

Full Length Research Paper

Comparing experimental deformations of steel-concrete-steel sandwich beams with full and partial interaction theories

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In this study, experimental deformation results (that is beam deflection and slip between layers) of steel-concrete-steel sandwich beams (double skin composite beam - DSC) are compared with full and partial interaction theories. The flexibility of shear stud connectors on both tension and compression faces is taken into account in the partial interaction analysis including the influence of frictional forces between the concrete and external steel plates at the supports and load points. Quasi-static test results on DSC beams are compared with the theoretical solutions based on partial interaction theory assuming realistic material and shear connector properties. The comparison of results indicates that the proposed theoretical method shows good correlation with real behaviour and may be reliably used for the analysis of simply supported DSC beams.

Key words: Steel-concrete-steel sandwich beams, double skin composite construction, partial interaction, full interaction, shear connectors, deflection, slip, frictional force, quasi-static loading.

INTRODUCTION

Double skin composite construction consists of a layer of a plain concrete, sandwiched between two layers of relatively thin steel plate, connected to the concrete by welded stud shear connectors. This construction acts in a similar way to doubly reinforced concrete elements but the flexibility of connection between the steel plates and concrete gives rise to interface slip and additional overall element deflection. This results in a strong and efficient structure with certain potential advantages over conventional forms of construction.

Steel-concrete composite systems generally consist of steel plate, concrete and reinforcement. Shear connectors are usually utilized to develop the composite action between steel and concrete. In steel-concrete composite members, the natural bonding, friction, and mechanical interlocking actions of shear connectors have a significant influence on the degree of interaction

(Veljkovic, 1996; Oehlers et al., 2000).

It is known that the degree of interaction between steel and concrete influences the shear flow and strain distribution. Also, it has an impact on the structural performance such as strength, stiffness, and failure mode. The degree of interaction in steel-concrete composite systems can be evaluated as full-interaction, partial-interaction, and no-interaction (Veljkovic, 1996; Oehlers et al., 2000). The assumption of full-interaction may result in an overestimation of the structural performance while the assumption of no-interaction may cause an underestimation of the structural performance. Therefore, the partial-interaction assumption and analysis with a degree of interaction becomes more practical and seems to be essential for a precise prediction of behaviour. Actually, the steel-concrete composite members generally show partial-interaction due to the deformation of shear connectors and slip at the interface under the applied loads (Johnson, 1994; Dogan, 1997; Roberts and Dogan, 1998; Oehlers and Bradford, 1999; Jeong et al., 2005; Ranzi et al., 2006; Gara et al., 2006; Queiroza et al., 2007; Ranzi and Bradford, 2007; Jeong,

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2008; Ranzi, 2008; Girhammar et al., 2009; Sousa Jr. et al., 2010).

In many situations, slip and its influence on the structural behaviour of steel-concrete composite systems may be small enough to be neglected in the analysis (that is, full interaction). However, in some cases it may be feasible to use either fewer connectors than are required for complete shear connection or connectors which possess a relatively low stiffness. In such situations the influence of slip may not be negligible and result in reduced stiffness of the system (that is, partial interaction). In general, the stiffness of the connectors has a significant influence on both the slip and deformations of a composite beam. The stiffness of the shear connectors may be determined experimentally from so called push-shear tests.

An important aspect of the design of DSC beams is the design of the so called shear connectors, which transfer both shear and normal forces between the concrete infill and external steel plates. The shear connection is sometimes defined as complete when the bending strength can not be increased by the provision of additional connectors. However, all connectors possess finite stiffness and therefore slip must occur between the concrete and steel plates if the shearing forces are to develop. Slip results in a strain discontinuity at the steel-concrete interface, with a corresponding reduction in flexural stiffness.

Analysis of the influence of slip in composite beams, assuming both linear and non-linear material and shear connector behaviour (Knowles, 1973; Yam, 1981; Newmark et al., 1951; Yam and Chapman, 1968; Yam and Chapman, 1972; Johnson, 1975; Johnson, 1981) has generally been based upon an approach attributed to Newmark et al. (1951). The equilibrium and compatibility equations for an element of the beam are reduced to a single second order differential equation in terms of either the resultant axial force in the concrete or the interface slip. Solutions for the axial force or interface slip are substituted back into the basic equilibrium and compatibility equations, which can then be solved to give displacements and strains throughout the beam.

Newmark et al. (1951) presented the results of tests and analysis for evaluating the load deflection behaviour of simply supported, partially interactive, composite concrete and steel T-beams. The theoretical analysis was based upon the assumption that a continuous imperfect connection existed between the two elements. A second order differential equation expressing the relationship between the longitudinal forces, transmitted through the shear connection from the concrete slab to the steel beam, and the applied bending moment, was derived and solved for the case of a beam loaded with a concentrated load.

Newmark et al. (1951) approach has been developed by Yam and Chapman (1968, 1972) and Yam (1981) to incorporate non-linear material and shear connector behaviour. The resulting non-linear differential equations

were solved iteratively and the influence of slip on the ultimate flexural strength of composite beams was studied.

Johnson (1975, 1981) has presented modified versions of Newmark et al.'s (1951) theory in which the differential equations are formulated in terms of interface slip. The equations were used as the basis of an extensive theoretical study of the loss of interaction in short-span composite beams and slabs.

An alternative approach to the analysis of composite beams with partial interaction has been presented by Roberts (1985), in which the basic equilibrium and compatibility equations are formulated in terms of the displacements of the layers. The resulting differential equations are then solved simultaneously by expressing the displacement derivatives in finite difference form. The development of this approach to incorporate non-linear material and shear connector behaviour has been described by Al-Amery and Roberts (1990). The resulting non-linear differential equations are expressed in finite difference form and solved iteratively.

Experimental and theoretical studies of the behaviour of DSC beams with partial interaction have been reported by Oduyemi and Wright (1989), Wright et al. (1991a, b) and Narayanan et al. (1982). Wright and Oduyemi (1991) presented closed form solutions for the partial interaction analysis of simply supported DSC beams. The analysis takes into account the flexibility of the connection on both the tension and compression faces, and incorporates the influence of concrete cracking and non-linear connector behaviour using a step-wise linearization technique. Two coupled differential equations for the axial forces in the tension and compression plates were formulated and closed form solutions found for various load combinations. The solutions were compared with the results of tests on several DSC beams and good agreement between the theory and experiment was found.

This paper aims to compare experimental deflections and slips in a total number of 12 DSC beams reported by Dogan (1997) with full and partial interaction theories originally introduced by Wright and Oduyemi (1991) and later modified by Dogan (1997). The partial interaction analysis is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points (Dogan, 1997; Dogan, 2010). The theoretical solutions derived by Dogan (1997) are compared with the experimental results including beam deflection, slip between steel plates and concrete on DSC beams reported by Dogan (1997).

GOVERNING DIFFERENTIAL EQUATIONS

Full interaction

Full interaction analysis of DSC beams is based on the following assumptions: both steel and concrete are

linearly elastic materials, concrete subjected to tensile strain is cracked and ineffective in resisting load, the shear connection between the concrete and steel is sufficiently stiff to ensure that slip is negligible, and plane sections remain plane.

The assumed linear strain distribution over the depth of a DSC section subjected to bending is shown in Figure 1a, together with the associated resisting forces in the concrete and steel plates. Figure 1b shows the assumed positive conventions for displacements u and v in the x and y directions, moments M , shear forces V and curvature k .

The general equation for the transverse displacement $v_s(x)$ of the beam in the y direction is written in the following form

$$v_s(x) = -\frac{1}{\sum EI(1+\alpha)} \int \left\{ \int M dx \right\} dx + C_1 x + C_2 \tag{1}$$

in which E is the Young's modulus of beam, I is moment of inertia, α is composite stiffness factor, M is moment, and C_1 and C_2 are constants of integration which can be determined from particular boundary conditions.

