as was used in Chiu et al. (2010c) as follows:

Q = production lot size, to be determined for each cycle,
n = number of fixed quantity installments of the finished batch to be delivered to customer, to be determined for each cycle,
D = number of finished items (fixed quantity) distributed to customer per delivery,
t1 = the production uptime for the proposed EPQ model,
t2 = time required for reworking of defective items,
t3 = time required for delivering all quality assured finished products,
H1 = maximum level of on-hand inventory in units when regular production process ends,
H = the maximum level of on-hand inventory in units when rework process finishes,
tn = a fixed interval of time (between each installment of finished products to be delivered to customer during production downtime t3),
T = cycle length,
K = setup cost per production run,
C = unit production cost,
h = unit holding cost,
K1 = fixed delivery cost per shipment,
CR = unit rework cost,
h1 = holding cost for each reworked item,
I(t) = on-hand inventory of perfect quality items at time t,
h2 = holding cost for each item kept by customer,
TC(Q,n) = total production-inventory-delivery costs per cycle for the proposed model,
E[TCU(Q,n)] = the long-run average costs per unit time for the proposed model.

Once again for the purpose of easing readability, this paper adopted the same basic formulations, as were used in Chiu et al. (2010c). Total production-inventory-delivery cost per cycle TC(Q,n) consists of variable manufacturing cost, setup cost, variable rework cost and disposal cost, fixed and variable shipping cost, holding cost at the manufacturer's end for items reworked and for all items produced during production uptime t1 and rework time t2, and holding cost for finished goods, kept by both manufacturer and customer during t3, when n fixed-quantity installments of the finished batch are delivered at a fixed interval of time. Using the same formulation procedures, one has TC(Q,n) as follows (Equation (11) in Chiu et al., 2010c).

$$\begin{aligned} TC(Q,n) &= CQ + K + C_R[xQ] + C_S[x\theta_1 Q] \\ &+ nK_1 + C_T[Q(1-\theta_1 x)] + h_1 \cdot \frac{P_1 \cdot t_2}{2} \cdot (t_2) \\ &+ h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2)\right] + h\left(\frac{n-1}{2n}\right)Ht_3 \\ &+ \frac{h_2}{2}\left[\frac{H}{n}t_3 + T(H - \lambda t_3)\right] \end{aligned} \quad (1)$$

The defective rate x is assumed to be a random variable with a known probability density function, taking into account of the randomness, one can use the expected values of x in cost analysis and obtains E[TCU(Q,n)] as follows (Equation (12) in Chiu et al., 2010c).

$$\begin{aligned} E[TCU(Q,n)] &= \frac{E[TC(Q,n)]}{E[T]} = \frac{C\lambda}{1-\theta_1 E[x]} + \frac{(K + nK_1)\lambda}{Q(1-\theta_1 E[x])} \\ &+ \frac{C_R E[x]\lambda}{(1-\theta_1 E[x])} + \frac{C_S E[x]\theta_1 \lambda}{(1-\theta_1 E[x])} + C_T \lambda + \frac{hQ\lambda}{2P(1-\theta_1 E[x])} \\ &+ \frac{hQ\lambda}{2P_1(1-\theta_1 E[x])}\left[\left(2E[x] - (E[x])^2 - \theta_1 (E[x])^2\right)\right] \\ &+ \left(\frac{n-1}{n}\right)\left[\frac{hQ(1-\theta_1 E[x])}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x]\lambda}{2P_1}\right] \\ &+ \frac{h_1 (E[x])^2 Q\lambda}{2P_1(1-\theta_1 E[x])} + \left(\frac{1}{n}\right)\frac{h_2 Q}{2}(1-\theta_1 E[x]) \\ &+ \left(1-\frac{1}{n}\right)\frac{h_2 Q\lambda}{2P} + \frac{h_2 Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x]\lambda}{P_1}\right] \end{aligned} \quad (2)$$

Production-shipment policy derived using algebraic derivations

Algebraic approach is employed in this section for deriving the optimal replenishment lot size as well as optimal number of deliveries. Because the decision variables are Q and n, we identify

