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Full Length Research Paper

Modeling of saturated salient pole synchronous machines in d-q axes

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This paper presents the modeling of saturated salient pole synchronous machines in standard d-q axes. First, a complete synthesis of possible d-q axes models is given. Second, the salient pole synchronous machine is transformed into an equivalent smooth air gap machine, in order to apply the main flux saturation method. Third, a description of a procedure to introduce magnetic saturation in found models is presented. Finally, transient simulation and related experimentation are carried out to ensure the validity and the accuracy of models.

Key words: Salient pole synchronous machines, models synthesis, saturation modeling.

INTRODUCTION

Today, it is well reorganised that magnetic saturation is an important feature either in steady state or in transient regimes for synchronous machines, especially isolated generators. The advent of the main flux theory, had resolved the problem related to the introduction of saturation in the characteristic equations of round rotor alternating current (ac) machines (Brown et al., 1983; Rehaoulia, 1983; Kovacs, 1984).

The problem of models synthesis in standard d-q axes proving the multiplicity of models is also treated in some papers (Levi, 1998, 2000; Rehaoulia et al., 2006, 2007). However, in spite of such valuable works, magnetic saturation is mostly modelled by using all the winding currents or all the winding fluxes as state variables.

In his interesting paper, Levi, (1998) has not explored all possible models. In addition, he did not adequately detail the transformation of the salient pole synchronous machine to a smooth air gap machine. Also, the method he adopted, based on inductance derivation, was unable to develop certain kind of models.

The present paper devoted to salient pole synchronous machines differs from its precedents by giving a complete and detailed synthesis of possible models. It gives the general expression of equivalent factors transforming a salient pole machine to a non-salient pole machine. It also encompasses a simple procedure to introduce the iron saturation in all found models.

It has to be emphasized that the theory of main flux saturation is especially applicable to round rotor machines. Therefore, an additional step was necessary to transform the anisotropic machine to isotropic one.

The study is validated by comparing simulation and experimental results taken in same conditions.

PRIMITIVE EQUATIONS

With the usual assumptions, transient operations of a salient pole synchronous machine are generally analyzed in a synchronous reference frame by the following electric equations.

$$
\begin{aligned}
v_{ds} &= R_s i_{ds} + \frac{d\lambda_{ds}}{dt} - w\lambda_{qs} \\
v_{qs} &= R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + w\lambda_{ds}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
v_{dr} &= R_{dr} i_{dr} + \frac{d\lambda_{dr}}{dt} \\
v_{qr} &= R_{qr} i_{qr} + \frac{d\lambda_{qr}}{dt}
\end{aligned}
\tag{2}
$$

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$$v_f = R_f i_f + \frac{d\lambda_f}{dt} \quad (3)$$

Where:

$$\lambda_{ds} = l_s i_{ds} + \lambda_{dm} \qquad \lambda_{qs} = l_s i_{qs} + \lambda_{qm} \quad (4)$$

$$\lambda_{dr} = l_{dr} i_{dr} + \lambda_{dm} \qquad \lambda_{qr} = l_{qr} i_{qr} + \lambda_{qm} \quad (5)$$

$$\lambda_f = l_f i_f + \lambda_{dm} \quad (6)$$

And:

$$\lambda_{dm} = L_{dm} i_{dm} \qquad \lambda_{qm} = L_{qm} i_{qm} \quad (7)$$

$$i_{dm} = i_{ds} + i_{dr} + i_f \qquad i_{qm} = i_{qs} + i_{qr} \quad (8)$$

Voltages, currents and fluxes are designated by v, i and λ. Subscripts m, s, r, f stand for magnetizing, stator, rotor and field. All rotor quantities are referred to the stator. If the shaft speed is not uniform, the electric equations must be completed by the mechanical equation.

MODELS SYNTHESIS IN TERMS OF SPACE VECTORS

In d-q axes, voltages, currents and fluxes can be considered as space vectors and may be expressed with complex quantities. The real and imaginary parts of each space vector are known as d and q components. Quantities v_f, i_f and λ_f are real elements because of the absence of a field excitation on the quadrature axis. Resolving equations (1)-(3) for any transient operation requires the selection of two vectors and a d-component of a third one, all from the set $(\bar{i}_s, \bar{i}_r, \bar{i}_m, i_f, \bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m, \lambda_f)$.