The particular solution of the governing differential equations for deflection along the left half of the beam is

$$v_{sA}(x) = \frac{1}{\sum EI(1+\alpha)} \left[-\frac{P}{12} x^3 - \left(\frac{Pu^2}{4} - \frac{PuL}{4} \right) x \right] \tag{2a}$$

$$v_{sB}(x) = \frac{1}{\sum EI(1+\alpha)} \left[-\frac{Pu}{4} x^2 + \frac{PuL}{4} x - \frac{Pu^3}{12} \right] \tag{2b}$$

where P is pointed load, u is distance between support and pointed load and L is span length of beam.

Partial interaction

The partial interaction analysis introduced by Wright and Oduyemi (1991) is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points. Theories of partial interaction are based on the following simplifying assumptions: (a) both steel and concrete are linearly elastic materials, (b) deflections are small, (c) shear deformations within each material are negligible, (d) the shear connection between the concrete and steel plates is continuous along the beam that is, the discrete stud connectors act as a continuous (smeared) connection, (e) the shear stiffness of the connection is linear, (f) the distribution of strain throughout the depth of each individual layer is linear, (g) at every section of the beam, each layer is bent to the same radius of curvature that is,

each layer deflects by the same amount and no buckling or separation of layers occurs, (h) the concrete subjected to tensile strain is cracked and ineffective in resisting load and (i) the depth of the neutral axis is constant and related to the beam geometry and material properties.

Compression and tension plate slips are related to the stud shear force and shear stiffness by the equations

$$s_{sc} = \frac{Q_{sc}}{K_{sc}} \tag{3a}$$

$$s_{st} = \frac{Q_{st}}{K_{st}} \tag{3b}$$

$$Q_{sc} = \frac{q_{sc} p_{sc}}{n_{sc}} \tag{3c}$$

$$Q_{st} = \frac{q_{st} p_{st}}{n_{st}} \tag{3d}$$

where s_{sc} is slip between concrete core and steel compression plate, s_{st} is slip between concrete core and steel tension plate, Q_{sc} is shear force in stud connector welded to steel compression plate, Q_{st} is shear force in stud connector welded to steel tension plate, K_{sc} is shear stiffness of stud connector welded to steel compression plate, K_{st} is shear stiffness of stud connector welded to steel tension plate, q_{sc} is shear force per unit length of steel compression plate, q_{st} is shear force per unit length of steel tension plate, p_{sc} is longitudinal spacing of stud connectors welded to steel compression plate, p_{st} is longitudinal spacing of stud connectors welded to steel tension plate, n_{sc} is number of stud connectors across the width of the compression plate and n_{st} is number of stud connectors across the width of the tension plate.

The general equation for the transverse displacement of a beam with partial interaction $v_p(x)$ is given by

$$v_p(x) = v_s(x) + \lambda_1 F_{sc} + \lambda_2 F_{st} \tag{4}$$

where $v_s(x)$ is given by Equation (1), λ_1 and λ_2 are axial force coefficients representing displacement and rigidity of beam, and F_{sc} and F_{st} are axial compressive and tensile forces in steel plates, respectively. The particular solutions for the transverse displacement with and without frictional forces at supports and load points are

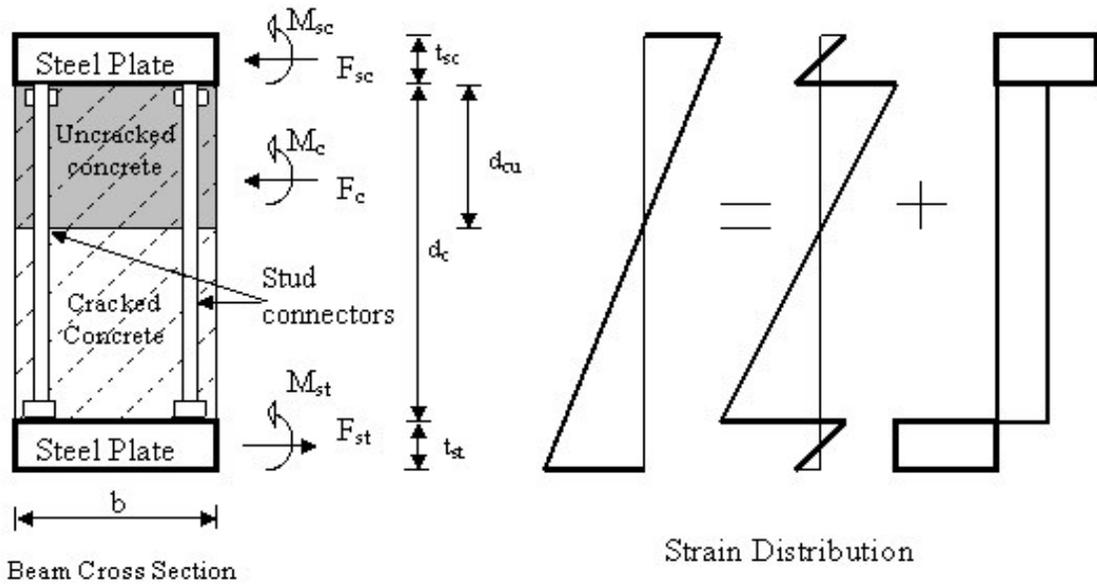


Figure 1a. Internal forces and strain distribution over the depth of a DSC section for full interaction.

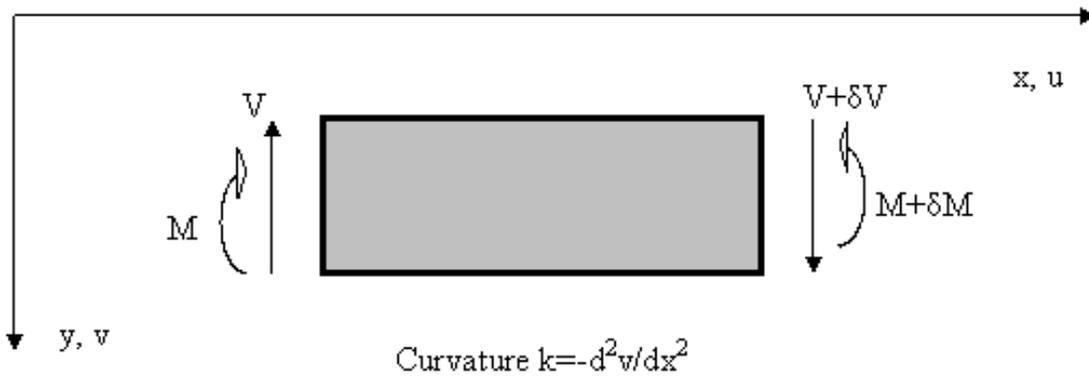


Figure 1b. The assumed positive sign conventions for displacements u and v in x and y directions.

$$v_{PA}(x) = -\frac{P}{2 \sum EI(1+\alpha)} \frac{x^3}{6} + \lambda_1 \frac{F_{scA}}{\sum EI} + \lambda_2 \frac{F_{stA}}{\sum EI} + C_1 x + C_2 \quad (5a)$$

$$v_{PB}(x) = -\frac{P}{2 \sum EI(1+\alpha)} \frac{x^2}{2} + \lambda_1 \frac{F_{scB}}{\sum EI} + \lambda_2 \frac{F_{stB}}{\sum EI} + C_3 x + C_4 \quad (5b)$$

where C_1, C_2, C_3 and C_4 are integration constants.

METHODOLOGY

The behaviour of DSC beams is extremely complex and therefore various assumptions are used in full and partial interaction analysis, to simplify the system as discussed before.

In this study, the distance between the symmetrical loads is reduced to zero, to obtain solutions for a simply supported beam

with a point load at midspan, as shown in Figures 1-3. Various parameters are investigated, in particular the stiffness of the shear connection and frictional forces between the steel plates and concrete infill. For the beam shown in Figures 1-3 the applied bending moment diagram is symmetrical about midspan and therefore only half of the beam need be considered.

For all the beams a frictional coefficient g of about 0.25 was found to give close agreement between theoretical and experimental results. In the analysis, the influence of studs outside of the supports was represented by an axial tensile force in the tension steel plate, deduced from the experimental results at the appropriate applied load level.

