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Full Length Research Paper

Optimization of design tolerance and asymmetric quality loss cost using pattern search algorithm

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Tolerances have a significant impact on manufacturing cost and product quality. In the design of discrete part shapes, the specification of tolerance constraints can have major consequences for product quality and cost. Traditional methods for tolerance analysis and synthesis are time consuming and have limited applicability. A more scientific approach is often desirable for better performance to overcome this difficulty. In this paper, the objective is to examine the optimal tolerance allocation by considering both tolerance cost and manufacturing cost so that the total assembly cost is minimized. A new global nonlinear optimization technique called Pattern search algorithm has been implemented to find the optimal tolerance allocation and asymmetric total cost to overcome the shortcomings in the conventional tolerance allocation problem.

Key words: Optimization, tolerance allocation, machining tolerance, pattern search.

INTRODUCTION

Design of tolerance is one of the key roles in Product design and development process as it affects both the product's functional requirements and manufacturing cost. Traditionally, this important phase of product development is accomplished intuitively to satisfy design constraints, based on handbooks' data and /or skill and experience of the designers. Tolerance design carried out in this manner does not necessarily lead to an optimum design (Singh et al., 2003, 2004).

The accuracy cost of a part dimension depends on the process and resources required for the production of this dimension within its tolerance limits. This cost is thus primarily determined by the part material, geometry and tolerances. Given the material, geometrical part characteristics such as shape, size, shape complexity, existence of internal surfaces and feature details has a direct impact on the accuracy cost as they are taken into consideration for the process planning and the required machines and tools (Dimitrellou et al., 2006). In a concurrent design environment, a robust optimum method is presented to directly determine the process tolerances from multiple correlated critical tolerances in an assembly (Jeang et al., 1997). Conventional tolerance analysis is tedious and time consuming where as the complex assembly problems are normally beyond the capabilities of most design and manufacturing engineering (Prabhaharan et al., 2004). The majority of the published articles on tolerance synthesis are based on optimization, most of which use the cost-tolerance models (Noorul et al., 2005).

In this paper, the new popular technique called the pattern search method is implemented to find the optimized asymmetric quality cost. To find the manufacturing cost, various methods are available. These methods are: (1) Exponential cost; (2) Power model; (3) Reciprocal square; (4) Reciprocal power; (5) Reciprocal power – Exponential Hybrid; (6) Combined reciprocal power and Exponential; (7) Combined linear exponential. In this work, a modified form of the exponential cost-tolerance model has been used.

Tolerance and quality loss function

Quality loss is one of the most important issues for quality engineering to evaluate the quality of the products or processes. Quality loss function is a quadratic expression estimating the cost of the average then comparing it to the customer identified target values and the variability of the product characteristics in terms of monetary loss due

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to product failure. The loss function L(y), indicates a monetary measure for the product characteristic value versus the target value. High quality and low cost are the two fundamental requirements for product design and manufacturing. In an assembly, critical tolerance must be guaranteed for functional requirements.

There is very important concept of quality engineering inherent in the loss function. In the usual practice of manufacturing quality control, the producer specifies a mean value of the performance characteristic and the tolerance interval around that value. Any value of the performance characteristic which falls within the interval is defined to be a quality product, even if it is barely inside the -3σ limit. With loss function as a definition of quality, the emphasis is on achieving the target value of the performance characteristic and any deviations from the target value, the greater the quality loss.

Types of loss function are expressed as:

i) Normal-the-best = $L(y) = \frac{A}{\Delta^2}(y-m)^2$ (1)

ii) Smaller-the–better = $L(y) = \frac{A}{\Delta^2} y^2$ (2)

iii) Larger-the-better = $L(y) = A\Delta^2 \left(\frac{1}{y^2}\right)$ (3)

In some situations, the quality loss resulting from deviation of quality characteristics in one direction is greater than the deviation in the other direction. In this case, various values will be assigned for the two directions of the target.

iv) The asymmetric quality loss function:

$$L(y) = K_1(y-m)^2 \quad y > m$$
$$L(y) = K_2(y-m)^2 \quad y < m \qquad (4)$$

In order to find the optimized manufacturing cost, the paper is organized as follows. Literature review on previous studies is discussed. A brief introduction of the pattern search algorithm, for nonlinear optimization is discussed follow by demonstration of the application of the parallel pattern search on an example problem. Then, a brief discussion on the results is given and lastly, conclusion.