Regarding models synthesis, the summary does not differ from that of smooth air gap synchronous machines detailed by Rehaoulia et al. (2007). The total combinations of possible state space variables for synchronous machines, in terms of space vectors, are sixty-four.

They are classified into three families: current, flux and mixed models.

Current models

$(\bar{i}_s, \bar{i}_r, i_f)$, $(\bar{i}_s, \bar{i}_m, i_f)$, $(\bar{i}_r, \bar{i}_m, i_f)$, $(\bar{i}_s, \bar{i}_r, i_{dm})$, $(\bar{i}_s, i_{dr}, \bar{i}_m)$, $(i_{ds}, \bar{i}_r, \bar{i}_m)$

Flux models

$(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_f)$, $(\bar{\lambda}_s, \bar{\lambda}_m, \lambda_f)$, $(\bar{\lambda}_r, \bar{\lambda}_m, \lambda_f)$, $(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_{dm})$, $(\bar{\lambda}_s, \lambda_{dr}, \bar{\lambda}_m)$, $(\lambda_{ds}, \bar{\lambda}_r, \bar{\lambda}_m)$

Mixed models

$(\bar{i}_s, \bar{i}_r, \lambda_{ds})$, $(\bar{i}_s, \bar{i}_r, \lambda_{dr})$, $(\bar{i}_s, \bar{i}_r, \lambda_{dm})$, $(\bar{i}_s, \bar{i}_r, \lambda_f)$, $(\bar{i}_s, \bar{i}_m, \lambda_{dr})$, $(\bar{i}_s, \bar{i}_m, \lambda_f)$, $(\bar{i}_r, \bar{i}_m, \lambda_{ds})$, $(\bar{i}_r, \bar{i}_m, \lambda_f)$.

$(\bar{\lambda}_s, \bar{\lambda}_r, i_{ds})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_{dr})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_{dm})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_f)$, $(\bar{\lambda}_s, \bar{\lambda}_m, i_{dr})$, $(\bar{\lambda}_s, \bar{\lambda}_m, i_f)$, $(\bar{\lambda}_r, \bar{\lambda}_m, i_{ds})$, $(\bar{\lambda}_r, \bar{\lambda}_m, i_f)$.

$(\bar{i}_s, \bar{\lambda}_s, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_s, i_f)$, $(\bar{i}_s, \bar{\lambda}_m, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_m, i_f)$, $(\bar{i}_r, \bar{\lambda}_r, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_r, i_f)$, $(\bar{i}_m, \bar{\lambda}_s, i_{dr})$, $(\bar{i}_m, \bar{\lambda}_s, i_f)$, $(\bar{i}_m, \bar{\lambda}_r, i_{ds})$, $(\bar{i}_m, \bar{\lambda}_r, i_f)$, $(\bar{i}_s, \bar{\lambda}_r, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_r, i_{dm})$, $(\bar{i}_s, \bar{\lambda}_r, i_f)$, $(\bar{i}_r, \bar{\lambda}_s, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_s, i_{dm})$, $(\bar{i}_r, \bar{\lambda}_s, i_f)$, $(\bar{i}_r, \bar{\lambda}_m, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_m, i_f)$.

$(\bar{i}_s, \bar{\lambda}_s, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_s, \bar{\lambda}_m, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_m, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_r, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_m, \bar{\lambda}_s, \lambda_{dr})$, $(\bar{i}_m, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_m, \bar{\lambda}_r, \lambda_{ds})$, $(\bar{i}_m, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_s, \bar{\lambda}_r, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_r, \lambda_{dm})$, $(\bar{i}_s, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_{dm})$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_m, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_m, \lambda_f)$.

TRANSFORMING THE SALIENT POLE MACHINE TO A SMOOTH AIR GAP MACHINE

Because $L_{qm} \neq L_{dm}$, the air gap flux and current in a salient pole machine are not aligned (Levi, 1998). In the following let:

$$\lambda'_{qm} = F_\lambda \lambda_{qm} \quad (9)$$

$$i'_{qm} = F_i\, i_{qm} \quad (10)$$

So that:

$$\lambda'_{qm} = L_{dm}\, i'_{qm} \quad (11)$$

Magnetizing flux and current are now expressed by:

$$\lambda_m = (\lambda^2{}_{dm} + \lambda'^2{}_{qm})^{1/2} \quad (12)$$

$$i_m = (i^2{}_{dm} + i'^2{}_{qm})^{1/2} \quad (13)$$

Incorporation of equation (11) and first equation of (7) in (12) gives with the help of equation (13):

$$\begin{aligned}\lambda_m &= L_{dm}\, i_m \\ &= L_m\, i_m\end{aligned} \quad (14)$$

Inductance L_m is a real element, even if it varies with magnetic saturation space vectors $\bar{\lambda}_m$ and $\bar{i}_m$ remain in phase. Therefore, with the new description, the machine seems to be transformed to an equivalent uniform air gap machine.