The assumed beam geometry for the comparison of full and partial interaction theories are span $L=1400$ mm, breadth $b=200$ mm, concrete core depth $d_c=150$ mm, top and bottom steel plate thicknesses $t_s=8$ mm and stud spacing on both plates $s_t=200$ mm. The Young's modulus of the steel E_s was assumed to be 210 kN/mm^2 . Due to variation in concrete compressive strength the Young's modulus of concrete E_c , determined from the following

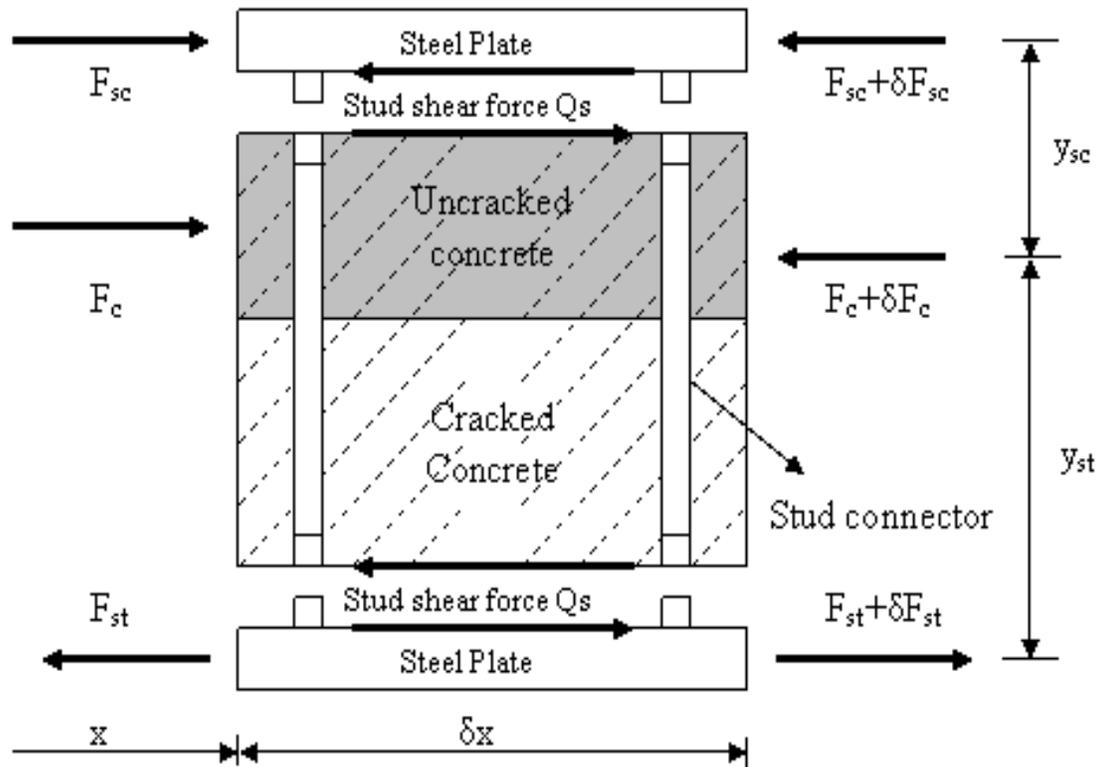


Figure 2a. Interface shearing forces of a DSC beam.

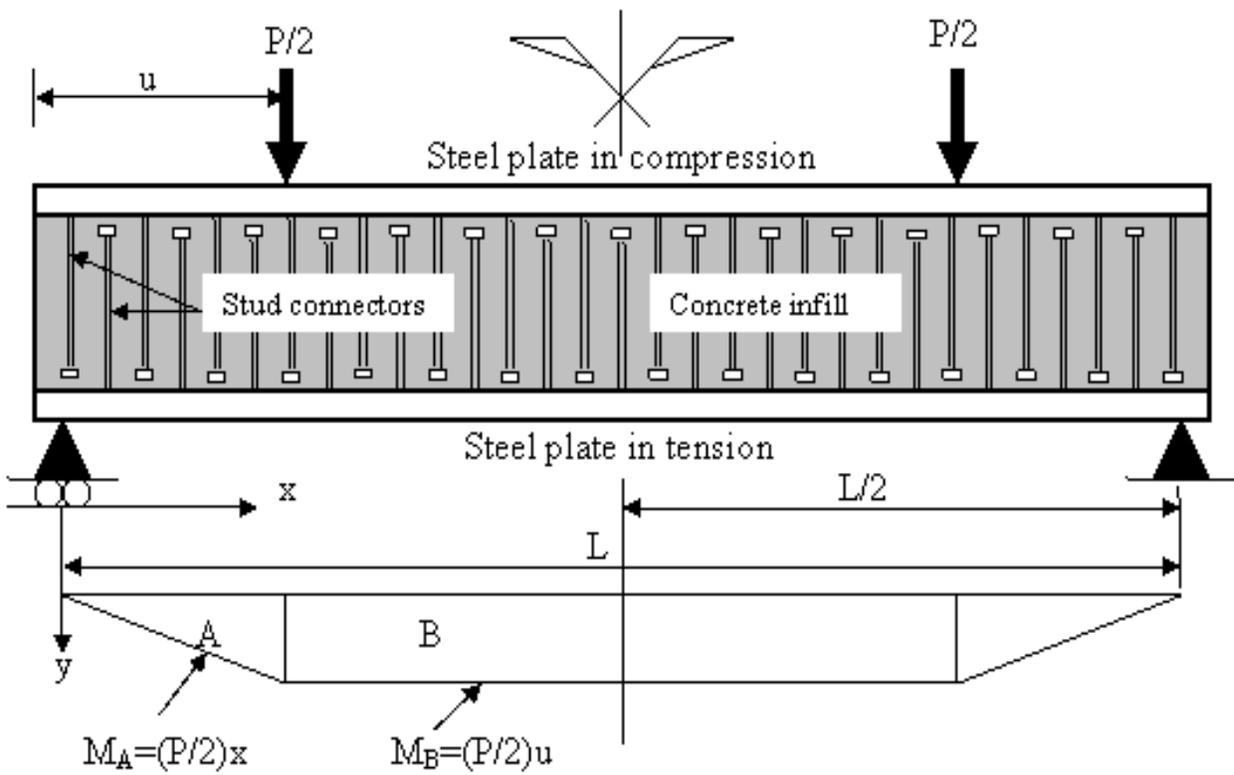


Figure 2b. Support, loading and bending moment diagram.

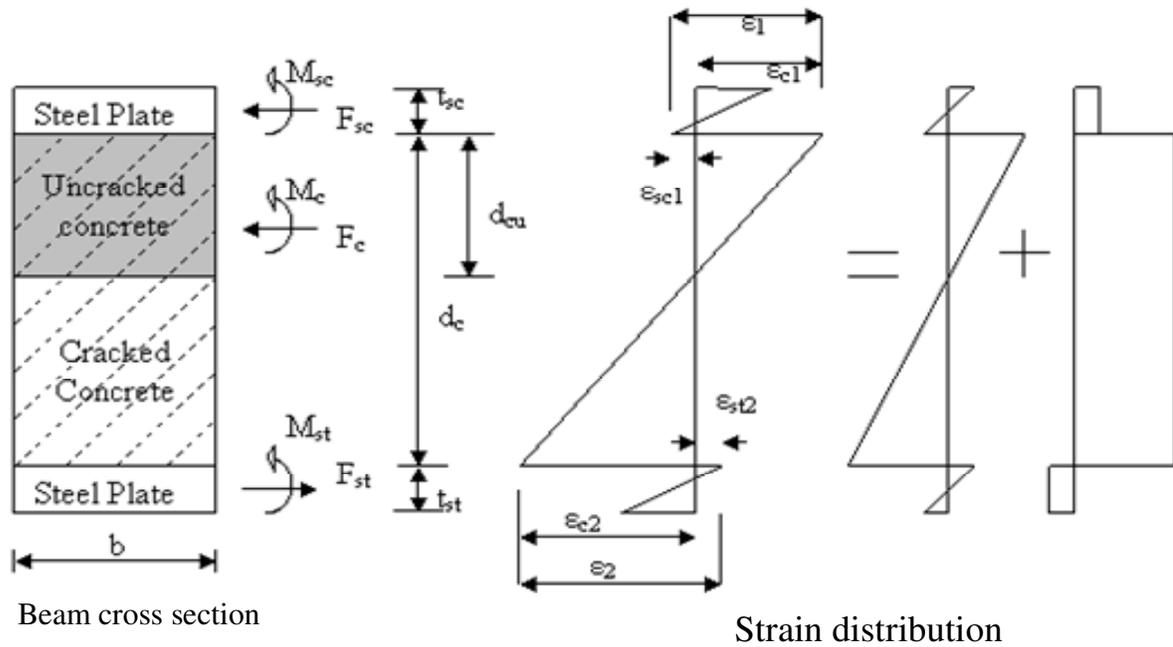


Figure 3a. Internal forces and strain distribution over the depth of a DSC section for partial interaction.

