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Full Length Research Paper

Modeling economic system with the use of matrix algebra (Leontief input-output model)

N. S. Amadi[1], C. amadi[2] and N. Amadi[3]

[1]Department of Science and Technical education, Rivers State University of Science and Technology, P. M. B. 5080, Port Harcourt, Nigeria.
[2]Department of Banking and Finance, Rivers State Polytechnic, Bori, Rivers State, Nigeria.
[3]Department of Economics, Rivers State College of Education, Port Harcourt, Nigeria.

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The purpose of the study was to determine the external industrial demand output, given a 3 x 3 matrix input values with prescribed demand values. The Leontief input-output model leads to a better understanding of modeling economic systems. The identity matrix and a 3 x 3 matrix was used to produce industry's output unit for external demand. Of the 8619 external demand output unit turned out by a farmer, 0.4 unit of farmer's product were used up by farmers, 0.5 unit of baker's product and 0.5 unit of Grocer's product were utilized by the farmers. Of the 4684 external demand output unit turned out by a Baker, 0.00 unit of Baker's product were used up by the Bakers, 0.30 unit of Farmer's product and 0.30 unit of Grocer's product were utilized by the Bakers. Of the 3661 external demand output unit turned out by a Grocer, 0.00 unit of Grocer's product were used up by Grocers, 0.20 unit of baker's product and 0.20 unit of Farmer's product were utilized by the Grocer. Leontief input-output model is an effective tool in modeling economic system in the industries.

Key words: Input-output model, matrix algebra.

INTRODUCTION

According to Larson, "Matrix algebra has proved effective in modeling economic systems with the use of input – output model's (Larson et al., 2000). The result of the model produces industry's output unit for external demand. The American Economic Wassily W. Leontief developed the economic input –output model. He was a Professor of Economics and Director of the Harward Economic Research Project. His input –output model was first published in 1936. In 1973 Leontief was awarded a Noble prize for his great work in economics. The Leontief input-output model leads to a better understanding of modeling economic systems. The identity matrix and a 3 x 3 matrix was used to produce industry's output unit for external demand. Input-output matrix describes needs Vs output. Inputs are raw materials like farm product, processed food, textiles, utilities etc. while output is the finished product. According to Ronald, analysis of production function in an industrial sector displayed seed, feed, fertilizer, irrigation, and insecticide as separable inputs, together with land, labor and physical capital, the traditional inputs. Output was measured as tonnage, fluid production, bales and bushels (Miller et al., 1989). Investigating the output of any sector, such as agriculture, reveals the need for using inputs and gross output as explicit variables. Input-output analysis remains as vigorous as ever and the strength of the input-output model is its versatility and adaptability.

According to Leontief, "to compute the input requirements of an industry for a prescribed output, one would have to know its input coefficient" (Leontief, 1964a). The input coefficient is the constant quantities of each of the various inputs absorbed per unit of its final product. With a given set of input coefficient describing the internal structure of all the productive sectors of the industries and a known value of final demand, the industries can make decision through the solution of simultaneous linear equations. Leontief stressed that "input absorbed by the industrial sectors should reflect their output" (Leontief, 1964b).

The world has changed greatly since Leontief published

*Corresponding author. E-mail: omadesope@yahoo.co.uk.

the first paper outlining the basic input-output model, but input-output analysis has kept up with this change. The future of input-output research and its use in industrial sector are without temporal or spatial limits. This model will be indispensable to future generations of input-output scholars as they in turn further extend, modify and find new application for this powerful and versatile tool of analysis.

Lawrence stated that input-output analysis is clearly a case of economic measurement and therefore solidly in the mainstream of econometrics (Klein, 1989). One of the most ambitious applications of the input-output approach was the construction of a multiregional, multispectral dynamic input-output model of the entire world economy. According to (Shun-ichi, 1982), the institute of developing economics of Tokyo recently completed the 1975 international input-output structure linking Japan, South Korea and United States with five Asian countries – Indonesia, Malaysia, the Philippines, Singapor and Thailand.

IMPLEMENTATION OF THE MODEL

In a small community consisting of a farmer, baker and a grocer there exists an input-output relationship among them. In order for the farmer to produce one unit of output, he requires the following: 0.4 unit of its own product, 0.5 unit of Baker's product and 0.5 unit of Grocer's product. To produce one unit worth of output, the Baker requires, 0.3 unit of Farmer's product, 0.00 unit of its own product and 0.3 unit of Grocer's product. To produce one unit worth of output, the Grocer requires 0.2 unit of Farmer's product, 0.2 unit of Baker's product and 0.00 unit of its own product.

F, B and G represent Farmer, Baker and Grocer respectively. What is the output matrix X using Leontief's model where X = DX +E?

D AND E MATRIX

D matrix is the over all values of Farmer's, Baker's and Grocer are input in the 3 x 3 matrix. From the implementation model on page 4 the input-output values for D matrix below represent the values of the Farmer, Baker and the Grocer's inputs needed to produce one unit of their product respectively. The E matrix represents the external demand.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Farmer} & \text{Baker} & \text{Grocer}\end{array} \\ D & \begin{bmatrix} 0.40 & 0.50 & 0.50 \\ 0.30 & 0.00 & 0.30 \\ 0.20 & 0.20 & 0.00 \end{bmatrix} \end{array}$$

Farmer		1000
Baker and	E =	1000
Grocer		1000

These values are derived from the implementation model in Figure 1.