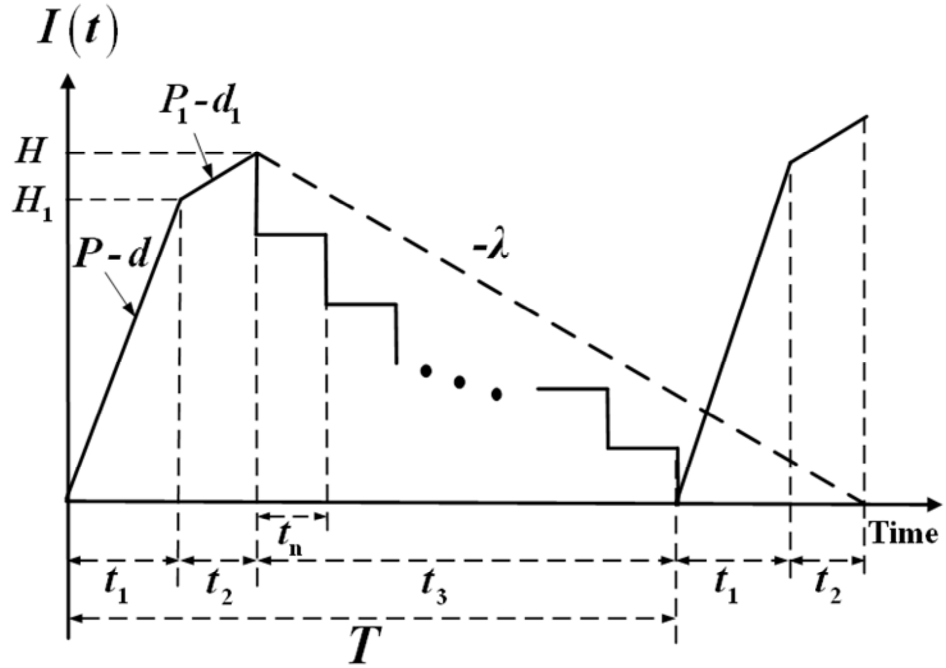

Figure 1. On-hand inventory of perfect quality items in an integrated production- inventory-delivery model with failure in repair (Chiu et al., 2010c).

that Equation (2) has terms for the constants, Q, Q-1, nQ-1, and Qn-1. First let q1, q2, q3, q4 and q5 denote the following:

$$q_1 = \frac{C\lambda}{1-\theta_1 E[x]} + \frac{C_R E[x]\lambda}{(1-\theta_1 E[x])} + \frac{C_S E[x]\theta_1\lambda}{(1-\theta_1 E[x])} + C_T\lambda \quad (3)$$

$$q_2 = \frac{K\lambda}{(1-\theta_1 E[x])} \quad (4)$$

$$q_3 = \frac{K_1\lambda}{(1-\theta_1 E[x])} \quad (5)$$

$$q_4 = \frac{h\lambda}{2P(1-\theta_1 E[x])} + \frac{h\lambda}{2P_1(1-\theta_1 E[x])}\left[\left(2E[x] - (E[x])^2 - \theta_1 (E[x])^2\right)\right] + \left[\frac{h(1-\theta_1 E[x])}{2} - \frac{h\lambda}{2P} - \frac{hE[x]\lambda}{2P_1}\right] + \frac{h_1 (E[x])^2\lambda}{2P_1(1-\theta_1 E[x])} + \frac{h_2\lambda}{2P} + \frac{h_2}{2}\left[\frac{E[x]\lambda}{P_1}\right] \quad (6)$$

$$q_5 = -\left[\frac{h(1-\theta_1 E[x])}{2} - \frac{h\lambda}{2P} - \frac{hE[x]\lambda}{2P_1}\right] + \frac{h_2}{2}(1-\theta_1 E[x]) - \frac{h_2\lambda}{2P} - \frac{h_2}{2}\left[\frac{E[x]\lambda}{P_1}\right] \quad (7)$$

Then Equation (2) can be expressed as:

$$E[TCU(Q,n)] = q_1 + q_2(Q^{-1}) + q_3(Q^{-1}n) + q_4(Q) + q_5(Qn^{-1}) \quad (8)$$