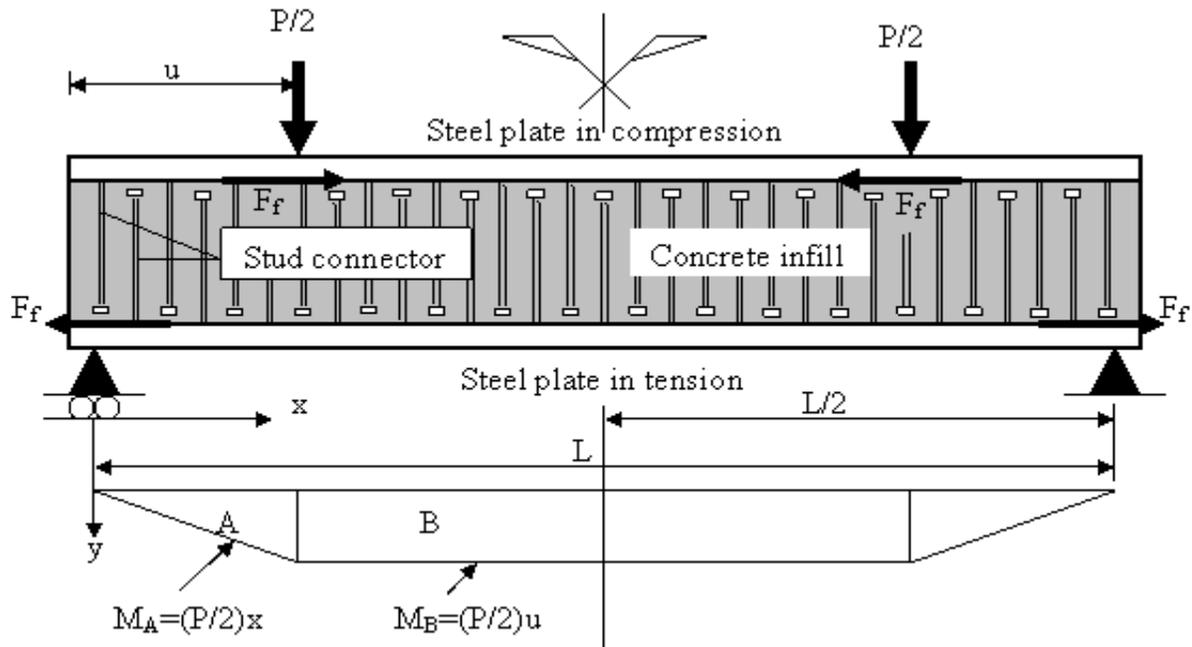


Figure 3b. Support, loading and frictional forces F_f at the supports and load points.

equation

$$E_c = 9.1(f_{cu})^{0.33} \tag{6}$$

where f_{cu} is the concrete cube compressive strength in N/mm^2 and

E_c is in kN/mm^2 . E_c varied between 25.2 and 30.2 kN/mm^2 . The test beams were therefore divided into four groups according to their estimated concrete Young's modulus (Group 1: B1 and B2 with $E_c=25.2$ kN/mm^2 , Group 2: B3 to B6 with $E_c=28.3$ kN/mm^2 , Group 3: B7 and B8 with $E_c=27.1$ kN/mm^2 and Group 4: B9 and B10 with $E_c=30.2$ kN/mm^2).

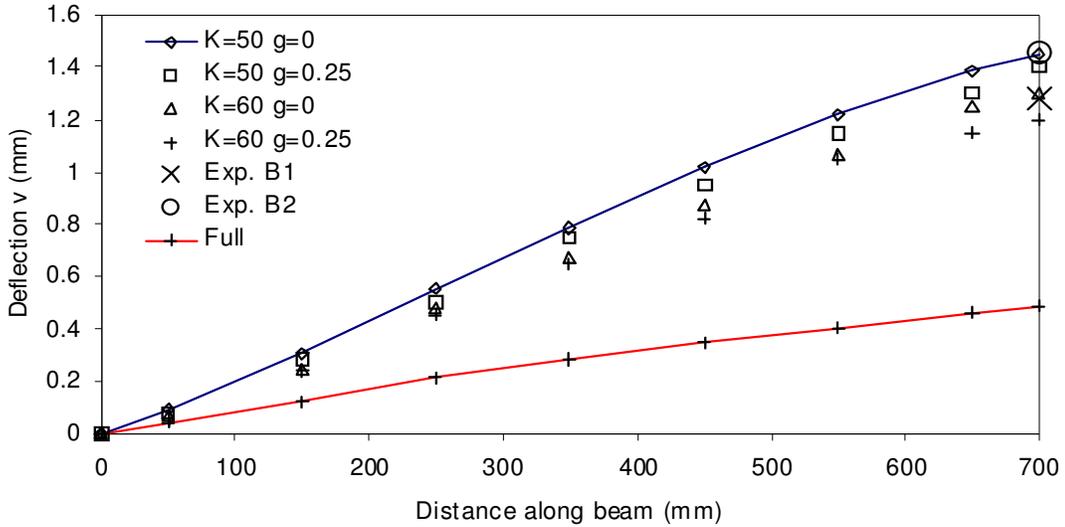


Figure 4. Comparison of experimental beam deflections for the first group of beams B1 and B2 (P=30 kN).

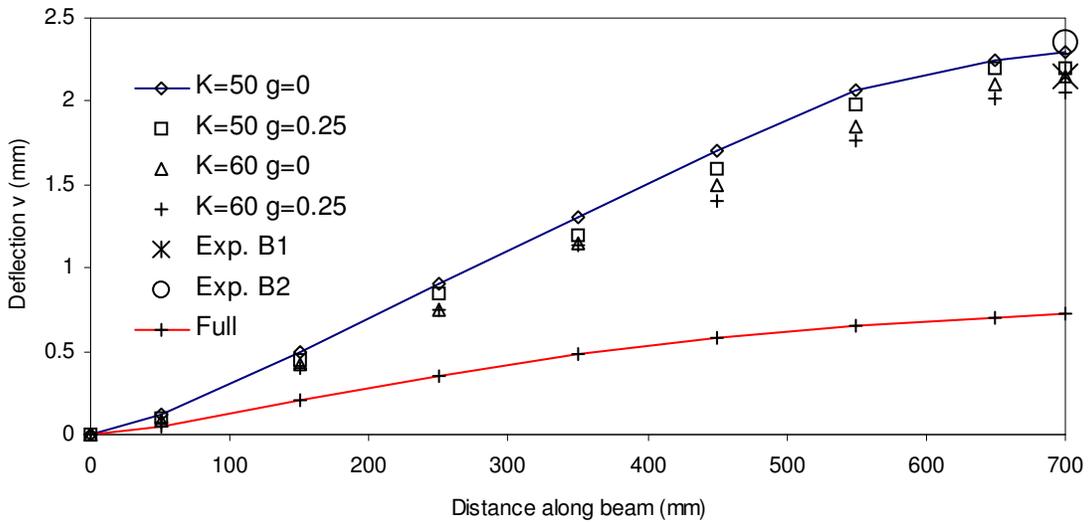


Figure 5. Comparison of experimental beam deflections for the first group of beams B1 and B2 (P=50 kN).