LITERATURE REVIEW

Many of the tolerance synthesis methods are based on conventional optimization methods, quality engineering methods and methods based on genetic algorithms, simulated annealing and Nero-fuzzy learning (Noorul et al., 2005). Majority of methods use the cost-tolerance models. Researches are mainly focused on the mathematical modeling of cost-tolerance relations and the optimization of related tolerances for minimum production cost (Dong et al., 1994; Sampath et al., 2009). The quality loss function of unbalanced tolerance design and an analytical approach for determining the process mean has been focused in many papers (Dimitrellou et al., 2006; Li, 2000; Ming-Hsien and Joseph, 2001).

Chase and Greenwood (1988) used data and obtained empirical functions for metal-removal process over the full range of nominal dimensions. The used original data was nevertheless old and may not be applicable to modern CNC machines. The assessment of tolerance allocation plays a vital role in product / process design. In this paper, Herbert et al. (2001) proposed both Parametric and nonparametric procedures that determine tolerance allocations. The parametric method determines tolerances that minimize the expected total loss, where loss consists of both internal (supplier processes) and external (loss to society) costs. The nonparametric method is a partial information procedure, since only information concerning a supplier process variance is required.

In a concurrent environment, the product tolerance design and process tolerance design can be integrated into one stage. Tolerance design has been extended directly from the product design to the manufacturing stage. In this work (Huang et al., 2005) robust optimization method in a concurrent tolerance environment has been presented. The production cost for a part includes the expense of achieving high quality and also manufacturing-related costs. The cost of producing a high quality product is estimated with a loss function, which represented the cost of variability of a chosen characteristic and staying close to the target. Manufacturing cost is represented by a tolerance-cost function (Herbert et al., 2001).

Normally, for an unbalanced tolerance design, a designer would choose the smaller tolerance for both sides. However, this work indicates that this method fails to minimize the expected quality loss. Two models of the quality loss function are investigated in this paper (Li, 2000). One, the quadratic loss function is assumed for the quality assessment of a target value. Secondly, the standard deviation is proportional to the process mean with the coefficient of variance held constant. Taguchies perspective of continuous quality loss incorporation of different robustness criteria and their applicability has also been discussed in the context of process engineering application (Ming-Hsien and Joseph, 2001).

More heuristic methods have also been applied, these are; Cagan and Kurfess and Zhaug and wang (Ta-Cheng and Gary, 2000) used simulated annealing to successfully solve tolerance process optimization problems. Montgomery applied the design of experiments approach.

The taguchi method for solving linear tolerance synthesis problems has been proposed by Kusiak and Feng. Dupinet solved a tolerance allocation problem in two stages by using Fuzzy logic and simulated annealing.

Recently, genetic algorithms (GA) have been widely applied to solve a variety of optimization problems, usually of a combinational nature. Painton and Campall have applied GA to nonlinear and mixed integer programming problems. Coit and Smith tried to solve the series - parallel reliability system problem. These models represent empirical production data directing another emerging research area in tolerance analysis and synthesis is computer automation. This was later extended to a feature based CAD environment as well as integrated concurrent engineering design. Current CAD software dealing with tolerancing belongs to two groups. The first group supports tolerance representation; known dimensional and geometrical tolerances are graphically assigned to the CAD model. The second group of software encounters with the problem of tolerance analysis and allocation.

