

*Full Length Research Paper*

# Generation of surface optical vortices by evanescent Bessel beams

S. Al-Awfi\* and S. Bougouffa

Department of Physics, Taibah University, P. O. Box 30002, Madinah, Saudi Arabia.

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**We present that a set of high order Bessel beams  $\ell > 0$  when internally reflected at a dielectric/vacuum interface can create distinct evanescent light modes in each of which the intensity distribution is restricted to a sub-wavelength region near the interface outside the dielectric. We believe that this could really facilitate lateral optical manipulation of nano-particles and neutral atoms along the interface. Similarly, the set-up could lead to the formation of two-dimensional surface optical vortices that possibly will give rise to attractive phenomena, including pattern rotation liken to a surface optical Ferris wheel. Applications are envisaged to be in atom lithography, optical surface tweezers, and spanners.**

**Key words:** Bessel beam, optical surface tweezers, optical surface spanners.

## INTRODUCTION

The rising use of light to guide the atomic systems and micro-particles has improved the attention in the mechanical properties of electromagnetic beams and their generations (Brousseau et al., 2011; Graham et al., 2008). In addition dielectric atomic mirrors were widely considered in which the repulsive force field owes its existence to an evanescent light (Kirk et al., 2002). Indeed, Cook and Hill (1982) first suggested that when light is totally reflected internally at a vacuum-dielectric interface, an atom in the thin transmitted evanescent wave experiences an optical force. This force tends to repel the atom from the dielectric surface; and hence the internally illuminated surface acts as a mirror for slow neutral atoms. Currently the investigation of atomic mirrors constructs an interesting area of the main flow of atom optics (Kirk et al., 2002), with atomic mirrors continuing to attract attention by both experiment and theory dealing with mechanisms for mirror action (Lembessis et al., 2009; Andrews et al., 2010; Al-Awfi et al., 2010; Lembessis et al., 2011).

However, optical forces at dielectric interface occur due

to light pressure and refraction, both of which can be used for the manipulation of small particles, neutral atoms, and molecules. The processes, basically known as optical manipulation, were initiated by Ashkin et al. (1986) and have advanced considerably in recent years with applications ranging from atom optics to biology and chemistry (Grier, 2003). In fact, in some application it may be useful to force in different way free object to be brought near a dielectric surface and made to reside at a well-defined region of space in the vicinity of the surface.

It is well established that laser light prepared as a Bessel beam shows little diffraction, typically exhibiting a number of concentric high intensity rings separated by dark rings (Durnin, 1987; McGloin et al., 2003). In particular, the central peak is considered to be remarkably stable against diffraction and it is this property that is behind the recent applications including atom guides, nonlinear optics and optical atom sorting (McGloin et al., 2003). Bessel beams of light were produced initially by Durnin et al. (1987) using an annular slit. Then, Bessel beams were also formed by other methods (Vasara et al., 1989; Jabczynski, 1990; Paterson and Smith, 1996; Erdelyi et al., 1997; Arif et al., 1998; Novitsky and Novitsky, 2008; Meltaus et al., 2003; De Angelis et al., 2003). A generation of high-order Bessel beam of arbitrary order by illuminating an axicon with the appropriate Laguerre-Gaussian light beam was

\*Corresponding author. E-mail: [alawfi99@hotmail.com](mailto:alawfi99@hotmail.com).

performed (Art and Dholakia, 2000) and later they were constructed in biaxial crystals (King, 2001). High-order Bessel beams offer distinct advantages over other hollow light beams for atom guiding and trapping charged particles (Bialynicki-Birula et al., 2005).

We indicate here that the non-diffractive feature also makes these beams suitable for the generation of surface optical vortices in which typically a Bessel beam propagating within an optically dense medium is totally internally reflected at a planar interface with vacuum. A light field possessing the rotational features of the Bessel beam, but which is tightly bound to the surface as an evanescent mode, is generated in the vacuum region, with its intensity in vacuum decaying exponentially with distance away from the surface. We present the theory exploring the surface mode due to a totally internally reflected Bessel beam, its optical angular momentum content, the interference of multiple beams and the possible applications that we envisage can be experimentally realized for systems in the vicinity of the surface, such as different possibilities for atom trapping and atom manipulation at a surface, and optical surface tweezers and spanners as well as in atom lithography.