RESULTS

General

The behavior of DSC beams is extremely complex and therefore various assumptions are used in full and partial interaction analysis in order to simplify the system as mentioned before. For comparison of theoretical solutions with the experimental test results, the system geometry and material properties assumed were the same as reported by Dogan (1997).

Here full and partial interaction theories are compared, firstly neglecting friction between the layers at the

supports and secondly taking frictional forces into consideration. Comparisons are also made with the test results at particular applied load levels. Results are presented for deflections along the beam, slip between the layers, axial forces in the steel plates and shear forces in the studs.

Deflections

Figures 4-9 show the variation of deflection along the beams for two values of shear connection stiffness $K=50$ and 60 kN/mm, with and without frictional forces between

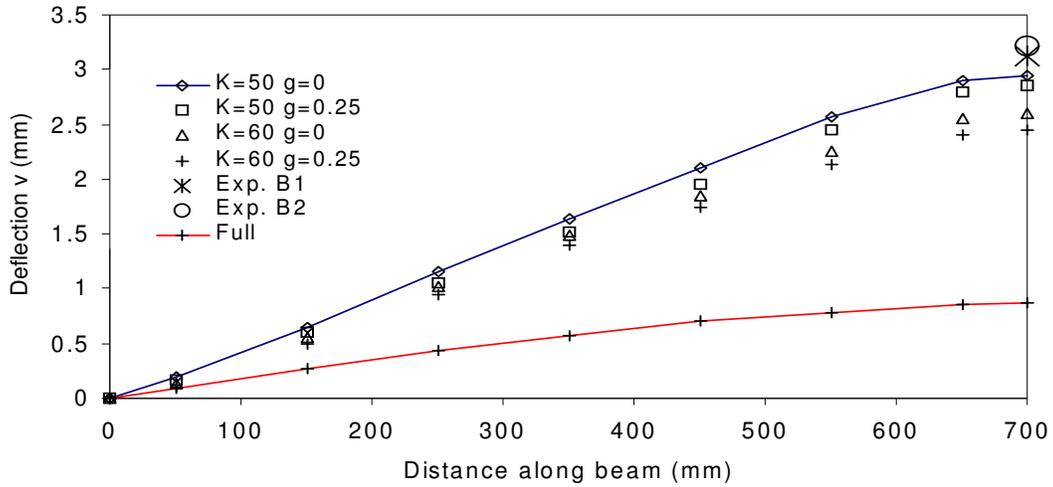
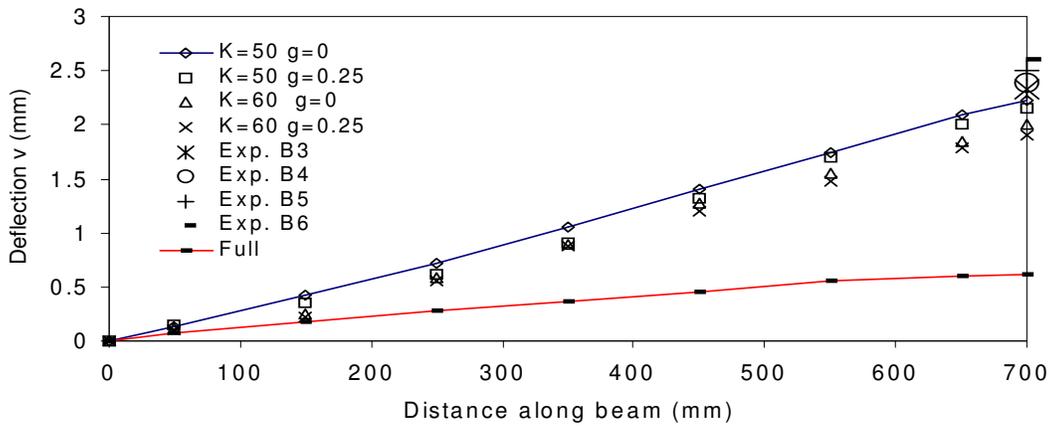


Figure 6. Comparison of experimental beam deflections for the first group of beams B1 and B2 (P=70 kN).



Figures 7. Comparison of experimental beam deflections for the second group of beams B3-B6 (P=50 kN).

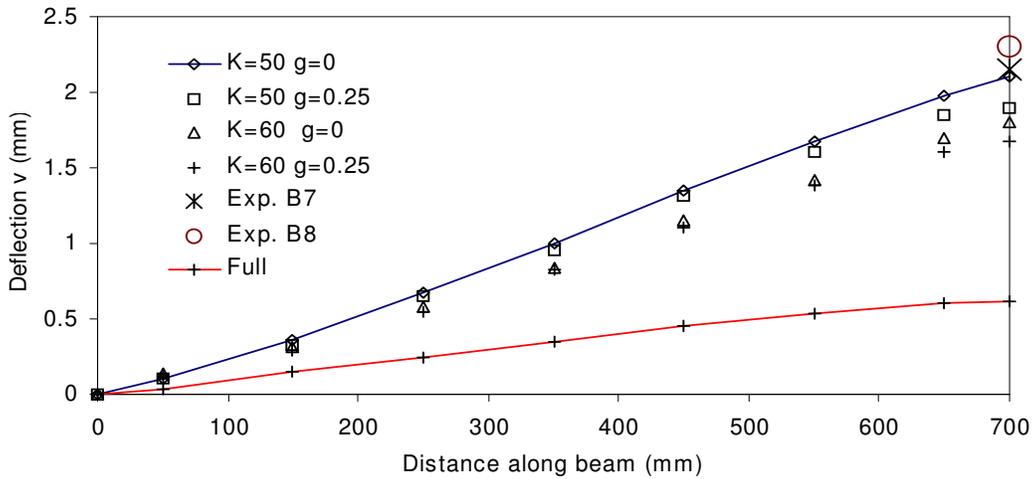


Figure 8. Comparison of experimental beam deflections for the third group of beams B7 and B8 (P=50 kN).

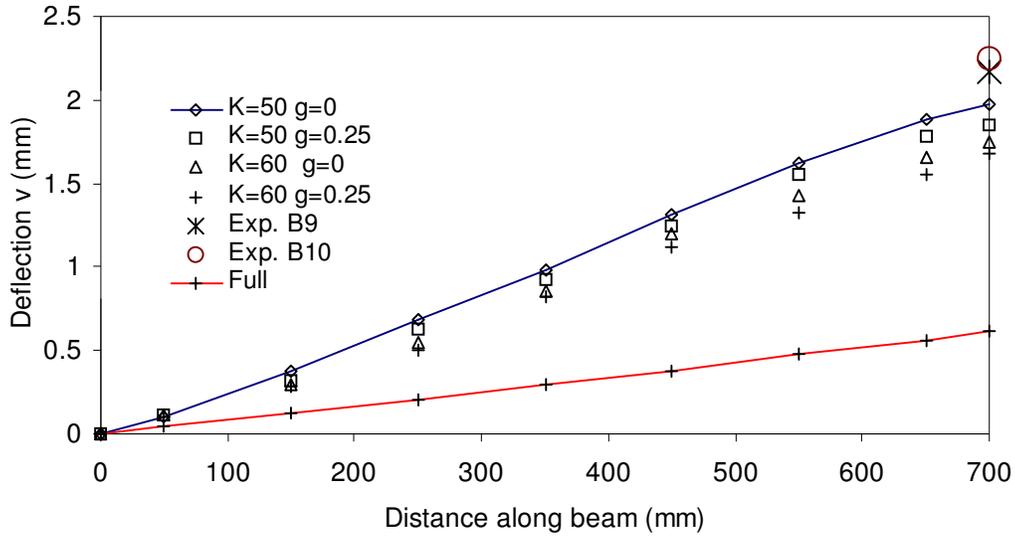


Figure 9. Comparison of experimental beam deflections for the fourth group of beams B9 and B10 (P=50 kN).