ROWS AND COLUMNS

The row entries above show what the Farmer, Baker and Grocer needed in order to produce one unit of their output. The columns entries show where the Farmer, Baker and Grocer's output go. Based on the Farmer's column, the farmer sends 0.4 unit of its output to itself, 0.3 units to the baker and 0.2 units to the grocer.

THE MODEL

X = DX + E; IX - DX = E; (1 - D) X=E

Where X is the output matrix, I is the matrix identity, D is 3 x 3 matrix and the E is the external demand.

THE MATRIX IDENTITY

$$I \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SOLUTION OF THE PROBLEM

Appling Gauss Jordan elimination

$$\begin{array}{ccccccc} & 1 & & D & & 1-D \\ 1-D & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & - & \begin{bmatrix} .40 & .50 & .50 \\ .30 & .00 & .30 \\ .20 & .20 & .00 \end{bmatrix} & \Rightarrow & \begin{bmatrix} .60 & -.50 & -.50 \\ -.30 & 1 & -.30 \\ -.20 & -.20 & 1 \end{bmatrix} \end{array}$$

Augmented

$$\begin{bmatrix} .60 & -.50 & -.50 & 10000 \\ -.30 & 1 & -.30 & 1000 \\ -.20 & -.20 & 1 & 1000 \end{bmatrix} 10 \frown 3$$

Let rl, r2 and r3 represent each rows.

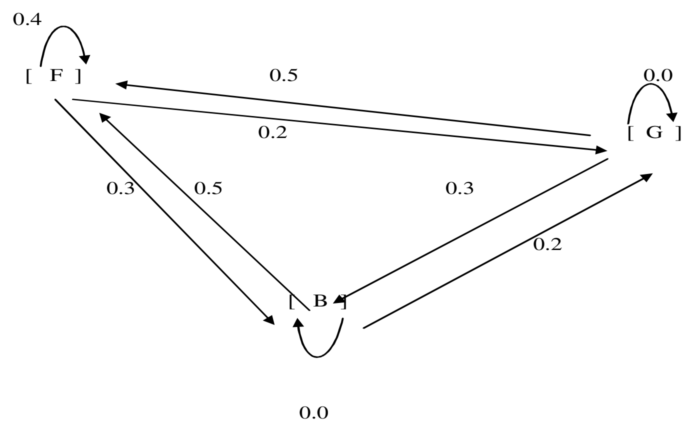

Figure 1. Diagrammatic representation of the intersectoral flows.

$$\begin{array}{c} \begin{bmatrix} .60 & -.50 & -.50 & 1 \\ -.30 & 1 & -.30 & 1 \\ -.20 & -.20 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Multiply r1 to r3 by 10

$$\begin{bmatrix} 6 & -.5 & -.5 & 10 \\ 0 & 15 & -11 & 30 \\ -2 & -2 & -10 & 10 \end{bmatrix} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Multiply r3 by 3 and add it to r1 to get new r3

$$\begin{bmatrix} 6 & -.5 & -.5 & 10 \\ 0 & 15 & -11 & 30 \\ -0 & -11 & -25 & 40 \end{bmatrix} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Divide r1 by 6 get new r1

$$\begin{bmatrix} 1 & -.833 & -.833 & 1.667 \\ 0 & 15 & -11 & 30 \\ 0 & -11 & -25 & 40 \end{bmatrix} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Divide r2 by 15 to get new r2

$$\begin{bmatrix} 1 & -.833 & -.833 & 1.667 \\ 0 & 1 & -.733 & 2 \\ 0 & -11 & 25 & 40 \end{bmatrix} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Multiply r2 by 11 and add it to r3 to get new r3

$$\begin{bmatrix} 1 & -.833 & -.833 & 1.667 \\ 0 & 1 & -.733 & 2 \\ 0 & 0 & 16.937 & 62 \end{bmatrix} \begin{array}{l} \ldots\ldots r1 \\ \ldots\ldots r2 \\ \ldots\ldots r3 \end{array}$$

Multiply r2 by 11 and add it to r3 to get new 3

X3 = G = 62/16.973 = 3.661 x 10^3 = 3661
X2 = B = 2 + (3.661) (.733)
= 2 + 2.684
= 4.684 x 10^3 = 4684
X1= F = 1.667 + (.833) (4.684) + (.833) (3.661)
= 1.667 + 3.902 + 3.050
= 8.619
= 8.619 x 10 ^3
= 8619

In conclusion, the industrial external demand output unit for a farmer, baker and a grocer are as follows:

	8619	Farmer
X =	4684	Baker
	3661	Grocer

CONCLUSION

Of the 8619 external demand output unit turned out by a farmer, 0.4 unit of farmer's product were used up by farmers, 0.5 unit of baker's product and 0.5 unit of Grocer's product were utilized by the farmers. Of the 4684 external demand output unit turned out by a Baker, 0.00 unit of Baker's product were used up by the Bakers, 0.30 unit of Farmer's product and 0.30 unit of Grocer's product were utilized by the Bakers. Of the 3661 external demand output unit turned out by a Grocer, 0.00 unit of Grocer's product were used up by Grocers, 0.20 unit of baker's product and 0.20 unit of Farmer's product were utilized by the Grocer. Leontief input-output model is an effective tool in modeling economic system in the industries.

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