About the pattern search algorithm

Pattern search methods belong to a class of optimization methods and a subclass of direct search algorithms. It is an evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods it was first introduced and analyzed by Torczon (Torczon, 1997) for unconstrained problems and extended by Torczon and Lewis, 1999) to problems with bound constraints (Torczon, 1997) and a finite number of linear constraints. In all three results, convergence of a subsequence of iterates to a limit point satisfying first-order necessary conditions was proven.

These methods have a long and rich history in the scientific and engineering communities where they have been applied to numerous problems. The main attraction of direct search methods is their ability to find optimal solutions without the need for computing derivatives in contrast to the more familiar gradient-based methods.

Pattern and direct search methods are one of the most popular classes of methods to minimize functions without the use of derivatives or of approximations to derivatives. They are based on generating search directions which positively span the search space. Direct search is conceptually simple and natural for parallelization. These methods can be designed to rigorously identify points satisfying stationary for local minimization (from arbitrary starting points). Moreover, their flexibility can be used to incorporate algorithms or heuristics for global optimization, in a way that the resulting direct or pattern search method inherits some of the properties of the imported global optimization technique, without jeopardizing the convergence for local stationary mention before.

A pattern is a set of vectors (v_i) that the pattern search algorithm uses to determine which points to search at each iteration. The set (v_i) is defined by the number of independent variables in the objective function, N, and the positive basis set. Two commonly used positive basis sets in pattern search algorithms are the maximal basis, with 2N vectors and the minimal basis, with N + 1 vector. With Generalized Pattern Search Algorithm (GPS), the collections of vectors that form the pattern are fixed-direction vectors. For example, if there are three independent variables in the optimization problem, the default for a 2N positive basis consists of the following pattern vectors:

$$\begin{array}{ll} V_1 = [1\ 0\ 0] & V_2 = [0\ 1\ 0] \\ V_3 = [0\ 0\ 1] & V_4 = [-1\ 0\ 0] \\ V_5 = [0\ -1\ 0] & V_6 = [0\ 0\ -1] \end{array} \tag{5}$$

An N + 1 positive basis consist of the following default pattern vectors.

$$\begin{array}{ll} V_1 = [1\ 0\ 0] & V_2 = [0\ 1\ 0] \\ V_3 = [0\ 0\ 1] & V_4 = [-1\ -1\ -1] \end{array} \tag{6}$$

With Mesh Adaptive Search Algorithm (MADS), the collections of vectors that form the pattern are randomly selected by the algorithm. Depending on the poll method choice, the number of vectors selected will be 2N or N + 1. As in GPS, 2N vectors consist of N vectors and their N negatives, while N + 1 Vectors consist of N vectors and one is the negative of the sum of the others.

Meshes

At each step, the pattern search algorithm searches a set of points, called a mesh, for a point that improves the objective function. The GPS and MADS algorithms form the mesh by

I) Generating a set of vectors (v_i) by multiplying each pattern vector vi by a scalar Δ_m. Where, Δ_m is called the mesh size.

ii) Adding the (di) to the current point-the point with the best objective function Value found at the previous step.

TOLERANCE DESIGN FOR A PISTON-CYLINDER ASSEMBLY

The problem consider in this work is a Piston and Cylinder assembly, which was given as a case study in (Al- Ansary et al., 1977; Singh et al., 2003). It is a simple linear mechanical assembly as shown in Figure 1 involving only two components. The study is an extension of the previous work on Concurrent optimization of design