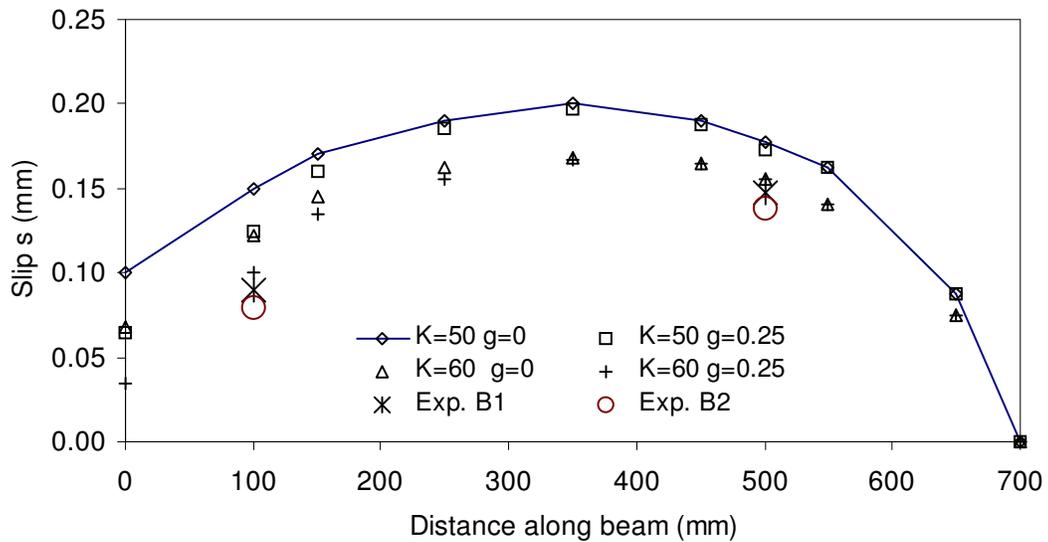


Figure 10a. Comparison of experimental tension plate slip for the first group of beams B1 and B2 (P=30 kN).

the layers at the supports (coefficient of friction $g=0.0$ and 0.25). The deflections decrease with an increase of shear connection stiffness and the results based on partial interaction theory tend to those based on full interaction theory. Comparison of partial interaction results and test results indicates the stiffness of the shear connection to be about 50 kN/mm , compared with values varying between 20 and 70 kN/mm determined from the study results of Dogan (1997).

Test results for beams B1 and B2, at load levels 30 , 50 and 70 kN , are compared with theoretical results in

Figures 4-6. Figures 7-9 show similar comparisons for beams B3 to B10. In general, there is reasonably close agreement between theoretical and experimental results for $K=50 \text{ kN/mm}$. It is also worth noting that partial interaction theory gives deflections approximately three times those based on full interaction theory.

Steel plate slip

Figures 10-15 show the variation of slip, between the

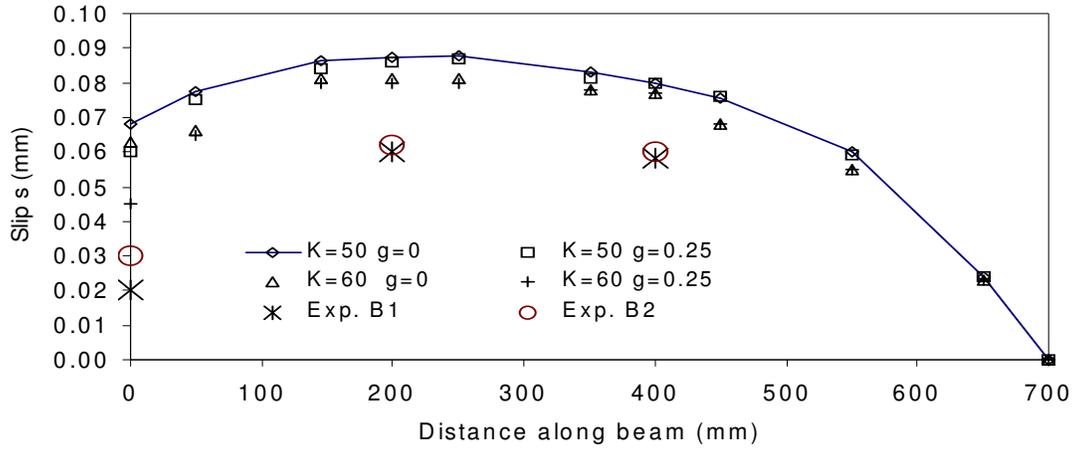


Figure 10b. Comparison of experimental compression plate slip for the first group of beams B1 and B2 (P=30 kN).

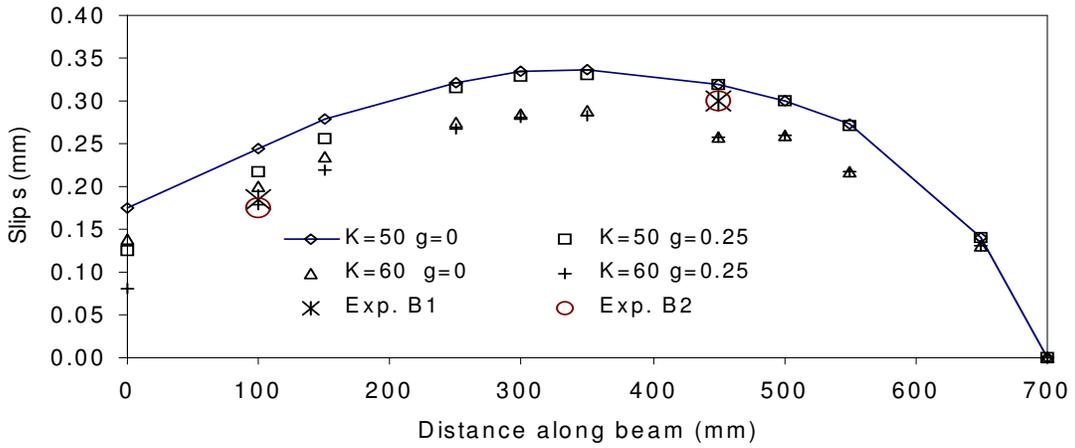


Figure 11a. Comparison of experimental tension plate slip for the first group of beams B1 and B2 (P=50 kN).

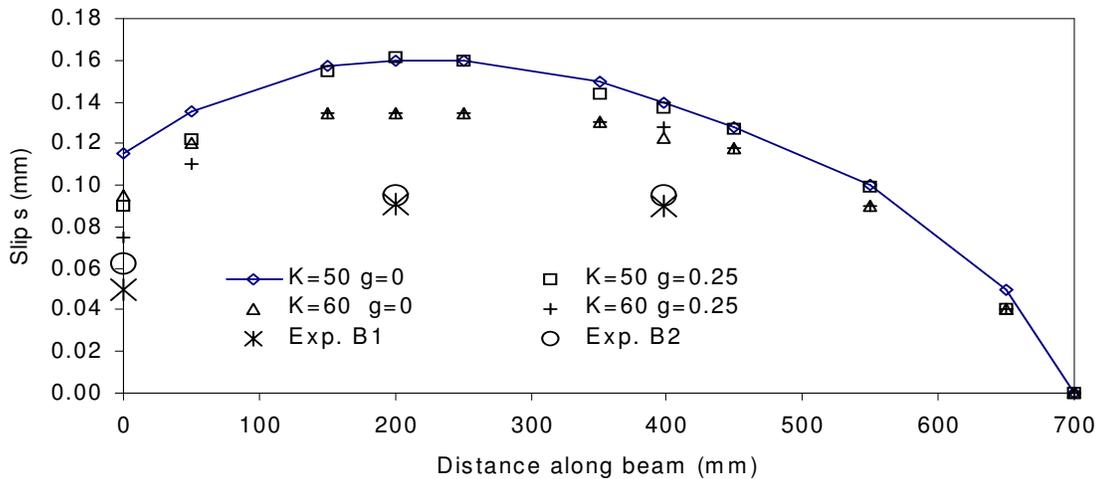


Figure 11b. Comparison of experimental compression plate slip for the first group of beams B1 and B2 (P=50 kN).

