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Full Length Research Paper

Modified Kudryashov method for generalized forms of the nonlinear heat conduction equation

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An analytic study on the two generalized forms of the nonlinear heat conduction equations is presented in this paper. The modified Kudryashov method is employed to construct exact solitary wave solutions of these equations. The results reveal that the modified Kudryashov method is a powerful mathematical tool to solve nonlinear partial differential equations (NPDEs) in terms of accuracy and efficiency.

Key words: Nonlinear heat conduction, modified Kudryashov method, travelling wave solutions.

INTRODUCTION

“The most incomprehensible thing about the world is that it is at all comprehensible” (Albert Einstein), but the question is how do we fully understand incomprehensible things? Nonlinear science provides some clues in this regard.

The world around us is inherently nonlinear. For instance, nonlinear evolution equations (NLEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, and biology. One of the basic physical problems for these models is to obtain their travelling wave solutions. In particular, various methods have been utilized to explore different kinds of solutions of physical models described by nonlinear partial differential equations (PDEs). In the numerical methods, stability and convergence should be considered, so as to avoid divergent or inappropriate results (Noor et al., 2011). However, in recent years, a variety of effective analytical and semi-analytical methods have been developed considerably to be used for solving nonlinear PDEs, such as the variational iteration method (VIM) (He et al., 2010; Jafari and Alipour, 2010), the homotopy perturbation method (HPM) (He, 2006; Biazar et al., 2011; Alam et al., 2011), the homotopy analysis method (HAM) (Mohyud-Din et al., 2011), the differential transform method (DTM) (Biazar and Eslami, 2011), the tanh-method (Malfliet and Hereman, 1996a, b; Fan, 2002; Wazwaz, 2005, 2008), the homogeneous balance method (Fan and Zhang, 1998), the Exp-function method (He and Wu, 2006; Kudryashov and Loginova, 2009; Kudryashov, 2009; Borhanifar and Kabir, 2009, 2010; Borhanifar et al., 2009; Kabir and Khajeh, 2009), the (G'/G)-expansion method (Abazari, 2010a, b; Neirameh et al., 2010; Kabir et al., 2011a), and others.

The generalized forms of the nonlinear heat conduction equation can be given as

$$u_t - a(u^n)_{xx} - u + u^n = 0, \qquad a > 0\,,\, n > 1 \tag{1}$$

and, in (2 + 1)-dimensional space,

$$u_t - a(u^n)_{xx} - a(u^n)_{yy} - u + u^n = 0. \tag{2}$$

Many authors have studied some types of solutions of these equations. Wazwaz (2005) used the tanh-method to find exact solitary solutions of these equations and a standard form of the nonlinear heat conduction equation (when $n = 3$ in Equation (1)). Also, Fan (2002) applied the solutions of Riccati equation in the tanh-method to obtain the travelling wave solution when ($n = 2$) in Equation (1). Lately, Kabir et al. (2009) implemented the Exp-function method to find exact solutions of Equation (1), and obtained more general solutions in comparison with Wazwaz’s results. In another study, Kudryashov and Chmykhov (2007) obtained approximate solutions to one-dimensional nonlinear heat conduction problems with the heat flux at the origin specified in the form of a power

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time dependence.

In this study, we use the modified Kudryashov method (Kabir et al., 2011b; Yusufoglu and Bekir, 2008; Demiray, 2004) to obtain the exact solitary wave solutions of Equations (1) and (2). The aim of this method is the modification of the approach by Kudryashov (1988); therefore it can be entitled - The modified Kudryashov method (Kudryashov, 2009; Kabir et al., 2011b).

THE MODIFIED KUDRYASHOV METHOD

We first consider a general form of nonlinear equation

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \ldots) = 0\,, \qquad (3)$$

By using the wave transformation

$$u = u(\eta)\,,\ \eta = k(x - ct) \qquad (4)$$

where k and c are constants to be determined later. Then the nonlinear partial differential equation (3) is reduced to a nonlinear ordinary differential equation (ODE)

$$P(u, -kc\,u', ku', k^2 u'', k^2 c^2 u'', -k^2 c u'', \ldots) = 0\,, \qquad (5)$$

Here, we shall seek a rational function type of solution for a given partial differential equation, in terms of $\exp(\eta)$, of the following form

$$U = \sum_{k=0}^{m} \frac{a_k}{[1 + \exp(\eta)]^k}\,, \qquad (6)$$

where $a_0, a_1, \ldots, a_m$ are some constants to be determined from the solution of (5).