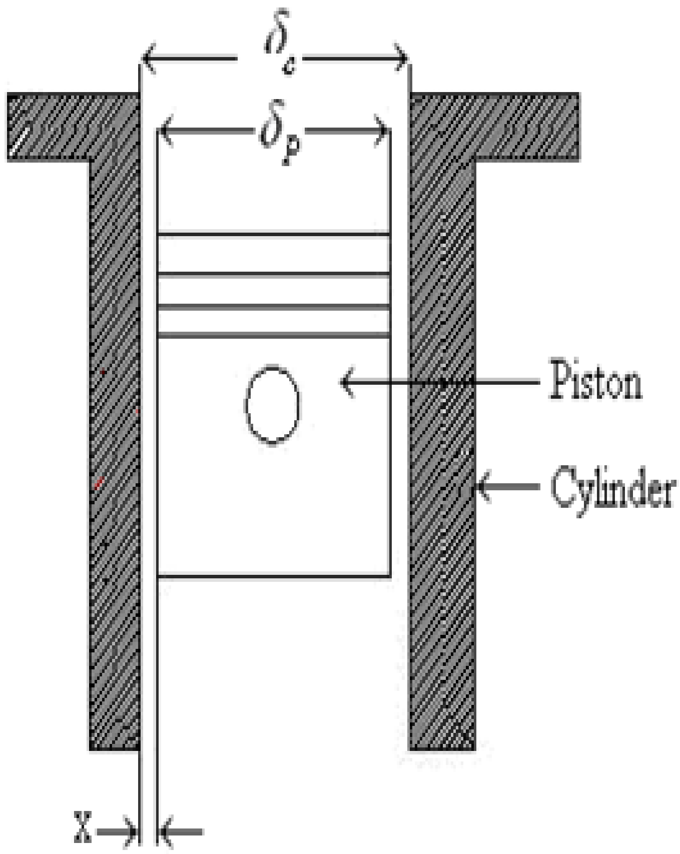

Figure 1. Piston and cylinder assembly.

and machining tolerances using the genetic algorithms method (Al- Ansary et al., 1977). This is also an extension of similar work using pattern search algorithm.

To determine the optimal design and manufacturing tolerances for the piston and cylinder assembly, the details of dimensions are:

Piston diameter (δ_p) = 50.8 mm,

Cylinder bore diameter (δ_c) = 50.856 mm,

Clearance (x) = 0.056 ± 0.025 mm.

The following process plan has been adapted for piston: Rough turning (δ_{11}), finish turning (δ_{12}), rough grinding (δ_{13}) and Finish grinding (δ_{14}). Similarly for cylinder bore, the following process was adapted: Drilling (δ_{21}), boring (δ_{22}), Semi-finish boring (δ_{23}) and grinding (δ_{24}).

In Table 1, the ranges of the principal machining tolerances in millimeters are given. Thus, there are only two design tolerance parameters, one for the piston diameter, the other for the cylinder bore diameter. Also, there are four machining tolerance parameters for machining each of the piston diameter and the cylinder bore diameter corresponding to the given process plans.

Constraints

The constraints on the principal design and machining tolerances are (Al- Ansary et al., 1977) the sum of the design tolerances of piston and cylinder bore diameter and should be less than (or) equal to the clearance tolerance

$$\delta_{11d} + \delta_{21d} \leq 0.001$$

The design tolerance for a given feature of a part is equal to the finial machining tolerance.

$\delta_{11d} = \delta_{14,}$ For the piston and $\delta_{21d} = \delta_{24}$

The sum of the machining tolerance for a process and the preceding process should be less than or equal to the difference of the nominal and minimum machining allowances for the process. For the piston;

$$\delta_{11} + \delta_{12} \leq 0.02, \delta_{12} + \delta_{13} \leq 0.005, \delta_{13} + \delta_{14} \leq 0.0018$$

Table 1. Machining tolerance ranges in millimeter.

S/No	Piston diameter in mm			Cylinder diameter in mm		
	Notation	Lower limit	Upper limit	Notation	Lower limit	Upper limit
1	δ_{11}	0.005	0.02	δ_{21}	0.007	0.02
2	δ_{12}	0.002	0.012	δ_{22}	0.003	0.012
3	δ_{13}	0.0005	0.003	δ_{23}	0.0006	0.005
4	δ_{14}	0.0002	0.001	δ_{24}	0.0003	0.002

Table 2. Cost-tolerance parameters for the eight machining process.