This paper is organized as follows. Subsequently, the eclectic field distributions of Bessel beams propagated within a non-dispersive medium is briefly outlined, after which the basic elements of the physical model that is employed to create surface modes on a planar dielectric is described. This is followed by a derivation of an expression for the Rabi frequency and optical dipole potential associated with an evanescent Bessel beam at the outer surface, after which an explanation is given for the time at which this product of the potential profile leads transparently to the presentation of a two-dimensional surface optical vortex. Finally, this study was concluded with the author's comments.

**BESSEL BEAMS FORMULATION**

Consider first the electric field of a Bessel beam traveling along a  $z$  -direction in a medium of constant refractive index  $n$ , characterized by the integer  $\ell$ , angular frequency  $\omega$ , and axial wavevector  $k = nk_0$ , where  $k_0 = \omega / c$  is the wavevector in a vacuum. Such a beam with the plane polarized along the field vector can be written in cylindrical coordinates as (McGloin et al., 2003)

$$\mathbf{E}_{k\ell}^I(\mathbf{r}) = \mathbf{F}_{k\ell}(r_\square, z) \exp^{i(kz - \omega t)} \exp^{i\ell\phi} \tag{1}$$

where  $\mathbf{F}_{k\ell}(r_\square, z)$  is the standard envelope function

$$\mathbf{F}_{k\ell}(r_\square, z) = \frac{\eta_{k0}}{2\pi k w_0^{-1} \left(\frac{z}{z_{\max}}\right)^{\ell+1/2}} \exp\left(-\frac{z^2}{z_{\max}^2}\right) \exp\left[i\left(\ell\hat{\phi} - \frac{\ell\pi}{2} - \frac{\pi}{4}\right)\right] J_\ell(k_r r_\square) \tag{2}$$

where  $w_0$  is the input beam waist,  $k_r$  is the radial wavevector and  $\phi$  is the azimuthal coordinate. The factor  $\eta_{k0}$  is the amplitude of a corresponding plane wave of intensity  $I$  propagating in the dielectric medium of refractive index  $n$

$$\eta_{k0} = \sqrt{2I / n^2 \epsilon_0 c} \tag{3}$$

while  $z_{\max}$  is the typical ring spacing. The last term  $J_\ell(k_r r)$  is the Bessel function of the order  $\ell$ . The mathematical formula of the Bessel beam that is given in equation (2) is only valid for the central region of the Bessel beam that has been produced. The main condition for this validity is  $2w(z)/w_0 \ll 1$  where  $w(z) = 1/k_r$  is a measure of the width of the central lobe in a  $J_0$  beam or the central dark fringe in a higher-order beam which can be called the beam size at axial coordinate  $z$ . This can be written as

$$w(z) = w_0 \sqrt{1 + \lambda z / \pi w_0^2} \tag{4}$$

Hence, the standard envelope function can simply be re-written as

$$\mathbf{F}_{k\ell}(r_\square, z) = \eta_{k0} \sqrt{\frac{2\pi w_0}{w(z)}} \left(\frac{z}{z_{\max}}\right)^{\ell+1/2} \exp\left(-\frac{z^2}{z_{\max}^2}\right) \times \exp i\Phi J_\ell\left(\frac{r_\square}{w(z)}\right) \tag{5}$$