With further rearrangement of Equation (8), one obtains:

$$E[TCU(Q,n)] = q_1 + Q^{-1}\left[q_2 + q_4 Q^2\right] + Q\left(n^{-1}\right)\left[q_3\left(Q^{-1}n\right)^2 + q_5\right] \quad (9)$$

$$\begin{aligned} E[TCU(Q,n)] = q_1 &+ Q^{-1}\left[\left(\sqrt{q_4}\cdot Q\right)^2 + \left(\sqrt{q_2}\right)^2 - 2\left(\sqrt{q_4}\cdot Q\right)\left(\sqrt{q_2}\right)\right] \\ &+ Q^{-1}\left[2\left(\sqrt{q_4}\cdot Q\right)\left(\sqrt{q_2}\right)\right] \\ &+ Q\cdot(n^{-1})\left[\left(\sqrt{q_3}\left(nQ^{-1}\right)\right)^2 + \left(\sqrt{q_5}\right)^2 - 2\left(\sqrt{q_3}\left(nQ^{-1}\right)\right)\sqrt{q_5}\right] \\ &+ Q\cdot\left(n^{-1}\right)\left[2\left(\sqrt{q_3}\left(nQ^{-1}\right)\right)\sqrt{q_5}\right] \end{aligned} \quad (10)$$

Therefore, one obtains:

$$\begin{aligned} E[TCU(Q,n)] = q_1 &+ Q^{-1}\left[\left(\sqrt{q_4}\cdot Q\right) - \left(\sqrt{q_2}\right)\right]^2 + 2\left(\sqrt{q_4}\right)\left(\sqrt{q_2}\right) \\ &+ Q\cdot(n^{-1})\left[\left(\sqrt{q_3}\left(nQ^{-1}\right)\right) - \left(\sqrt{q_5}\right)\right]^2 + 2\left(\sqrt{q_3}\right)\sqrt{q_5} \end{aligned} \quad (11)$$

E[TCU(Q,n)] is minimized, if the second and the fourth square terms in Equation (11) equal zeros. That is:

$$Q^{-1}\left[\left(\sqrt{q_4}\cdot Q\right) - \left(\sqrt{q_2}\right)\right]^2 = 0 \quad (12)$$

and

$$Q\cdot(n^{-1})\left[\left(\sqrt{q_3}\left(nQ^{-1}\right)\right) - \left(\sqrt{q_5}\right)\right]^2 = 0 \quad (13)$$

or

$$Q^* = \sqrt{\frac{q_2}{q_4}} \quad (14)$$

and

$$n^* = \sqrt{\frac{q_5}{q_3}} \cdot Q^* \tag{15}$$

Substituting Equations (4) and (6) in Equation (14), with further derivations, one has the optimal lot size as:

$$Q^* = \sqrt{\frac{2K\lambda}{\frac{h\lambda}{P} + \frac{h\lambda}{P_1}\left[\left(2E[x] - (E[x])^2 - \theta_1 (E[x])^2\right)\right] + (1-\theta_1 E[x])\left[h(1-\theta_1 E[x]) - \frac{h\lambda}{P} - \frac{hE[x]\lambda}{P_1}\right] + \frac{h_1 (E[x])^2 \lambda}{P_1} + (1-\theta_1 E[x]) h_2 \lambda \left[\frac{1}{P} + \frac{E[x]}{P_1}\right]}} \tag{16}$$

Substituting Equations (5) and (7) in (15), one has the optimal number of deliveries as:

$$n^* = \sqrt{\frac{-\left[\frac{h(1-\theta_1 E[x])}{2} - \frac{h\lambda}{2P} - \frac{hE[x]\lambda}{2P_1}\right] + \frac{h_2}{2}(1-\theta_1 E[x]) - \frac{h_2\lambda}{2P} - \frac{h_2}{2}\left[\frac{E[x]\lambda}{P_1}\right]}{\frac{K_1\lambda}{(1-\theta_1 E[x])}} \cdot Q^*} \tag{17}$$

With further derivations, one has:

$$n^* = \sqrt{\frac{\frac{1}{2}(h_2 - h)\left[(1-\theta_1 E[x]) - \frac{\lambda}{P} - \frac{E[x]\lambda}{P_1}\right]}{\frac{K_1\lambda}{(1-\theta_1 E[x])}} \cdot Q^*} \tag{18}$$