S/N	Coefficients	$g_{11}\,\delta_{11}$	$g_{12}\,\delta_{12}$	$g_{13}\,\delta_{13}$	$g_{14}\,\delta_{14}$	$g_{21}\,\delta_{21}$	$g_{22}\,\delta_{22}$	$g_{23}\,\delta_{23}$	$g_{24}\,\delta_{24}$
1	C_0	5	9	13	18	4	8	10	2
2	C_1	309	790	3196	8353	299	986	3206	9428
3	C_2	5.0×10^{-3}	2.04×10^{-3}	5.30×10^{-4}	2.19×10^{-4}	7.02×10^{-3}	2.97×10^{-3}	6.0×10^{-4}	3.6×10^{-4}
4	C_3	1.51	4.36	7.48	11.99	2.35	5.29	9.67	13.12

For the Cylinder;

$$\delta_{21}+\delta_{22}\le 0.02,\delta_{22}+\delta_{23}\le 0.005,\delta_{23}+\delta_{24}\le 0.0018$$

The Design tolerances are framed by four stacked up conditions: Worst case, RSS, Spotts and estimated mean shift criteria. These stack-up conditions yield a set of design constraints as below:

Worst case criteria: $\delta_{14}+\delta_{24}\le 0.001$;

RSS criteria: $\delta^2{}_{14}+\delta^2{}_{24}\le (0.001)^2$;

Spotts: $\frac{1}{2}\left[(\delta_{14}+\delta_{24})+\sqrt{(\delta^2{}_{14}+\delta^2{}_{24})}\right]\le 0.001$;

Estimated mean shift criteria:

$$(m_1\delta_{14}+m_2\delta_{24})+\frac{z}{3}\sqrt{\left[(1-m_1)^2\delta^2{}_{14}+(1-m_2)^2\delta^2{}_{24}\right]}\le 0.001 \quad (7)$$

The total machining cost is determined by sum of the machining cost-tolerance model equations for the eight machining processes of the piston-cylinder assembly. In general, many cost tolerance models are in use, in this example the modified form of the exponential cost-tolerance model was used to find the machining cost.

The form of exponential cost-tolerance mode (Al-Ansary et al., 1977) used, g (δ), is expressed

as $$g(\delta)=\frac{c_0}{e^{c_1(\delta-c_2)}}+c_3 \quad (8)$$

To find the total machining cost (c_m) by using the below given empirical relation, a more coefficient values is required (Al- Ansary et al., 1977) which is given in Table 2 and its corresponding machining tolerance as given in Table 1.

$$c_m = g_{11}(\delta_{11})+g_{12}(\delta_{12})+g_{13}(\delta_{13})+g_{14}(\delta_{14})+g_{21}(\delta_{21})+g_{22}(\delta_{22})+g_{23}(\delta_{23})+g_{24}(\delta_{24}) \quad (9)$$

By giving the ranges of these tolerance values (which is available in Table 1) one by one into the empirical relation g (δ) at same time satisfying the machining tolerance constraints (Which is given above) on can obtain the total machining cost c_m. For different value of the input (Ranges of tolerance in Table 1) different total machining cost will be arrived. To find the least total machining cost (c_m), the Pattern search algorithms method was used as an optimization engine. In the optimization process binary representation was employed with an individual length of 80 bits for the eight variables of the design space. Populations size of 20 with 1000 generations were used with the binary tournament selection method. The results obtained in different stock up conditions are shown in Tables 3 - 6.

DISCUSSION ON THE RESULTS

The above said total machining cost g(δ), empirical rela-relation was expanded and expressed as follows;

Table 3. Optimal tolerances allocated using Genetic algorithm – Worst case method.

Tolerances for Piston		Tolerances for Cylinder bore		Least total cost (Manufacturing cost + Asymmetric quality loss)
δ_{11}	0.0170	δ_{21}	0.0170	55.77
δ_{12}	0.0040	δ_{22}	0.0045	
δ_{13}	0.0021	δ_{23}	0.0024	
δ_{14}	0.0012	δ_{24}	0.0010	

Table 4. Optimal tolerances allocated using Genetic algorithm – RSS Method.