Differentiating (6) with respect to η, introducing the result into Equation (5), and setting the coefficients of the same power of e^{η} equal to zero, we obtain algebraic equations. The rational function solution of the equation (3) can be solved by obtaining $a_0, a_1, \ldots, a_m$ from this system (Kudryashov, 1988, 2009; Kabir et al., 2011b).

A GENERALIZED FORM OF THE NONLINEAR HEAT CONDUCTION EQUATION

By introducing a complex variable η defined as Equation (4), then Equation (2) becomes an ordinary differential equation (ODE), which can be written as

$$-kc\,U' - ak^2 (U^n)'' - U + U^n = 0, \qquad a > 0 \qquad (7)$$

or, equivalently,

$$-kcU' - ak^2 n(n-1)U^{n-2}U'^2 - ak^2 nU^{n-1}U'' - U + U^n = 0, \qquad (8)$$

To get a closed-form analytic solution, we use the transformation (Kabir and Khajeh, 2009)

$$U(\eta) = V^{\frac{-1}{n-1}}(\eta), \qquad (9)$$

which will convert Equation (8) into

$$kc(n-1)V'V^2 + ak^2n(1-2n)V'^2 + ak^2n(n-1)VV'' - (n-1)^2V^3 + (n-1)^2V^2 = 0, \qquad (10)$$

By using the rational function in $\exp(\eta)$, we may choose the solution of (10) in the form

$$V(\eta) = a_0 + \frac{a_1}{1 + \exp(\eta)}, \qquad (11)$$

where a_0, a_1 are some constants to be determined from the solution of (10). Differentiating (11) with respect to η introducing the result into Equation (10), and setting the coefficients of the same power of e^{η} equal to zero, we obtain these algebraic equations:

$[\exp(\eta)]^4$ coefficient:

$$n^2a_0^2 + a_0^2 - 2na_0^2 - a_0^3 + 2na_0^3 - n^2a_0^3 = 0,$$

$[\exp(\eta)]^3$ coefficient:

$$\begin{aligned}&-3n^2a_0^2a_1 - 3a_0^2a_1 + ak^2n^2a_0a_1 - 4na_0a_1 + kca_0^2a_1 - 4n^2a_0^3 - 4a_0^3 - 8na_0^2\\&-kcna_0^2a_1 + 8na_0^3 - ak^2na_0a_1 + 2a_0a_1 + 2n^2a_0a_1 + 4a_0^2 + 6na_0^2a_1 + 4n^2a_0^2 = 0,\end{aligned}$$

$[\exp(\eta)]^2$ coefficient:

$$\begin{aligned}&6a_0a_1 + 6a_0a_1n^2 - ak^2n^2a_1^2 - 6n^2a_0^3 + 12na_0^3 - 2kcna_0a_1^2\\&+18na_1a_0^2 + 6n^2a_0^2 - 12na_0^2 - 9n^2a_1a_0^2 - 2na_1^2 + a_1^2 - 9a_1a_0^2\\&-3a_0a_1^2 - 2kcna_1a_0^2 + 6na_0a_1^2 + 2kca_1a_0^2 + 2kca_0a_1^2 - 6a_0^3\\&-3n^2a_0a_1^2 + n^2a_1^2 - 12na_0a_1 + 6a_0^2 = 0,\end{aligned} \qquad (12)$$

$[\exp(\eta)]^1$ coefficient:

$$\begin{aligned}&kca_1^3 + kca_1a_0^2 - ak^2n^2a_1^2 + ak^2na_0a_1 + ak^2na_1^2 - ak^2n^2a_0a_1 - kcna_1a_0^2\\&-2kcna_0a_1^2 + 2a_1^2 + 2kca_0a_1^2 - 4a_0^3 - a_1^3 - kcna_1^3 - 9a_1a_0^2 - 6a_0a_1^2 - 4n^2a_0^3\\&-n^2a_1^3 + 8na_0^3 + 2na_1^3 + 4n^2a_0^2 + 2n^2a_1^2 - 9n^2a_1a_0^2 - 6n^2a_0a_1^2 + 18na_1a_0^2\\&+12na_0a_1^2 + 6n^2a_0a_1 - 12na_0a_1 - 4na_1^2 + 4a_0^2 + 6a_0a_1 - 8na_0^2 = 0,\end{aligned}$$

$[\exp(\eta)]^0$ coefficient:

$$-a_1^3-3a_1a_0^2-3a_0a_1^2-n^2a_0^3-n^2a_1^3+2na_0^3+2na_1^3-4na_0a_1-2na_1^2+a_0^2$$
$$+2a_0a_1-2na_0^2+a_1^2+2n^2a_0a_1-3n^2a_1a_0^2-3n^2a_0a_1^2+n^2a_0^2+n^2a_1^2+6na_0a_1^2$$
$$+6na_1a_0^2-a_0^3=0.$$

With the aid of Maple 12, the solutions of these algebraic equations are found to be in the following.