Elsewhere, relationship between factors F_i and F_λ can be achieved using equations (7), (9), (10) and (11).

$$\lambda_{qm} = \frac{\lambda'_{qm}}{F_\lambda} = \frac{L_{dm}\, i'_{qm}}{F_\lambda} = L_{dm}\frac{F_i}{F_\lambda} i_{qm} \quad (15)$$

Equating second equation of (7) with (15) results in:

$$\frac{F_i}{F_\lambda} = \frac{L_{qm}}{L_{dm}} \quad (16)$$

INTRODUCTION OF MAGNETIC SATURATION

Further, it will be shown that the proposed common approach to introduce magnetic saturation in any d-q existing model relies only on the knowledge of the winding currents model. For that reason, it will be first shown how to obtain it.

Developing the winding currents model

The leakage inductances in equations (4)-(6) are assumed to be constant, only the main flux $\bar{\lambda}_m$ is subject to saturation. Deriving stator and rotor linkage fluxes, in equations (4)-(6), leads to the time derivative of the magnetizing flux $\bar{\lambda}_m$.

$$\frac{d\bar{\lambda}_{s,r,f}}{dt} = l_{s,r,f}\frac{d\bar{i}_{s,r,f}}{dt} + \frac{d\bar{\lambda}_m}{dt} \quad (17)$$

Therefore, $\frac{d\bar{\lambda}_m}{dt}$ has to be described by means of the winding currents.

$$\frac{d\bar{\lambda}_m}{dt} = \frac{d(\lambda_m e^{i\alpha})}{dt} = e^{i\alpha}\left(\frac{d\lambda_m}{dt} + \lambda_m\frac{d\alpha}{dt}\right) \quad (18)$$

Angular α designates the position of $\bar{\lambda}_m$ with respect to the d-axis. It also characterizes the position of the magnetizing current $\bar{i}_m$ in the air gap, since the machine saliencies were already eliminated.

$$\alpha = \tan^{-1}\frac{F_i\, i_{qm}}{i_{dm}} \quad (19)$$

$$\frac{d\lambda_m}{dt} = \frac{d\lambda_m}{di_m}\frac{di_m}{dt} = L_{mdy}\frac{di_m}{dt} \quad (20)$$

Notice that equations (12)-(13) define unique static and dynamic mutual inductances, (21). They are evaluated from the open circuit d-axis magnetizing curve $\lambda_m = f(i_m)$.

$$L_m = \frac{\lambda_m}{i_m} \qquad L_{mdy} = \frac{d\lambda_m}{di_m} \quad (21)$$

After some mathematical manipulations, we get:

$$\frac{d\lambda_{dm}}{dt} = L_{d1}\frac{di_{dm}}{dt} + L_{dq1}\frac{di_{qm}}{dt} \quad (22)$$

$$\frac{d\lambda_{qm}}{dt} = L_{dq1}\frac{di_{dm}}{dt} + L_{q1}\frac{di_{qm}}{dt} \quad (23)$$

Where:

$$L_{dq1} = \frac{1}{F_\lambda}(L_{mdy} - L_m)\cos\alpha\sin\alpha \quad (24)$$

$$L_{d1} = L_m + (L_{mdy} - L_m)\cos^2\alpha \quad (25)$$

$$L_{q1} = \frac{F_i}{F_\lambda}(L_m + (L_{mdy} - L_m)\sin^2\alpha) \quad (26)$$

Use of previous expressions leads to:

$$\frac{d\lambda_{ds}}{dt} = (l_s + L_{d1})\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \quad (27)$$

$$\frac{d\lambda_{qs}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + (l_s + L_{q1})\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + L_{q1}\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \quad (28)$$

$$\frac{d\lambda_{dr}}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + (l_{dr} + L_{d1})\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \quad (29)$$

$$\frac{d\lambda_{qr}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + L_{q1}\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + (l_{qr} + L_{q1})\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \quad (30)$$

$$\frac{d\lambda_f}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + (l_f + L_{d1})\frac{di_f}{dt} \quad (31)$$