where the phase  $\Phi$  can be written as

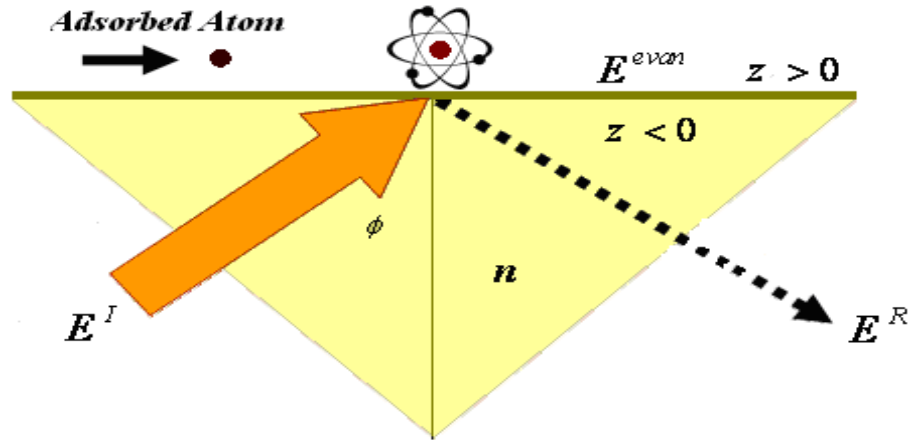
$$\Phi = \ell\hat{\phi} - \frac{\ell\pi}{2} - \frac{\pi}{4} = \ell \tan^{-1}\left(\frac{x}{y}\right) - \frac{2\pi(2\ell - 1)}{8} \tag{6}$$

The plane  $z = 0$  corresponds to the minimum beam waist  $w(0) = w_0$  and on this plane the phase in Equation (6) vanishes.

**OBLIQUE INCIDENCE**

As a vector Bessel beam falls onto the interface between two media, the total internal reflection can occur. The Bessel beam that refracts the interface exponentially decreases, while moving from the boundary. Such a wave is called the evanescent one. Such a light field can be arranged, as shown in Figure 1. Consequently, the evanescent Bessel beam corresponds to the situation

$$k \rightarrow i\sqrt{k_r^2 - (\omega/c)^2} \tag{7}$$



**Figure 1.** Schematic diagram of the experimental set up in which Bessel Beam is internally reflected at an angle greater than the critical angle.

It is clear from this transformation that the field distribution associated with the evanescent light is confined to the region close to the surface, rather than spread out along the axis, as in the normal Bessel light in an unbounded space. The oscillatory behaviors of Bessel functions imply an opportunity of trap-ping the atoms between some-adjacent maxima. Of course, a Bessel beam is far from behaving like static potential. On the other hand, it does construct some what like ring shaped barriers in the transverse direction.

On the other hand, evanescent light suitable to conventional beams has been fruitfully used in atomic mirrors (Kirk et al., 2002) and its exploit in surface manipulation has been used for micro-particles at surfaces (Garces-Chavez et al., 2005; Reece et al., 2006; Jia et al., 2008), rather than near-resonance atoms and molecules. Recently, the generation of the Bessel beams makes them interesting to be used experimentally in the context of surface-atom manipulation. In addition, we will here that for Bessel beam, the evanescent light that appears in the vacuum region is exponentially decaying with the distance normal to the surface, however transmits the in-plane distribution of the incident beam and its angular momentum properties. Placed over on this is a plane wave propagating along the surface with a wave vector equal to the component of the axial wavevector of the incident beam (Lembessis et al., 2009; Andrews et al., 2010; Al-Awfi et al., 2010; Lembessis et al., 2011). Then, the created field distribution forms a surface optical vortex, with a well-defined orbital angular momentum.

In order to simplify the formalism, consider here the electric field of a Bessel beam propagating along  $z$  in a medium of a constant refractive index  $n$ , characterized by the integers  $\ell$ , angular frequency  $\omega$ , and axial wave vector  $k = nk_0$ , where  $k_0 = \omega/c$  is the wave vector in vacuum. Assuming the Bessel light plane polarized along