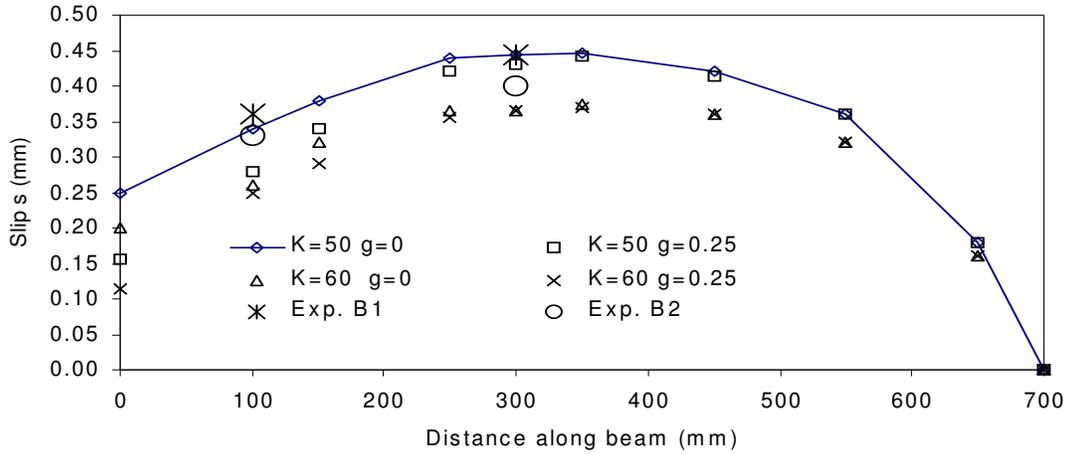


Figure 12a. Comparison of experimental tension plate slip for the first group of beams B1 and B2 (P=70 kN).

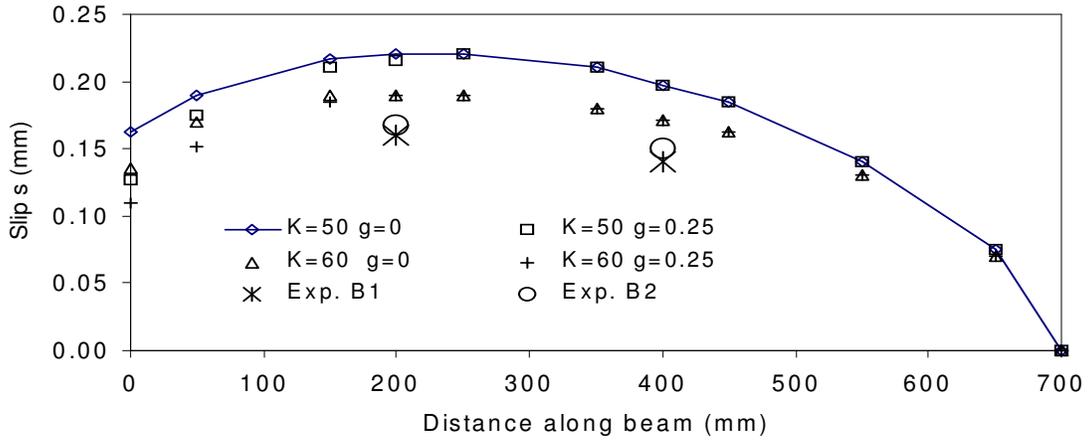


Figure 12b. Comparison of experimental compression plate slip for the first group of beams B1 and B2 (P=70 kN).

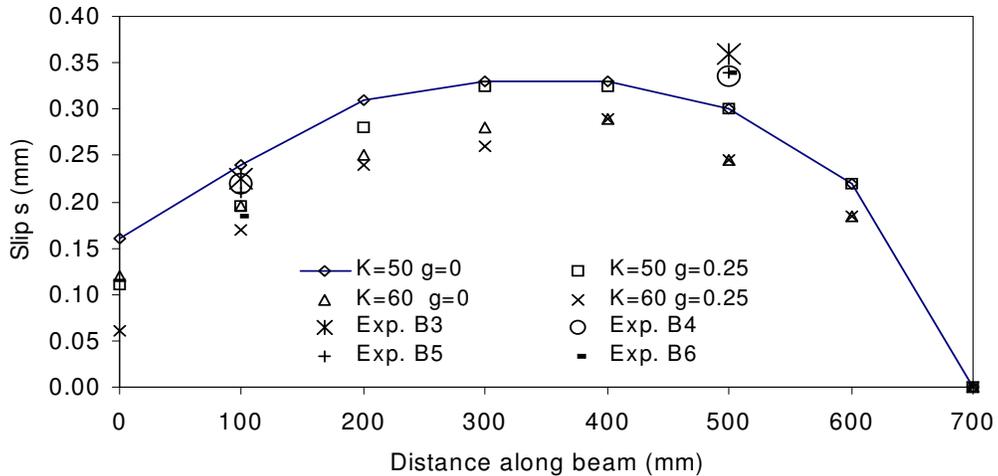


Figure 13a. Comparison of experimental tension plate slip for the second group of beams B3-B6 (P=50 kN).

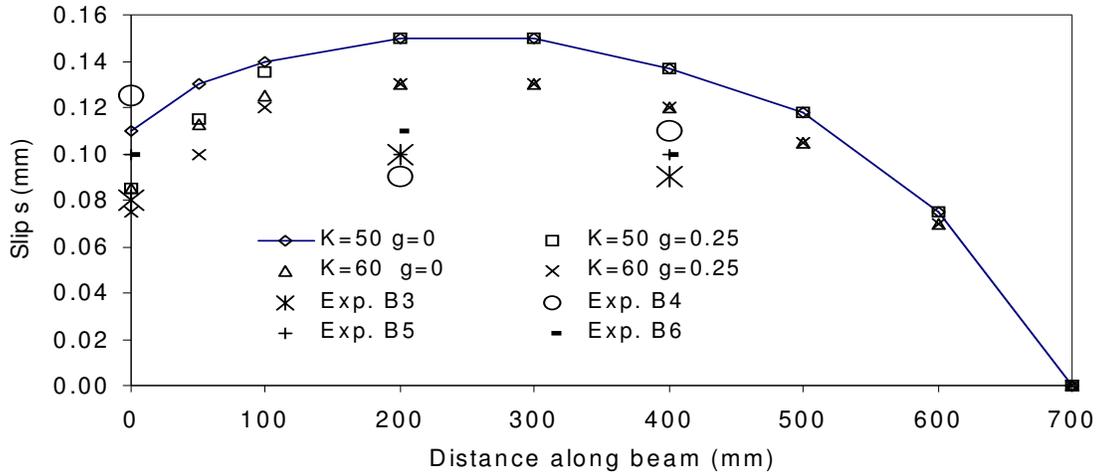


Figure 13b. Comparison of experimental compression plate slip for the second group of beams B3-B6 ($P=50$ kN).

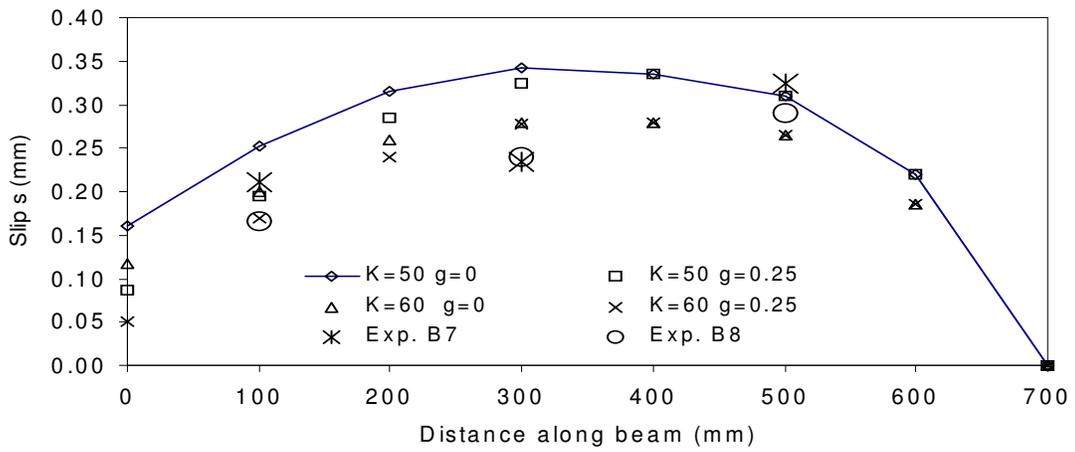


Figure 14a. Comparison of experimental tension plate slip for the third group of beams B7 and B8 ($P=70$ kN).