The main differential system can be represented by $[V] = [A][\dot{X}] + [B][X]$

With:

$$[V] = [v_{ds}\ v_{qs}\ 0\ 0\ v_f\ ]^t \quad (32)$$

$$[X] = [i_{ds}\ i_{qs}\ i_{dr}\ i_{qr}\ i_f\ ]^t \quad (33)$$

$$[A]=\begin{bmatrix} l_s+L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & L_{d1} \\ L_{dq1} & l_s+L_{q1} & L_{dq1} & L_{q1} & L_{dq1} \\ L_{d1} & L_{dq1} & l_{dr}+L_{d1} & L_{dq1} & L_{d1} \\ L_{dq1} & L_{q1} & L_{dq1} & l_{qr}+L_{q1} & L_{dq1} \\ L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & l_f+L_{d1} \end{bmatrix} \quad (34)$$

$$[B]=\begin{bmatrix} R_s & -wL_{qs} & 0 & -wL_{qm} & 0 \\ wL_{ds} & R_s & wL_{dm} & 0 & wL_{dm} \\ 0 & 0 & R_{dr} & 0 & 0 \\ 0 & 0 & 0 & R_{qr} & 0 \\ 0 & 0 & 0 & 0 & R_f \end{bmatrix} \quad (35)$$

With

$$\begin{aligned} L_{ds} &= l_s + L_{dm} \qquad L_{dr} = l_{dr} + L_{dm} \\ L_{qs} &= l_s + L_{qm} \qquad L_{qr} = l_{qr} + L_{qm} \end{aligned} \quad (36)$$

Developing the remaining models

The proposed approach to develop the remaining models consists of three stages (Rehaoulia et al., 2007). First, a combination of state-space vectors is chosen among the sixty-three remaining possibilities. Second, the d-q components of linkage fluxes and winding currents are described in terms of these selected variables using equations (4)-(8). Third, by ordinary manipulations of equations (27)-(31) time derivatives of the d-q components of the linkage fluxes are written as functions of the chosen variables.

Evidently, it is not possible to report in one paper the results related to all models. In return, the model $(\overline{i_r}, \overline{i_m}, i_{ds})$ is selected as illustration.

Since $[X]=[i_{dr}, i_{qr}, i_{dm}, i_{qm}, i_{ds}]$ in the primitive d-q equations only i_f and i_{qs} has to be eliminated. For that purpose, equation (8) permits the description of i_f and i_{qs} in terms of the selected variables and then matrix [B].

$$i_f = i_{dm} - i_{ds} - i_{dr} \quad (37)$$

$$i_{qs} = i_{qm} - i_{qs} \quad (38)$$

Next, derivatives of equations (37) and (38) when introduced in equations (27)-(31) give immediately matrix [*A*]. The resulting matrices [*A*] and [B] are:

$$[A]=\begin{bmatrix} 0 & 0 & L_{d1} & L_{dq1} & l_s \\ 0 & -l_s & L_{dq1} & l_s+L_{q1} & 0 \\ l_{dr} & 0 & L_{d1} & L_{dq1} & 0 \\ 0 & l_{qr} & L_{dq1} & L_{q1} & 0 \\ -l_f & 0 & l_f+L_{d1} & L_{dq1} & -l_f \end{bmatrix} \quad (39)$$

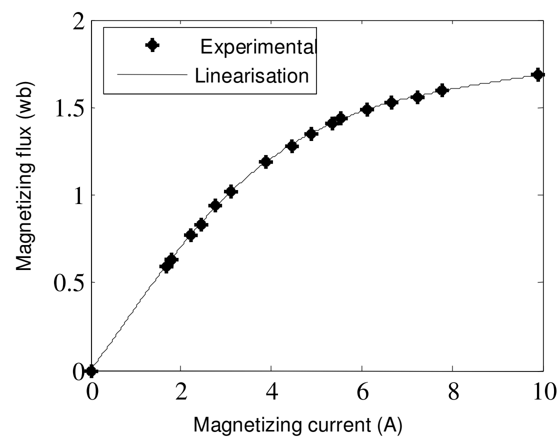

Figure 1. Magnetizing characteristic of the alternator.