Tolerances for Piston		Tolerances for Cylinder bore		Least total cost (Manufacturing cost+ Asymmetric quality loss)
δ_{11}	0.0174	δ_{21}	0.0174	54.85
δ_{12}	0.0020	δ_{22}	0.0045	
δ_{13}	0.0007	δ_{23}	0.0023	
δ_{14}	0.0004	δ_{24}	0.0010	

Table 5. Optimal tolerances allocated using genetic algorithm – Spotts Method

Tolerances for Piston		Tolerances for Cylinder bore		Least total cost (Manufacturing cost+ Asymmetric quality loss)
δ_{11}	0.0173	δ_{21}	0.0170	55.22
δ_{12}	0.0020	δ_{22}	0.0046	
δ_{13}	0.0006	δ_{23}	0.0022	
δ_{14}	0.0003	δ_{24}	0.0009	

Table 6. Optimal tolerances allocated using Genetic algorithm – Greenwood and Chases Method.

Tolerances for Piston		Tolerances for Cylinder bore		Least total cost (Manufacturing cost+ Asymmetric quality loss)
δ_{11}	0.0171	δ_{21}	0.0169	55.12
δ_{12}	0.0039	δ_{22}	0.0044	
δ_{13}	0.0020	δ_{23}	0.0023	
δ_{14}	0.0011	δ_{24}	0.0009	

$$g(\delta)=\left\{\frac{c_0}{e^{c_1(\delta_{11}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{12}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{13}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{14}-c_2)}}+c_3\right\}+$$
$$\left\{\frac{c_0}{e^{c_1(\delta_{21}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{22}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{23}-c_2)}}+c_3\right\}+\left\{\frac{c_0}{e^{c_1(\delta_{24}-c_2)}}+c_3\right\} \quad (10)$$

From the equation 9, the total machining cost g(δ) was obtained by substituting the appropriate coefficient values taking from Tables 1 and 2. Then optimize these total machining costs with Tolerance of the piston and cylinder assembly, for each case of the design constraint was carried out for 1000 generations on a P-IV personal com-