Substituting Equation (16) in (18) and with further derivations, one obtains:

$$n^* = \sqrt{\frac{K(h_2 - h)(1-\theta_1 E[x])\left[(1-\theta_1 E[x]) - \left(\frac{\lambda}{P} + \frac{E[x]\lambda}{P_1}\right)\right]}{K_1\left\{\frac{h\lambda\theta_1 E[x]}{P} + \frac{h\lambda E[x]}{P_1}(1-E[x]) + h(1-\theta_1 E[x])^2 + \frac{h_1 (E[x])^2 \lambda}{P_1} + h_2\left(\frac{\lambda}{P} + \frac{E[x]\lambda}{P_1}\right)(1-\theta_1 E[x])\right\}}} \tag{19}$$

One notes that Equation (19) is identical to the optimal number of deliveries n* given in Equation (20) of Chiu et al. (2010c), which is derived by the use of the conventional differential calculus method. It follows that the long-run average cost E[TCU(Q*,n*)] is:

$$E\left[TCU\left(Q^*, n^*\right)\right] = q_1 + 2\sqrt{q_2}\sqrt{q_4} + 2\sqrt{q_3}\sqrt{q_5} \tag{20}$$

RESULTS AND DISCUSSION

Numerical example and verification

The research results obtained previously were verified by using the same example as in Chiu et al. (2010c). Consider the annual demand rate of a manufactured item is 3,400 units. This product can be produced at an annual rate of 60,000 units. The random defective rate x, is assumed during production uptime, which follows a uniform distribution over the interval [0, 0.3]. All nonconforming items produced are considered to be rework-able with a rate of rework P1 = 2,200 units per year. Failure in rework rate is θ1 = 0.1. Other values of parameters are:

h = $20 per item per year,
h1 = $40 per item reworked per unit time (year),
h2 = $80 per item kept at the customer's end per unit time,
C = $100 per item,
K = $20,000 per production run,
CR = $60, repaired cost for each item reworked,
CS = $20, disposal cost for each scrap item,
CT = $0.1 per item delivered,
K1 = $4,350 per shipment, a fixed cost.

By using Equations (19), (16) and (20), one obtains the optimal number of shipments n* = 2, the optimal production lot size Q* = 1,693, and the long-run average cost E[TCU(Q*,n*)] = $494,631. One notes that, these numbers are identical to that in Chiu et al. (2010c).

It is also noted that, since n only takes on integer values, one should use Equation (2) to determine the optimal integer value of number of deliveries, then plug it into Equation (16) to derive optimal replenishment lot size. Finally, one uses the resulting optimal production-shipment policy to calculate (that is, Equation (20)) the long-run average cost for such a specific EPQ model with quality assurance.

Conclusions

Chiu et al. (2010c) used differential calculus, along the Hessian matrix equations (that is, the conventional methods) to derive the economic production lot size and the optimal number of shipments for an EPQ model with failure in rework. This paper re-examines their model by using an alternative algebraic approach in lieu of their differential calculus. As a result, the optimal production-shipment policy in terms of lot size and number of deliveries is derived without derivatives. Such a straightforward algebraic approach allows students or practitioners, who lack sufficient training in calculus, to learn or deal with the real world integrated production-delivery systems with ease.

REFERENCES

Abolhasanpour M, Khosrojerdi G, Sadeghi A, Lotfi E, Jafari AA, Arab S, Hojabri H, Kimiagari AM (2009). Supply chain planning with scenario based probabilistic demand. Afr. J. Bus. Manage., 3(11): 705-714.

Banerjee A (1986). A joint economic-lot-size model for purchaser and vendor. Decis. Sci., 17(3): 292- 311.

Baten A, Kamil AA (2009). Analysis of inventory-production systems with Weibull distributed deterioration. Int. J. Phys. Sci. 4(11): 676-682.

Cheung KL, Hausman WH (1997). Joint determination of preventive maintenance and safety stocks in an unreliable production environment. Nav. Res. Logistics, 44: 257-272.