$\hat{y}$ , from Equation (1), the electric field can be read as

$$\mathbf{E}^{evan} = \hat{y} \frac{\eta_{k_0}}{2\pi k_0 w_0^{-1}} \left( \frac{z}{z_{max}} \right)^{\ell+1/2} \exp\left(-\frac{z^2}{z_{max}^2}\right) \exp\left(i\ell \tan^{-1}\left(\frac{x}{y}\right) - \frac{2\pi(2\ell-1)}{8}\right) \times J_\ell \left( \frac{1}{w_0} \sqrt{\frac{x^2+y^2}{1+\lambda z/\pi w_0^2}} \right) \times \exp -zk_0 \sqrt{n^2 \sin^2(\phi)-1} \times \exp -ik_0 n x \sin \phi \quad (8)$$

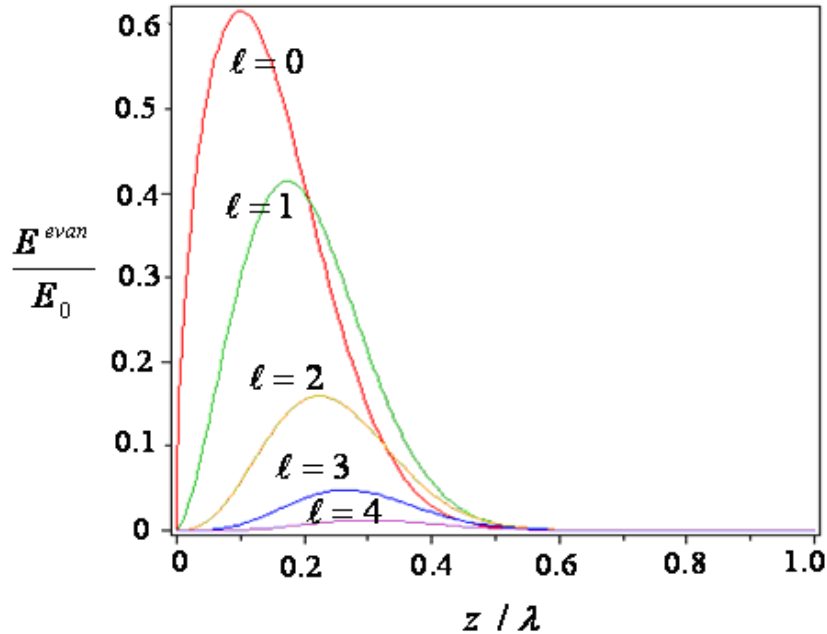
Using the continuity formalism as presented by Lembessis et al. (2009); Andrews et al. (2010), Al-Awfi et al. (2010), Lembessis et al. (2011) of the electric field vector tangential to the surface, along with exponential decay with the coordinate  $z$ , the structure of the evanescent field in the vacuum region can be given as

$$\mathbf{E}^{evan} = 2E_{k_\ell}^I(x \rightarrow x \cos \phi, y; z \rightarrow -x \sin \phi) \times \exp -zk_0 \sqrt{n^2 \sin^2 \phi - 1} \times \exp -ik_0 n x \sin \phi \quad (9)$$

The significant point to stand in mind in this situation is that the field distribution associated with this vortex is concentrated on the surface, rather than axially, as in normal Bessel light beams in an unbounded space. The typical exponential decay of the evanescent light field along  $z$  is shown in Figure 2. The length scale of the exponential decay along perpendicular to the interface is seen, from Figure 2, to span a small fraction of the wavelength, indicating that the evanescent light does not play a significant role in the trapping normal to the surface. Adsorbed atoms are generally subject to a much more strongly attractive van der Waal potential.