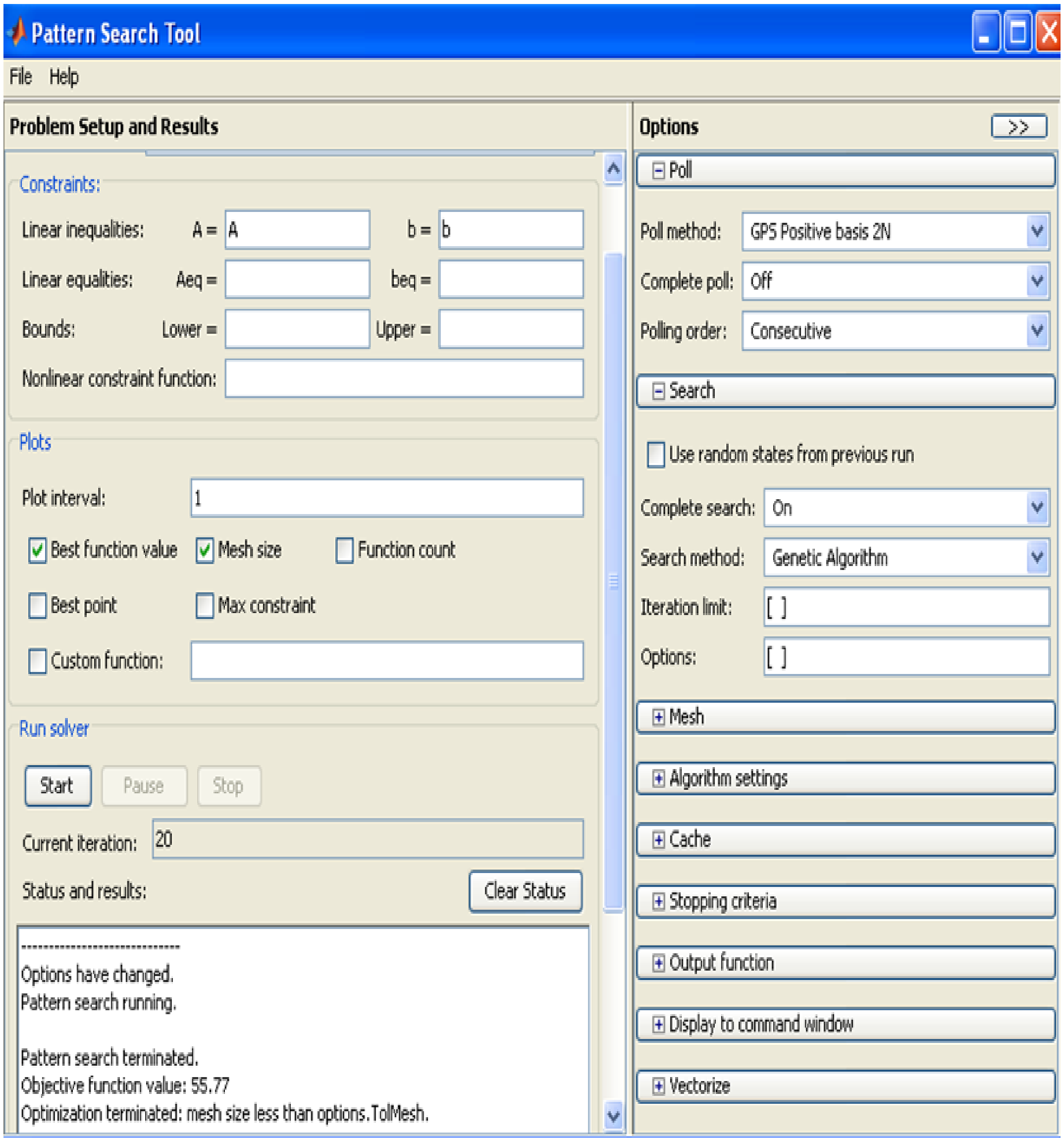

Figure 2. Pattern search representation of problem setup.

computer using MATLAB 7.5 version. The following results were obtained which is shown in Figure 2 and 3. Figure 2 shows the detailed representation of the pattern search tool setup.

By optimization, using various input values of the tolerance and its corresponding least manufacturing cost was found.

The values of manufacturing cost are compared in the Table. The result indicates that the minimum total cost of the assembly is lowest with Greenwood and Chase Method and highest with RSS method. Table 7 shows the comparison of total machining cost with total cost for Worst case method in Pattern search optimization methods. The total cost obtained in this research is very less; even it includes asymmetric quality loss with machining cost. It is because of the following reasons, one of the reason was the work done by Al-Ansary (Al-Ansary et al., 1977), who adopted the penalty function approach for constraint handling, which increases the number of parameters to be selected by training the algorithm. Second, normally in GA, starting point values are selected randomly from the limits, when the starting point values is constantly changed, the corresponding output will be changed. In general, the number of generations keeps on increasing up to 1000 optimal which is the least value that can be achieved.

Concluding remarks

This paper presents an optimization of Machining tolerance and manufacturing cost for simple linear assembly.