Chen KK, Chiu YSP, Hwang MH (2010). Integrating a cost reduction delivery policy into an imperfect production system with repairable items. Int. J. Phys. Sci. (in press).

Chiu SW, Chiu YSP (2006). Mathematical modeling for production system with backlogging and failure in repair. J. Sci. Ind. Res., 65(6): 499-506.

Chiu SW (2008). Production lot size problem with failure in repair and backlogging derived without derivatives. Eur. J. Oper. Res., 188(2): 610-615.

Chiu YSP, Chiu SW, Li CY, Ting CK (2009). Incorporating multi-delivery policy and quality assurance into economic production lot size problem. J. Sci. Ind. Res., 68(6): 505-512.

Chiu YSP, Cheng FT, Chang HH (2010a). Remarks on optimization process of manufacturing system with stochastic breakdown and rework. Appl. Math. Lett., 23(10): 1152-1155.

Chiu YSP, Chen KK, Cheng FT, Wu MF (2010b). Optimization of the finite production rate model with scrap, rework and stochastic machine breakdown, Comput. Math. Appl., 59(2): 919-932.

Chiu YSP, Lin CAK, Chang HH, Chiu V (2010c). Mathematical modeling for determining economic batch size and optimal number of deliveries for EPQ model with quality assurance. Math. Comp. Model. Dyn. Sys., 16(4): 373-388.

Diponegoro A, Sarker BR (2006). Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries. Comput. Oper. Res., 33: 2387-2404.

Goyal SK (1977). Integrated inventory model for a single supplier-single customer problem. Int. J. Prod. Res., 15: 107-111.

Goyal SK, Gupta YP (1989). Integrated inventory models: The buyer-vendor coordination. Eur. J. Oper. Res., 41(3): 261-269.

Grosfeld-Nir A, Gerchak Y (2002). Multistage production to order with rework capability. Manage. Sci., 48(5): 652-664.

Grubbström RW, Erdem A (1999). The EOQ with backlogging derived without derivatives. Int. J. Prod. Econ., 59: 529-530.

Henig M, Gerchak Y (1990). Structure of periodic review policies in the presence of random yield. Oper. Res., 38(4): 634-643.

Kim S-L, Banerjee A, Burton J (2008). Production and delivery policies for enhanced supply chain partnerships. Int. J. Prod. Res., 46: 6207-6229.

Lin HD, Chiu YSP, Ting CK (2008). A note on optimal replenishment policy for imperfect quality EMQ model with rework and backlogging. Comput. Math. Appl., 56(11): 2819-2824.

Mak KL (1985). Inventory control of defective products when the demand is partially captive. Int. J. Prod. Res., 23(3): 533-542.

Rardin RL (1998) Optimization in Operations Research. Int. Ed., Prentice-Hall, New Jersey, USA.

Rosenblatt MJ, Lee HL (1986). Economic production cycles with imperfect production processes. IIE Trans., 18: 48-55.

Sarker BR, Parija GR (1994). An optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy. J. Oper. Res. Soc., 45(8): 891-900.

Schwarz LB (1973). A simple continuous review deterministic one-warehouse N-retailer inventory problem. Manage. Sci., 19: 555-566.

Swenseth RS, Godfrey RM (2002). Incorporating transportation costs into inventory replenishment decisions. Int. J. Prod. Econ., 77(2): 113-130.

Taft EW (1918). The most economical production lot. Iron Age, 101: 1410-1412.

Tersine RJ (1994). Principles of Inventory and Materials Management. PTR Prentice-Hall, New Jersey, USA, 120-131.

Viswanathan S (1998). Optimal strategy for the integrated vendor-buyer inventory model. Eur. J. Oper. Res., 105: 38-42.

Wazed MA, Ahmed S, Nukman Y (2010a). Impacts of quality and processing time uncertainties in multistage production system. Int. J. Phys. Sci., 5(6): 814-825.

Wazed MA, Ahmed S, Yusoff N (2010b). Impacts of common processes in multistage production system under machine breakdown and quality uncertainties. Afr. J. Bus. Manage., 4(6): 979-986.

Wee HM, Yu J, Chen MC (2007). Optimal inventory model for items with imperfect quality and shortage back ordering. Omega, 35: 7-11.