## SURFACE OPTICAL VORTEX

To show the effects of the evanescent light on matter



**Figure 2.** The radial component of the evanescent electric  $E^{evan}(x, y) / E_0$  variations near surface (arbitrary units) with distance normal to the surface (in units of wavelength) for different values of  $\ell$ .

localized near the surface, we consider adsorbed atoms that possess a transition frequency  $\omega_0$  such that  $\omega_0$  is at near-resonance with the frequency  $\omega$  of the light. The evanescent field has significant intensity distribution in the vicinity and so the adsorbed atom will be subject to corresponding optical forces that influence its in-plane motion. In particular, the dipole potential to which the atom is subject is responsible for lateral trapping and is given by (Andrews et al., 2010; Al-Awfi et al., 2010; Lembessis et al., 2011)

$$U(x, y) = \frac{\hbar\Delta}{2} \ln \left[ 1 + \frac{2\Omega_\ell^2}{\Delta^2 + \Gamma^2} \right] \quad (10)$$

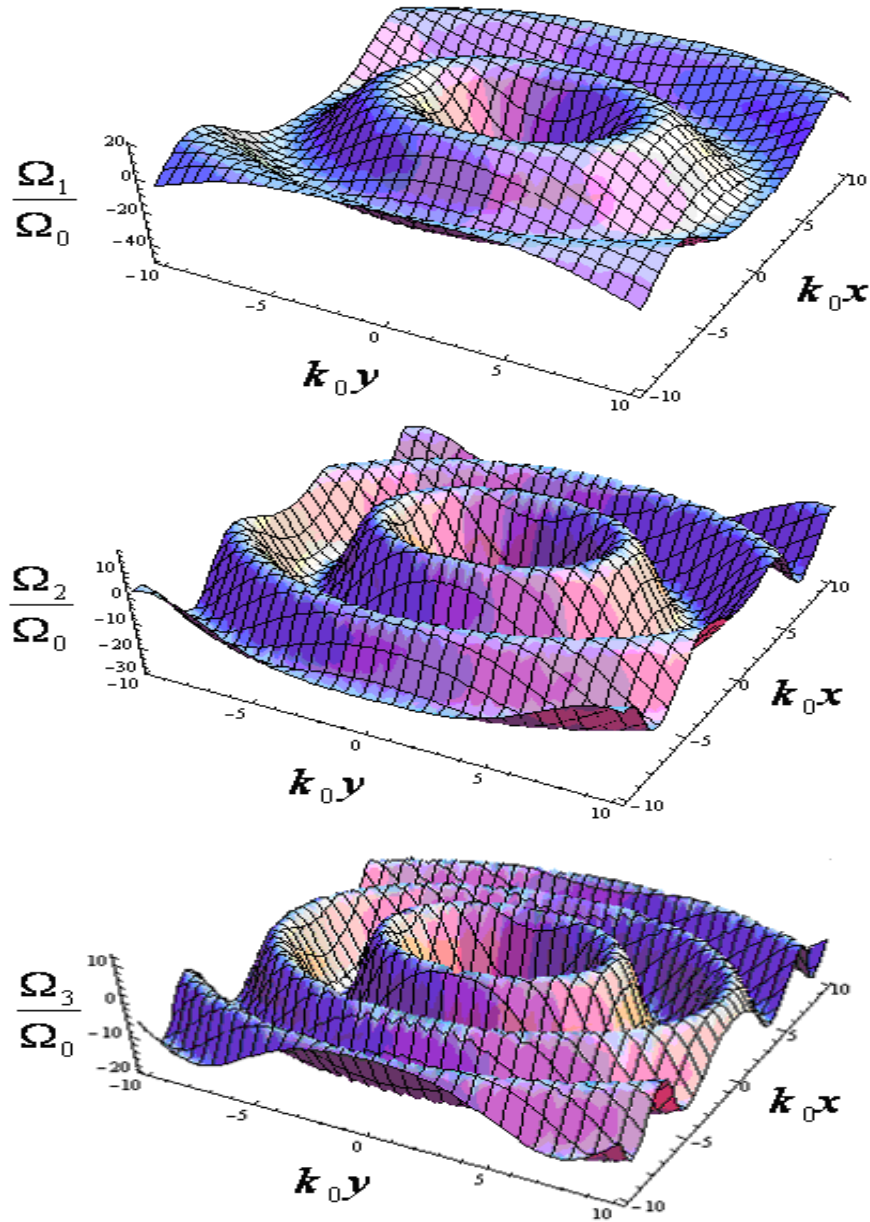
where  $\Gamma$  is the decay emission rate,  $\Delta = \omega - \omega_0$  is the detuning of the laser light from the atomic transition, and  $\Omega_\ell$  is the Rabi frequency defined by