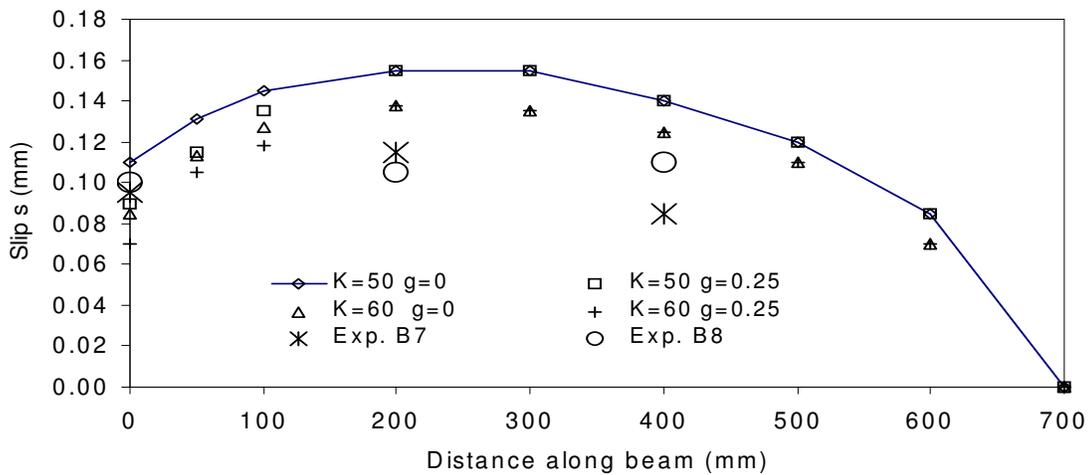


Figure 14b. Comparison of experimental compression plate slip for the third group of beams B7 and B8 ($P=50$ kN).

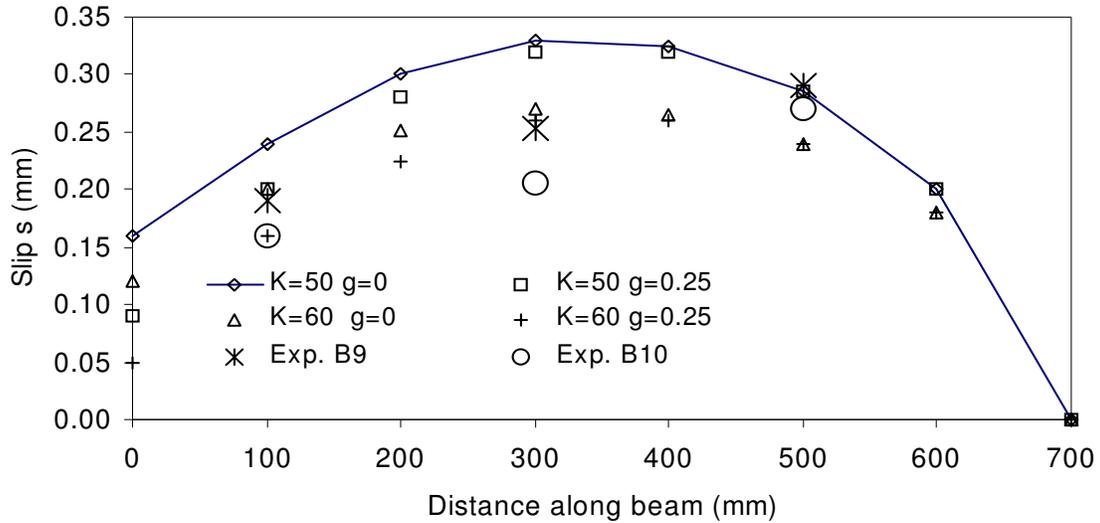


Figure 15a. Comparison of experimental tension plate slip for the fourth group of beams B9 and B10 (P=70 kN).

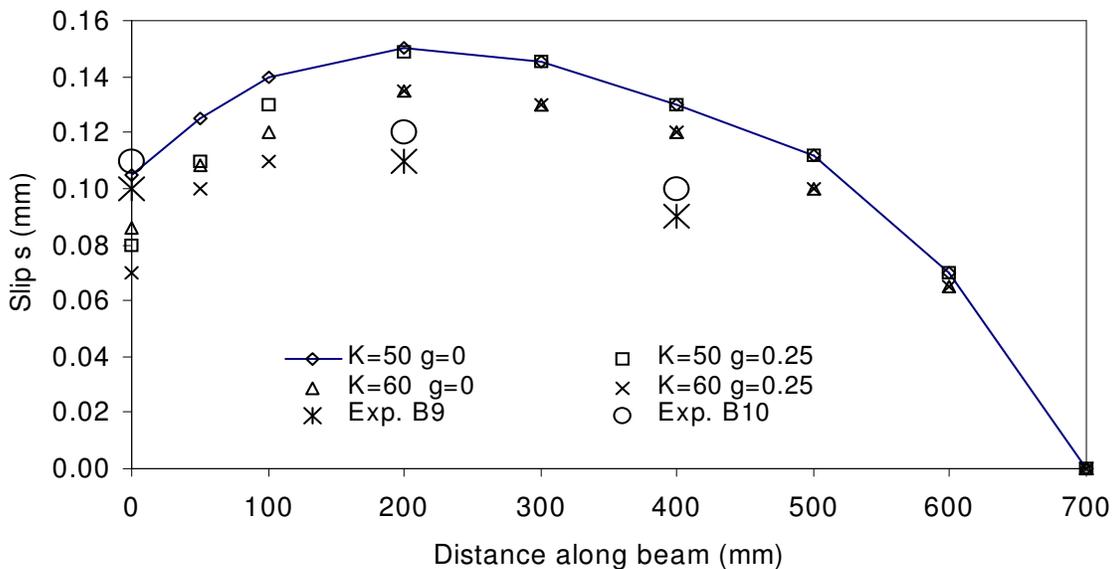


Figure 15b. Comparison of experimental compression plate slip for the fourth group of beams B9 and B10 (P=50 kN).

steel plates and concrete infill, along beams B1 to B10, for two values of the connection stiffness $K=50$ and 60 kN/mm, with and without frictional forces between the layers at the supports. The slip decreases with an increase of shear connection stiffness and tends to zero as the shear connection stiffness tends to infinity.

A comparison of theoretical and experimental tension and compression plate slip for beams B1 and B2, at load levels 30, 50 and 70 kN, is shown in Figures 10-12. The partial interaction results for deflection indicate that the stiffness of the stud connectors was about 50 kN/mm at

low applied load levels. At higher load levels cracking of the concrete at the fourth group of studs from the end of the beam increased the slip locally. Slip generally decreased towards the end of the beam due to the influence of frictional forces at the supports and the additional studs outside the supports.

A comparison of theoretical and experimental tension and compression plate slip for the other groups of beams B3-10 is shown in Figures 13-15. In general, there is reasonable correlation between theoretical and experimental results.

DISCUSSION AND CONCLUSION

Comparisons have been made between experimental results and theoretical predications of the behaviour of DSC beams, based on full and partial interaction analysis. Because of the variation in concrete cube strength and elastic modulus of the test beams, they have been divided into four groups for which comparisons between experimental and theoretical beam deflection and slip between the layers are presented.

Due to the variation of concrete crack depths along the beams and separation between the tension steel plates and concrete infill, experimental results differed slightly from the theoretical results. Local concrete cracking at the fourth group of studs from the end of the beam resulted in a discontinuity in slip.

The partial interaction analysis indicates that frictional forces at the supports and studs outside of the supports have a significant influence on the behaviour of DSC beams.

The theoretical results based on partial interaction theory, assuming realistic material and shear connector properties and incorporating the influence of interface frictional forces, show satisfactory correlation with test results.

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