$$[B]=\begin{bmatrix} 0 & wl_s & 0 & -wL_{qs} & R_s \\ 0 & -R_s & wL_{dm} & R_s & wl_s \\ R_{dr} & 0 & 0 & 0 & 0 \\ 0 & R_{qr} & 0 & 0 & 0 \\ -R_f & 0 & R_f & 0 & -R_f \end{bmatrix} \quad (40)$$

SIMULATION AND EXPERIMENTAL VALIDATION

The most popular solution of equation (16) corresponds to (Levi, 2000):

$$F_i = \frac{1}{F_\lambda} = F = \sqrt{\frac{L_{qm}}{L_{dm}}} \quad (41)$$

F is assumed to be constant at all saturation levels. Hence, it can be easily calculated from the unsaturated region of the d and q-magnetizing curves.

The test machine is a salient pole alternator which ratings and parameters are: 1KVA, 400/230V, 2.66/1.52A, 50Hz, 4poles, R_s = 11.3Ω, R_f = 2.4Ω, R_{dr} = 40.17Ω, R_{qr} = 113.92Ω, l_s = 0.0705Ω, l_f = 0.0152Ω, l_{dr} = 0.0702Ω, l_{qr} = 0.039Ω. The non linear magnetizing characteristic is provided by Figure 1.

Computer simulations were conducted with the factor F defined in (41), $F = 0.707$ for the alternator under investigation.

The aim of such application is to initially show the equivalence between the sixty for models and then evaluate their capability to predict transient operations of salient pole synchronous machine. For that purpose, the typical example of the three-phase short circuit of the alternator is performed here. The alternator, initially shorted and unexcited, was actuated at his speed of synchronism.

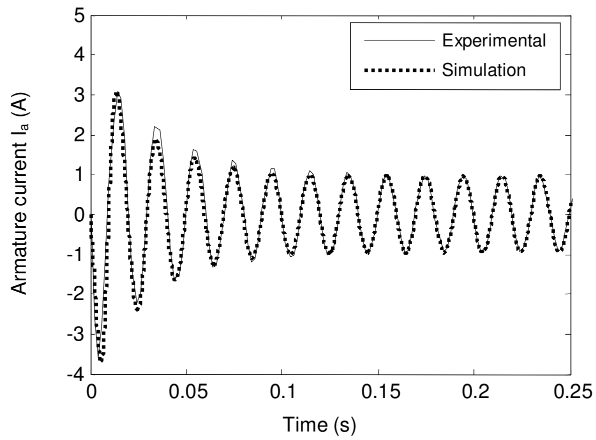

Figure 2. Transient behavior of armature current I_a following the short circuit of the alternator.

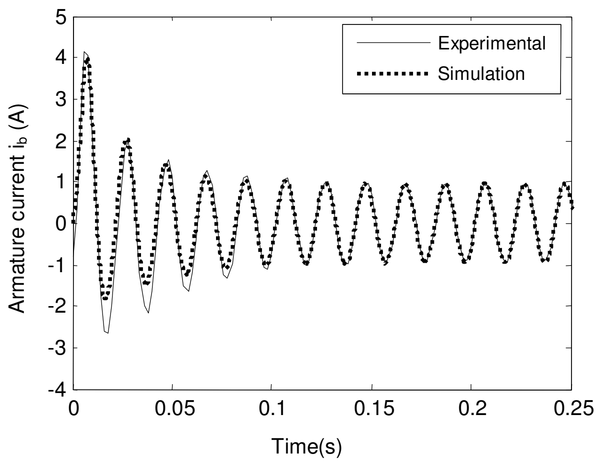

Figure 3. Transient behavior of armature current I_b following the short circuit of the alternator.

Figures 2 and 3 give, respectively the transient evolution of armature currents i_a and i_b. Simulation results can be obtained using any of the sixty-four found models, especially with those fully developed in this paper. It was not necessary to report results separately for each model because they gave exactly the same ones.

Comparison between the experimental and simulation traces for stator currents, in Figures 2 and 3, show that they are practically identical and thus proves the validity of both approaches to saturation modelling and models derivation.

CONCLUSION

The modeling of saturated salient pole synchronous machines is discussed. A complete and detailed synthesis of possible d-q models in standard d-q axes, of a saturated salient pole synchronous machine, is established by changing the state space variables. A simple procedure for introducing magnetic saturation in any found model is described. The study also establishes the relationship between flux and current equivalent factors (F_λ and F_i) defined in order to convert a salient pole machine to a smooth air gap machine. All approaches and results are validated by experimental tests.

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