$$\Omega_\ell(x, y, z) = \left| \frac{\wp E^{evan}(x, y, z)}{\hbar} \right| \quad (11)$$

where  $\wp$  is the transition electric dipole matrix element. Since adsorption is due to a potential that depends on the coordinate  $z$  normal to the surface, we shall only be concerned with the motion of the atom in the parallel adsorption plane close to the surface. Then we consider

a Bessel beam propagating along the  $z$ -axis and which is polarized along the  $y$ -axis. On the other hand, to avoid the calculations of the numerical constants of normalization, which are not of importance in our actual treatment, we can see that the potential  $U$  is related to the Rabi frequency through the logarithmic functions Equation (10). Then these two functions have the same behavior except that the initial values are different. The Rabi frequencies, corresponding to the evanescent light created by an incident Bessel beam for which  $\ell = 1, 2, 3$  are shown in Figure 3 plotted in  $x - y$  plane at a fixed value of  $z > 0$  close to the surface. It is noticed that Rabi frequency, that is, the potential of the evanescent light possesses well-defined maxima and minima that can be used to trap adsorbed atoms that have transition frequencies approximately detuned from the frequency  $\omega$  of the light. The oscillatory behavior of Bessel beam functions implies a chance of trapping the atoms between two adjacent maxima.

Certainly, a Bessel beam is far from performing like static potential. However, it does generate something like ring-shaped barriers in the evanescent wave. From Figure 3, we can see that the evanescent Bessel beam forms concentrated ring potential wells of kind where the atoms may be kept on orbits. The shapes of the various ring maxima in the Rabi frequency which correspond to the maxima of the potential of the evanescent light, are elliptic and the orbit of the central ring increases with  $\ell$ .



**Figure 3.** The Rabi frequency distribution  $\Omega_\ell(x, y)/\Omega_0$  on the surface for which  $\ell = 1, 2, 3$ . Distances are measured in units of  $k_0$  and the angle of internal reflection is taken as  $\phi = 7\pi/30$ , which is greater than the critical angle.

## Conclusion

In this paper we have presented the crucial formalism for the realization of surface optical vortices and optical lattices for cold atoms using Bessel beam. In general, cold atom optical lattices have been of much interest recently since they assisted explorations of dynamical processes, such as strong atom-atom interactions leading to quantum phase transitions as well as Bloch

oscillations. Furthermore, the Berezinskii-Kosterlitz-Thouless transition has been identified for atoms trapped in a three-dimensional optical lattice (Trombettoni et al., 2005). On the other hand, The optical vortices of Bessel basis represent an entire set for the description of vortices of any desired order defined by  $\ell = 1$ , so that the phenomenon is relatively general. Further work is needed to expose the properties of surface optical vortices. Valuable information can be increased by finding phase

profiles, wave-front structures, and energy-momentum flow. Optical vortices suggest an exceptional potential for the management of adsorbed atoms collecting in intense regions of intensity on the surface.

The principle can be used to produce patterned surfaces by utilizing carefully planned sets of incident beams to create a lattice of evanescent light wells. The scheme also gives the chance to manipulate larger particles on the surface, translation and rotation being concerned, by moving the spot at which the incident beam strikes the interface. Finally, the proposed formalism can be used to trap atoms in a planar collection of optical wells. As we know the atoms trapped in finite array at the surface may be used as qubits for the practical performance of quantum computing, seeing as their positions are well-defined by the minima of the intensity distributions and they can also be dealt with optically through an intervening vacuum environment. The formation of a two-dimensional lattice is quite different from the case in three dimensions and therefore forms an interesting field of investigation. The diminution in the dimensionality and the nearness of the system to the dielectric surface present original physical circumstances that we expect will yield a means of new physics.

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