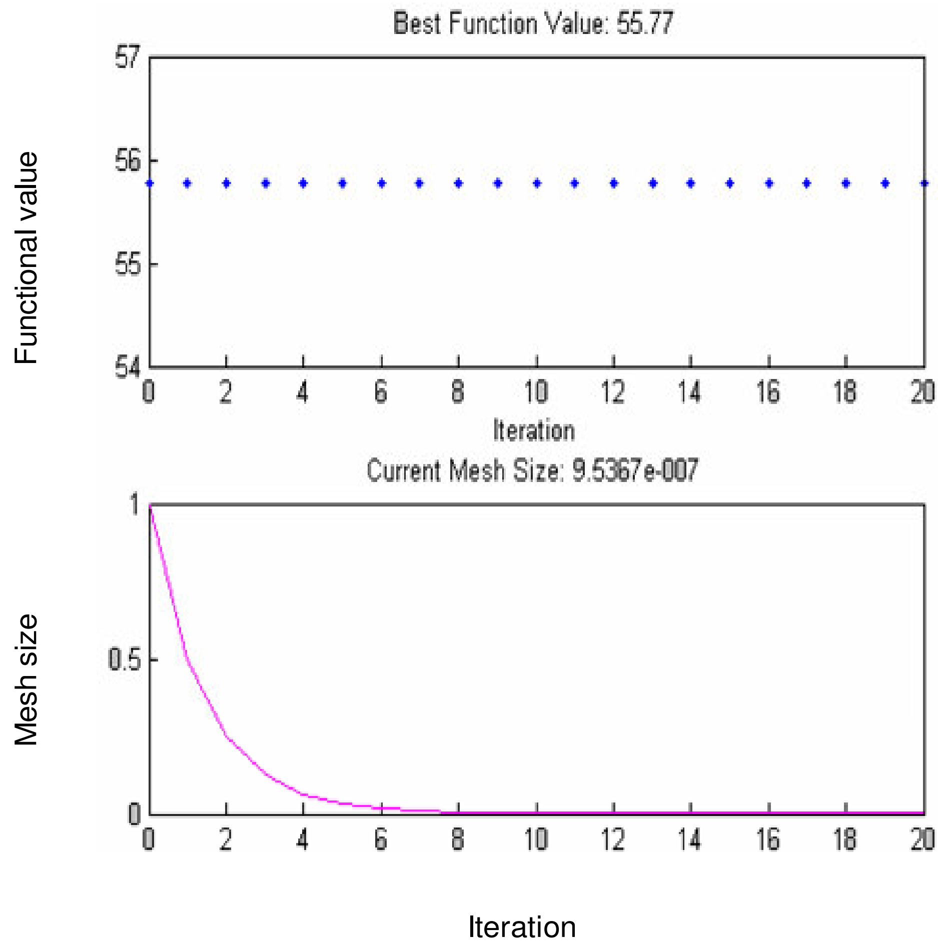

Figure 3. Comparison of Iteration with function value and mesh size.

Table 7: Comparison of Optimum machining cost with total cost for worst case method.

Run No	Total Machining cost [1]			Total Cost (Machining cost + Asymmetric quality loss)
	Genetic Algorithms	Simulated annealing	Complex method	Genetic algorithm
1	66.91	67.24	71.09	55.77
2	66.91	67.23	71.06	55.84
3	66.91	67.21	71.00	55.79
4	66.91	67.27	71.06	55.70
5	66.91	67.25	71.03	55.75

Exponential model was used to find the machining tolerance and cost simultaneously. Least manufacturing cost was optimized by using pattern search algorithm.

Our evaluation shows that the pattern search method exhibits a better performance than the genetic algorithm (GA) in terms of efficiency and ability to locate the optimal solutions, at least for the relatively local search that the earlier GA tests were limited to. In addition, pattern search produces far fewer invalid (that is unphysical) structures and requires far fewer function evaluations than Genetic algorithm.

Generally, pattern search has the advantage of being very simple in concept, easy to implement and computationally efficient. In future, better model may be approached for different assembly problems and try to find the Optimized cost using different techniques may give better results.

Notation

c_i; Coefficients used in Machining cost function, c_m; total machining cost for the components, c_r; cost of casting process, $g(\delta)$; machining cost function, $g_{ijk}(\delta)$; machining cost – tolerance model, δ; Machining tolerance parameter, δ_{ij}; Tolerance for part 'i' and process 'j' and Dimension 'k', δ_{ijd}; design tolerance on dimension chain, m_1, m_2; Mean shift value, Z; Process capability (value will be 3).

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