Case 1

$$k=\pm\frac{n-1}{n\sqrt{a}},\; c=\pm\sqrt{a},\; a_0=0,\; a_1=1,\quad a>0 \tag{13}$$

Substituting Equations (13) into (14) and inserting the result into the transformation (9), we get the exact solitary wave solution of Equation (1) as follows:

$$u(x,t)=\left[\frac{1}{1+\exp\left(\pm\frac{n-1}{n\sqrt{a}}\left(x\mp\sqrt{a}t\right)\right)}\right]^{\frac{-1}{n-1}} \tag{14}$$

Case 2

$$k=\pm\frac{n-1}{n\sqrt{a}},\; c=\mp\sqrt{a},\; a_0=1,\; a_1=-1,\quad a>0 \tag{15}$$

Similar to the previous case, we can find the following exact solution:

$$u(x,t)=\left[1-\frac{1}{1+\exp\left(\pm\frac{n-1}{n\sqrt{a}}\left(x\pm\sqrt{a}t\right)\right)}\right]^{\frac{-1}{n-1}} \tag{16}$$

THE GENERALIZED NONLINEAR HEAT CONDUCTION EQUATION IN TWO DIMENSIONS

The wave variable $\eta=k(x+y-ct)$ transforms Equation (2) to the ODE

$$-kcU'-2ak^2(U^n)''-U+U^n=0,\qquad a>0 \tag{17}$$

or, equivalently,

$$-kcU'-2ak^2n(n-1)U^{n-2}U'^2-2ak^2nU^{n-1}U''-U+U^n=0, \tag{18}$$

then we use the transformation (9) which will convert Equation (18) to

$$kc(n-1)V'V^2+2ak^2n(1-2n)V'^2+2ak^2n(n-1)VV''-(n-1)^2V^3+(n-1)^2V^2=0, \tag{19}$$

By the same manipulation as illustrated previously in 'A generalized form of the nonlinear heat conduction equation', we obtain the following sets of solutions.

Case 1

$$k=\pm\frac{n-1}{n\sqrt{2a}},\; c=\pm\sqrt{2a},\; a_0=0,\; a_1=1,\quad a>0 \tag{20}$$

Substituting Equations (20) into (11) and inserting the result into the transformation (9), we get the exact travelling wave solution of Equation (2) as follows:

$$u(x,y,t)=\left[\frac{1}{1+\exp\left(\pm\frac{n-1}{n\sqrt{2a}}\left(x+y\mp\sqrt{2a}t\right)\right)}\right]^{\frac{-1}{n-1}} \tag{21}$$

Case 2

$$k=\pm\frac{n-1}{n\sqrt{2a}},\; c=\mp\sqrt{2a},\; a_0=1,\; a_1=-1,\quad a>0 \tag{22}$$

Similar to the aforementioned, we can gain the following solitary wave solution:

$$u(x,y,t)=\left[1-\frac{1}{1+\exp\left(\pm\frac{n-1}{n\sqrt{2a}}\left(x+y\pm\sqrt{2a}t\right)\right)}\right]^{\frac{-1}{n-1}} \tag{4.7}$$

Remark

We have verified all the obtained solutions by putting them back into the original equations (1) and (1) with the aid of Maple 12.

CONCLUSION AND FUTURE RESEARCH

In summary, the purpose of the study is to show that exact solutions of two generalized forms of the nonlinear heat conduction equation can be obtained by the modified Kudryashov method. The solution procedure is very simple and straightforward. Also, the obtained solutions have very concise and explicit forms. Overall, the results reveal that the modified Kudryashov method is a powerful mathematical tool to solve nonlinear partial differential equations (NPDEs) in the terms of accuracy and efficiency. We would like to mention that the proposed method or an extended kind of this technique will be used in further works to establish new and more general exact solutions of other kinds of nonlinear evolution equations in mathematical physics. Also, the physical interpretation of these solutions and actual applications in reality will be investigated in future papers.

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